Vibration Control for Chatter Suppression with Application to Boring Bars

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A mechatronic system of actuators, sensors, and analog circuits is demonstrated to control the self-excited oscillations known as chatter that occur when single-point turning a rigid workpiece with a flexible tool. The nature of this manufacturing process, its complex geometry, harsh operating environment, and poorly understood physics, present considerable challenges to the control system designer. The actuators and sensors must be rugged and of exceptionally high bandwidth and the control must be robust in the presence of unmodeled dynamics. In this regard, the qualitative characterization of the chatter instability itself becomes important. Chatter vibrations are finite and recognized as limit cycles, yet modeling and control efforts have routinely focused only on the linearized problem. The question naturally arises as to whether the nonlinear stability is characterized by a jump phenomenon. If so, what does this imply for the “robustness” of linear control solutions?

To answer our question, we present an advanced hardware and control system design for a boring bar application. Initially, we treat the cutting forces merely as an unknown disturbance to the structure which is essentially a cantilevered beam. We then approximate the structure as a linear single-degree-of-freedom damped oscillator in each of the two principal modal coordinates and seek a control strategy that reduces the system response to general disturbances. Modal-based control strategies originally developed for the control of large flexible space structures are employed; they use second-order compensators to enhance selectively the damping of the modes identified for control.

To attack the problem of the nonlinear stability, we seek a model that captures some of the behavior observed in experiments. We design this model based on observations and
intuition because theoretical expressions for the complex dynamic forces generated during cutting are lacking. We begin by assuming a regenerative chatter mechanism, as is common practice, and presume that it has a nonlinear form, which is approximated using a cubic polynomial. Experiments demonstrate that the cutting forces couple the two principal modal coordinates. To obtain the jump phenomena observed experimentally, we find it necessary to account for structural nonlinearities. Gradually, using experimental observation as a guide, we arrive at a two-degree-of-freedom chatter model for the boring process. We analyze the stability of this model using the modern methods of nonlinear dynamics. We apply the method of multiple scales to determine the local nonlinear normal form of the bifurcation from static to dynamic cutting. We then find the subsequent periodic motions by employing the method of harmonic balance. The stability of these periodic motions is analysed using Floquet theory.

Working from a model that captures the essential nonlinear behavior, we develop a new post-bifurcation control strategy based on quench control. We observe that nonlinear state feedback can be used to control the amplitude of post-bifurcation limit cycles. Judicious selection of this nonlinear state feedback makes a supplementary open-loop control strategy possible. By injecting a harmonic force with a frequency incommensurate with the chatter frequency, we find that the self-excited chatter can be exchanged for a forced vibratory response, thereby reducing tool motions.
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In this Dissertation, extensive use has been made of the so-called “active voice” writing style. The motivation behind this being to engage the readers by inviting their participation. A statement such as “We now investigate...” is intended to make the reader feel as though he or she is an active participant in the research. Of course, it also serves a more practical purpose. It acknowledges, in a subtle way, the contributions of the many colleagues, friends, and family members who made this work possible.

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November, 1997
In memory of my greatgrandmother Lola, and to her legacy, Musetta
Morraine, Aurora Emmanuel, and Wynnona Esmé
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Chapter 1

Introduction

1.1 Background and Motivation

Extensive research has been devoted to the characterization, modelling, and control of vibrations that occur when machine tools operate at the limit of their dynamic stability. These vibrations, known as machine-tool chatter, must be avoided to maintain machining tolerances, preserve surface finish, and prevent tool breakage. To avoid chatter, machine-tool users limit material removal rates in order to stay within the dynamic stability boundary of their machines. From a manufacturing standpoint, chatter is a constraint on the machine-tool user that limits the available production capacity. Thus, vibration-control methods for extending machine-tool operating envelopes are highly desirable.
1.1.1 Chatter Mechanisms

Machine-tool chatter is thought to occur for a variety of reasons. The excellent monographs by Tobias (1965) and Koeingsburger and Thusty (1970) document much of the pioneering work in the field. These authors were the first to identify the mechanisms known as regeneration (Tobias and Fishwick, 1958) and mode coupling (Koeingsburger and Thusty, 1970).

Briefly, regenerative chatter occurs whenever cuts overlap and the cut produced at time $t$ leaves small waves in the material that are regenerated with each subsequent pass of the tool. It is considered to be the dominant mechanism of chatter in turning operations. If regenerative tool vibrations become large enough that the tool loses contact with the workpiece, then a type of chatter known as multiple regenerative chatter occurs. This mechanism has been the subject of studies by Shi and Tobias (1984), Kondo, Kawano, and Sato (1981), and Thusty and Ismail (1982).

Mode coupling occurs whenever the relative vibration between the tool and the workpiece exists simultaneously in at least two directions in the plane of the cut. In this case, the tool traces out an elliptic path that varies the depth of cut in such a fashion as to feed the coupled modes of vibration. It is considered to be a factor when chatter develops in slender nearly symmetric tools, such as boring bars. We note the similarity between this mechanism and the phenomenon of aeroelastic flutter.

Other mechanisms have been postulated. Arnold (1946) suggested that the cutting forces depend on the velocity in such a fashion as to produce negative damping. This chatter mechanism is essentially a frictional effect and has characteristics similar to that of the well-known Rayleigh oscillator (Nayfeh and Mook, 1979).

The foregoing are all mechanisms that lead to self-excited oscillations. Forced vibrations
also occur. A common source of such vibrations in turning operations is rotating imbalance or misalignment of the workpiece. Tool runout and spindle errors also cause forced vibrations. Milling operations generally produce interrupted cuts as the cutters rotate in and out of the workpiece. These so-called interrupted cuts lead to impact oscillations, a form of forced machine-tool vibration that has been studied by Davies and Balachandran (1996).

1.1.2 Existing Chatter Mitigation Strategies

It is generally accepted that stiff highly damped tools have a lower tendency to chatter. Rivin and Kang (1989) substantially increased the damping of a lathe tool by using a sandwich of steel plates and hard rubber viscoelastic material to form a laminated clamping device to hold the tool. Kelson and Hsueh (1996) also achieved an increase in stability by redesigning the tool holder for added stiffness and damping. A patent for a damping sandwich of plural layers of steel and plural alternate layers of viscoelastic solid material was awarded to Seifring (1991) and assigned to the Monarch Machine Tool Company.

Tobias (1965) cites a number of instances where passive vibration absorbers of various configurations have been applied with success. He describes applications using a Lanchester absorber, a dynamic vibration absorber, and an impact absorber. Boring bars with passive vibration absorbers incorporated into the tool shank have been available from Kennametal for some time (see, for instance, Kosker, 1975), and a patent for a similar device was awarded to Hopkins (1974) and assigned to the Valeron Corporation.

Researchers have also found that the cutting speed can be modulated to enhance stability (Sexton, Milne, and Stone, 1977; Takemura, et al 1974). Parametric variation of the tool stiffness has also been proposed as a method for suppressing regenerative chatter (Segalman
A variety of patents exist for devices that detect chatter and then adjust the process parameters, such as speed and feed, to produce a stable cut. A patent for a system that detects the “lobe precession” angle during single-point turning and adjusts the cross feed to maintain stability was awarded to Thompson (1986) and assigned to the General Electric Company. A fundamental assumption of this type of chatter mitigation technique is that a supercritical bifurcation exists, or in other words, the transition from static to dynamic cutting is smooth and free of jump phenomena. Drawing an analogy to aerodynamic flutter, one assumes that by simply decreasing the aircraft/workpiece velocity the flutter/chatter will go away.

More aggressive active control solutions have been sought. Nachtigal, Klein, and Maddux (1976) patented an “apparatus for controlling vibrational chatter in a machine-tool utilizing an updated synthesis circuit”. By sensing tool motion, the authors claimed they could model the cutting forces in their circuit and apply appropriate counteracting forces via an actuator. Recently, Tewani, Rouch, and Walcott (1995) demonstrated chatter control of a boring bar by using what they call an active dynamic absorber. They also patented the device (Rouch et al, 1991).

The active control scheme of Tewani, Rouch, and Walcott (1995) is the nearest in spirit to the control scheme demonstrated in this Dissertation and, so, deserves a closer scrutiny. The device which they patented uses a piezoelectric reaction-mass actuator mounted inside the boring bar. The reaction-mass actuator is modelled as an additional degree of freedom coupled to the boring bar through a spring, dashpot, and control force. The bar itself is modelled using a single-degree-of-freedom lumped-parameter approximation of the first mode of a cantilever beam. Accelerations of the boring bar and reaction mass are sensed and conditioned to yield signals proportional to the four state variables of the system.
The combination of bar and actuator has the classic configuration of the two-degree-of-freedom dynamic vibration absorber analyzed by Den Hartog (1985), but with the added functionality of the active dynamic absorber analysed by Tewani, Walcott, and Rouch (1991). The authors use optimal control techniques to obtain state-variable feedback gains. The resulting system has a highly-damped driving-point frequency-response function that produces a substantial increase in the predicted stable width of cut. Cutting tests using this system reveal stability problems for length-to-diameter ratios of 9 or more (Tewani, Switzer, Walcott, Rouch, and Massa, 1993). The authors state that, because of nonlinear actuator dynamics, the control was at times incapable of maintaining a stable cut.

The instability described by Tewani et al. (1993) seems to be similar to the subcritical cutting stability reported by Hooke and Tobias (1964) for turning operations on a lathe and by Hanna and Tobias (1974) for face milling operations on a vertical mill. To address this stability, Hanna and Tobias (1974) developed a nonlinear single-degree-of-freedom model based on experimental identification of the structure and cutting force. This model was studied recently by Nayfeh, Chin, and Pratt (1997) who found an analytical expression for the normal form of the bifurcation, used harmonic balance to reveal the true nature of the subcritical stability, and discovered that the model possesses a torus-doubling route to chaos. Pratt and Nayfeh (1996) confirmed this analysis using analog computer simulations.

All previous attempts at chatter control in boring bars have assumed that chatter may be characterized by a single mode of vibration. We will show in this Dissertation, through experiment, simulation, and theory, that multiple modes of chatter vibration can coexist, and that to insure a robust and effective control system it is necessary to apply control forces in two-orthogonal directions.
1.2 Organization of the Dissertation

The Dissertation is divided into two parts. In Part I, we treat the linear chatter control problem, whereas in Part II we consider complications that arise due to nonlinearity.

We begin Chapter 2 with a brief overview of the machine-tool structure and cutting process, reviewing the nomenclature and geometry of single-point turning. Working from these definitions, we review the standard approaches to static cutting-force modeling and some of the ways these techniques are extended to the dynamic condition. Next, we take up the linear regenerative chatter theory, which is essentially the problem of time-delay feedback. We make the important observation that the regenerative cutting force acts like a negative damping. The linear stability is developed as a function of depth of cut and workpiece rotational speed. We find that the machine-tool structure becomes marginally stable when the negative damping of the cutting force balances the positive damping inherent in the flexible tool. The point at which stability is lost is a Hopf bifurcation point for the nonlinear system, a topic that is taken up in Part II of the Dissertation.

In Chapter 3, we seek an active control solution to increase the overall system damping and thereby extend the operating envelope of the boring bar. We consider various vibration control options and select an active strategy that uses 2nd-order feedback compensation. We demonstrate through an example from the literature how the addition of the compensator can greatly extend the limit width of cut.

Part I of the Dissertation comes to a close in Chapter 4. In this chapter, a prototype chatter control system is presented. Actuators and sensors are selected and the issues surrounding their placement are discussed. A block diagram of the control system is developed. Modal parameters are determined for the combined boring-bar-actuator system. The frequency-response characteristics are determined experimentally and a mathematical
model for the closed-loop vibration control system is presented and compared with the measured data. Working from the block diagram we formulate a method for designing the feedback compensators.

In Part II, we begin to examine the nonlinear aspects of the chatter problem. Results from cutting experiments using the chatter controller are presented in Chapter 5. Chatter signatures for a variety of operating conditions with and without control are examined using time traces, autospectra, and crossspectral analyses. We find experimentally that stable cutting and large-amplitude chatter coexist for the same nominal cutting conditions.

Motivated by the results of the cutting experiments in Chapter 5, we present and analyze a nonlinear single-degree-of-freedom chatter model in Chapter 6. We first develop analytical techniques to explore the model stability. The method of multiple scales is used to arrive at the so-called nonlinear normal form for the Hopf bifurcation. The resulting periodic solutions are determined using the method of harmonic balance. The stability of these solutions is determined using a combination of Floquet theory and Hill’s determinant. With these techniques at our disposal, we consider the stability of the nonlinear system with linear control. We find that the linear control can shift the bifurcation point by a substantial amount. Results from analog computer simulation are reported.

In Chapter 7, we couple the two principal modal coordinates through a nonlinear cutting force. We explore the complex dynamic response of the system in two cutting regimes. Numerical simulations are employed and compared with measured chatter signatures.

In Chapter 8, we consider nonlinear control strategies. First, we show how nonlinear feedback can be used to change the global stability of a single-degree-of-freedom cutting model. We then look at the forced response of a single-degree-of-freedom nonlinear cutting model and find that a quench phenomenon analogous to that observed in the Rayleigh and Van Der Pol oscillators can be achieved for time-delay systems.
We close the Dissertation by summarizing our findings and making recommendations for future research.
Part I

Linear Machine-Tool Dynamics and Control
Chapter 2

The Statics and Dynamics of Single-Point Turning

The conversion of raw material into manufactured products usually requires that some sort of material removal process be performed. By far the most common material removal processes are the so-called chip-forming types. Chip forming, or the act of shaving metal from a workpiece to produce a desired geometric shape, is carried out using a machine tool. The type of machine tool used to manufacture a product, its general shape and orientation of cutting surfaces, is dictated to a large extent by the eventual geometry and surface finish desired for the end product.

Milling machines, engine lathes, twist drills, and shaping and planing machines are but a few examples of the diverse types of machine tools that exist. One way in which machine tools are distinguished from one another is by the number of cutting surfaces, or cutters, employed to remove the metal. In this respect, machine tools are referred to as either single-point, or multipoint, according to the common convention.
We will be concerned with single-point tools. Specifically, we will consider a boring operation on an engine lathe. Boring bars, as will be seen, are particularly prone to dynamic instability due to their relatively high flexibility and low damping. This particular tool and manufacturing process are amenable for study because the structural dynamics are reasonably well characterized using a low-order model.

In the remainder of this chapter we summarize the background material necessary to an understanding of the problem. This is a mature subject area, and a number of fine text books are available that treat the material in greater detail (Tobias, 1965; King, 1985; Armarego and Brown, 1969; Boothroyd, 1975). The following is intended as an introduction to the terminology and physics of the problem for the reader who may be new to the subject.

### 2.1 Boring Bars and Engine Lathes

An engine lathe is illustrated in Fig. 2.1. The machine consists of a headstock (A) mounted on the lathe bed (B). The headstock contains the spindle (C) that rotates the cylindrical workpiece (D) that is gripped in the chuck (E). The single-point cutting tool (F), in this case a boring bar, is held in the toolholder (G) mounted on the cross slide (H). The cross slide is in turn mounted to the carriage (I).

Boring produces an internal cylindrical surface. Generally, one begins with a cylindrical workpiece of some nominal inner diameter that must be bored out to a larger diameter of specified tolerance.

The machining parameters controlled by an operator are the cutting speed \( V \), the feedrate \( r \), and the depth of cut \( w \) that are illustrated in Fig 2.2 for a single-point turning operation. The cutting speed refers to the relative velocity between the tool and the workpiece. It is
a function of the spindle revolutions per minute and workpiece diameter and has units of velocity (f.p.m. or m s$^{-1}$). The speed is sometimes referred to as the primary motion and is measured along the $y$ direction. The feedrate is a measure of the carriage motion. It is expressed in terms of the distance the carriage moves towards the headstock (the “feed” $s_o$) per spindle revolution (i.p.r. or mm rev$^{-1}$) and is measured along the $z$ direction. The carriage and spindle are connected by gearing and a lead screw (item (J) of Fig. 2.1) so that the two motions are synchronized. Thus, as the workpiece rotates, the machine feeds the tool in the $z$ direction. Finally, the depth of cut is a measure of the amount of material to be removed, has units of length (in or mm), and is measured along the $x$ direction.

As the chip forms, it produces the transient and machined surfaces of Fig. 2.2 (a). We see that the machined surface is of a diameter that is decreased by twice the depth of cut for
single-point turning (analogously, the diameter is increased by twice the depth of cut for single-point boring). Section AA of Fig. 2.2 shows the chip flowing over the tool in the $y - z$ plane, sometimes referred to as the working plane.

In Fig. 2.3, we see an exploded plan view of the tool point while it is engaged in cutting. The cross section of the uncut chip is seen to depend on the feed, the lead angle, and the corner radius. For steady cutting, the undeformed chip thickness $s$ is simply the feed $s_0$. The area of the uncut chip influences the power required to carry out the operation and is often used to characterize the cutting forces.

### 2.2 Mechanics of Metal Removal

The forces that arise during metal removal are classified as either static or dynamic. There is a well established literature concerning the modelling and analysis of both classes. In this section we begin with the classic shear plane models for static orthogonal cutting and then review how researchers have tried to develop dynamic cutting force models.

#### 2.2.1 Static Orthogonal Cutting with a Single Edge

A thin-shear-plane model is often used as the basis for the characterization of static cutting forces. This type of solution is referred to as a shear-angle solution, or Merchant analysis when crediting its originator (Merchant, 1945). The theory of Merchant is popular because it purportedly results in an upper bound solution for the cutting forces. Thus, it is conservative and provides a simple way to deal qualitatively with what is at present a difficult if not intractable problem.
Figure 2.2: Single-point turning. (a) Oblique view, (b) plan view, (c) side view, and (d) section AA.
First, we define an orthogonal cutting process as one where a single, straight cutting edge is oriented perpendicular to the cutting speed. This condition is approximated when \( w \) is at least \( 5s \) and the inclination angle \( i \) of Fig 2.2 (c) is zero (Shaw, 1984). Orthogonal cutting geometries are contrived so that the determination of cutting forces is a two-dimensional problem. In practice, cutting geometries tend to be oblique, a topic we will explore a little later.

Conditions are assumed to be such that a continuous chip is removed. The primary deformation zone is then taken to be a well-defined plane known as the shear plane. Consequently, the chip is assumed to act like a rigid body held in equilibrium by the forces created at the chip-tool interface and shear plane. A free-body diagram illustrating the forces acting on the chip is shown in Fig. 2.4(a).

Following Merchant (1945), we construct a diagram of forces in Fig. 2.4(b). Using the diagram, the resultant force \( R \) due to the shearing action of the tool on the workpiece for a given feed \( s = s_o \) and surface speed \( V = v_o \) is resolved into a normal force component
From purely geometric considerations, the cutting and thrust forces are:

\[ F_c = \frac{sw\tau_s \cos (\beta - \alpha)}{\sin \phi \cos (\phi + \beta - \alpha)} \]  
\[ F_t = \frac{sw\tau_s \sin (\beta - \alpha)}{\sin \phi \cos (\phi + \beta - \alpha)} \]  

where \( \alpha \) is the tool rake angle (slope of the front face of the tool), \( \tau_s \) is the shear stress, \( \phi \) is the shear angle, \( w \) is the width of cut, \( s \) is the undeformed chip thickness or feed \( s = s_o \) for steady turning operations, and \( \beta \) is the friction angle.

A continuity relation is used to relate the cutting speed \( v_o \) and the undeformed chip thickness \( s_o \) to the chip velocity \( v_c \) and the chip thickness \( s_c \). The result is

\[ s_o v_o = s_c v_c \]

where it is assumed that the chip does not spread during formation (i.e., \( w \) is a constant).
From the continuity relation, the chip thickness ratio is defined as

$$\rho = \frac{s_o}{s_c} = \frac{v_c}{v_o}$$  \hspace{1cm} (2.4)

and from the geometry of Fig. 2.4 (b)

$$\tan \phi = \frac{\rho \cos \alpha}{1 - \rho \sin \alpha}$$  \hspace{1cm} (2.5)

A number of thin-zone models have been proposed that are variants of the Merchant analysis (Stabler, 1951; Lee and Schaffer, 1951; Oxley, 1961; Kobayashi and Thomsen, 1962; Hastings, Mathew, and Oxley, 1980). Discussions of the assumptions and relative merits of these and other models for static cutting can be found in Shaw (1984), Boothroyd (1975), Armarego and Brown (1969), Kalpakjian (1992), and Oxley (1989). It is important to note, as all the authors point out, that none of these models matches the experimental data outside of the narrow range of values for which it has been adapted. However, it is generally accepted that for a given set of cutting conditions, an empirical relation of the form

$$\phi = C_1 - C_2(\beta - \alpha)$$  \hspace{1cm} (2.6)

can be found among the angles where $C_1$ and $C_2$ are constants. Thus, for steady-state orthogonal cutting conditions, the cutting force is a constant and proportional to the area of the uncut chip for a fixed speed.

### 2.2.2 Oblique Cutting

One seldom encounters a truly orthogonal cut in practice. Nearly all practical cutting processes are oblique; that is, the tool’s cutting edge is inclined to the relative velocity between the tool and the workpiece as shown in Fig. 2.5. Furthermore, most tools engage
two or more cutting edges at a time, as occurs along the toolnose in the exploded view of the tool point shown in Fig. 2.3.

The mechanics of oblique cutting may be determined using a thin-shear-plane model. Descriptions of this type of cutting theory are presented by Armarego and Brown (1969) and Shaw (1984). The original work is most often credited to Stabler (1951). The problem is worked out by finding the inclination angle for the given tool geometry and using this information, along with the chip flow direction, to construct an equivalent orthogonal cutting condition. Techniques for working out the geometry and deriving the forces are explained in Armarego and Brown (1969) and in Shaw (1984).
Armarego and Brown (1969) suggest the following relations for the cutting and thrust forces:

\[ F_c = \frac{sw\tau_s}{\sin \phi_n} \left\{ \frac{\cos (\beta_n - \alpha_n) + \tan i \tan \eta_c \sin \beta_n}{\sqrt{\cos^2 (\phi_n + \beta_n - \alpha_n) + \tan^2 \eta_c \sin^2 \beta_n}} \right\} \]  (2.7)

\[ F_t = \frac{sw\tau_s}{\sin \phi_n} \left\{ \frac{\cos (\beta_n - \alpha_n) \tan i - \tan \eta_c \sin \beta_n}{\sqrt{\cos^2 (\phi_n + \beta_n - \alpha_n) + \tan^2 \eta_c \sin^2 \beta_n}} \right\} \]  (2.8)

where \( \phi_n, \beta_n, \) and \( \alpha_n \) are the shear angle, friction angle, and rake angle in the plane normal to the cutting edge. They are determined by consideration of the oblique geometry as characterized by the angle of inclination \( i \) and the chip-flow direction \( \eta_c \).

A theory for oblique cutting geometries that involves multiple edges is still being developed. For instance, an upper bound cutting model for oblique cutting tools possessing a nose radius, essentially the problem of Fig. 2.3, was recently proposed by Seethaler and Yellowley (1997). It supposes the existence of multiple shear planes that may be treated as a series of single-edge cutting problems subject to the constraint that the chip leave the tool as a rigid body.

### 2.2.3 Summary of Static Cutting Force Models

A review of the literature reveals a dizzying array of static cutting-force models. Though none of the models can be said to accurately predict cutting forces for the most general cases, a consensus does seem to exist that a thin-shear-zone model, as proposed by Merchant (1945), can provide an upper bound solution.

For the purposes of this Dissertation, the most important observation that can be made is that the cutting forces are dependent on the undeformed chip area. This conclusion holds for both orthogonal and oblique geometries. It appears that, for a single-edged cut of fixed speed and width, the cutting forces will depend entirely on the undeformed chip thickness.
2.2.4 Dynamic Orthogonal Cutting

The problem of tool motion while machining with an orthogonal cutting geometry is illustrated in Fig. 2.6. The undeformed chip thickness is variable due to the outer and inner chip modulations. Inner chip modulations are the result of tool motions $x(t)$ and $y(t)$ that generate a wavy surface during the present tool pass. During the next tool pass, this wavy surface is removed and becomes the outer surface of the chip. Hence, for the case of an orthogonal turning operation with a single cutting edge, the outer chip modulations are due to tool motions that occurred during the previous tool pass and may be characterized by the tool displacements $x_o(t) = x_\tau + s_\tau$ and $y_o(t) = y_\tau$, where $x_\tau = x(t - \tau)$, $y_\tau = y(t - \tau)$, and $\tau$ is the period of one workpiece revolution.

![Figure 2.6: Dynamic orthogonal cutting.](image)

The production of an inner chip modulation by the tool is commonly referred to as wave generation. In a similar fashion, the removal of the outer chip modulation is often referred to as wave removal. Clearly, the chip thickness varies due to wave generation and removal. Furthermore, it seems that the shear angle should vary as well.
Numerous approaches have been devised to account for the forces generated as a result of wave generation and removal. Some investigators work from Merchant’s model and seek an explicit expression for the fluctuation of the shear angle in order to obtain the dynamic forces. Other researchers take a more empirical approach. Space considerations preclude a detailed discussion of all of these approaches, but some representative treatments follow.

The model of Tobias and Fishwick (1958)

Tobias and Fishwick hypothesised that, under dynamic cutting conditions, the cutting force $P$ is a function of three independent factors, or $P(s, r, \Omega)$ where $s$ is the undeformed chip thickness, $r$ is the feedrate, and $\Omega$ is the spindle rotational speed. Hence, the cutting-force variation for small changes in these factors is

$$dP = k_1 ds + k_2 dr + k_3 d\Omega$$

where $k_1$, $k_2$, and $k_3$ are dynamic cutting coefficients such that

$$k_1 = \left(\frac{\partial P}{\partial s}\right)_{dr=d\Omega=0}, \quad k_2 = \left(\frac{\partial P}{\partial r}\right)_{ds=d\Omega=0}, \quad k_3 = \left(\frac{\partial P}{\partial \Omega}\right)_{dr=ds=0}$$

Tobias (1965) related the dynamic cutting coefficients to the static force coefficients to obtain

$$dP = k_1 ds + \frac{2\pi K}{\Omega} dr + \left[k_\Omega - \frac{2\pi K}{\Omega} s_0\right] d\Omega$$

where $k_1$ is a dynamic coefficient termed the chip thickness ratio, $K = k_s - k_1$ is the penetration coefficient, $k_s$ and $k_\Omega$ are static force coefficients relative to the undeformed chip thickness and speed, respectively, and $s_0$ is the steady-state undeformed chip thickness. The values for these coefficients are determined at a nominal speed $v_o$ and feedrate $r_o$. The force $dP$ depends linearly on the width of cut $w$, and it has become customary to factor out this dependence and write

$$dP = w \left\{ k_1 ds + \frac{2\pi K}{\Omega} dr + \left[k_\Omega - \frac{2\pi K}{\Omega} s_0\right] d\Omega \right\}$$
Restricting the tool motion to the direction normal to the cut and taking $x$ to be positive into the workpiece,

$$ds = x(t) - x(t - \tau) \quad (2.12)$$

$$dr = \dot{x} \quad (2.13)$$

where $x(t)$ is the tool vibration superimposed on the steady feed. Then, assuming that changes in the speed $d\Omega$ can be ignored, one finds that

$$dP_x = wk_1[x(t) - x(t - \tau)] + w\frac{2\pi K}{\Omega} \dot{x} \quad (2.14)$$

This basic model shows that the cutting-force fluctuations have a component that is proportional to the undeformed chip thickness and a component due to the “rate-of-penetration” which is sometimes referred to as the plowing effect. The component due to the undeformed chip thickness depends on the displacement at a previous time and for this reason is termed a *regenerative* cutting force. The component due to the rate of penetration is a velocity dependent term that can appear as either positive or negative damping, depending on the geometry of the cut. Tobias (1965) tends to differentiate rate-of-penetration effects from other damping-type forces that occur during the cutting process, though the distinction would appear difficult to make in practice.

**The model of Nigm, Sadek, and Tobias (1977a,b)**

Nigm, Sadek, and Tobias (1977a) used dimensional analysis of the steady-state orthogonal cutting process to derive explicit mathematical expressions for the chip-thickness ratio and the force ratio in terms of the rake angle, cutting speed, and the feed. They (1997b) then considered an incremental oscillation of the shear plane in response to dynamic variation of the cutting parameters such that

$$d\phi = n_1 ds + n_2 d\alpha + n_2 d\phi + n_3 dv \quad (2.15)$$
They found the following expressions for the incremental force components:

\[
dP_x = wk_{1c}\left\{C_1(x - x_r) + C_2\frac{\dot{x}}{v_0} + C_3\left(\frac{\dot{x}}{v_0} - \frac{\dot{x}_r}{v_0}\right)\right\} \tag{2.16}
\]

\[
dP_y = wk_{1c}\left\{T_1(x - x_r) + T_2\frac{\dot{x}}{v_0} + T_3\left(\frac{\dot{x}}{v_0} - \frac{\dot{x}_r}{v_0}\right)\right\} \tag{2.17}
\]

where the \(x\) direction is taken to be positive into the material, \(x\) and \(x_r\) are the deviations of the tool from its prescribed path during the present and former tool passes, respectively, and \(C_1, C_2, C_3, T_1, T_2, \) and \(T_3\) are cutting coefficients determined by the geometry of the cut.

**The model of Wu and Liu (1985a,b)**

Wu and Liu (1985a) begin with the Merchant Eqs. (2.1) and (2.2) and assume that an exponential form for the mean friction coefficient \(\mu = \tan \beta\) can be obtained from static cutting measurements. They use an approximate form of the continuity relationship of Eq. (2.5) in conjunction with the shear-angle formula

\[
\phi = \frac{1}{2}C_m - \frac{1}{2}(\beta - \alpha) \tag{2.18}
\]

to derive the following dynamic shear-angle relation:

\[
\cot \phi = (A_\phi - C_\phi v_0) + \frac{B_\phi}{2}(\dot{x} - \dot{x}_o) - \frac{C_\phi}{2}(\dot{y} - \dot{y}_o) \tag{2.19}
\]

where \(A_\phi, B_\phi,\) and \(C_\phi\) are the dynamic shear-angle coefficients evaluated at a given cutting condition with mean shear angle \(\phi_0\) and cutting speed \(v_0\), see Wu and Liu (1985a) for the explicit expressions. The relationship represents a first-degree approximation for shear-angle oscillations about the mean cutting condition when the tool is free to oscillate both normal and tangential to the machined surface.

Wu and Liu substitute the dynamic shear-angle relation into Eqs. (2.1) and (2.2) and obtain

\[
P_x = -2\tau(x_o - x)[(A_x - C_x v_0) + \frac{C_x}{2}(\dot{x} - \dot{x}_o) - \frac{1}{2}C_x(\dot{y} - \dot{y}_o)] + f_p \tag{2.20}
\]
\[ P_y = 2w\tau(x_o - x)[(A_y - C_yv_0) + \frac{1}{2}B_y(\dot{x} - \dot{x}_o) - \frac{1}{2}C_y(\dot{y} - \dot{y}_o)] \] (2.21)

where \(A_x, B_x,\) and \(C_x\) and \(A_y, B_y,\) and \(C_y\) are the dynamic cutting coefficients, explicit expressions for which are derived by Wu and Liu (1985a), and \(f_p\) is the ploughing force, which they approximate as

\[ f_p = \frac{Kw}{v_0}\dot{x} \] (2.22)

They arrive at a reduced form of the dynamic cutting force and shear angle relations by assuming a constant length of the shear plane. The result is

\[ \cot \phi = (A\phi - C\phi v_0) + B_x(\dot{x} - \dot{x}_o) \] (2.23)

\[ P_x = -2w\tau(x_o - x)[(A_x - C_xv_0) + B_x(\dot{x} - \dot{x}_o)] - f_p \] (2.24)

\[ P_y = 2w\tau(x_o - x)[(A_y - C_yv_0) + B_y(\dot{x} - \dot{x}_o)] \] (2.25)

where the dynamic coefficients are the same as those previously mentioned. We see that, in this case, the cutting forces arise wholly as a function of tool motions normal to the cut surface, which was the assumption of Nigm, Sadek, and Tobias (1977). Linearizing about a static cutting condition, Minis, Magrab, and Pandelidis (1990) report that the model of Wu and Liu reduces to that employed by the CIRP (International Institute for Production Engineering Research), which is the form proposed by Nigm, Sadek, and Tobias (1977b). The linearized single-degree-of-freedom version of the model was also used in the experimental verification reported by Wu and Liu (1985b).

**The model of Lin and Weng (1991)**

Lin and Weng obtain a third-order weak nonlinear form for the shear-angle variation; that is,

\[ d\phi = \frac{\dot{x}_o - \dot{x}}{v_0} - \frac{1}{3} \left[ \left( \frac{\dot{x}_o}{v_0} \right)^3 - \left( \frac{\dot{x}}{v_0} \right)^3 \right] \] (2.26)
Then they expand Eqs. (2.1) and (2.2) about the mean shear angle \( \phi_o \) and obtain

\[
F_x(\phi_o + d\phi) = F_x(\phi_o) + \left( \frac{\partial F_x}{\partial \phi} \right)_{\phi_o} d\phi + \left( \frac{\partial^2 F_x}{\partial \phi^2} \right)_{\phi_o} d\phi^2 + \ldots \tag{2.27}
\]

\[
F_y(\phi_o + d\phi) = F_y(\phi_o) + \left( \frac{\partial F_y}{\partial \phi} \right)_{\phi_o} d\phi + \left( \frac{\partial^2 F_y}{\partial \phi^2} \right)_{\phi_o} d\phi^2 + \ldots \tag{2.28}
\]

Substituting Eq. (2.26) into Eqs. (2.27) and (2.28), taking the appropriate derivatives while making use of the shear-angle relation

\[
\phi = C_1 - \frac{1}{2}(\beta - \alpha) \tag{2.29}
\]

and keeping terms up to third order, they obtain

\[
dP_x = A_x w \Delta s - B_x w s \frac{\dot{s}}{v_o} + w s \left[ C_x \left( \frac{\dot{s}}{v_o} \right)^2 + B_x \left( \frac{x_o y_o - \dot{x} \dot{y}}{v_o^2} \right) \right] \tag{2.30}
\]

\[
+ w s \left\{ B_x \left[ \frac{(\dot{x}_o^3 - \dot{x}^3) - 3(\dot{x}_o y_o^2 - \dot{x} \ddot{y})}{3v_o^3} \right] - C_x \left[ \frac{2s(x_o y_o - \dot{x} \dot{y})}{v_o^3} \right] - D_x \left( \frac{\dot{s}}{v_o} \right)^3 \right\}
\]

\[
dP_y = A_y w \Delta s - B_y w s \frac{\dot{s}}{v_o} + w s \left[ C_y \left( \frac{\dot{s}}{v_o} \right)^2 + B_y \left( \frac{x_o y_o - \dot{x} \dot{y}}{v_o^2} \right) \right] \tag{2.31}
\]

\[
+ w s \left\{ B_y \left[ \frac{(\dot{x}_o^3 - \dot{x}^3) - 3(\dot{x}_o y_o^2 - \dot{x} \ddot{y})}{3v_o^3} \right] - C_y \left[ \frac{2s(x_o y_o - \dot{x} \dot{y})}{v_o^3} \right] - D_y \left( \frac{\dot{s}}{v_o} \right)^3 \right\}
\]

where \( \Delta s = s(t) - s_o \), and \( s(t) = x_o - x \).

We see that, to the second-order approximation, these equations have the form employed by Wu and Liu (1985a)).

**The model of Grabec (1988)**

Grabec uses empirical relations derived from the work of Hastings, Oxley, and Stevenson (1971) and Hastings, Mathew, and Oxley (1980) to develop dynamic cutting-force relations. First, the thrust force \( F_x \) is related to the main cutting force \( F_y \) through a friction coefficient \( K \) as

\[
F_x = K F_y \tag{2.32}
\]
He considers the case where cuts do not overlap and models the main cutting force as a function of the cutting speed $v$ and chip thickness $s$ in the form

$$ F_y = F_{yo} \frac{s}{s_o} \left[ C_1 \left( \frac{v}{v_o} - 1 \right)^2 + 1 \right] \quad (2.33) $$

where $F_{yo}$ is the steady-state main cutting force at the nominal cutting condition. The friction coefficient is treated in a similar fashion. The result is

$$ K = K_o \left( C_2 \left( \frac{v_f R}{v_o} - 1 \right)^2 + 1 \right) \left( C_2 \left( \frac{s}{s_o} - 1 \right)^2 + 1 \right) \quad (2.34) $$

where $K_o = F_{xo}/F_{yo}$ is a constant determined at the nominal cutting condition, $v_f$ is the friction velocity or the velocity along the tool rake face, and $\rho$ is the chip thickness ratio.

Grabec assumes also that the chip thickness ratio is a function of the cutting speed; that is,

$$ \rho(t) = \rho_o \left( C_4 \left( \frac{v}{v_o} - 1 \right)^2 + 1 \right) \quad (2.35) $$

Finally, he makes the following substitutions in order to obtain dynamic cutting relations:

$$ s(t) = s_o - x(t) \quad (2.36) $$
$$ v(t) = v_o - \dot{y}(t) \quad (2.37) $$
$$ v_f(t) = \frac{v(t)}{\rho t} - \dot{y}(t) \quad (2.38) $$

**The model of Moon (1994)**

Moon assumes that a shear-plane model is applicable and writes

$$ F_x = N \sin \alpha - F \cos \alpha \quad (2.39) $$
$$ F = \mu N \quad (2.40) $$
He assumes that $N$ is a linear function of the chip thickness so that

$$ N = C_1 w(x - x_o) \quad (2.41) $$

where $C_1$ is a chip-thickness coefficient. To investigate stick-slip behavior, Moon employs the continuous-friction law

$$ \mu = \left[ \mu_k + (1 - \mu_k) \sec (\beta v) \right] \tanh \alpha v \quad (2.42) $$

where $v = v_c + \dot{x} \cos \alpha$ is the relative velocity between the tool face and the chip, $v_c = \rho V$ is the chip velocity, and $\rho = s/s_c < 1$ is the chip thickness ratio.

### 2.2.5 Dynamic Oblique Cutting

A dynamic cutting-force model for oblique cutting has yet to be developed. An approach has been proposed by Gilsinn (1997) for diamond turning operations using a rounded nose tool. He assumes the cutting force to be a function of the uncut chip area. He defines the geometry of this area as a function of a variable depth of cut and constant feedrate. He finds that the resulting area has a decidedly quadratic dependence on the depth of cut. Thus, it seems reasonable that the cutting force for oblique cutting may be approximated by assuming a polynomial expansion in terms of the tool displacements.

### 2.2.6 Summary of Dynamic Cutting-Force Models

A great deal of effort has been focused on the development of suitable cutting-force models for the orthogonal cutting problem. The three primary mechanisms that the models seek to account for are the regeneration of surface waviness, the penetration, or ploughing effect, and the oscillation of the shear plane. Examining the models, we see that regeneration
gives rise to terms that are linear in the chip-thickness modulation $ds$. The penetration, or ploughing effect, produces a damping-type force proportional to the tool oscillation normal to the work surface. Finally, the oscillation of the shear plane leads to nonlinear terms that are proportional to the product of the chip thickness and its derivative. Oscillation of the shear plane is also responsible for coupling the main and thrust cutting forces.

### 2.3 Cutting Stability for Simple Machine-Tool Structures

The dynamic forces that arise during cutting can cause the machine-tool structure to lose stability. When this happens, the machine-tool vibrates and is said to chatter. Chatter occurs at the point when relative motion between the tool and workpiece results in a negative damping force that overcomes the dissipation inherent in the system. Chatter is a so-called self-excited oscillation because the energy that creates the vibration is generated by the vibration itself.

In this section, we consider the stability of a flexible tool in the presence of a dynamic cutting force

$$dP = wk_1 \left\{ ds + \frac{2\pi C_1}{\Omega} \dot{x} + \frac{2\pi C_2}{\Omega} d\dot{s} \right\}$$

(2.43)

where $w$ is the width of cut, $k_1$ is the “chip thickness coefficient”, $C_1 = K/k_1$ and $K$ is the “penetration rate coefficient”, $C_2$ is a cutting-force constant, and $x$ is the tool displacement normal to the machined surface in an orthogonal cutting geometry. This is the model considered by Nigm, Sadek, and Tobias (1977) and has the same form as the linearized model of Wu and Liu (1986b) and Lin and Weng (1991). The cutting force is seen to depend on the chip-thickness modulation $ds$, its derivative $d\dot{s}$, and the penetration rate $\dot{x}$. 
2.3.1 Stability of a Single-Degree-of-Freedom Cutting Model

In many practical instances, the structural modes of vibration of the machine tool are well spaced and may be considered as separate, single-degree-of-freedom linear oscillators. A modal approximation for the receptance of such a structure can be formulated as (Ewins, 1986)

\[ G_{ab}(\omega) = \frac{h_n}{\omega_n} \left( 1 - \omega^2/\omega_n^2 \right) + j(2\zeta_n\omega/\omega_n) + C \]  

(2.44)

where \( \omega_n, \zeta_n, \) and \( h_n \) are the natural frequency, damping, and modal factor, respectively, and \( C \) is a constant representing the contribution of higher modes. The receptance \( G_{ab} \) characterizes the response of the structure in the \( a \) direction to a force in the \( b \) direction.

Now consider the system of Fig. 2.7, which is an example taken from Tlusty (1985). Tlusty observes that only the projection \( P \cos (\theta - \psi) \) of the cutting force acts in the principal modal coordinate \( X' \). Furthermore, only the projection \( x' \cos \psi \) of the vibration of the mode in the direction \( X \) will modulate the chip, hence

\[ G_{xx} = \frac{X(\omega)}{P(\omega)} = \frac{X'(\omega)}{P(\omega)} \cos (\theta - \psi) \cos \psi = u G_{x'x'} \]  

(2.45)

where \( u = \cos (\theta - \psi) \cos \psi \) is the directional factor of the mode and \( G_{xx} \) is the so-called oriented transfer function or operative receptance locus.

The response of the machine tool to a harmonic cutting force \( dP(\omega) \) in a direction normal to the machined surface is

\[ X(\omega) = G_{xx}(\omega)dP(\omega) \]  

(2.46)

To assess the stability using the operative receptance locus, we assume that the cutting process has been disturbed, causing tool oscillations \( x(t) \) to be superimposed on the steady feed \( s_o \) normal to the machined surface. The material was machined on a previous tool
pass, which left a wavy surface that is the outer chip modulation $x_o(t)$. The instantaneous chip thickness is thus

$$s(t) = x_o(t) - x(t)$$  \hspace{1cm} (2.47)

We are interested in determining the point where the system becomes marginally stable. At this threshold, we assume that the system oscillates freely at the chatter frequency $\omega_c$ so that

$$x(t) = a_o \cos(\omega_c t)$$  \hspace{1cm} (2.48)

$$x_o(t) = a_o \cos(\omega_c t - \beta) + s_o$$  \hspace{1cm} (2.49)

where $\beta$ is the phase angle between the inner and outer chip modulations. For a single-point turning operation,

$$\beta = \frac{2\pi \omega_c}{\Omega}$$  \hspace{1cm} (2.50)
where $\Omega$ is the spindle rotational speed in rad/s.

The variation of the chip thickness is

$$ ds = s(t) - s_0 = Ax(t) + \frac{B}{\omega_c} \dot{x}(t) $$

(2.51)

and the derivative of its variation is

$$ d\dot{s} = \dot{s} = A \dot{x}(t) - \omega_c Bx(t) $$

(2.52)

where

$$ A = \cos(\beta) - 1 \quad (2.53) $$

$$ B = -\sin(\beta) \quad (2.54) $$

Substituting the expressions for the chip thickness and its derivative into the incremental cutting force

$$ dP = wk_1 \left\{ (A - \beta C_2 B)x(t) + \left[ \frac{2\pi}{\Omega} (C_1 + C_2 A) + \frac{B}{\omega_c} \right] \dot{x}(t) \right\} $$

(2.55)

we see that the cutting force acts in such a fashion as to produce both displacement and velocity feedback to the machine-tool structure.

For the single-degree-of-freedom problem

$$ G_{xx} = u \frac{\frac{1}{m}}{\omega_n^2 - \omega_c^2 + i2\zeta \omega_c \omega_n} $$

(2.56)

We let $dP(t) = dP(\omega_c)e^{i\omega_c t}$ and $x(t) = X(\omega_c)e^{i\omega_c t}$ and substitute Eqs. (2.55) and (2.56) into Eq. (2.46). Then, separating real and imaginary terms, we obtain the linear stability condition

$$ \omega_n^2 - \omega_c^2 - \frac{uwk_1}{m} (A - \beta C_2 B) = 0 $$

(2.57)

$$ 2\zeta \omega_c \omega_n - \frac{uwk_1}{m} \left( \frac{2\pi}{\Omega} (C_1 + C_2 A) + \frac{B}{\omega_c} \right) = 0 $$

(2.58)
The stability boundary can be plotted as a function of width of cut and spindle rotational speed.

As an example, we generate the stability lobes for the single-degree-of-freedom system considered by Hanna and Tobias (1974)

\[ \ddot{x} + 2\xi \dot{x} + p^2(x + \beta_2x^2 + \beta_3x^3) = -p^2w[x - x_\tau + \alpha_2(x - x_\tau)^2 + \alpha_3(x - x_\tau)^3] \] (2.59)

The parameters as determined experimentally by Hanna and Tobias (1974) are \( p = 1088.56 \text{ rad/sec}, \xi = 24792/\omega \text{ rad}^2, \beta_2 = 479.3 \text{ 1/in}, \beta_3 = 264500 \text{ 1/in}^2, \alpha_2 = 5.668 \text{ 1/in}, \text{ and } \alpha_3 = -3715.2 \text{ 1/in}^2. \) This model does not take into account penetration or shear-plane effects.

For the moment, we are concerned only with the linear problem

\[ \ddot{x} + 2\xi \dot{x} + p^2x + p^2w(x - x_\tau) = 0 \] (2.60)

where it is evident that \( w \) is, in this case, an effective width of cut that encompasses the directional factor \( u \), the chip thickness coefficient \( k_1 \), and the modal stiffness of the structure \( k \).

To study the neutral stability, we substitute

\[ x = a \cos \omega_c t \] (2.61)

into Eq. (2.60), set each of the coefficients of \( \cos \omega_c t \) and \( \sin \omega_c t \) equal to zero, and obtain

\[ p^2 - \omega_c^2 + p^2w(1 - \cos \omega_\tau) = 0 \] (2.62)

\[ 2\xi + p^2w \sin \omega_\tau = 0 \] (2.63)

We establish the linear stability boundary in Fig. 2.8 by solving Eqs. (2.62) and (2.63) for \( w, \tau, \text{ and } \omega_c \). It follows from Eqs. (2.62) and (2.63) that when \( w < 2\xi \omega_c/p^2 \), which
is 0.0418, the cutting process remains stable. We note that this occurs when the phasing between subsequent cuts is \( \beta = \omega_c \tau = 2n\pi + 3\pi/2 \) where \( n = 1, 2, 3, \ldots \).

The line \( w = 0.0418 \) on the stability plot is known as the tangent boundary to the lobed stability curve. This tangent boundary takes on different shapes as we include the effects of the penetration rate and shear plane-oscillations. Tobias (1965) shows that, as the cutting speed is increased, the tangent boundary asymptotically approaches the straight line boundary, even if one considers the effects of both positive and negative penetration-rate coefficients.

\[
\beta = \omega_c \tau = 2n\pi + 3\pi/2 \quad \text{where} \quad n = 1, 2, 3, \ldots
\]

Wu and Liu (1986b) explored the effect of shear-plane oscillations in addition to the penetration rate on the tangent boundary. Using energy considerations they show that \( \beta = \omega_c \tau = 2n\pi + 3\pi/2 \) where \( n = 1, 2, 3, \ldots \) is the approximate phasing between the inner and outer chip modulations along the tangent boundary to the lobed stability curves. They then solve for \( w \) as a function of the rotational speed \( \Omega \) along the tangent boundary. They find that the penetration effect tends to have a stabilizing influence at low cutting speeds, whereas friction along the rake face tends to have a destabilizing influence. Both effects
diminish at high speeds.

2.3.2 Stability of a Two-Degree-of-Freedom System

The stability of multi-degree-of-freedom cutting tools is complicated by the combined presence of regenerative and the so-called mode-coupling effects. An early investigation of this problem was due to Saljé (1956) whose experimental results reveal some of the complex motions of a two-degree-of-freedom tool. The most simple practical system that can be considered is the case of two orthogonal modes of vibration considered by Tlusty (1985) and illustrated in Fig. 2.9. Following Tlusty (1985) we ignore the penetration and friction effects.

First, the modal receptances for the two modes of vibration are found in the principal modal coordinates to be

$$G_{x_1x_1}(\omega) = \frac{b_1}{\omega_1^2} \left(1 - \omega^2/\omega_1^2\right) + j(2\zeta_1 \omega/\omega_1) + C_1$$

(2.64)

Figure 2.9: Two-degree-of-freedom cutting system.
\[ G_{x_2x_2}(\omega) = \frac{h_n}{(1 - \omega^2/\omega_n^2) + j(2\zeta_2\omega/\omega_2)} + C_2 \] (2.65)

where \( \omega_n, \zeta_n, \) and \( h_n \) are the natural frequency, damping, and modal factor, respectively, of mode \( n \), and \( C_n \) is a constant representing the contribution of higher modes to mode \( n \). The directional factors are computed by taking a unit cutting force vector \( dP \) and projecting it onto \( X_1 \) and \( X_2 \) and finally onto \( X \). The result is

\[
\begin{align*}
    u_1 &= \cos(\theta - \psi_1) \cos \psi_1 \\
    u_2 &= -\cos\left(\frac{\pi}{2} + \psi_1 - \theta\right) \sin \psi_1 
\end{align*}
\] (2.66) (2.67)

The operative response locus is then

\[ G_{xx} = u_1G_{x_1x_1} + u_2G_{y_2y_2} \] (2.68)

For the simplest regenerative cutting condition,

\[ dP(t) = -k_1[w(x(t) - x(t - \tau))] \] (2.69)

The stability is determined as before. Let \( dP(t) = dP(\omega_c)e^{i\omega_ct} \) and \( x(t) = X(\omega_c)e^{i\omega_ct} \) in Eq. (2.69) and substitute Eqs. (2.68) and (2.69) into Eq. (2.46). Then, by separating real and imaginary parts, the characteristic equation of the system.

It is well-known that the stability of a system with two orthogonal modes is strongly dependent on the orientation of these modes (Thusty, 1985; Tobias, 1965) with respect to the machined surface. For instance, if \( \psi_1 = 0 \) then the linear stability is determined wholly by the stability of the mode normal to the machined surface. The result is quite different, however, if the principal modal coordinates are rotated with respect to the normal to the machined surface, as shown by Kuchma (1957) who performed tests using boring bars of nonuniform cross-section to reveal this dependence.
To illustrate these concepts, we consider a boring bar with well spaced modes of vibration and determine its stability by supposing that the dynamics are well described by a reduced-order model of the first two transverse modes. For a plunge type of cut, where the tool feeds directly into the tube wall along the $x$ direction, the schematic diagram of Fig. 2.9 is appropriate.

We assume that the bar is essentially symmetric, so that the modal mass is the same for each of the first transverse modes, but that the stiffnesses differ owing to the manner in which the tool is supported by the fixture. We further assume that the principal modal coordinates are rotated with respect to the normal to the machined surface, as indicated by the angle $\psi_1$ due to an oblique cutting geometry.

The equations of motion expressed in terms of the modal coordinates are

\[
\begin{align*}
 m\ddot{x}_1 + c_1\dot{x}_1 + \lambda_1 x_1 &= -dP \cos (\theta - \psi_1) \\
 m\ddot{x}_2 + c_2\dot{x}_2 + \lambda_2 x_2 &= -dP \cos \left(\frac{1}{2}\pi + \psi_1 - \theta\right)
\end{align*}
\]

where $x_1$ and $x_2$ are the tool modal coordinates, respectively, $m$ is the modal mass, $c_1$ and $c_2$ are linear viscous damping coefficients, $\lambda_1$ and $\lambda_2$ are linear stiffness coefficients, and $dP$ is the differential regenerative cutting force

\[
dP = k_s w(x - x_\tau)
\]

where $k_s$ is the cutting force coefficient associated with the workpiece material, $x$ is the displacement component normal to the machined surface, and $x_\tau = x(t - \tau)$ is the displacement during the previous workpiece revolution, $w$ is the width of cut, and $\tau$ is the period of the revolution.

We assume that the stiffness of the tool in the $x_1$ direction is that of a simple cantilever beam of length $l$, modulus $E$, and moment of inertia $I$ about its midplane so that

\[
\lambda_1 = \frac{3EI}{l^3}
\]
Dividing through by the modal mass $m$, we obtain

$$
\ddot{x}_1 + 2\zeta_1 \omega_1 \dot{x}_1 + \omega_1^2 x_1 = -dP \frac{\omega_1^2}{\lambda_1} \cos(\theta - \psi_1) \quad (2.74)
$$

$$
\ddot{x}_2 + 2\zeta_2 \omega_2 \dot{x}_2 + \omega_2^2 x_2 = dP \frac{\omega_1^2}{\lambda_1} \sin(\theta - \psi_1) \quad (2.75)
$$

where $\zeta_1$ and $\zeta_2$ are linear viscous damping ratios, and $\omega_1$ and $\omega_2$ are the natural frequencies.

From Fig. 2.9 we find that

$$
x = x_1 \cos(\psi_1) - x_2 \sin(\psi_1) \quad (2.76)
$$

so that

$$
dP = k_s w [\cos(\psi_1)(x_1 - x_{1r}) - \sin(\psi_1)(x_2 - x_{2r})] \quad (2.77)
$$

Let $\gamma_1 = \omega_1^2 \frac{k_s}{\lambda_1} \cos(\theta - \psi_1)$, $\gamma_2 = -\omega_1^2 \frac{k_s}{\lambda_1} \sin(\theta - \psi_1)$, $\eta = \cos(\psi_1)$, and $\nu = -\sin(\psi_1)$, then the equations of motion can be rewritten as

$$
\ddot{x}_1 + 2\zeta_1 \omega_1 \dot{x}_1 + \omega_1^2 x = -\gamma_1 w [\eta(x_1 - x_{1r}) + \nu(x_2 - x_{2r})] \quad (2.78)
$$

$$
\ddot{x}_2 + 2\zeta_2 \omega_2 \dot{x}_2 + \omega_2^2 x = -\gamma_2 w [\eta(x_1 - x_{1r}) + \nu(x_2 - x_{2r})] \quad (2.79)
$$

To analyze the stability of this system, we assume zero initial conditions, take the Laplace transform, and obtain

$$
(s^2 + 2\zeta_1 \omega_1 s + \omega_1^2)X_1(s) = -\gamma_1 w [\eta(1 - e^{-s\tau})X_1(s) + \nu(1 - e^{-s\tau})X_2(s)] \quad (2.80)
$$

$$
(s^2 + 2\zeta_2 \omega_2 s + \omega_2^2)X_2(s) = -\gamma_2 w [\eta(1 - e^{-s\tau})X_1(s) + \nu(1 - e^{-s\tau})X_2(s)] \quad (2.81)
$$

Then, solving Eq. (2.81) for $X_2(s)$ yields

$$
X_2(s) = \frac{-\gamma_2 w \eta(1 - e^{-s\tau})X_1(s)}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2 + \gamma_2 w \nu(1 - e^{-s\tau})} \quad (2.82)
$$
Substituting Eq. (2.82) into Eq. (2.80), we find the characteristic equation

\[
[s^2 + 2ζ_1ω_1s + ω_1^2 + γ_1w(1 − e^{-st})][s^2 + 2ζ_2ω_2s + ω_2^2 + γ_2wν(1 − e^{-st})]\]

\[-γ_1γ_2w^2ν(1 − e^{-st})^2 = 0 \quad (2.83)
\]

To obtain the familiar lobed stability boundary, we assume \( s = iω_c \), separate real and imaginary terms, and solve the resulting pair of equations for \( w \) and \( ω_c \) as a function of the delay \( τ \).

We now consider the predicted cutting stability when the modal properties are \( f_2 = 493 \) Hz and \( ζ_2 = 0.03 \) and \( f_1 = 365 \) Hz and \( ζ_1 = 0.02 \).

The boring bar itself is approximated as a circular cylinder of length 9” (length of bar measured from tool tip to clamped end) and diameter 1”.

We recall that the moment of inertia for a circular cross section is \( I = \pi/4(d/2)^4 \), then \( λ_1 = 6060 \) psi for a steel of modulus \( E = 30000 \) kpsi. A representative value for the cutting stiffness of aluminum is \( k_s = 145000 \) psi according to Kalpakjian (1992).

We consider two cases. The first case is a two-degree-of-freedom system where \( ψ_1 = 15^\circ \). In the second case, \( ψ_1 = 0^\circ \) and the stability reduces to that of a single-degree-of-freedom system. The resulting stability lobes are plotted for a range of spindle speeds around 170 rpm in Fig. 2.10. We see from the plot that the stability is affected by the presence of the second degree of freedom.
Figure 2.10: Stability lobes for two-degree-of-freedom cutting system.
Chapter 3

A Linear Chatter Control Scheme

The linear stability of regenerative chatter was shown in Chapter 2 to be a complex function of workpiece revolutions per minute and depth of cut for a fixed feedrate. We saw that the maximum stable width of cut was directly related to the amount of positive damping in the machine-tool structure for a fixed feedrate and cutting speed. Thus, the greater the damping in the system is, the larger the region is of stable machining operations. Introducing some terminology from nonlinear dynamics, we refer to the transition from static machining to chatter as a bifurcation. Thus, the stability diagram developed in Chapter 2, or stability lobes as they are often referred to in the literature, represents the locus of the bifurcation points encountered for a fixed feedrate as the speed and depth of cut are allowed to vary. Our goal for a linear control system is to delay the onset of these bifurcations. In other words, we seek to alter the system dynamics such that the stability lobes are shifted up and bifurcation occurs at larger depths of cut.

In this chapter we develop a vibration control system to enhance the tool damping and thereby shift the stability lobes. To begin, we briefly consider some of the available control
options and settle on second-order feedback compensation, in essence an active vibration absorber. We then take up the topic of active vibration absorbers for vibration control, comparing and contrasting semi-active and fully active approaches and reviewing the formulations found in the literature. Next, we consider the effect of an absorber on the stability of the single-degree-of-freedom model for machine-tool dynamics that was analyzed in Chapter 2. We find that an absorber is quite effective in shifting the bifurcation in the single-degree-of-freedom case.

3.1 Selecting a Vibration Control Strategy

Our goal for the boring-bar application is to achieve stable cuts while accommodating long tool overhangs. It is clear that in order to achieve this goal we must enhance the tool’s dynamic stiffness and damping. Obviously, we may achieve this via a number of methods. In this section we briefly review some of the known options and present the design considerations that lead to our choice of 2nd-order feedback compensation.

3.1.1 Passive Vibration Control Solutions

Passive vibration control is achieved by incorporating design elements such as damping materials and tuned mass absorbers into a structure to modify its response to initial conditions and/or forced excitations. The phrase passive control arises because no external energy source is associated with its operation. Passive methods have been, and continue to be, the dominant choice for attacking vibration problems because of there simplicity and economy.

For boring bars, one course of action, common on the shop floor, is to add a damping
treatment. A good machinist will often wrap the boring bar with a rubber strap when chatter causes problems, or merely grip the tool firmly with his hand (not an entirely safe practice, but common nonetheless) in order to increase the tool’s damping and stiffness. Other common passive approaches include modifications to the tool holder (Rivin and Kang, 1989; Kelson and Hsueh, 1996) and the addition of a tuned mass vibration absorber (Hopkins, 1974; Kosker, 1975).

Passive vibration control solutions can be quite effective for specific applications and are desirable from the standpoint of simplicity and cost. Unfortunately, they have their limitations. For instance, heat adversely affects viscoelastic performance, so that as the tool heats up, the performance of conventional damping treatments tends to degrade. Alternatively, a properly tuned damped vibration absorber can provide good narrow bandwidth vibration control. But if the tool is to be used for a variety of overhung conditions, the absorber must be “retuned” (i.e., its mass or stiffness modified) for each operation in order to be effective. A self-tuning, or even an easily tunable absorber can overcome this limitation, but is difficult to achieve with passive elements. This leads us to consider active control.

3.1.2 Active Vibration Control Solutions

Active vibration control is typically achieved by incorporating sensor and actuator pairs in the structural design to modify the response via feedback control. Obviously, once active elements are incorporated into the structure, any type of feedback control may be used, with the caveat that the dynamics we wish to modify are both observable and controllable.

There are two recent examples of active vibration control schemes specifically for boring bars in the literature. Both methods seek to add damping to a single axis and vibration mode of the structure, though the actual control methodologies differ. Matsubara, Ya-
mamoto, and Mizumoto (1989) used piezoelectric actuators to apply control moments to the boring bar. Their system uses time delay to phase shift an accelerometer signal and achieve narrowband velocity feedback control. In this way, they were able to add damping to the first bending mode in a direction normal to the cutting surface. Tewani, Rouch, and Walcott (1995) mounted a piezoelectric reaction mass actuator in the bar itself and created what they termed an active dynamic absorber. Using optimal control methods, they designed a feedback control system to add damping to the bar. The orientation of the actuator in this scheme may be adjusted so that control forces are oriented in the plane of chatter. We point out that the cutting force direction and the direction of chatter do not necessarily coincide and that the direction of the cutting force vector can vary as a function of time under dynamic cutting conditions.

¿From a structural viewpoint, a boring bar is simply a cantilevered beam, and one can find examples of various active control schemes for this type of elements in the literature. Our performance goal is to enhance the damping of targeted vibration modes. Robust and efficient methods for this type of control, such as the positive position feedback of Goh and Caughey (1985) and the active vibration absorber of Juang and Phan (1992), already exist in the literature. In the remainder of this chapter we adapt these techniques to the boring-bar chatter problem.

### 3.2 Vibration Absorption via 2nd-Order Compensators

For our purposes, vibration-absorber control strategies may be described as either passive, or semiactive, or fully active, the distinction being made according to how the actuators and sensors are used. We assume that the reader is familiar with passive absorption techniques and proceed to describe some of the relevant active schemes.
Figure 3.1: Vibration-absorber schemes: (a) semi-active and (b) fully active. Dashed box represents a “virtual absorber”.

In Fig. 3.1(a), a semiactive scheme for a single-degree-of-freedom mechanical oscillator is illustrated. The actuator and secondary mass $m_a$ are designed as a passive absorber, or reaction mass actuator, to counteract the motion of the primary mass $M$ of the plant in response to a disturbance force $d(t)$. The active part of this system produces a control force $f_c(t)$ to enhance the passive system performance. Details of an optimal control strategy using linear state-variable feedback to enhance the passive absorber, now termed an active dynamic absorber, can be found in Tewani, Walcott, and Rouch (1991). We note that this approach relies on the interaction of two structural modes; namely, that of the reduced-order model of the plant and that of the reaction-mass actuator. This implies that a separate reaction-mass actuator equipped with collocated sensors for state-feedback control must be employed for each mode of a multi-degree-of-freedom plant.

In Fig. 3.1(b), fully active structural control is illustrated. The control force is applied directly without the need of a secondary passive system. The plant dynamics in this case can be modified using state-variable feedback. To supplement, or as an alternative to state-
variable feedback, many authors have also suggested the use of second-order compensators in the feedback loop.


We observe that these fully active control schemes do not require a physical absorber but instead achieve a similar result via 2nd-order feedback compensation. The compensator is a “virtual absorber”, to borrow the terminology of Juang and Phan (1992), and, as such, its “virtual” mass, stiffness, and damping are at the user’s discretion. These methods are fairly robust in the presence of uncertain plant dynamics, are easily designed using root-locus techniques, and can be made immune to spillover effects provided that the actuator and sensor are collocated, a topic we will take up again later. They may be implemented using either digital or analog control. In digital control, the “virtual absorber” is simply a second-order difference equation or an infinite impulse response digital filter. For analog systems, a resonant second-order compensator circuit will suffice.

### 3.3 Application to Chatter Control

Mathematically, as in Chapter 2, we consider a structural model similar to that of Hanna and Tobias (1974). Thus, we consider:

$$\ddot{x} + 2\xi \dot{x} + p^2(x + \beta_2 x^2 + \beta_3 x^3) = d(t) + f_c(t)$$  \hspace{1cm} (3.1)
where $x$ is the coordinate to be controlled, $\xi$ is the hysteretic damping factor, $p$ is the
natural frequency, $\beta_2$ and $\beta_3$ are the nonlinear stiffness coefficients, $d(t)$ is the general
disturbance force due to the cutting process, and $f_c(t)$ is the control force exerted by the
actuator. Philosophically, we take the standpoint that the form of the disturbance $d(t)$
is unimportant for the design of a simple controller. We are interested in developing a
loop-shaping compensator to enhance damping of the structure.

By introducing a second-order compensator as an auxiliary, or absorber, coordinate and
coupling it to the plant, we obtain the following general system:

\[ \ddot{x} + 2\xi \dot{x} + p^2(x + \beta_2 x^2 + \beta_3 x^3) = d(t) + f_c(t) \quad (3.2) \]
\[ \ddot{y} + c_a \dot{y} + \omega_a^2 y = C(y, \dot{y}, x, \dot{x}) \quad (3.3) \]
\[ f_c(t) = f_c(y, \dot{y}, x, \dot{x}) \quad (3.4) \]

where $y$ is the absorber coordinate, $c_a$ and $\omega_a$ are the damping factor and frequency of the
absorber, and the coupling is defined by the functions $C(y, y, x, \dot{x})$ and $f_c(y, y, x, \dot{x})$.

Ignoring the structural nonlinearity and assuming that all of the plant states can be sensed,
we obtain the active vibration absorber of Lee and Sinha (1986) if the following coupling
terms are used:

\[ C(y, \dot{y}, x, \dot{x}) = cx \quad (3.5) \]
\[ f_c(y, \dot{y}, x, \dot{x}) = dy + \lambda_1 x + \lambda_2 \dot{x} \quad (3.6) \]

where $c_a = 0$ and $\lambda_1$ and $\lambda_2$ are linear state-variable feedback gains obtained by solving an
optimal regulator problem.

Similarly, with a displacement measure, such as a strain-gage signal, the positive-position-
feedback control of Goh and Caughey (1985) can be implemented using

\[ C(y, \dot{y}, x, \dot{x}) = cx \quad (3.7) \]
\[ f_c(y, \dot{y}, x, \dot{x}) = dy \quad (3.8) \]
By sensing the acceleration instead of the displacement, one can implement the active vibration absorber of Bruner et al. (1992) using

\[ C(y, \dot{y}, x, \dot{x}) = -c \ddot{x} \]  
\[ f_c(y, \dot{y}, x, \dot{x}) = d(c_a \dot{y} + \omega_0^2 y) \]

where \( c \) and \( d \) are control gains. Both of these approaches have been shown to produce robust control, even for multimode systems with multiple compensators.

All of these methods assume that the actuator and sensor are collocated. Collocation implies a certain structure for the frequency-response function between actuator input and sensor output and becomes of increasing importance as one considers the effect of the control in the presence of higher modes of the structure. A collocated frequency-response function, or FRF for short, should have the same form as a driving point FRF that is typically measured during an experimental modal analysis. In other words, the FRF should be characterized by alternating resonances and antiresonances. Authors, such as Fanson and Caughey (1987), Fanson, Blackwood, and Chu (1989), and Campbell and Crawley (1997) have discussed the importance of having well spaced poles separated by zeros in the actuator/sensor FRF. As stressed by Campbell and Crawley (1997), the placement of actuators and sensors has a direct bearing on the pole/zero structure, and this issue must be taken into account in the design.

In our initial theoretical developments, we will assume that the actuator dynamics are negligible and that, within a certain bandwidth, the actuator and sensor can be treated as a collocated pair. The compensation scheme that we will consider is the one proposed by Bruner et al (1992) that uses acceleration feedback. This problem is entirely analogous to adding a mechanical vibration absorber and will be treated as such in the subsequent analysis. Later, in Chapter 4, we will relax the restrictions on collocation and develop a design strategy that employs additional filtering to cope with actuator dynamics present...
in the actual experimental system.

3.3.1 Linear Machine-Tool Stability Analysis with an Absorber

In this section we consider the stability of the machine tool model of Hanna and Tobias (1974) with the addition of a fully-active vibration absorber. Thus, our complete system may be described by

\[
\ddot{x} + 2\xi \dot{x} + p^2(x + \beta_2 x^2 + \beta_3 x^3) = -p^2 w[x - x_\tau + \alpha_2(x - x_\tau)^2 + \alpha_3(x - x_\tau)^3] + d(c_a \dot{y} + \omega_a^2 y) \tag{3.11}
\]

\[
\ddot{y} + c_a \dot{y} + \omega_a^2 y = -c \ddot{x} \tag{3.12}
\]

where all variables are as previously defined. Linearizing, we obtain

\[
\ddot{x} + 2\xi \dot{x} + p^2 x = -p^2 w(x - x_\tau) + d(c_a \dot{y} + \omega_a^2 y) \tag{3.13}
\]

\[
\ddot{y} + c_a \dot{y} + \omega_a^2 y = -c \ddot{x} \tag{3.14}
\]

To design the absorber, we neglect the regenerative terms and note that the resulting system is entirely analogous to a passive vibration absorber, as in Bruner et al (1992). It can be shown that for \(c = 1\), the controller gain \(d\) takes on the role of an effective mass ratio \(d = m_a/M\) if one considers \(y\) as the relative motion between a single-degree-of-freedom plant of mass \(M\) and a damped vibration absorber of mass \(m_a\). In practice, the value of \(d\) will be constrained by the limitations of the actuator system. For our purposes, we chose \(d = 0.35\) because it gave reasonable results for the scaling limitations of subsequent analog-computer simulations.

Having fixed the effective mass ratio, one can use the design criteria of Den Hartog (1985) to arrive at a frequency and damping for the absorber. According to Den Hartog, the
frequency ratio \( f = \omega_a/p \) that will produce an “optimal” absorber is

\[
f = \frac{1}{1 + d}
\]  

(3.15)

while the absorber damping should be

\[
\zeta_a = \sqrt{\frac{3d}{8(1 + d)^3}}
\]  

(3.16)

Thus, for \( d = 0.35 \) the design parameters according to Den Hartog’s method are \( \omega_a/p = 0.74 \) and \( \zeta = 0.23 \). We find through trial and error using an analog simulator that a frequency ratio of \( \omega_a/p = 0.7 \) and an absorber damping factor of \( c_a = 2\zeta_a\omega_a \) where \( \zeta_a = 0.57 \) produce a satisfactory frequency-response curve. The frequency ratio is in near agreement with the design value while the damping is substantially higher than suggested. The discrepancy in damping is not wholly unexpected since Den Hartog assumed that the damping of the primary structure was negligible.

We now consider the stability of the controlled system in the presence of the regenerative terms. To begin, we take the Laplace transform of Eqs. (3.13) and (3.14), assume zero initial conditions, and obtain

\[
(s^2 + 2\zeta s + p^2 + p^2w(1 - e^{-s\tau}))X(s) = d(c_a s + \omega_a^2)Y(s)
\]  

(3.17)

\[
(s^2 + 2c_a s + \omega_a^2)Y(s) = -s^2 X(s)
\]  

(3.18)

Solving for \( Y(s) \) in terms of \( X(s) \) yields

\[
Y(s) = \frac{-s^2 X(s)}{s^2 + 2c_a s + \omega_a^2}
\]  

(3.19)

Substituting Eq. (3.19) into Eq. (3.17), we obtain the characteristic equation of the system as

\[
s^4 + (2\zeta + c_a(1 + d))s^3 + [p^2(1 + w(1 - e^{-s\tau})) + 2\zeta c_a + \omega_a^2(1 + d)]s^2
\]  

\[+ [c_a p^2(1 + w(1 - e^{-s\tau})) + 2\zeta \omega_a^2]s + \omega_a^2 p^2(1 + w(1 - e^{-s\tau})) = 0
\]  

(3.20)
We establish the linear stability boundary by substituting $s = i\omega$ into Eq. (3.20), separating real and imaginary parts, and solving the resulting pair of equations for $w$, $\tau$, and $\omega$. Using the parameter values from Hanna and Tobias (1974) and letting $d = 0.35$, $\omega_a/p = 0.7$, and $\zeta = 0.57$, we plot the limit width of cut with and without the absorber as a function of the time delay in Fig. 3.2. Clearly, the absorber has substantially increased the limit width of cut.
Chapter 4

Control-System Hardware Design, Integration, and Modeling

Having established that an absorber type control approach can improve cutting stability, we design a prototype system for a specific boring bar and engine lathe setup. A model for the prototype is generated using experimental modal analysis techniques. The frequency-response characteristics of individual control elements are examined and used to build a block diagram that yields the open-loop response of the actuator/sensor pairs. We find from the structure of the plant frequency-response function characterizing an actuator/sensor pair that the system is not collocated. Actuator impedance effects must be accounted for. A third-order compensator is suggested and a single-degree-of-freedom design methodology is presented for selecting controller gains. Good agreement between modeling and experiment is obtained.
4.1 Design Criteria

From a structural control viewpoint, the boring bar is a cantilevered beam. It has lateral vibration modes in two orthogonal directions and is subject to disturbances in the form of cutting forces at its free end that can couple these two coordinates, as we observed in Chapter 2.

From Chapter 2, we also know that it is a common assumption in cutting force analysis that the shearing of material from the workpiece occurs along a thin plane. When this is the case, the resultant cutting force is planar with a magnitude dependent on the variation of the chip thickness. This is the so-called dynamic orthogonal cutting condition. Based on the assumption of dynamic orthogonal cutting, an argument can be made that it is only necessary to control the tool motion in the direction perpendicular to the machined surface, because these motions modulate the chip thickness. This notion is embedded in the determination of an operative receptance locus to assess machine-tool stability and is a guiding design principle in the single-axis, semi-active vibration-absorber scheme that was developed and used by Tewani, Walcott, and Rouch (1991) with success.

Orthogonal cutting with a single edge is seldom the case in practice, and it has been our experience that for shallow, finish type cuts, where chatter is most common and detrimental, a complex oblique-cutting condition exists along multiple cutting surfaces. Furthermore, it seems likely that, as the linear stability of cutting is extended by controlling tool motions normal to the machined surface, one is likely to encounter a destabilizing effect in the uncontrolled coordinate due to shear-plane oscillation effects or plowing, even for an orthogonal condition.

Consider the nonlinear model of Wu and Liu (1985 a) Eqs. (2.20) and (2.21). If we assume that the motion normal to the machined surface is constrained by the control and that the
tool has a vibrational mode tangent to the machined surface, the following single-degree-of-freedom cutting problem results:

\[ m\ddot{y} + c\dot{y} + ky = -2\omega_{so} \left( A_y - C_y y_o \right) - \frac{1}{2} C_y (\dot{y} - \dot{y}_o) \]  \hspace{1cm} (4.1)

where the term \( \dot{y} - \dot{y}_o \) due to shear plane oscillation can result in negative damping. Thus, the mode can become unstable even though the oriented receptance locus is zero. It appears, then, that cutting forces can destabilize the tool in either of its principal directions independent of one another. So the ability to control tool motions in both in-plane and out-of-plane directions becomes our first design criterion and is a natural extension of the work of previous researchers.

As a second design criterion, we desire the resulting system to be compatible with a variety of different cutting tools, not simply the boring bar. Active control systems are expensive, and the greater the range of applicabilities, the more easily the cost is justified. Third, wherever possible, we wish to use standard actuator and sensor products already available on the market.

### 4.2 Selection and Placement of Actuator/Sensor Pairs

The selection and placement of actuator/sensor pairs is determined by a number of factors. On the one hand, we desire an actuator and sensor pair that can be easily incorporated with existing tooling while able to withstand the harshness of a machining environment. On the other hand, we wish to ensure that our placement of the pair results in a system capable of observing and controlling the desired dynamics given the type of feedback control we wish to implement.

As a first step, we choose between a fully-active and a semi-active control approach. In
principle, one could design a bidirectional semi-active absorber system using two actuators and appropriate sensors. Such a system would ideally be attached near the free end of the bar to achieve the greatest mechanical advantage and hence controllability. This means placing the actuators inside the tool to avoid interaction with the cutting process, as in the system of Tewani, Walcott, and Rouch (1991). Such a strategy places severe geometric constraints on the design, involves modifying the tool, and necessitates custom actuator designs. For these reasons the approach was not considered for this Dissertation. Instead, we elected to develop a fully-active scheme using actuators located near the tool base. This removes the geometric constraints on actuator size, keeps the tool modifications to a minimum, and facilitates the use of standard actuators.

A common approach for actuating a cantilevered beam near its base is to incorporate piezoelectric benders, essentially the approach taken by Matsubara, Yamamoto, and Mizumoto (1989). Such a solution has merit, but results in a custom, one size fits all tool. Rather than creating such a specialized tool, we opted to create a special tool holder. In this way, we could accommodate vibration control for other tooling. A schematic of the resulting design is shown in Fig. 4.1.

In this first design iteration, the actuators are positioned so that they sting the tool near the tool base. This sacrifices some potential tool overhang. In future work, we plan to explore similar control using actuation nearer to the fixed boundary, thus maximizing the usage of the tool.

At present, actuators available for structural control consist of electrodynamic, induced strain (the so-called smart materials), and hydraulic or pneumatic types. Of these, the hydraulic and pneumatic types have tremendous force and stroke capabilities but lack sufficient bandwidth for effective chatter control. Electrodynamic actuators have sufficient bandwidth and stroke, are inexpensive, and are a proven reliable technology. However,
they tend to be more bulky than their induced strain counterparts for comparable force capabilities. The force required to actuate the beam near its base is substantial while the stroke required is small, so we chose to use induced strain actuators.

Of the available induced strain actuators, those fashioned from magnetostrictive materials, such as Terfenol-D, have the advantage that they can be driven using standard audio-type power amplifiers, because their electrical behavior is comparable to electrodynamic shakers. Terfenol-D actuators in particular also have been shown to have a high energy density

Figure 4.1: Boring bar control system layout.
in comparison to comparable piezoelectric stack-type actuators. And, in the author’s experience, Terfenol-D actuators are more rugged than piezo-stack actuators, because small localized crack-type failures in the actuator material do not automatically render the actuator useless as is often the case in piezoceramic materials. Thus, we chose Terfenol-D actuators for our prototype.

4.3 Description of the Smart-Tool System

The layout of components is shown in Fig. 4.1. The control system is comprised of two actuator/sensor pairs; one pair is used to control motion in the direction normal to the machined surface, the other pair is used to control motions in the direction tangential to the machined surface.

The lathe used in this study is a 10-horsepower, Yamazaki Mazak, two-axis engine lathe with a 14-inch four-jaw chuck. The tool is a 12-inch long and 1-inch diameter boring bar with an ISO TNMG 160304 insert. The side rake angle is $-5^\circ$, the back rake angle is $-12^\circ$, and the lead angle is $0^\circ$. The bar is tapped with a series of 10/32 holes to accommodate the attachment of actuators through flexural elements known as “stingers”. We note that the orientation of the insert in the boring bar yields an oblique cutting geometry.

The tool holder is a custom designed clamp bolted to a dynamometer that is itself bolted to a mounting block on the cross slide. The bar is overhung with a length-to-diameter ratio of 8, as measured from the tool holder to the cutting edge, though the clamped length is best approximated as 9" through consideration of the set screw location that holds the bar in the tool holder.

Two ETREMA Products, Inc., 75/12 MP Terfenol-D actuators oriented in the $x$ and $y$
directions sting the boring bar at a point 1.75 inches from the tool holder. The tangential control actuator mounts directly to the boring bar through a 10/32 stud. The actuator oriented along the normal to the machined surface attaches through a tensioned wire stinger. This arrangement makes the tangential direction much stiffer than the normal direction.

Clamps and fixtures are designed so that the actuators are “grounded” to the machine-tool cross slide. The motions are detected using two PCB model 303 A02 accelerometers oriented along the $x$ and $y$ axes and mounted to the boring bar using a 0.25” cubic mounting block glued to the bar at a point 3” from the tool holder.

Voltage signals from each of the accelerometers are filtered through Krohn-Hite signal conditioners with ac coupling and seven-pole elliptic low-pass filters set for 2.5 kHz and a 20 dB gain throughout the passband. The amplified and filtered acceleration signals are then fed to their respective compensator circuits. This extra filtering is used to compensate for the lack of collocation between the actuator/sensor pairs, a topic we take up later.

The compensator circuits, or electronic vibration absorbers, are fabricated on breadboards using standard operational amplifiers, resistors, and capacitors. A circuit diagram is included in Fig. 4.2 along with nominal component values. The outputs from the compensator circuits are amplified using MB Dynamics SS250 amplifiers to drive the actuators.

### 4.4 System Identification and Compensator Design

In this section we begin by modeling the smart tool in terms of the frequency response measured using the individual actuator-sensor pairs. The approach differs somewhat from standard modal practice, because we tacitly incorporate the actuator dynamics. We use the model obtained to represent a single-input/single-output plant and consider the problem
of designing a loop-shaping compensator.

4.4.1 The Plant Response Function

We desire a model for the frequency-response function or FRF between the voltage input to the actuator and the voltage output by its corresponding accelerometer, or the so-called collocated response function. This name is unfortunate, because it is generally applied whether the actuator-sensor pair is collocated or not. Thus, for this Dissertation we will simply refer to the frequency response between an external voltage applied to the actuator and the output voltage at the sensor as the plant response function, because it characterizes the single-input/single-output system we wish to control.

The normal and tangential vibration-control loops are aligned with the principal axes of the smart tool so that they may be treated as uncoupled systems in the absence of cutting...
forces. We develop a model for the tangential control loop as an example of the procedure.

The actuator is provided with a random voltage excitation and the corresponding output acceleration is sensed as a voltage from the accelerometer signal conditioner. The actuator power amplifier is set at a midrange gain and a random voltage input is selected to avoid amplifier and actuator saturation. Both the input and output voltages are measured using a digital frequency analyzer that computes the corresponding FRF, shown as the dashed line in the Bode plot of Fig. 4.3.

The phase plot indicates first-order dynamics, as evidenced by the roll off from 180° to approximately 90° between 0 and approximately 300 Hz. This is to be expected because the actuator is magnetostrictive, and its electrical impedance should resemble that of an inductive load. Between 450 and 600 Hz the response exhibits a resonance peak accompa-
nied by a 180° shift in phase. Clearly, this mode is observable and controllable. However, we note evidence of a noncollocated response in the continued phase roll off near the first antiresonance. In a collocated actuator-sensor pair, the input is in phase with each modal response and there is phase recovery associated with each antiresonance. If one thinks in terms of the associated transfer function, the antiresonance is the result of a zero. The fact that the frequency continues to roll off near this zero is a clue that the system has what is referred to as a nonminimum phase zero.

Based on the preceding observations, we construct the tentative block diagram shown in Fig. 4.4 for the control system, where $K_1$ is the power amplifier gain, $K_2$ is the electromechanical gain of the actuator, the $d(s)$ are unknown cutting force disturbances, $K_s$ is the gain due to the accelerometer sensitivity and amplification of its signal conditioner, $Z(s) = (R + Ls)^{-1}$ is the actuator electrical impedance, $A(s)$ is the Laplace transform of the cross-accelerance between the transduction “force” input at the actuator location and the acceleration measured at the sensor location, and $C(s)$ is a compensator to be determined.

Following standard procedures (e.g., Ewins, 1986), we assume a modal model for the structural response in a partial-fraction expansion of the cross-accelerance so that its Laplace transform $A(s)$ is

$$A(s) = \frac{s^2h_1}{s^2 + 2\zeta_1\omega_1s + \omega_1^2} + s^2 \sum_{n=2}^{N} \frac{h_n}{s^2 + 2\zeta_n\omega_ns + \omega_n^2}$$  \hspace{1cm} (4.2)$$

where $\omega_n$, $\zeta_n$ and $h_n$ are the frequency, damping ratio, and modal factors, respectively, of the $n^{th}$ mode.
Table 4.1: Estimated dynamic properties of the tangential control system.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Hz</th>
<th>damping ratio</th>
<th>modal factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>523.4</td>
<td>0.028</td>
<td>0.018</td>
</tr>
<tr>
<td>2</td>
<td>750</td>
<td>0.1</td>
<td>-0.0013</td>
</tr>
<tr>
<td>3</td>
<td>2535</td>
<td>0.05</td>
<td>0.223</td>
</tr>
<tr>
<td>4</td>
<td>3000</td>
<td>0.05</td>
<td>0.223</td>
</tr>
</tbody>
</table>

We assume that the frequencies and dampings associated with the structural response are global properties that can be determined using FRF’s obtained anywhere along the structure provided nodal points are avoided. Since the resonant peaks are fairly well separated, we use single-degree-of-freedom techniques to obtain estimates of the natural frequencies, damping ratios, and modal factors of the dominant modes directly from the measured plant response function. The results of the tangential control-system identification are summarized in Table 4.1. The second-mode parameters are the result of trial and error curve fits where the frequency, damping ratio, and modal factor were varied in order to produce the appropriate total phase response. The second mode is not observable from the magnitude plot of the measured plant response, yet its effect on the phase is dramatic.

Next, the actuator’s electrical properties $L$ and $R$ are determined. The resistance $R$ is measured and found to be 5.7 Ohms. The manufacturer provides a plot of the electrical impedance as a function of frequency from which we estimate an $L$ of approximately 6 mH. We note that our model does not account for back emf or eddy current losses.

The force generated by the actuator is assumed to be proportional to the current by a constant electromechanical gain factor $K_2$ and the actuator power amplifier is simply characterized as a constant gain $K_1$. Taking into account the sensor gain $K_s$, we obtain the
plant transfer function

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = K_1 K_2 K_s Z(s) A(s)
\]  

(4.3)

Rather than determine each of the gains separately, we lump them together as a single parameter \( K \). Setting \( K = 48 \) and letting \( s = i\omega \), where \( \omega \) is the frequency of excitation, we plot the model plant response as the solid line in Fig. 4.3.

### 4.4.2 Closing the Loop

To achieve effective chatter control, we must minimize the tool vibratory response to arbitrary disturbance forces. In terms of closed-loop performance goals, this translates into an output regulation problem. However, there is little to be gained by controlling all of the states, or modes, present in the plant response functions because the chatter vibration at the tool tip is clearly dominated by the first mode in each case. Thus we seek a compensation scheme that targets this problematic mode for regulation.

Second-order compensators, as discussed in Chapter 3, have been proposed in the literature to provide loop-shaping control of structures (Goh and Caughey, 1985; Fanson and Caughey, 1987; and Juang and Phan, 1992) using collocated sensor-actuator pairs. In these approaches, the compensator is used to increase the damping of a targeted structural mode in a fashion analogous to a mechanical vibration absorber. For a single-degree-of-freedom structure, these controllers take the form

\[
\ddot{x} + 2\zeta_1 \omega_1 \dot{x} + \omega_1^2 x = d(t) + f_c(y, \dot{y}, x, \dot{x})
\]  

(4.4)

\[
\ddot{y} + 2\zeta_c \omega_c \dot{y} + \omega_c^2 y = C(y, \dot{y}, x, \dot{x})
\]  

(4.5)

where \( x \) is the generalized structural coordinate, \( y \) is the compensator coordinate with damping factor \( \zeta_c \) and frequency \( \omega_c \), and the coupling is defined by the functions \( C(y, \dot{y}, x, \dot{x}) \).
and \( f_c(y, \dot{y}, x, \dot{x}) \).

When a collocated strain or position measurement is available, positive position feedback (see Goh and Caughey, 1985) is often used so that

\[
C(y, \dot{y}, x, \dot{x}) = \omega_c^2 x \quad \text{and} \quad f_c(y, \dot{y}, x, \dot{x}) = g\omega_1^2 y
\]

(4.6)

Similarly, when a collocated acceleration measurement is available, a “virtual absorber” (see Juang and Phan, 1992) scheme can be devised using

\[
C(y, \dot{y}, x, \dot{x}) = -\ddot{x} \quad \text{and} \quad f_c(y, \dot{y}, x, \dot{x}) = g(c_1\dot{y} + \omega_c^2 y)
\]

(4.7)

where \( g \) in both cases is the feedback gain.

In practice, collocation is not often easy to obtain. In the present situation, the accelerometer is not collocated physically with the actuator, resulting in a nonminimum phase zero near the first mode. Furthermore, the first mode occurs at a frequency greater than the pole characterizing the actuator electrical dynamics. Thus, we find that the actuator is no longer collocated for a reason other than the typically considered spatial relationship between the applied force and the sensed motion. This shortcoming, sometimes referred to as lack of temporal collocation, can be alleviated in a number of ways.

A current-mode amplifier can be used to drive the actuator instead of a voltage-mode amplifier. This is a standard practice for electrodynamic shakers used in vibration testing (McConnell, 1997). It would leave us with a virtual absorber design because the amplifier itself would compensate for the actuator impedance. Alternatively, we can add another pole to the compensator and arrive at a system that is analogous to positive position feedback. The latter approach is considered here so that the compensator has the general form

\[
C(s) = \frac{g\omega_c^2}{(\tau_c s + 1)(s^2 + 2\zeta_c \omega_c s + \omega_c^2)}
\]

(4.8)
where $g$ is the feedback gain, $\omega_c$ and $\zeta_c$ are the frequency and damping ratio associated with the second-order dynamics of the compensator, and $\tau_c$ is the time constant associated with its first-order dynamics.

### 4.4.3 Compensator Design

We develop initially a compensator design strategy based on a single-degree-of-freedom structural model with first-order actuator dynamics. The single-input, single-output plant to be controlled is

$$G(s) = \frac{Ks^2}{(s^2 + 2\zeta_1\omega_1 s + \omega_1^2)(s + \alpha_p)} \quad (4.9)$$

where $K = K_1K_2K_s h_1/L$ and $\alpha_p = R/L$.

Assuming positive feedback through the previously defined compensator $C(s)$ yields the characteristic equation

$$1 - G(s)C(s) = s^6 + c_5s^5 + c_4s^4 + c_3s^3 + c_2s^2 + c_1s + c_0 = 0 \quad (4.10)$$

where the coefficients $c_n$ are

\[
\begin{align*}
c_0 &= \omega_1^2 \omega_c^2 \alpha_p \alpha_c \\
c_1 &= \omega_1^2 \omega_c^2 (\alpha_p + \alpha_c) + (2\zeta_1 \omega_1 \omega_c^2 + 2\zeta_c \omega_c \omega_1^2) \alpha_p \alpha_c \\
c_2 &= \omega_1^2 \omega_c^2 + (2\zeta_1 \omega_1 \omega_c^2 + 2\zeta_c \omega_c \omega_1^2) (\alpha_p + \alpha_c) + (\omega_1^2 + \omega_c^2 + 4\zeta_1 \zeta_c \omega_1 \omega_c) \alpha_p \alpha_c - gK \alpha_c \omega_c^2 \\
c_3 &= (2\zeta_1 \omega_1 \omega_c^2 + 2\zeta_c \omega_c \omega_1^2) + (\omega_1^2 + \omega_c^2 + 4\zeta_1 \zeta_c \omega_1 \omega_c) (\alpha_p + \alpha_c) + (2\zeta_1 \omega_1 + 2\zeta_c \omega_c) \alpha_p \alpha_c \\
c_4 &= (\omega_1^2 + \omega_c^2 + 4\zeta_1 \zeta_c \omega_1 \omega_c) + (2\zeta_1 \omega_1 + 2\zeta_c \omega_c) (\alpha_p + \alpha_c) + \alpha_p \alpha_c \\
c_5 &= (2\zeta_1 \omega_1 + 2\zeta_c \omega_c) + \alpha_p + \alpha_c
\end{align*}
\]

and $\alpha_c = 1/\tau$. We seek a desired characteristic equation of the form

$$\delta(s) = (s + \beta_1)(s + \beta_2)(s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0) = 0 \quad (4.11)$$
where $\beta_1$ and $\beta_2$ are arbitrary positive constants, and the coefficients $p_n$ are selected to produce any desired complex poles. We choose to match the poles to an equivalent damped vibration-absorber system and employ Den Hartog’s (1985) design criterion for the sake of simplicity. Thus, the coefficients are determined by selecting a virtual mass ratio $\mu$ and using the relations

$$
\begin{align*}
 p_0 &= \frac{\omega_1^4}{(1 + \mu)^2}, \\
p_1 &= \frac{2\omega_1^3\zeta_2}{(1 + \mu)}, \\
p_2 &= \omega_1^2 \frac{2 + \mu}{1 + \mu}, \\
p_3 &= 2\zeta_2\omega_1, \\
\zeta_2 &= \sqrt{\frac{3\mu}{8(1 + \mu)^3}}
\end{align*}
$$

Knowing the $p_n$, we equate the coefficients of Eqs. (4.10) and (4.11) and solve the resulting set of six nonlinear algebraic equations for the unknown compensator values. The results of this procedure using the parameters identified for the first mode of the system are shown in Fig. 4.5 for the “effective” mass ratios $\mu = 0.1, 0.2, \text{ and } 0.35$. The resulting FRF’s display the classic, “double hump” damped vibration-absorber structure.

Next, we examine the performance of a single-degree-of-freedom equivalent absorber-type compensator design when the complete plant model (consisting of the four previously identified modes) is employed. We see in Fig. 4.6 that the “humps” of the response labeled SDOF are no longer equal and that the performance is degraded. This mistuning can be simply adjusted by reducing the feedback gain.

Consider the root locus of Fig. 4.7 where we focus on the behavior of the first structural pole and that of the compensator. The root locus provides a map of the individual closed-loop pole behaviors as a function of the feedback gain. Looking at the root locus, we might choose to modify the design feedback gain such that the structural pole and compensator pole migrate to points that lie along the same line radiating from the origin, so that the poles have matched damping ratios. This design results in the curve labeled DAMP in Fig. 4.6. We see that the performance of such a design varies little from the SDOF equivalent
absorber design. Another approach is to select the feedback gain such that the projections of these poles along the real axis are equal. This design is labeled REAL in Fig. 4.6. We see that this design has more of the classic absorber profile that we desire for chatter control. The locations of the poles resulting from such a design are emphasized in the root locus plot.

### 4.4.4 Structural Control Results

For our compensator, we use a 2nd-order analog circuit in conjunction with an off-the-shelf, low-pass filter. The filter time constant is approximated by measuring the rate of change of its phase angle with frequency; the result is $\tau = 0.0003175$ seconds. This is a crude approximation, even within the filter passband (0-2500 Hz), because it is actually a
Figure 4.6: Closed-loop performance; single-degree-of-freedom design (SDOF), matched damping design (DAMP), and matched projections along the real axis (REAL).
seven-pole elliptic filter; however, the accuracy is sufficient for purposes of illustration.

The assembled 2nd-order compensator has a natural frequency of 367 Hz, a damping ratio of 0.707, and a static gain of $0.6\omega^2$. The design is not like those previously described but was arrived at in an ad hoc fashion on the shop floor by trial and error. Nevertheless, we plug the compensator values into our model and show a Bode plot of the predicted closed-loop performance in Fig. 4.8. The actual performance is shown in Fig. 4.9.
Figure 4.8: Bode plot of the predicted performance.

Figure 4.9: Bode plot of the in situ tangential plant response function.
Part II

Nonlinear Machine-Tool Dynamics and Control
Chapter 5

Chatter Observations and Smart Tool Performance Testing

In this chapter, the Smart Tool of Chapter 4 is used to machine tubular aluminum specimens. A variety of chatter signatures are recorded for cases with and without feedback control. Time traces, autospectra, cross spectra, and coherence are used to characterize the tool motions during chatter conditions. We find evidence of coupling between the structural modes normal and tangential to the machined surfaces. Also, a jump phenomenon is observed for a case without feedback control and also a case when control is applied only in the direction tangent to the machined surface. A time trace of tool motions first without feedback control and then with control demonstrates the ability of the smart machine-tool system to suppress chatter.
5.1 Description of Experiments

Tubular aluminum specimens of nominal inner diameter 4.5”-5.25”, outer diameter 6.1”, and length 5” are machined to test the Smart Tool and to examine boring-bar chatter dynamics. An example of a typical specimen is shown in the photo of Fig. 5.1.

A schematic of the measurement system is shown in Fig. 5.2. Acceleration time responses normal and tangential to the machined surface are recorded using LabView software operating on a MacIntosh II data acquisition computer equipped with a National Instruments NBMIO16x board. A Krohn Hite seven-pole elliptic low-pass filter with cutoff frequency of 2.5 kHz was used as an anti-aliasing filter. Unless otherwise stated, the data is sampled at 5 kHz for 10 seconds and subsequently broken into ten one-second duration ensembles that are averaged in the frequency domain for the autospectra, cross spectrum, and coherence plots. Post processing of the data is accomplished using routines written in MATLAB. In some instances, spectral information is obtained using an HP 3562A two-channel signal analyzer.

5.2 Results

The results reported here are from a variety of different test sessions conducted in the Manufacturing Systems Laboratory of the Industrial Systems Engineering Department during school holidays in August and December of 1996. Where appropriate, the nominal dynamic properties of the Smart Tool are included in the description of individual results because these properties varied due to differences in the assembly of the components from one test period to the next.
We investigate the Smart Tool dynamics for a variety of cutting speeds and depths of cut at the minimum feedrate of the lathe, which is 0.0024'' per revolution (ipr). Results fall into four categories:

- experimental bifurcation data,
- transient tool responses,
- “steady-state” chatter signatures, and
- controlled tool motions.

### 5.2.1 Bifurcation Data

In Fig. 5.3 the amplitude of the fundamental harmonic of the tool’s acceleration tangent to the machined surface, as measured using the analyzer, is plotted as a function of the workpiece rotational speed in revolutions per minute (rpm) for the case when \( w_0 = 0.03'' \). For the data in this plot, the first lateral vibration modes have the frequencies and dampings.
Figure 5.3: Bifurcation diagram showing the amplitude of the fundamental harmonic of chatter for various spindle speeds when $w = 0.03''$ and feedrate = 0.0024 ipr.

$f_t = 524$ Hz and $\zeta_t \approx 0.03$ and $f_n = 365$ Hz and $\zeta_n \approx 0.02$, respectively, as determined by impact testing and circle fits of the resulting frequency-response functions.

Two observations can be made from the results in Fig. 5.3. First, the presence of stable cutting for all workpiece rotational speeds below 165 rpm indicates that the cutting forces are dependent on the mean cutting velocity and contribute a positive damping at low speeds, in agreement with Tlusty’s results (Tlusty, 1985) for aluminum. Second, a subcritical-type instability occurs between 165 and 195 rpm, where both chatter and stable cutting conditions coexist.
5.2.2 Transient Data

Further evidence of subcritical-type instability is offered in the time record of Fig. 5.4, where we record the progression from stable cutting to steady-state chatter in the tool accelerations normal to the machined surface. The tool frequencies and dampings are $f_t = 493$ Hz and $\zeta_t \approx 0.03$ and $f_n = 365$ Hz and $\zeta_n \approx 0.02$ and feedback control is inactive.

We see that, for the first five seconds of recorded data, the cut appears to be stable. After approximately five seconds have elapsed, a hammer blow is administered to the tool in a manner reminiscent of the experiments of Hooke and Tobias (1964). Clearly, chatter ensues, though even after 80 seconds it has failed to reach a discernible steady state. The tool is allowed to continue chattering in this fashion, and another data set is recorded after the clearly audible (during the experiment) once per revolution variation in the tool vibrations has subsided. This data is presented in Fig. 5.5 as an example of “steady-state” chatter and will be more closely scrutinized in the following section on chatter signatures.

In Fig. 5.6 we consider the transient tool response to a hammer blow during stable cutting. An exponentially decaying function is plotted along with the data to illustrate the relative difference in damping between the motions in the normal and tangential directions for this width and speed of cut. The exponential function is

$$a = a_o e^{-\zeta \omega t}$$

where $a_o = 200$, $\zeta = 0.15$, and $\omega = 2\pi(365)$ for the normal direction, and $a_o = 35$, $\zeta = 0.028$, and $\omega = 2\pi(524)$ for the tangential direction. Interestingly, the tool response in the normal direction appears to be heavily damped by the cutting forces, while the tool response in the direction tangent to the cut appears to be unaffected.
Figure 5.4: Evolution of chatter in the tool response normal to the machined surface when $w = 0.002''$, speed = 90 rpm, and feedrate = 0.0024 ipr.

Figure 5.5: The time trace of the steady-state chatter normal to the machined surface when $w = 0.002''$, speed = 90 rpm, and feedrate = 0.0024 ipr.
5.2.3 Chatter Signatures

A great variety of chatter signatures were obtained in the two testing periods. Because the tool dynamic properties were not identically the same in each period, we shall consider the results separately.

Signatures from the test period of December, 1996.

We consider the time histories and autospectra of the tangential and normal accelerations as well as the cross spectra and coherence between these signals for some typical chatter signatures obtained when the tool frequencies and dampings are \( f_t = 524 \text{ Hz} \) and \( \zeta_t \approx 0.03 \) and \( f_n = 365 \text{ Hz} \) and \( \zeta_n \approx 0.02 \).

The tool dynamic properties during this set of tests are the same as those observed during the bifurcation test, and we begin by contrasting two responses that were recorded during
that experiment. The cutting conditions were maintained as nearly identical as possible for both data records (same specimen, internal diameter, speed, feedrate, and depth of cut, but at a different location along the tube). In Fig. 5.7 we see a typical spectral response for a “stable” cutting condition. The tool response is a few tenths of a g in each direction and the spectral content is indicative of a forced response due to “random” fluctuations in the mean cutting force. Each spectrum shows evidence of the independent modal responses. Comparing the tool response of Fig. 5.7 with the large chatter response of Fig. 5.8, we see clearly that both stable and chattering motions coexist for the same cutting parameters. In the chatter of Fig. 5.8 the spectral content is similar in each coordinate, with the tool clearly undergoing a large-amplitude, harmonic mode coupled vibration.

Next, we see that it is possible to observe qualitatively different chatter signatures for the same nominal cutting conditions. For instance, Fig. 5.9 shows large-amplitude chatter of 497 Hz that is dominated by tangential motions, while Fig. 5.10 shows large-amplitude chatter of 416 Hz that is dominated by normal motions. These results are from the largest width of cut $w = 0.1''$ considered in the study. We note that the two experiments were not precisely identical since the internal diameter of the specimen represented by the chatter of Fig. 5.9 is $0.5''$ less than the specimen of Fig. 5.10.

Finally, the chatter signature of Fig. 5.11 is typical of tool responses obtained for shallow cuts where the depth of cut is less than the nominal tool nose radius. These cuts are dominated by tool motions normal to the machined surface.

*Signatures from the test period of August, 1996.*

Next, we consider the time histories and autospectra of the tangential and normal accelerations as well as the cross spectra and coherence between these signals for chatter signatures obtained when the tool frequencies and dampings are $f_t = 493$ Hz and $\zeta_t \approx 0.03$ and $f_n = 365$ Hz and $\zeta_n \approx 0.02$. For this series of tests, the tool appeared far more prone to
Figure 5.7: Stable tool response normal (bottom trace) and tangential (top trace) to the machined surface when $w = 0.03''$, speed = 190 rpm, and feedrate = 0.0024 ipr. Units are g’s.

Figure 5.8: Chattering tool response normal (bottom trace) and tangential (top trace) to the machined surface when $w = 0.03''$, speed = 190 rpm, and feedrate = 0.0024 ipr. Units are g’s.
Figure 5.9: Tangentially oriented chatter signature when \( w = 0.1'' \), speed = 235 rpm, and feedrate = 0.0024 ipr.

Chatter is observed at spindle rotational speeds as low as 90 rpm and for widths of cut as small as 0.002'', as seen in Fig. 5.12, which is the same data as the steady-state response shown in Fig. 5.5. We note that for the shallow cut of 0.002'', chatter tends to be localized in a direction normal to the machined surface, whereas for the deeper cut of 0.03'', chatter is oriented more along a line tangent to the machined surface, as is evident in Fig. 5.13. In both cases, the fundamental of the motion and its harmonics are seen to be linearly coherent, suggesting that mode coupling is present.
Figure 5.10: Normally oriented chatter signature when $w = 0.1''$, speed = 235 rpm, and feedrate = 0.0024 ipr.

Figure 5.11: Chatter signature normal (bottom trace) and tangential (top trace) to the machined surface when $w = 0.005''$, speed = 195 rpm, and feedrate = 0.0024 ipr. Units are g’s.
5.2.4 Controlled Responses

Control performance was evaluated both in August and December of 1996 with similar results. The data reported here is from August, and the dynamic properties of the structure are those previously cited for that time period.

In Fig. 5.14, we show an example of a jump-type phenomenon in the time record of the accelerations normal to the machined surface when only the tangential control loop is active. Initially, the control is only active in the tangential direction. The cut appears to be stable. A disturbance is introduced by hitting the bar in the normal direction with a hammer. Clearly, the disturbance grows due to regeneration. After the chatter reaches steady state, control is added in the normal direction, thereby stabilizing the system. Another disturbance is applied but the control maintains stability. In Fig. 5.15, we show the time
Figure 5.13: Time traces, autospectra, cross-spectra, and coherence when $w = 0.03''$, speed = 210 rpm, and feedrate 0.0024 ipr.

traces, autospectra, cross-spectrum, and coherence of the normal and tangential accelerations during steady-state chatter of this type. We see that though chatter is predominantly oriented in the normal direction, a substantial tangential component exists.

Finally, we demonstrate biaxial control in Fig. 5.16. Large-amplitude chatter is allowed to develop before applying feedback control in both directions that stabilizes the system. We observe a dramatic improvement in the surface finish, as shown in the photo of Fig. 5.17.

### 5.3 Summary

In this chapter we have explored the dynamics of boring by observing the Smart Tool’s behavior in a wide range of machining conditions. Transient responses, bifurcation experi-
Figure 5.14: Time traces of the tool accelerations normal to the machined surface; initially, only tangential control is active; then, in addition, control is applied in the normal direction to quench the chatter and eliminate the jump-type instability; \( w = 0.002'' \), speed = 170 rpm, and feedrate = 0.0024 ipr.

In general, we observe that shallow cuts, where the primary cutting edge is the tool nose radius, give rise to chatter that is characterized by motions normal to the machined surface. Deeper cuts that use more of the tool insert give rise to chatter that is characterized by motions tangential to the machined surface. This behavior is most clear in the testing period of August, 1996, where distinctions between the widths of cut \( w = 0.002'' \) versus \( w = 0.03'' \) were consistent and obvious.

The distinction is less clear in the data obtained in the second testing period. Both tangentially and normally oriented modes of chatter are witnessed in the very wide cuts of Figs. 5.9 and 5.10. In Fig. 5.18 we see results for the intermediate width of cut \( w = 0.01'' \)
Figure 5.15: Time traces, autospectra, cross-spectrum, and coherence when $w = 0.002''$, speed = 180 rpm, feedrate = 0.0024 ipr, and tangential control is active where both modes seem to be interacting to produce chatter of varying frequency content as we progress in time from one ensemble to the next. Even in the light cut of Fig. 5.11, we can see a higher-frequency chatter mode struggling to make its presence known in the tangential acceleration response. We observe that the development of chatter can exhibit a very long transient behavior, and because of this it is difficult to say conclusively whether or not the complex, multi-mode chatter signatures were indicative of a “steady-state” attractor or simply transient motions that, given enough time (and workpiece material!), would eventually settle into a single periodic-type motion.

Nevertheless, the coexistence of multiple chatter modes only serves as a stronger motivation for a biaxial control approach. In all of the operations considered, the Smart Tool was capable of returning stability to the cutting process provided feedback was active in both directions.
Figure 5.16: Time traces of the normal and tangential tool accelerations when $w = 0.03''$, speed = 230 rpm, and feedrate = 0.0024 ipr, before and after application of biaxial control.

Figure 5.17: An example of surface finish before and after application of the biaxial Smart Tool control. External Figure
Figure 5.18: Waterfall plot of autospectra obtained from ten consecutive ensembles.
Chapter 6

Nonlinear Dynamics Techniques for Machine-Tool Stability Analysis

In this chapter we develop the techniques necessary to explore the dynamics of nonlinear machine-tool and cutting force models and then confirm our results via analog-computer simulations. Once again, our example is the model of Hanna and Tobias (1974).

Having previously established the stability boundary for this system, we first undertake a perturbation analysis to determine the amplitudes and phases of the periodic solutions (limit cycles) created as a result of the instability. This analysis yields the so-called normal form, which is in essence a closed-form solution for the dynamics near the predicted linear stability. This closed-form solution will also ascertain whether the created limit cycles are stable or unstable and hence whether the accompanying bifurcation is supercritical or subcritical. To study the periodic solutions and their stability for progressively larger widths of cut, we construct a harmonic-balance solution in Section 6.2. In Section 6.3, we compare these predictions with the results of numerical integration. We verify a subcritical
instability, the inadequacy of a two-term approximation, and the stability-limit of periodic solutions. Having exhausted our purely analytical tools, we then explore the route to chaos using computer simulation.

The material to this point is from Nayfeh, Chin, and Pratt (1997). The author gratefully acknowledges the work of his coauthors, who performed the original bifurcation analysis, and particularly the contributions of Dr. Char-Ming Chin, who produced the figures used in this portion of the chapter.

In Section 6.4 we develop an analog-computer simulation to verify our analytical techniques. We first confirm in Section 6.4.2 the global subcritical instability that was predicted via harmonic-balance solutions and numerical integration. We revisit the route to chaos in Section 6.4.2. We close Section 6.4 by considering the pointwise dimension of the experimentally measured attractor using a reconstructed phase space. Finally, in Section 6.5 we consider the effect of an absorber on the nonlinear stability of the model of Hanna and Tobias (1974).

6.1 Normal Form of the Hopf Bifurcation

The equation of motion for the model of Hanna and Tobias (1974) was introduced in Chapter 2 and is repeated here for convenience:

$$\ddot{x} + 2\xi \dot{x} + p^2(x + \beta_2 x^2 + \beta_3 x^3) = -p^2 w[x - x_\tau + \alpha_2(x - x_\tau)^2 + \alpha_3(x - x_\tau)^3]$$ (6.1)

where the parameters as determined experimentally by Hanna and Tobias (1974) are \(p = 1088.56\) rad/sec, \(\xi = 24792/\omega^2\) rad², \(\beta_2 = 479.3\) 1/in, \(\beta_3 = 264500\) 1/in², \(\alpha_2 = 5.668\) 1/in, and \(\alpha_3 = -3715.2\) 1/in².

The nonlinearities considered in Eq. (6.1) include the nonlinear stiffness of the machine tool
and the nonlinear cutting force generated in the cutting process. According to the linear analysis, a Hopf bifurcation occurs, leading to the birth of a limit cycle as \( w \) increases past \( w_c \). The qualitative behavior in the neighborhood of this bifurcation point can be predicted by studying the corresponding normal form. Due to the presence of both cubic and quadratic nonlinearities, the perturbation analysis must be performed using either the method of multiple scales or the generalized method of averaging (Nayfeh, 1973, 1981). In this Dissertation, we choose a multiple-scales approach.

Using the method of multiple scales (Nayfeh, 1973, 1981), we introduce a fast time scale \( T_0 = t \) and a slow time scale \( T_2 = \epsilon^2 t \) and seek a third-order expansion in the form

\[
x = \epsilon x_1(T_0, T_2) + \epsilon^2 x_2(T_0, T_2) + \epsilon^3 x_3(T_0, T_2) + \cdots
\]

Moreover, we let

\[
w = w_c + \epsilon^2 w_2 + \cdots
\]

Substituting Eqs. (6.2) and (6.3) into Eq. (6.1) and equating coefficients of like powers of \( \epsilon \), we obtain

**Order \( \epsilon \):**

\[
D_0^2 x_1 + 2\xi D_0 x_1 + p^2 x_1 + p^2 w_c(x_1 - x_{1r}) = 0
\]

**Order \( \epsilon^2 \):**

\[
D_0^2 x_2 + 2\xi D_0 x_2 + p^2 x_2 + p^2 w_c(x_2 - x_{2r}) = -p^2 \beta_2 x_1^2 - p^2 w_c \alpha_2(x_1 - x_{1r})^2
\]

**Order \( \epsilon^3 \):**

\[
D_0^2 x_3 + 2\xi D_0 x_3 + p^2 x_3 + p^2 w_c(x_3 - x_{3r}) = -2D_0 D_2 x_1 - 2\xi D_2 x_1 - p^2 w_2(x_1 - x_{1r})
\]
\[
-2p^2 \beta_2 x_1 x_2 - 2p^2 w_c \alpha_2(x_1 - x_{1r})(x_2 - x_{2r})
\]
\[
-p^2 \beta_3 x_1^3 - p^2 w_c \alpha_3(x_1 - x_{1r})^3
\]
The solution of Eq. (6.4) can be expressed as

\[ x_1 = A(T_2)e^{i\omega_c T_0} + cc, \quad \text{and} \quad x_{1\tau} = A(T_2)e^{i\omega_c (T_0 - \tau)} + cc \]  

(6.7)

where \( \omega_c \) is the chatter frequency at the Hopf bifurcation point. Substituting Eq. (6.7) into Eq. (6.5) and solving for \( x_2 \), we obtain

\[ x_2 = -p^2 A^2 \Gamma_1 e^{2i\omega_c T_0} - 2\Gamma_2 A \bar{A} - p^2 \bar{A}^2 \Gamma_1 e^{-2i\omega_c T_0} \]  

(6.8)

where

\[ \Gamma_1 = \frac{\beta_2 + w_c \alpha_2 (1 - e^{-i\omega_c \tau})^2}{p^2 - 4\omega_c^2 + 4i\omega_c \xi + p^2w_c(1 - e^{-2i\omega_c \tau})} \]  

(6.9)

\[ \Gamma_2 = \beta_2 + w_c \alpha_2 (1 - e^{-i\omega_c \tau})(1 - e^{i\omega_c \tau}) \]  

(6.10)

Substituting Eqs. (6.7) and (6.8) into Eq. (6.6) and eliminating the terms that lead to secular terms, we obtain the solvability condition

\[ 2(\xi + i\omega_c)A' + p^2 w_2 (1 - e^{-i\omega_c \tau})A + \Lambda A^2 \bar{A} = 0 \]  

(6.11)

where

\[ \Lambda = \Lambda_r + i\Lambda_i \]

\[ = -2p^2 \beta_2 (p^2 \Gamma_1 + 2\Gamma_2) + 3p^2 \beta_3 - 2p^4 w_c \alpha_2 \Gamma_1 (1 - e^{i\omega_c \tau})(1 - e^{-2i\omega_c \tau}) \]

\[ + 3p^2 w_c \alpha_3 (1 - e^{-i\omega_c \tau})^2 (1 - e^{i\omega_c \tau}), \]  

(6.12)

and \( \Lambda_r \) and \( \Lambda_i \) are real constants.

Introducing the polar transformation

\[ A = \frac{1}{2}ae^{i\beta} \]  

(6.13)

into Eq. (6.11) and separating real and imaginary parts, we obtain the normal form

\[ a' = c_1 w_2 a + c_2 a^3 \]  

(6.14)
\[ a\beta' = c_3 w^2 a + c_4 a^3 \] (6.15)

where

\[ c_1 = -\frac{p^2}{2(\xi^2 + \omega_c^2)}[\xi(1 - \cos \omega_c \tau) + \omega_c \sin \omega_c \tau] \] (6.16)
\[ c_2 = -\frac{1}{8(\xi^2 + \omega_c^2)}(\xi \Lambda_r + \omega_c \Lambda_i) \] (6.17)
\[ c_3 = -\frac{p^2}{2(\xi^2 + \omega_c^2)}[\xi \sin \omega_c \tau - \omega_c (1 - \cos \omega_c \tau)] \] (6.18)
\[ c_4 = -\frac{1}{8(\xi^2 + \omega_c^2)}(\xi \Lambda_i - \omega_c \Lambda_r) \] (6.19)

The qualitative behavior near a Hopf bifurcation point can therefore be determined by the sign of \( c_2 \). The bifurcation is supercritical when \( c_2 < 0 \) and subcritical when \( c_2 > 0 \). For example, a supercritical Hopf bifurcation occurs at \((\tau, w_c, \omega_c) = (1/75, 0.117072, 1205.51)\) because \((c_1, c_2, c_3, c_4) = (159, -787737, 953, -2608438)\). A supercritical Hopf bifurcation is also predicted at \((\tau, w_c, \omega_c) = (1/60, 0.075037, 1095.48)\) because \((c_1, c_2, c_3, c_4) = (300, -195328, 98, 3678391)\). Hence, local disturbances decay for \( w < w_c \) and result in small limit cycles (periodic motions) for \( w > w_c \).

### 6.2 Periodic Solutions and Their Stability

To construct analytically the steady-state periodic solutions of Eq. (6.1), we note that they can be expressed in Fourier series. Hanna and Tobias (1974) truncated the series by using two terms. It turns out that two terms in the series may not be enough. We find that including up to the third harmonics yields accurate results when compared with the results of numerical integration. Thus, we consider three harmonics and let

\[ x = a \cos \omega t + a_0 + a_2 \cos 2\omega t + a_3 \sin 2\omega t + a_4 \cos 3\omega t + a_5 \sin 3\omega t \] (6.20)
Substituting Eq. (6.20) into Eq. (6.1) and equating the coefficient of each harmonic on both sides, we obtain a system of seven nonlinear algebraic equations. The algebra was carried out by using a symbolic manipulator. We numerically calculate the roots of this system of equations and hence $\omega$ and the $a_i$. Due to the lengthy expressions in these equations, we only show the numerical results in the figures.

To determine the stability of a periodic solution $x_0(t)$, we use Floquet theory (Nayfeh and Mook, 1979; Nayfeh and Balachandran, 1995). We first perturb the periodic solution $x = x_0(t)$ by introducing a disturbance term $u(t)$ and obtain

$$x(t) = x_0(t) + u(t) \tag{6.21}$$

Substituting Eq. (6.21) into Eq. (6.1) and linearizing in the disturbance $u$, we obtain

$$\ddot{u} + 2\xi \dot{u} + p^2(1 + 2\beta_2 x_0 + 3\beta_3 x_0^2)u = -p^2 w[1 + 2\alpha_2(x_0 - x_{0r}) + 3\alpha_3(x_0 - x_{0r})^2](u - u_r) \tag{6.22}$$

According to Floquet theory, Eq. (6.22) admits solutions of the form

$$u(t) = e^{\gamma t} \phi(t) \tag{6.23}$$

where $\phi(t)$ is a periodic function with period $T = 2\pi/\omega$, which is equal to the period of $x_0(t)$. Substituting Eq. (6.23) into Eq. (6.22) yields

$$\ddot{\phi} + 2\gamma \dot{\phi} + \gamma^2 \phi + 2\xi \gamma \phi + 2\xi \dot{\phi} + p^2(1 + 2\beta_2 x_0 + 3\beta_3 x_0^2)\phi$$

$$= -p^2 w[1 + 2\alpha_2(x_0 - x_{0r}) + 3\alpha_3(x_0 - x_{0r})^2](\phi - e^{-\gamma t} \phi_r) \tag{6.24}$$

To solve Eq. (6.24), we expand $\phi$ in a Fourier series and keep up to the third harmonics for consistency with Eq. (6.20). Thus, we let

$$\phi = b_1 \cos \omega t + b_2 \sin \omega t + b_0 + b_3 \cos 2\omega t + b_4 \sin 2\omega t + b_5 \cos 3\omega t + b_6 \sin 3\omega t \tag{6.25}$$

Substituting Eq. (6.25) into Eq. (6.24) and equating the coefficient of each harmonic on both sides, we obtain a system of seven linear homogeneous algebraic equations governing
the $b_m$. Setting the determinant of the coefficient matrix of this system equal to zero, we obtain the so-called Hill’s determinant governing $\gamma$. This determinant is more complex than a common polynomial form due to the time delay $\tau$. Again, the algebra was carried out by using a symbolic manipulator.

It follows from Eq. (6.23) that a given periodic solution $x_0(t)$ is asymptotically stable if and only if all of the eigenvalues $\gamma$ lie in the left-half plane and it is unstable if at least one eigenvalue lies in the right-half plane. Starting with a width of cut $w$ for which periodic solutions are stable and then increasing $w$, we find that these periodic solutions will undergo a bifurcation and hence lose stability. The response subsequent to the bifurcation depends on the manner in which the eigenvalues move from the left- to the right-half plane. If a real eigenvalue moves from the left- to the right-half plane along the real axis, the system response will jump to another solution, which in this case may be periodic or trivial. On the other hand, when a pair of complex conjugate eigenvalues moves transversely from the left- to the right-half plane, the resulting response will be either a two-period quasiperiodic motion or a periodic motion with a large period (phase-locked motion).

### 6.3 Numerical Results

As a first step, we validate the analytical results by numerically integrating Eq. (6.1). For the numerical integration, we use a fifth-order Runge-Kutta scheme. Then, we start with some initial conditions and, as in the actual cutting process, let $x_\tau = 0$ for $0 \leq t < \tau$ and switch on $x_\tau$ for $t \geq \tau$. The integration is carried out long enough for the transients to die out. The numerically obtained periodic solutions are very close to those obtained analytically so much so that they are indistinguishable when plotted together. Therefore, we show in Fig. 6.1 two-dimensional projections of the phase portraits of the periodic
solutions (limit cycles) obtained with the two approaches for the two widths of cut $w = 0.06$ and $0.136$ when $\tau = 1/75$. The agreement is excellent and indicates that our harmonic-balance solution is accurate for periodic motions. Numerical integration is used exclusively once the periodic solutions lose stability.

### 6.3.1 Subcritical Instability

In Fig. 6.2, we show variation of the amplitude $a$ of the fundamental harmonic with the width of cut $w$. Such a variation is called a bifurcation diagram. The trivial solution is stable for $w < w_c$ and it is a sink; and the trivial solution is unstable for $w > w_c$ and it is a saddle. The nontrivial solutions represented by solid curves are stable and correspond to stable limit cycles. A limit cycle is an isolated periodic solution. The nontrivial solutions represented by the dotted curves correspond to unstable limit cycles with a real eigenvalue
\( \gamma \) lying in the right-half plane; hence they correspond to limit cycles of the saddle type. The nontrivial solutions represented by the dotted-dashed curve correspond to unstable limit cycles with a pair of complex conjugate eigenvalues lying in the right-half plane; the response in this case is either a two-period quasiperiodic motion or a chaotic motion. There are four bifurcations labeled as \( H, CF_1, CF_2, \) and \( SH \). In the neighborhood of \( H \), there exists a branch of stable trivial solutions for \( w < w_c \) and a branch of unstable trivial solutions and a branch of stable limit cycles for \( w > w_c \). Such a bifurcation is called supercritical. The lower branch of stable limit cycles meets the branch of saddle limit cycles at \( CF_1 \), resulting in the destruction of both limit cycles. Such a bifurcation is called cyclic fold. Similarly, the upper branch of stable limit cycles meets the branch of saddle limit cycles at \( CF_2 \), resulting in the destruction of both limit cycles in a cyclic-fold bifurcation. In the neighborhood of \( SH \), there is a branch of stable limit cycles on one side and a branch of unstable limit cycles and a branch of stable two-period quasiperiodic motions on the other side. Such a bifurcation is called supercritical secondary Hopf bifurcation.

In the absence of large disturbances, performing an experiment starting with a small value of \( w \) and slowly increasing \( w \), one finds that the response is trivial and hence there is no chatter until \( w = w_c \). Increasing \( w \) past \( w_c \), one finds that the trivial solution loses stability, giving way to small limit cycles. As \( w \) is increased further, the limit cycle deforms and grows until it collides with an unstable limit cycle (the dotted branch at point \( CF_1 \)), resulting in the destruction of both limit cycles. Consequently, the system response jumps to point \( C \), which is a large-amplitude limit cycle. Increasing \( w \) past point \( C \) results in a slow deformation and growth of the limit cycle until the secondary Hopf bifurcation point \( SH \) is reached. Starting to the left of \( SH \) and slowly decreasing \( w \), one finds that the response continues to be a large-amplitude limit cycle past \( CF_1 \) and \( H \) until the cyclic-fold bifurcation \( CF_2 \) is reached. Decreasing \( w \) below \( CF_2 \) results in the destruction of the limit cycle and a jump to the trivial solution. We note that in between \( CF_2 \) and \( CF_1 \), the
system response may be trivial (no chatter) or a large-amplitude limit cycle (chatter with a large amplitude), depending on the initial conditions. This phenomenon is usually referred to as a subcritical instability. As a consequence, the nonlinearity reduces the instability limit $w = w_c \approx 0.117072$ predicted by the linear theory, to $w \approx 0.0431$, corresponding to $CF2$, a reduction of about 63%. The subcritical instability was observed experimentally by Hanna and Tobias (1974).

In Fig. 6.3 for $\tau = 1/60$, which is referred to as the left side of the lobe (see Fig. 2.8), the small periodic solution born as a result of the supercritical Hopf bifurcation, instead of quickly suffering a jump via a cyclic-fold bifurcation, grows smoothly and remains stable as $w$ increases until it undergoes a cyclic-fold bifurcation at $w > 4.0$. Another perspective of this type of behavior is shown in Fig. 6.4 for $w = 0.1$. All of the Hopf bifurcations are supercritical. As the rotational speed decreases, every Hopf bifurcation occurring at the right end of an unstable branch of trivial solutions (or the right side of the shaded area in Fig. 2.8) gives birth to a small periodic solution, which immediately suffers a cyclic-fold bifurcation, resulting in a jump to an upper branch of stable periodic solutions. On the other hand, every Hopf bifurcation occurring at the left end leads to a small periodic solution whose amplitude grows smoothly with increasing rotational speed.

### 6.3.2 Inadequacy of the Two-Term Approximation

We compare in Fig. 6.2 the analytical results obtained with the two-term and six-term approximations in the Fourier series when $\tau = 1/75$. Except for the small-amplitude limit cycles for values of $w$ near $w_c$, as shown in the inset, the two approximations yield results that are very close. However, whereas the six-term approximation predicts a secondary Hopf bifurcation at $w \approx 0.141$, the two-term approximation predicts stable limit cycles. When $\tau = 1/60$, the results in Fig. 6.3 show that the two-term approximation predicts
Figure 6.2: Comparison of the nonlinear response curves obtained using the method of harmonic balance for the case when $\tau = 1/75$: thin lines obtained with a six-term approximation and thick lines obtained with a two-term approximation. Both show supercritical Hopf bifurcations. Solid lines denote stable trivial or periodic solutions, dotted lines denote unstable trivial or periodic solutions, and dotted-dashed lines denote the region of quasiperiodic and chaotic solutions. $a$ is the amplitude of the fundamental harmonic of vibration and $w$ is the width of cut.
Figure 6.3: Comparison of the nonlinear response curves obtained using the method of harmonic balance for the case when \( \tau = 1/60 \): thin lines obtained with a six-term approximation and thick lines obtained with a two-term approximation. The two-term approximation erroneously predicts a subcritical rather than a supercritical Hopf bifurcation. Solid lines denote stable trivial or periodic solutions and dotted lines denote unstable trivial or periodic solutions. \( a \) is the amplitude of the fundamental harmonic of vibration and \( w \) is the width of cut.
Figure 6.4: Nonlinear response curves for a six-term approximation when $w = 0.1$. Solid lines denote stable trivial or periodic solutions and dotted lines denote unstable trivial or periodic solutions. $a$ is the amplitude of the fundamental harmonic of vibration and $w$ is the width of cut.
qualitatively erroneous results; it predicts a subcritical rather than a supercritical Hopf bifurcation, as shown in the inset.

Because of the unsymmetrical nature of the quadratic stiffness and cutting force, at least the second-harmonic terms need to be included in the method of harmonic balance. As shown in the previous section, the normal form, obtained by using the method of multiple scales, qualitatively predicts accurate results because it accounts for the effects of both the quadratic and cubic nonlinearities.

### 6.3.3 Route to Chaos

Next, we investigate the behavior of the motions for values of \( w \) exceeding that corresponding to the secondary Hopf bifurcation \( SH \) by using a numerical integration of the governing equation (6.1) for a time long enough for the transients to die out. To characterize the motions, we use four tools, namely, phase portraits, time traces, Poincaré sections, and power spectra. For the Poincaré sections, we collected all of the points of intersection of the trajectory with the surface of section \( x(t - \tau) = 0 \) when \( \dot{x}(t - \tau) > 0 \).

In Fig. 6.5, we show characteristics of the motion when \( w = 0.136 \). As expected, the motion is periodic: the phase portrait is a closed curve, the Poincaré section consists of a finite number of points (one point), and the power spectrum consists of a fundamental frequency and its even and odd harmonics, including a constant component (usually referred to as \( dc \) component). The magnitudes of the peaks are in good agreement with those obtained by using the method of harmonic balance.

As \( w \) increases, the limit cycle increases in size, deforms, and loses stability via a secondary Hopf bifurcation \( SH \) with a pair of complex conjugate eigenvalues \( \gamma \) crossing transversely from the left-half to the right-half plane. At \( w \approx 0.141 \), a pair of the eigenvalues is
\[ \gamma = 0.1166 \pm 267.372i, \] the chatter frequency \( \omega_1 = 1391.07 \), and the bifurcated solution can be characterized by two independent frequencies \( \omega_1 \) and \( \omega_2 \), which are, in general, incommensurate (i.e., \( \omega_2/\omega_1 \), is not a rational number). Characteristics of this motion are shown in Fig. 6.6. Such a motion is called two-period quasiperiodic or two-torus. The time trace suggests that the motion contains two periods and the power spectrum clearly contains another frequency besides the original chatter frequency. The ratio of the chatter frequency \( \omega_1 = 1391.07 \) to the modulated frequency \( \omega_2 = 267.37 \) is 5.203 approximately. As indicated by the power spectrum, the two peaks on both sides of the fundamental frequency component are about 0.4\( \omega_1 \) apart. The phase portrait seems not to close on itself, suggesting a nonperiodic motion. The collected points on the one-sided Poincaré section fill up uniformly and densely a closed curve, confirming the two-period quasiperiodic nature of the motion. Such a motion can be calculated by using a spectral analysis. Keeping up to the second harmonics and sum and difference of the harmonics, one can approximate such a motion with

\[
x(t) = a_0 + a_1 \cos \omega_1 t + a_2 \cos 2\omega_1 t + a_3 \sin 2\omega_1 t + b_1 \cos \omega_2 t + b_2 \cos 2\omega_2 t + b_3 \sin 2\omega_2 t + c_1 \cos[(\omega_2 + \omega_1)t] + c_2 \sin[(\omega_2 + \omega_1)t] + c_3 \cos[(\omega_2 - \omega_1)t] + c_4 \sin[(\omega_2 - \omega_1)t]
\]

Substituting Eq. (6.26) into Eq. (6.1) and equating the coefficient of each harmonic on both sides, one obtains a system of 13 nonlinear algebraic equations for the \( a_i, b_i, c_i, \omega_1, \) and \( \omega_2 \). This we leave for future research, and the remaining results are obtained via numerical integration.

As \( w \) is increased substantially above 0.141, the Poincaré section gets distorted and the collected points on the surface of section cover the closed curve nonuniformly, as shown in Fig. 6.7 when \( w = 0.230 \). The power spectrum contains many more frequencies. As \( w \) increases further, the torus grows and doubles, as shown in Fig. 6.8 for \( w = 0.240 \). The post-bifurcation state is a new torus that forms two loops around the original torus. The power spectrum of the doubled torus has many more frequencies than that of the torus
in Fig. 6.7. Instead of undergoing a complete cascade of period-doubling bifurcations, the doubled torus deforms into a wrinkled one, followed by a fractal torus, and finally a destruction of the torus and the emergence of a chaotic attractor, as shown in the Poincaré sections of Figs. 6.9 and 6.10. This transition to chaos through a two-period quasiperiodic attractor is often described as chaos via torus breakdown (Nayfeh and Balachandran, 1995).

6.3.4 Influence of Nonlinear Stiffness

In this section, we ascertain the importance of the nonlinear stiffness terms $\beta_2 x^2$ and $\beta_3 x^3$ on the response of the machine tool. To this end, we compare in Fig. 6.11 the nonlinear responses for the cases with nonlinear stiffness and those without nonlinear stiffness ($\beta_2 = \beta_3 = 0$). Clearly the nonlinear stiffness terms have a profound influence on the system behavior. Neglecting the nonlinear stiffness terms eliminates the cyclic-fold and hence eliminates the subcritical instability.

6.4 Analog-Computer Simulation Results

In order to solve Eq. (6.1) on the analog computer, we first assume that the highest derivative term is known and write the equation as follows:

$$\ddot{x} = -2\xi \dot{x} - p^2 (x + \beta_2 x^2 + \beta_3 x^3) - p^2 w [x - x_\tau + \alpha_2 (x - x_\tau)^2 + \alpha_3 (x - x_\tau)^3] \quad (6.26)$$

We note, that the coefficients of Eq. (6.26) are poorly scaled for the analog computer. To begin, we let

$$x = u/\beta_2 \quad (6.27)$$

in Eq. (6.26) and obtain

$$\ddot{u} = -2\xi \dot{u} - p^2 (u + u^2 + 1.15u^3)$$
Figure 6.5: The phase portrait (a) the Poincare section (b) the time history (c) and the FFT (d) of the attractor obtained when $\tau = 1/75$ and $w = 0.136$. 
Figure 6.6: The phase portrait (a) the Poincare section (b) the time history (c) and the FFT (d) of the attractor obtained when $\tau = 1/75$ and $w = 0.141$.

Figure 6.7: The phase portrait (a) the Poincare section (b) the time history (c) and the FFT (d) of the attractor obtained when $\tau = 1/75$ and $w = 0.230$. 
Figure 6.8: The phase portrait (a) the Poincare section (b) the time history (c) and the FFT (d) of the attractor obtained when $\tau = 1/75$ and $w = 0.240$.

$$-p^2 w[u - u_\tau + 0.012(u - u_\tau)^2 - 0.016(u - u_\tau)^3] \quad (6.28)$$

Next, we rescale time to obtain a linear natural frequency of 10 rad/s and rewrite Eq. (6.28) as

$$u'' = -0.41886u'' - 100(u + u^2 + 1.15u^3)$$
$$-100[u - u_\tau + 0.012(u - u_\tau)^2$$
$$-0.016(u - u_\tau)^3] \quad (6.29)$$

where the prime denotes the derivative with respect to

$$\hat{t} = 108.856t \quad (6.30)$$
Figure 6.9: The Poincare sections obtained when $\tau = 1/75$ and $w = (a) 0.250$ (b) 0.255 (c) 0.260 (d) 0.265 (e) 0.266 and (f) 0.267.

Figure 6.10: The Poincare sections obtained when $\tau = 1/75$ and $w = (a) 0.270$ (b) 0.30 (c) 0.32 and (d) 0.35.
Figure 6.11: Comparison of the nonlinear response curves obtained with a six-term approximation with nonlinear stiffness (thin lines) and linear stiffness (thin lines) when (a) \( \tau = 1/60 \) and (b) \( \tau = 1/75 \). Solid lines denote stable trivial or periodic solutions and dotted-dashed lines denote unstable trivial or periodic solutions. \( a \) is the amplitude of the fundamental harmonic of vibration and \( w \) is the width of cut.

### 6.4.1 Experimental Setup

Equation (6.29) was programmed into a pair of analog computers. The desired time delay in the feedback loop was implemented using a digital controller consisting of an analog input/output board installed in a personal computer.

The delay procedure was as follows. A signal proportional to \( u \) was sampled by the controller and stored in a shift register. At the next sample, a new value of \( u \) was read into the shift register, and the previous value was mapped into a new location. In this fashion the controller continuously updated a series of shift registers, acquiring a sampled sequence of \( u \) values of length \( \tau \). After obtaining a sample string of length \( \tau \), the controller output was updated on a first-in-first-out basis.

The signal is sampled at approximately 15 millisecond intervals (\( \approx 0.0154 \text{sec/shift register} \)) with some variation due to program execution speed. The nominal sample rate of 65 Hz permits representation of signals within a bandwidth of approximately 32 Hz according to Shannon’s sampling criterion. This corresponds to approximately five times the bandwidth...
required to reproduce the fundamental and first two harmonics of the combined system response. As such, the sampling interval seems small enough to approximate a continuous time controller. However, in the author’s experience, there exists no reliable criterion as to what constitutes “real” time discretization when the plant is nonlinear. As in numerical integration, the time step is small enough when the results make sense.

To evaluate the timing of the digital delay, we tested the controller using a square wave input from a signal generator. The input and output of the delay were captured using a dual-channel, digital-storage oscilloscope, and the time lag between the signals then was recorded. Ten measurements of the delay revealed that the actual experimental value of $\tau$ varied approximately 0.5 percent around a mean value. This variation was attributed to fluctuations in the program execution speed.

In the following sections, we present and discuss results obtained using the analog computer to integrate Eq. (6.29). All integrations were carried out long enough for transients to die out. Data were recorded using both a spectrum analyzer and a multichannel data acquisition board. The specifics of the data-acquisition will be discussed where appropriate.

### 6.4.2 Bifurcation Diagrams

As a first step in the experimental analysis, we sought to reproduce the bifurcation diagrams that were developed using the six-term harmonic balance. An experiment was performed beginning with a small value of $w$ and observing the system response to prescribed initial conditions. After the system reached steady state, a spectrum analyzer was used to compute the average power spectrum of $u$ using a Hanning window and 5 averages. The amplitude $a$ of the fundamental frequency component was recorded in units of peak Volts and then scaled accordingly. The frequency resolution was 0.012 Hz and a typical funda-
mental frequency was approximately 2.0 Hz (analog-computer time scale). We note that in hindsight, a flattop window would have been more appropriate for precise measurement of the amplitudes. The frequency of the measured periodic motions was a continuously varying function of the control parameter, and, in the course of generating the bifurcation set, measurements were made of periodic responses that had frequencies between the available spectral lines. This problem of spectral resolution explains some of the data fluctuation that will be observed.

In the bifurcation diagram of Fig. 6.12 (a), we compare the measured amplitudes for $\tau = 1/75$ as a function of the control parameter $w$ with the predicted values. For values less than the predicted $w_c$ of 0.117, the response spiraled into the trivial solution, as expected. Values of $w$ were increased in successive experiments until the limit value $w_c$ was determined. We observed the Hopf bifurcation at a gain level of between $w = 0.115$ and $w = 0.118$, in excellent agreement with theory. The gain was gradually increased, and the cyclic fold predicted by theory to occur at $w \approx 0.119$ was observed to occur at a value of $w \approx 0.120$. Again the agreement is excellent. In general, we observe that the experiment produces smaller amplitudes for the large limit-cycle and quasiperiodic motions than predicted by theory. Also, the subcritical instability extends only to $w \approx 0.060$ instead of the predicted $w \approx 0.043$. However, the structures are in qualitative agreement. Differences between theory and experiment may be attributed to the presence of noise, discretization of the feedback signal, variations in the time delay, or combinations of these.

In Fig. 6.12 (b), we consider a similar bifurcation diagram for $\tau = 1/60$. The analysis predicts a supercritical Hopf bifurcation leading to a small-amplitude limit cycle that grows smoothly and remains stable as $w$ increases until it undergoes a cyclic-fold bifurcation at $w > 4$; a value that is outside the parameter range considered in this study. Once again, the experimentally determined amplitudes are smaller than those predicted, but the general structure is in qualitative agreement for most of the range of $w$. Experimental data along
the branch of small-amplitude limit cycles for \( w < 0.120 \) was in the noise base of the analog computer and unreliable. The general trend indicates a Hopf bifurcation for \( w \) larger than approximately 0.090. This is larger than the predicted value of \( w \approx 0.060 \). Experiment and theory also diverge noticeably as \( w \) increases beyond \( w \approx 0.125 \) for the branch of solutions corresponding to large-amplitude limit cycles. According to the analysis, this branch corresponds to stable limit cycles that grow smoothly and remain stable as \( w \) increases. However, in Fig. 6.13, we show the power spectrum of the response of the system in this region. It does not correspond to a periodic motion and suggests that the system experienced a bifurcation(s) along this branch, which the theory did not predict.

The obvious discrepancy between theory and experiment for \( \tau = 1/60 \) may be explained by considering the linear stability diagram, see Fig. 2.8. We note that the linear stability boundary for this value of the delay is very steep when compared with the boundary at \( \tau = 1/75 \). We believe that the system is highly sensitive to parameter variation in this region, and that the sample rate was not sufficiently rapid or accurate enough for these experiments to faithfully reproduce the dynamics. In hindsight, considering the nature of the stability boundary in this region, we might have expected that our sample rate would not be adequate.

### 6.4.3 Route to Chaos Revisited

Next we investigate the behavior of large-amplitude motions along the branch of solutions predicted to progress from limit cycles to chaos via torus breakdown when \( \tau \approx 1/75 \). To characterize these motions, we use the same four tools used in our previous numerical study, namely, phase portraits, time traces, Poincaré sections, and power spectra.

For the Poincaré sections, phase portraits, and time traces presented in this section, data
were collected using a multichannel data-acquisition board that sampled values of $u$ and $u'$ at a frequency of 1403 Hz for 20 seconds and wrote the recorded signal values into an ascii file. Twenty such files were created for each simulation, taking approximately one hour. Drift was not apparent. For the Poincaré sections, we needed to collect points of intersection of the trajectory with the surface of section $u(t - \tau) = 0$ when $u'(t - \tau) > 0$. Because the delay coordinates were not directly accessible during the experiment, they were created from the sampled data. The sample frequency was such that 2048 samples approximated $\tau = 1/75$. Thus, delay coordinates were created by truncating the last 2048 points in the data set, while real-time coordinates were created by truncating the first 2048 samples of the set. The high sample rate made it possible to detect accurately when the trajectory intersected the desired surface; however, it was unsatisfactory for producing power spectra of sufficient frequency resolution. For the power spectra plots of this section, data were collected using the same data acquisition system, but sampled at 16 Hz for 64 seconds. Power spectra were computed using an FFT algorithm with rectangular windowing and 10 ensemble averages.

In Figs. 6.14 (a-d), we show characteristics of the motion when $w = 0.060$. As expected, the motion is periodic: the phase portrait is a closed curve, the Poincaré section consists of a single point (within experimental accuracy), and the power spectrum consists of a fundamental frequency and its even and odd harmonics, including a constant component.

In Figs. 6.14 (e-h), we show characteristics of the motion when $w = 0.136$. The limit cycle appears to have grown, deformed, and lost stability via a secondary Hopf bifurcation. This bifurcation was predicted to occur at $w \approx 0.141$, however, the noise present in the experiment could easily account for the variation from theory. The phase portrait no longer appears to close on itself, suggesting a nonperiodic motion. Side bands on either side of the fundamental frequency component suggest the presence of another frequency. The collected points on the one-sided Poincaré section appear to fill up uniformly and densely
a closed curve. Motions with these characteristics are known as two-period quasiperiodic or two-torus motions.

As $w$ was increased further, the two-torus grew and became more pronounced. It follows from Figs. 6.15 (a-d) that at $w = 0.185$ the motion is now clearly a two-torus. The time trace suggests the motion contains two periods, as is evident in the power spectrum. Further confirmation is provided by the Poincaré section. Our previous analysis found that the two-torus doubled for $w = 0.240$. In Figs. 6.15 (e-h) we show the experimental results. The motion is clearly more complex than a two-torus, but the Poincaré section does not provide conclusive evidence of a doubled torus. Finally, Fig. 6.16 shows the Poincaré section of the response for $w = 0.350$; this motion is apparently chaotic.

### 6.4.4 Pointwise Dimension

In this section, we reconstruct the chaotic attractor using the method of delays and then estimate the pointwise dimension of the attractor (Nayfeh and Balachandran, 1995). The dimension calculation is performed to provide more confidence in our assessment that the motion is chaotic at $w = 0.350$ and $\tau = 1/75$.

We first construct a two-dimensional pseudo-state space and compare it with the measured phase portrait in Fig. 6.17 in order to evaluate our selection of a time delay for the embedding. The pseudo-state space is created from a sampled scalar time variable ($u$ in this case) using delayed coordinates. In theory, the so-called reconstructed space exhibits the same invariants as the original state space, and the dimension can be calculated for the motion without actually measuring the independent coordinates required to construct a state space. In our case, because of the time delays inherent in the governing equation, it is not clear what constitutes a “proper” state space, so it is advantageous to avail ourselves
of the systematic approaches available using the reconstructed space.

A time series of 28067 points was collected at a sampling frequency of 1403 Hz. In constructing the pseudo-state space from this time series, we tried a variety of delay times that were integer multiples of the sample interval. Initially, a delay was selected based on the first zero crossing of the autocorrelation function; it yielded a delay time $\hat{\tau}$ of approximately 0.1625 seconds. Eventually, we selected a $\hat{\tau}$ of approximately 0.107 seconds (150 sample intervals) because it produced a pseudo-state space that bore the most resemblance to the recorded phase portrait.

Next, a 10-dimensional pseudo-state space was constructed for a delay time $\hat{\tau} = 0.10691$. The embedding dimension $d = 10$ was selected because it was anticipated that this would be well in excess of $2d_a + 1$, the embedding dimension necessary to preserve the geometric structure of the original system dynamics, where $d_a$ is the actual attractor dimension. The probability of finding a point in a sphere of radius $r$ was calculated for each of 100 randomly selected points within the 10-dimensional pseudo-state space. In Fig. 6.18 we show a plot of $\ln P(r)$ vs $\ln(r)$ for an embedding dimension of 10. The slope of this graph for the region between $r = 0.5$ and 1.5 yields an estimated fractal dimension of 2.88. The fractional nature of the dimension is a further evidence of chaos.

### 6.5 Nonlinear Stability with Control

In this section, we look over the global stability of the Hanna and Tobias model (1974) coupled to a linear vibration absorber. We considered the linear stability of this system in Chapter 3.

Using a six-term harmonic balance solution, we plot the bifurcation diagram for this system
in Fig. 6.19. Along with the results of the harmonic balance solution, we plot the results of an analog simulation. Both results are in excellent agreement and suggest that the addition of the absorber has both increased the limit width of cut and eliminated the globally subcritical behavior, at least at the rotational speed considered. In Chapter 8, we will find that for higher rotational speeds the global subcritical behavior persists, but that it can be controlled via nonlinear feedback.
Figure 6.12: Comparison of nonlinear response curves obtained from analog simulation with the results obtained from a six-term harmonic balance solution when (a) $\tau = 1/75$ and (b) $\tau = 1/60$. Solid lines denote stable periodic solutions and dotted lines denote unstable periodic solutions determined in the previous analysis. Diamonds denote points obtained via analog computer.
Figure 6.13: Power spectrum of the response for $\tau = 1/60$ and $w = 0.200$. 
Figure 6.14: The phase portraits (a) and (e), the Poincaré sections (b) and (f), the time histories (c) and (g), and the power spectra (d) and (h) of the attractors obtained when $\tau = 1/75$ and $w = 0.060$ and $w = 0.136$, respectively.
Figure 6.15: The phase portraits (a) and (e), the Poincaré sections (b) and (f), the time histories (c) and (g), and the power spectra (d) and (h) of the attractors obtained when $\tau = 1/75$ and $w = 0.185$ and $w = 0.240$, respectively.
Figure 6.16: Poincaré section for \( w = 0.350 \) and \( \tau = 1/75 \).

Figure 6.17: Comparison of phase portrait and pseudo-state space for \( w = 0.350 \) and \( \tau = 1/75 \).
Figure 6.18: Pointwise dimension for an embedding dimension of 10.

Figure 6.19: Bifurcation diagram showing the amplitude of the fundamental chatter harmonic $a$ as a function of the effective width of cut $w$. The solid curve was obtained using a six-term harmonic balance. The circles indicate points obtained via analog simulation.
Chapter 7

A Two-Degree-of-Freedom Boring-Bar Chatter Model

We now take up the two-degree-of-freedom model for boring-bar dynamics that was presented in Chapter 2.

Recall, that the proposed system can be expressed mathematically as

\[
\ddot{x}_1 + 2\zeta_1\omega_1\dot{x}_1 + \omega_1^2 x_1 = -\gamma_1 w\left[\eta(x_1 - x_{1r}) + \nu(x_2 - x_{2r})\right] \\
\ddot{x}_2 + 2\zeta_2\omega_2\dot{x}_2 + \omega_2^2 x_2 = -\gamma_2 w\left[\eta(x_1 - x_{1r}) + \nu(x_2 - x_{2r})\right]
\]  

(7.1) (7.2)

where the coordinates \(x_1\) and \(x_2\) correspond to tool motions directed in the principal modal directions and the constants are as defined previously. In the remainder of this chapter, we develop this model more thoroughly, incorporate nonlinearities in the fashion of Hanna and Tobias (1974), explore the local stability of the combined system, and illustrate the existence of multiple chatter modes, as observed experimentally, via numerical integration of the complete system.
7.1 Modeling

So far, the system lacks any nonlinearity that would clamp the chatter oscillations and produce limit-cycle type behavior. Taking our cue from Hanna and Tobias (1974), we include both structural and cutting force nonlinearities and modify the linear model as follows:

\[
\ddot{x}_1 + 2\xi_1\omega_1\dot{x}_1 + 2\xi_1\dot{x}_1 + \omega_1^2[x_1 + f_1(x_1)] = -\gamma_1 w \{\eta [(x_1 - \mu_1 x_1) + g_1(x_1 - \mu_1 x_1)] + \nu [(x_2 - \mu_2 x_2) + g_2(x_2 - \mu_2 x_2)]\} \tag{7.3}
\]

\[
\ddot{x}_2 + 2\xi_2\omega_2\dot{x}_2 + 2\xi_2\dot{x}_2 + \omega_2^2[x_2 + f_2(x_2)] = -\gamma_2 w \{\eta [(x_1 - \mu_1 x_1) + g_1(x_1 - \mu_1 x_1)] + \nu [(x_2 - \mu_2 x_2) + g_2(x_2 - \mu_2 x_2)]\} \tag{7.4}
\]

where \(f_n(x_n)\) is a function describing the structural nonlinearity in the \(n^{th}\) modal coordinate, \(g_n(x_n - \mu_n x_{n'})\) is a function describing the regenerative cutting force nonlinearity in each coordinate, \(\xi_n\) is the damping due to the cutting process in each coordinate, \(\gamma_n\) is an influence coefficient describing the component of the differential cutting force projected onto each coordinate, \(\eta\) is a coefficient describing the degree to which motions in the \(x_1\) direction contribute to the differential cutting force, \(\nu\) is a coefficient describing the degree to which motions in the \(x_2\) direction contribute to the differential cutting force, and \(\mu_n\) describes the degree of cut overlap in each direction.

7.1.1 Geometric Considerations

The model is based on knowledge of the direction of the mean cutting force, the angles that it makes with the tool coordinates, and the angles these coordinates make with an
orthogonal cutting plane. We have noted in the experimental work that the orientations of the mean cutting force and the orthogonal cutting plane tend to change with the width of cut and will attempt to illustrate this phenomenon heuristically via consideration of two limiting cases.

In Fig. 7.1 (b), we illustrate the case encountered for extremely light cuts where the width of cut is less than the tool nose radius. Under these conditions, the chip flows smoothly from the tool in a direction oriented between 60 and 90 degrees to the machined surface in the plane of the tool for small feedrates. Tool motions normal to the machined surface have a component that modulates the chip thickness while the affect of motions in the tangential direction is small, but significant.

In Fig. 7.1 (a), we illustrate the case where the width of cut is much greater than the tool nose radius. Under these conditions, the chip flows smoothly away from the tool in a direction normal to the transient surface in the plane of the tool for small feedrates. The orthogonal cutting plane is now rotated by as much as 90 degrees from that of the shallow cutting condition, and tool motions normal to the machined surface no longer modulate the chip thickness. Instead, tool motions tangential to the machined surface indirectly modulate the chip thickness and give rise to regenerative chatter of type B as defined by Tobias (1965).

The seeming change in the orientation of the plane in which cutting might be characterized as nearly “orthogonal” is due to the finite nose radius of the insert. Clearly, the effective tool lead angle can vary from $0^\circ$ to $\approx 90^\circ$ as the chip width is decreased below a value corresponding to the radius of the insert (0.01" in this case). Similar observations regarding the chip flow in oblique, multi-edged cutting geometries were originally reported by Stabler (1964).

For certain cutting geometries, it might appear that the two transverse vibratory modes of
the tool could be treated as decoupled, and the stability can be analyzed using a single-degree-of-freedom approximation. However, it seems that, for cutting conditions which exist between the two extremes delineated above, the tool stability at a given width of cut will depend in a complex fashion on the interaction of the two modal coordinates, and that this behavior may persist to fairly substantial cuts.

Consider Fig. 7.2, where we show the spectrum of the tool responses in the modal directions when the boring bar is used in a truly orthogonal cut with \( w = 0.1'' \). This cut was obtained by machining a tube of 0.1" wall thickness. The tool was wider than the cut and only the primary cutting edge was engaged. We observe that the response is stable and that the motions in the modal coordinates are independent of each other. The accelerations are small and “random” with the exception of humps in the spectrum near the tool modal frequencies. In contrast, the responses of Figs. 5.9 and 5.10 obtained while boring at the same speed, feedrate, and depth of cut exhibit large-amplitude chatter, and we conclude that the influence of the secondary cutting edge on the machine-tool stability is profound.

### 7.1.2 A Model for Light Cuts

To model the tool accelerations of Fig. 5.12, we let \( \psi_1 = 15^\circ, \theta = 45^\circ, \xi_1 = 0, \xi_2 = 0, \gamma_1 = \omega_1 k \cos (\theta - \psi_1), \gamma_2 = -\omega_1 k \sin (\theta - \psi_1), \eta = \cos (\psi_1), \nu = -\sin (\psi_1), \mu_1 = 1, \mu_2 = 1, \) and assume the following functional relations for the structural and cutting force nonlinearities:

\[
f_n(x_n) = 10x_n^2 + 300x_n^3 \tag{7.5}
\]

\[
g_n(x_n - \mu_n x_{nr}) = 5.7(x_n - \mu_n x_{nr})^2 + \chi(x_n - \mu_n x_{nr})^3 \tag{7.6}
\]

The structural nonlinearity is identical in form to that used by Hanna and Tobias (1974), though the magnitudes of the coefficients have been reduced. The influence of process damping is neglected in the linear stability analysis of Chapter 2, and the resulting limit
width of cut of $\approx 0.002''$ agrees well with that obtained in the experiments, thus we choose $\xi_1 = \xi_2 = 0$. The overlap coefficients $\mu_1 = \mu_2 = 1$ are selected to capture the fact that motions in both the $x_1$ and $x_2$ directions modulate the undeformed chip thickness.

Finally, we need to consider a form for the cutting force nonlinearity $g_n$. Clearly, our model must possess a subcritical instability given the experimental evidence of Chapter 5. In Chapter 6 we found that global subcritical behavior can be expected for a nonlinearity such as that considered by Hanna and Tobias (1974). However, we also found that the global instabilities associated with Hanna and Tobias-like models occur between the tangential and lobed stability boundaries (see Nayfeh and Pratt, 1997). For the cutting speeds of the experiments, the predicted tangential stability boundary differs little from the lobed stability boundary, as seen in Fig. 2.10. It therefore seems unlikely that a subcritical behavior would be observed if it were of this type. Rather, the observed instability of
Figure 7.2: Tool response for a “heavy” orthogonal cut when $w = 0.1''$, speed is 230 rpm, and the feedrate is 0.0024 ipr.

the light cutting experiments seems to be more like a locally subcritical one capable of producing jumps for widths of cut below the tangential boundary. Such a behavior was encountered by Shi and Tobias (1986) in some of their face milling experiments and was recently analyzed by Stepan (1997) in his reconsideration of Shi and Tobias’s work.

Stepan (1997), working from the results of Shi and Tobias (1986), suggested that a local subcritical-type instability can exist when the sign of the cubic term in the polynomial expression for the cutting-force nonlinearity proposed by Hanna and Tobias (1974) is changed from negative to positive. Physically, the results of Stepan (1997) suggest that a softening-type cutting-force characteristic can produce locally subcritical behavior in a single-degree-of-freedom model for dynamic cutting.

In Figure 7.3 we show the static cutting forces measured using a dynamometer that was incorporated as part of the smart-tool fixture. These results were obtained by shortening the boring-bar overhang, disconnecting the actuators, and measuring the cutting forces for a series of widths of cut at a fixed speed of 200 surface feet per minute (approximately
180 rpm) and a feedrate of 0.0024 ipr. The zero point is imposed to indicate loss of tool engagement.

We see a nearly linear dependence on cut width in the direction of the tool speed, as is typical, however the forces normal (radial) to the machined surface appear to have a more complex structure. A “knee” in the radial cutting force occurs as the width of cut exceeds 0.01”. This is precisely the point where the width of cut begins to exceed the tool nose radius. It provides a further corroboration of our hypothesized mean force variation with width of cut. We also note that the data indicates an inflection point near a width of cut 0.002”. Radial force measurements in this region were inconsistent, but it appears that the force achieves a plateau in a fashion that might be modeled with a locally softening characteristic.

It seems, on the basis of the observed stability and the measured static cutting forces, that our model for light cutting conditions below 0.005” should employ some form of a softening-type cutting-force characteristic.

### 7.1.3 A Model for Heavy Cuts

To model the tool accelerations of Fig. 5.13, we let $\xi_1/\omega_1 + \zeta_1 = 0.2$, $\xi_2 = 0$, $\gamma_1 = \cos(\pi/3)\omega_1^2 k_s \lambda_1$, $\gamma_1 = \cos(\pi/6)\omega_1^2 k_s \lambda_1$, $\eta = 0.495$, $\nu = -0.24$, $\mu_1 = 1$, $\mu_2 = 1$, and assume the following functional relations for the structural and cutting force nonlinearities:

$$f_1(x_1) = \begin{cases} 0 & \text{if } x_1 < 0 \\ 5x_1 & \text{otherwise} \end{cases}$$

(7.7)

$$f_2(x_2) = 10x_2^2 + 300x_2^3$$

(7.8)

$$g_n(x_n - \mu_n x_{nt}) = 5.7(x_n - \mu_n x_{nt})^2 - 3700(x_n - \mu_n x_{nt})^3$$

(7.9)
The regenerative cutting force as well as the structural nonlinearities are identical in form to those used by Hanna and Tobias (1974). In this case we ignore the issue of the type of instability and focus merely on producing an accurate representation of the tool chatter.

The damping in the $x_1$ direction was observed to be qualitatively “high” in the time traces of the impact response of the tool measured during stable cutting at lower speeds (see Fig. 5.6); hence, a very substantial effect of process damping is incorporated by using a large value of $\xi_1$. The overlap coefficient $\mu_1 = 1$ was selected to capture the fact that motions in the $x_1$ direction modulate the undeformed chip cross-section in a complex fashion that depends on the previous tool position. On the other hand, motions in the $x_2$ direction modulate the chip thickness via a penetration effect labeled type-B chatter by Tobias (1965). The sign of the coefficient $\nu$ was chosen to be negative based on the tendency of the tangential chatter frequency to be lower than $\omega_2$, which would not be the case if $\nu > 0$. Perhaps the most unconventional feature of the model is the bilinear structural stiffness.
represented by the function $f_1(x_1)$. This function was chosen to produce the unsymmetric tool motions that were evident in the time traces at this width of cut and speed. We hypothesize that the tool does not so much cut the workpiece in the $x_1$ direction as plow into it, giving rise to the near impact-type oscillations observed in this coordinate during the experiments.

7.2 Stability

We consider the conditions prevalent during the light cutting scenario because this will define the lower bound of stable widths of cut. Furthermore, from a practical standpoint, chatter is most detrimental during a finish cut, where the emphasis is on bringing a part to a precise dimensional tolerance and surface finish.

The linear stability boundary for this case was determined in Chapter 2 and plotted for the two-degree-of-freedom model in Fig. 2.10. In this section, we consider the transition from stable to unstable cutting by deriving the normal form for the Hopf bifurcation point using the methods described in Chapter 6.

First, we write the equations of motion. Based on the light cutting model, we have

$$
\ddot{x}_k + 2\zeta_k \omega_k \dot{x}_k + \omega_k^2 F_k(x_k) = -\gamma_k w F_c[(x_1 - x_{1r}), (x_2 - x_{2r})] 
$$

(7.10)

where $k = 1, 2$,

$$
F_k(x_k) = x_k + \beta_{k2} x_k^2 + \beta_{k3} x_k^3,
$$

(7.11)

and

$$
F_c[(x_1 - x_{1r}), (x_2 - x_{2r})] = \eta[(x_1 - x_{1r}) + \alpha_{12}(x_1 - x_{1r})^2 + \alpha_{13}(x_1 - x_{1r})^3]
$$

$$
+ \nu[(x_2 - x_{2r}) + \alpha_{22}(x_2 - x_{2r})^2 + \alpha_{23}(x_2 - x_{2r})^3]
$$

(7.12)
Due to the presence of both cubic and quadratic nonlinearities, the perturbation analysis must be performed using either the method of multiple scales or the generalized method of averaging, or a center manifold technique (Stepan, 1989). In this Dissertation, we choose a multiple-scales approach (Nayfeh, 1973, 1981).

Following the developments of Chapter 6, we introduce a fast time scale $T_0 = t$ and a slow time scale $T_2 = \epsilon^2 t$ and seek a third-order expansion in the form

$$x_k = \epsilon x_{k1}(T_0, T_2) + \epsilon^2 x_{k2}(T_0, T_2) + \epsilon^3 x_{k3}(T_0, T_2) + \cdots \quad (7.13)$$

We express the time derivatives as

$$\frac{d}{dt} = D_0 + \epsilon^2 D_2 + \cdots \quad \text{and} \quad \frac{d^2}{dt^2} = D_0^2 + 2\epsilon^2 D_0 D_2 + \cdots \quad (7.14)$$

where

$$D_n = \frac{\partial}{\partial T_n} \quad (7.15)$$

Moreover, we let

$$w = w_c + \epsilon^2 w_2 + \cdots \quad (7.16)$$

Substituting Eqs. (7.13)-(7.16) into Eqs. (7.10) and equating coefficients of like powers of $\epsilon$, we obtain

**Order $\epsilon$:**

$$D_0^2 x_{k1} + 2\zeta_k \omega_k D_0 x_{k1} + \omega_k^2 x_{k1} + \gamma_k w_c [\eta(x_{11} - x_{11r}) + \nu (x_{21} - x_{21r})] = 0 \quad (7.17)$$

**Order $\epsilon^2$:**

$$D_0^2 x_{k2} + 2\zeta_k \omega_k D_0 x_{k2} + \omega_k^2 x_{k2} + \gamma_k w_c [\eta(x_{12} - x_{12r}) + \nu (x_{22} - x_{22r})] =$$

$$-\omega_k^2 \beta_k x_{k1}^2 - \gamma_k w_c [\eta \alpha_{12} (x_{11} - x_{11r})^2$$

$$+ \nu \alpha_{22} (x_{21} - x_{21r})^2] \quad (7.18)$$
Order $\epsilon^3$:

\[ D_0^2x_{k3} + 2\zeta_k\omega_kD_0x_{k3} + \omega_k^2x_{k3} + \gamma_kw_c[\eta(x_{13} - x_{13\tau}) + \nu(x_{23} - x_{23\tau})] = \]

\[-\omega_k^2(2\beta_kx_{k1}x_{k2} + \beta_kx_{k3}^3) - 2\zeta_k\omega_kD_2x_{k1} - 2D_0D_2x_{k1} \]

\[-\gamma_kw_c\{\eta[2\alpha_2(x_{11} - x_{11\tau})(x_{12} - x_{12\tau}) + \alpha_3(x_{11} - x_{11\tau})^3] \]

\[+\nu[2\alpha_2(x_{21} - x_{21\tau})(x_{22} - x_{22\tau}) + \alpha_3(x_{21} - x_{21\tau})^3]\}

\[-\gamma_kw_2[\eta(x_{11} - x_{11\tau}) + \nu(x_{21} - x_{21\tau})] \]

(7.19)

We now define a delay operator

\[ L_D[x(t), \tau] = x(t - \tau) \] (7.20)

and rewrite the equations making use of matrix operator notation as

Order $\epsilon$:

\[ \mathcal{L}\{x_{11} \ x_{12}\}^T = 0 \] (7.21)

Order $\epsilon^2$:

\[ \mathcal{L}\{x_{12} \ x_{22}\}^T = \begin{cases} 
-\omega_k^2\beta_{12}x_{11}^2 - \gamma_1w_c\{\eta\alpha_2[(1 - L_D)x_{11}]^2 + \nu\alpha_2[(1 - L_D)x_{21}]^2 \} \\
-\omega_k^2\beta_{22}x_{21}^2 - \gamma_2w_c\{\eta\alpha_2[(1 - L_D)x_{11}]^2 + \nu\alpha_2[(1 - L_D)x_{21}]^2 \} 
\end{cases} \] (7.22)
Order $\epsilon^3$:

\[
L\{x_{13} \ x_{23}\}^T = \begin{cases} 
-\omega_1^2(2\beta_{12}x_{11}x_{12} + \beta_{13}x_{11}^3) - 2\zeta_1\omega_1D_2x_{11} - 2D_0D_2x_{11} \\
-\gamma_1w_c\eta[2\alpha_{12}((1 - L_D)x_{11})(1 - L_D)x_{12}] + \alpha_{13}((1 - L_D)x_{11})^3] + \nu[2\alpha_{22}((1 - L_D)x_{21})(1 - L_D)x_{22}] + \alpha_{23}((1 - L_D)x_{21})^3] \\
-\gamma_1w_2\eta((1 - L_D)x_{11}) + \nu((1 - L_D)x_{21}) \end{cases} 
\]

(7.23)

where

\[
L = \begin{bmatrix} D_0^2 & 0 \\
0 & D_0^2 \end{bmatrix} + \begin{bmatrix} 2\zeta_1\omega_1D_0 & 0 \\
0 & 2\zeta_2\omega_2D_0 \end{bmatrix} + \begin{bmatrix} \omega_1^2 + \gamma_1w_c\eta(1 - L_D) & \gamma_1w_c\nu(1 - L_D) \\
\gamma_2w_c\eta(1 - L_D) & \omega_2^2 + \gamma_2w_c\nu(1 - L_D) \end{bmatrix} 
\]

(7.24)

We note that the asymmetry of the operator is reminiscent of aeroelastic flutter or gyroscopically coupled systems.

The solution of Eq. (7.21) can be expressed as

\[
x_{11} = A_1(T_2)e^{i\omega_cT_0} + cc, \quad \text{and} \quad x_{11\tau} = A_1(T_2)e^{i\omega_c(T_0 - \tau)} + cc 
\]

(7.25)
\[ x_{21} = A_1(T_2)b_1 e^{i\omega_c T_0} + cc, \quad \text{and} \quad x_{21}\tau = A_1(T_2)b_1 e^{i\omega_c (T_0 - \tau)} + cc \] (7.26)

where \( b_1 \) can be obtained by various methods through consideration of the linear problem.

Previously, we used a Laplace transform approach, and we stick with that method here.

We take the Laplace transform of Eq. (7.21) and, assuming zero initial conditions, express \( x_{21} \) in terms of \( x_{11} \) via the transfer function

\[
H(s) = -\frac{\gamma_2 w_c \eta(1 - e^{s\tau})}{s^2 + 2\zeta_2 \omega_2 s + \omega_2^2 + \gamma_2 w_c \nu(1 - e^{s\tau})} \tag{7.27}
\]

At the Hopf bifurcation point,

\[
b_1 = H(i\omega_c), \quad \text{and} \quad \bar{b}_1 = H(-i\omega_c) \tag{7.28}
\]

Substituting Eqs. (7.25) and (7.26) into Eq. (7.22) we obtain

\[
\mathcal{L}\{x_{12} x_{22}\} = -A_1^2 \begin{cases}
-A_1 \{\omega_1^2 \beta_{12} + \gamma_1 w_c (1 - e^{-i\omega_c \tau})^2 [\eta \alpha_{12} + \nu \alpha_{22} b_1^2]\} e^{i2\omega_c T_0} \\
-A_1 \bar{A}_1 \{\omega_1^2 \beta_{22} + \gamma_1 w_c (1 - e^{-i\omega_c \tau}) (1 - e^{i\omega_c \tau}) [\eta \alpha_{12} + \nu \alpha_{22} \bar{b}_1 \bar{b}_1]\}
\end{cases} + cc \tag{7.29}
\]

which we express in a more compact fashion as

\[
\mathcal{L}\{x_{12} x_{22}\} = -A_1^2 \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} e^{i2\omega_c T_0} - A_1 \bar{A}_1 \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} + cc \tag{7.30}
\]

where

\[
p_k = \omega_k^2 \beta_k + \gamma_k w_c (1 - e^{-i\omega_c \tau})^2 [\eta \alpha_{12} + \nu \alpha_{22} b_1^2], \quad k = 1, 2 \tag{7.31}
\]

\[
q_k = \omega_k^2 \beta_k + \gamma_k w_c (1 - e^{-i\omega_c \tau}) (1 - e^{i\omega_c \tau}) [\eta \alpha_{12} + \nu \alpha_{22} \bar{b}_1 \bar{b}_1], \quad k = 1, 2 \tag{7.32}
\]

We observe that the inhomogeneous terms are nonsecular and either constant or proportional to \( e^{2i\omega_c T_0} \). To obtain a solution at this order, we assume a particular solution of the
form

\[ x_{k2} = A_1^2 P_k e^{i2\omega_c T_0} + Q_k A_1 \tilde{A}_1 + cc, \quad k = 1, 2 \]  

(7.33)

and substitute it into Eq. (7.30). Equating coefficients of like terms from either side of the resulting equation yields

\[
G(2i\omega_c) \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = - \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}
\]

(7.34)

\[ Q_k = - \frac{q_k}{\omega_k^2} \]

(7.35)

where in general

\[
G(i\omega) = \begin{bmatrix}
\omega_1^2 - \omega^2 + 2i\zeta_1 \omega_1 \omega \\
+\gamma_1 w_c \eta(1 - e^{-i\omega_c \tau}) & \gamma_1 w_c \nu(1 - e^{-i\omega_c \tau}) \\
\gamma_2 w_c \eta(1 - e^{-i\omega_c \tau}) & \omega_2^2 - \omega^2 + 2i\zeta_2 \omega_2 \omega \\
+\gamma_2 w_c \nu(1 - e^{-i\omega_c \tau})
\end{bmatrix}
\]

(7.36)

Solving for the coefficients \( P_k \), we obtain

\[
\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -G^{-1}(2i\omega_c) \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}
\]

(7.37)

Substituting Eq. (7.33) into Eq. (7.23), performing some algebraic manipulations, and defining some new variables, we have
\[
\mathcal{L}\{x_{13}, x_{23}\}^T = \begin{cases} 
-2(\zeta_1 \omega_1 + i \omega_c) A'_1 \\
-\gamma_1 w_2 (\eta + \nu c_1) (1 - e^{-i \omega_c \tau}) A_1 \\
-\Lambda_1 A_f^2 \tilde{A}_1 \\
-2(\zeta_2 \omega_2 + i \omega_c) A'_1 \\
-\gamma_2 w_2 (\eta + \nu c_1) (1 - e^{-i \omega_c \tau}) A_1 \\
-\Lambda_2 A_f^2 \tilde{A}_1 
\end{cases} e^{i \omega_c T_0} + cc + NST 
\] (7.38)

where the prime denotes differentiation with respect to \(T_2\) and

\[
\Lambda_1 = \omega_1^2 (2 \beta_{12} P_1 + 3 \beta_{13} + 2 \beta_{12} Q_1) \\
+ \gamma_1 w_c \{ \eta [2 \alpha_{12} P_1 (1 - e^{-2i \omega_c \tau})(1 - e^{i \omega_c \tau})] \\
+ 3 \alpha_{13} (1 - e^{-i \omega_c \tau})^2 (1 - e^{i \omega_c \tau}) \} \\
+ \nu [2 \alpha_{22} P_2 Q_1 (1 - e^{-2i \omega_c \tau})(1 - e^{i \omega_c \tau})] \\
+ 3 \alpha_{23} \tilde{b}_1^2 Q_1 (1 - e^{-i \omega_c \tau})^2 (1 - e^{i \omega_c \tau}) \} 
\] (7.39)

\[
\Lambda_2 = \omega_2^2 (2 \beta_{22} P_2 \bar{c}_1 + 3 \beta_{23} c_f^2 \bar{c}_1 + 2 \beta_{22} c_1 Q_2) \\
+ \gamma_2 w_c \{ \eta [2 \alpha_{12} P_1 (1 - e^{-2i \omega_c \tau})(1 - e^{i \omega_c \tau})] \\
+ 3 \alpha_{13} (1 - e^{-i \omega_c \tau})^2 (1 - e^{i \omega_c \tau}) \} \\
+ \nu [2 \alpha_{22} P_2 \bar{Q}_1 (1 - e^{-2i \omega_c \tau})(1 - e^{i \omega_c \tau})] \\
+ 3 \alpha_{23} \tilde{b}_1^2 \bar{Q}_1 (1 - e^{-i \omega_c \tau})^2 (1 - e^{i \omega_c \tau}) \} 
\] (7.40)

To determine the solvability conditions, we seek a particular solution free of secular terms in the form

\[
x_{13} = K_1 e^{i \omega_c T_0}, \quad \text{and} \quad x_{23} = K_2 e^{i \omega_c T_0} 
\] (7.41)
Substituting Eq. (7.41) into Eq. (7.38), we have

\[ G(i\omega_c) \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -2(\zeta_1 \omega_1 + i\omega_c)A'_1 - \gamma_1 w_2(\eta + \nu c_1)(1 - e^{-i\omega_c \tau})A_1 - \Lambda_1 A_1^2 \bar{A}_1 \\ -2(\zeta_2 \omega_2 + i\omega_c)A'_1 - \gamma_2 w_2(\eta + \nu c_1)(1 - e^{-i\omega_c \tau})A_1 - \Lambda_2 A_1^2 \bar{A}_1 \end{bmatrix} \]  

(7.42)

Equations (7.42) constitute a system of two inhomogeneous algebraic equations for \( K_1 \) and \( K_2 \). Their homogeneous parts have a nontrivial solution as we learned in Chapter 2. Hence, the solvability condition is obtained by setting the determinant of the following augmented matrix equal to zero. The result is

\[
\begin{vmatrix}
\omega_1^2 - \omega_c^2 + 2i\zeta_1 \omega_1 \omega_c + \gamma_1 w_c \eta(1 - e^{-i\omega_c \tau}) & -2(\zeta_1 \omega_1 + i\omega_c)A'_1 - \gamma_1 w_2(\eta + \nu c_1)(1 - e^{-i\omega_c \tau})A_1 - \Lambda_1 A_1^2 \bar{A}_1 \\
\gamma_2 w_c \eta(1 - e^{-i\omega_c \tau}) & -2(\zeta_2 \omega_2 + i\omega_c)A'_1 - \gamma_2 w_2(\eta + \nu c_1)(1 - e^{-i\omega_c \tau})A_1 - \Lambda_2 A_1^2 \bar{A}_1
\end{vmatrix} = 0
\]

(7.43)

This condition yields

\[ \Gamma_1 A'_1 + \Gamma_2 A_1 + \Gamma_3 A_1^2 \bar{A}_1 = 0 \]

(7.44)

where

\[
\begin{align*}
\Gamma_1 &= \Gamma_{1r} + i\Gamma_{1i} \\
&= 2(\zeta_1 \omega_1 + i\omega_c)(\gamma_2 w_c \eta(1 - e^{-i\omega_c \tau})) \\
&\quad - 2c_1(\zeta_2 \omega_2 + i\omega_c)(\omega_1^2 - \omega_c^2 + 2i\zeta_1 \omega_1 \omega_c + \gamma_1 w_c \eta(1 - e^{-i\omega_c \tau})) \\
\Gamma_2 &= \Gamma_{2r} + i\Gamma_{2i} \\
&= \gamma_1 \gamma_2 w_2 w_c \eta(1 - e^{-i\omega_c \tau})^2(\eta + \nu c_1) \\
&\quad - \gamma_2 w_2(\eta + \nu c_1)(1 - e^{-i\omega_c \tau})(\omega_1^2 - \omega_c^2 + 2i\zeta_1 \omega_1 \omega_c + \gamma_1 w_c \eta(1 - e^{-i\omega_c \tau}))
\end{align*}
\]

(7.45)
\[ \Gamma_3 = \Gamma_{3r} + i\Gamma_{3i} \]
\[ = \Lambda_1 \gamma_2 w_c \eta (1 - e^{-i\omega_c \tau}) - 2\Lambda_2 (\omega_1^2 - \omega_c^2 + 2i\zeta_1 \omega_1 \omega_c + \gamma_1 w_c \eta (1 - e^{-i\omega_c \tau})) \]  
(7.47)

Introducing the polar transformation
\[ A_1 = \frac{1}{2} a e^{i\beta} \]  
(7.48)

into the solvability condition Eq. (7.44) and separating real and imaginary parts, we obtain the normal form
\[ a' = c_1 w_2 a + c_2 a^3 \]  
(7.49)
\[ a\beta' = c_3 w_2 a + c_4 a^3 \]  
(7.50)

where
\[ c_1 = -\frac{\Gamma_{1r} \Gamma_{2r} + \Gamma_{1i} \Gamma_{2i}}{\Gamma_{1r}^2 + \Gamma_{1i}^2} \]  
(7.51)
\[ c_2 = -\frac{1}{4} \frac{\Gamma_{1r} \Gamma_{3r} + \Gamma_{1i} \Gamma_{3i}}{\Gamma_{1r}^2 + \Gamma_{1i}^2} \]  
(7.52)
\[ c_3 = -\frac{\Gamma_{1r} \Gamma_{2i} - \Gamma_{1i} \Gamma_{2r}}{\Gamma_{1r}^2 + \Gamma_{1i}^2} \]  
(7.53)
\[ c_4 = -\frac{1}{4} \frac{\Gamma_{1r} \Gamma_{3i} - \Gamma_{1i} \Gamma_{3r}}{\Gamma_{1r}^2 + \Gamma_{1i}^2} \]  
(7.54)

As discussed in Chapter 6, the qualitative behavior of the local instability is determined by the sign of the coefficient \( c_2 \) in the above normal form. Plugging in the model values from Section 7.1.2 we find that a transition from a locally supercritical behavior to a locally subcritical behavior occurs as \( \chi_i \) changes from negative to positive.

Stepan (1997) obtained a similar normal form and results for a single-degree-of-freedom nonlinear cutting model. He considered a softening-type cutting force characteristic, essentially changing the sign of coefficient \( \alpha_3 \) in the Hanna and Tobias model, and used center manifold reduction to obtain the normal form. He did not address the global behavior of
the system in the post-bifurcation region in a rigorous fashion, but argued that at some point the tool must leave the cut, and that one could reasonably expect that the region where jump phenomena could occur would be limited to a parameter space where widths of cut produced unstable limit cycles of amplitudes less than the mean chip thickness. He obtained an expression for the amplitude of the unstable limit cycles from consideration of his normal form, an exercise we carry out now for our system.

We seek the amplitude of the limit cycles as predicted by the local analysis. This may be simply obtained by setting \( a' = 0 \) in Eq. (7.49). Then the steady state amplitude of the resulting limit cycles can be determined as a function of width of cut from the expression

\[
w = w_c - \frac{c_2 a^2}{c_1}
\]  

(7.55)

We let \( \chi = -4.25\alpha_3 \) and use Eq. (7.55) to generate the bifurcation diagram plotted in Fig. (7.4).

In the simulations, we confirm that a subcritical bifurcation due to cutting force nonlinearity exists and that it requires another substantial nonlinearity if bounded periodic motions are to result. We hypothesize that, in order to predict the post-bifurcation behavior, it is necessary to extend the model to incorporate the effect of the tool leaving the cut, as implied by Stepan (1997) and considered by Tlusty (1985), Shi and Tobias (1986), and Kondo et al. (1981).

### 7.3 Simulated Responses

In the following simulations we use the dynamic tool properties that were identified for the Smart Tool during the experimental testing period of August, 1996. Thus, the frequencies and dampings are \( f_2 = 493 \) Hz and \( \zeta_2 \approx 0.03 \) and \( f_1 = 365 \) Hz and \( \zeta_1 \approx 0.02 \).
7.3.1 Simulated Smart-Tool Response for a Light Cutting Condition

The model parameters for a light cutting condition are programmed into a simulation with the aid of MATLAB and Simulink (1993). The integration scheme uses a 5th-order Runge-Kutta method and the maximum and minimum step sizes are both set for $\Delta t = 0.0002$ seconds to produce time series at a fixed sample rate of 5000 Hz, just as in the experiments.

The nonlinear coefficient $\chi = -4.25\beta_3$ is selected based on experiments with the simulation. We find in these experiments that this value produces a nice example of the subcritical Hopf bifurcation. In the time trace of Fig. 7.5 we see the simulated tool accelerations in response to a small initial disturbance. The width of cut is $w = 0.002''$ which is less than the limit width of cut $w = 0.0020136''$ predicted by carrying out the linear stability analysis described in Chapter 2. We see that for a small disturbance, the tool motions decay and stable cutting resumes.
In Fig. 7.6 a larger initial disturbance is introduced with \( x_1 = 0.008'' \) (which is really outside of the cut) and \( x_2 = 0.0005'' \). Tool oscillations are seen to grow in amplitude with time. The solution is obviously diverging, confirming our analytical prediction of a locally subcritical-type instability.

We speculate that for light cuts with locally subcritical-type instabilities, the dominant nonlinearity necessary to produce bounded tool motions is that produced by the tool leaving the cut, as has been commonly cited in the literature. The question arises as to whether or not the nonlinearity produced by the tool leaving the cut is sufficient by itself to account for the subcritical behavior of machine tools, as was once speculated in Hooke and Tobias (1964), or whether it is necessary to include a softening characteristic in the cutting force, as is speculated here and by Stepan (1997). At present, we have no answer for this question, but we believe it can be determined by looking at the stability of periodic solutions that result from models such as those suggested by Tlustý, where the only nonlinearity considered is that of the tool leaving the cut. Presumably, a harmonic-balance approach like that undertaken by Shi and Tobias (1986) for the discontinuous cutting force could be employed to obtain a periodic solution and then the stability could be ascertained using Floquet theory, as was demonstrated in Chapter 6. Alternately, we could attack the problem numerically by extending the shooting method to retarded dynamical systems.

### 7.3.2 Simulated Smart Tool Response for a Heavy cutting Condition

The model parameters for a heavy cutting condition are programmed into a simulation with the aid of MATLAB and Simulink (1993). The integration scheme uses a 5th-order Runge-Kutta method and the maximum and minimum step sizes are both set for \( \Delta t = 0.0001 \) seconds to produce time series at a fixed sample rate of 10000 Hz. The simulation is started
from initial conditions and allowed to continue until a steady state is observed. The results are plotted in Fig. 7.7. We see that the simulated response has much of the same character as the experimentally measured tool response of Fig. 5.13.

### 7.4 Summary

In this chapter we have developed a nonlinear modeling and stability of boring-bar chatter based on insights gained from experimental observations. We have proposed that the general orientation of the cutting forces varies with depth of cut owing to the finite nose radius of the tool based on observations of chip flow during cutting experiments. Based on this insight, we have developed evolutionary cutting models that appear to capture some of the complicated nonlinear dynamics present in two cutting regimes. For light cuts,
Figure 7.6: Simulated time trace of the normal tool acceleration when $w = 0.002''$, speed = 90 rpm, and feedrate = 0.0024 ipr for a small initial disturbance.

we have presented a perturbation analysis of a two-degree-of-freedom nonlinear model of machine-tool dynamics. We determined the so-called normal form of the instabilities and explored the range of parameters for which it produces locally subcritical-type behavior. From the normal form, we obtained an expression for the amplitudes of the unstable limit cycles and plotted them as a function of width of cut for the model parameters identified in Section 7.1.2. For heavy cuts, we obtained a simulation which bares an astonishing resemblance to the chatter signatures recorded during experimentation. To achieve this match, we proposed a “pseudo-impact” model for tool plowing motions normal to the machined surface, where the plowing was modeled using a bilinear stiffness.
Figure 7.7: Simulated time traces, autospectra, cross-spectrum, and coherence of the normal and tangential tool accelerations when $w = 0.03''$, speed = 210 rpm, and feedrate = 0.0024 ipr.
Chapter 8

Nonlinear Control Techniques for Chatter Reduction

Working from nonlinear single-degree-of-freedom models for chatter, we develop two ideas that show promise as alternatives or supplements to linear chatter mitigation, provided that a structural control system of sufficient bandwidth and dynamic range can be devised.

The first idea is based on the observation that the amplitude of post-bifurcation limit cycles due to regenerative chatter is determined completely by the nonlinearities present in the combined machine-tool and cutting-process dynamics. Thus, by applying proper nonlinear feedback through the structural control, one can effectively limit the size of limit-cycle vibrations. Also, for a system such as that considered by Hanna and Tobias (1974), the nonlinear feedback can be used to achieve a global bifurcation control. We demonstrate, by feeding back a term proportional to the cube of the tool velocity, that one can change the structure of the machine-tool stability from globally subcritical, or unsafe, to globally supercritical, or safe.
The second idea is an open-loop control strategy based on the well-known “quench” phenomenon that occurs in the forced response of self-excited systems (see Nayfeh and Mook 1979). Here, for the first time, we show that a quench phenomenon exists for time-delay systems. We inject a signal with frequency incommensurate to the chatter frequency into the bifurcation control feedback signal. The combination of the high-frequency forced response with cubic velocity feedback is shown to quench chatter vibrations to acceptably small amplitudes, thereby extending the operating range of a Hanna and Tobias-like machine tool.

8.1 Global Bifurcation Control via Nonlinear State Feedback

We observed experimentally in Chapter 5 and theoretically in Chapters 6 and 7 that the local and global dynamics of machining processes can exhibit subcritical, or unsafe instabilities, characterized by jump phenomena. In this section, we propose a method for eliminating this behavior by ensuring that an appropriate nonlinear term is included in the equation of motion. For this Dissertation, we suppose that the force represented by this term is available via direct nonlinear feedback control using an actuator, though clearly it would be desirable to design the tool in such a fashion that the force arises naturally as part of the cutting process.

The stability we wish to alter is, in some instances, part of the global dynamics. Recall, that locally, the stability of a hardening-type cutting force is characterized by a supercritical Hopf bifurcation, but that the subsequent small-amplitude periodic motions quickly lose stability via a cyclic-fold bifurcation that cannot be detected using a local analysis. Nevertheless, we begin our theoretical consideration of the problem by examining the effect
of cubic-velocity feedback on the structure of the nonlinear normal form for such a system. In essence, we seek clues as to how much more supercritical we must make the local bifurcation in order to preserve its supercritical structure in the global response. We then use the harmonic-balance technique developed in Chapter 6 to generate nonlinear response curves, or bifurcation diagrams, for various levels of nonlinear feedback to ascertain the minimum gain required to achieve the desired change in the form of the bifurcation. We then present digital-computer simulation results for comparison with the stability predictions of the harmonic-balance analysis.

8.1.1 Theory for High-Speed Machining

The current thrust in machine-tool research is towards ever higher spindle speeds; the so-called high-speed machining centers. This trend is motivated by two factors. First, higher speeds and feeds result in greater production capacity and faster time to market. Second, tool forces tend to decline with increasing speed while the stability against regenerative chatter increases, as evidenced by the spreading of the stability lobes. However, the perceived gain in cutting stability with increased speed (and hence reduced time delay) may be dubious in the presence of subcritical instabilities. Here, we consider a hypothetical high-speed machining operation and develop a theoretical framework for implementing bifurcation control via nonlinear feedback.

The model is that of Hanna and Tobias (1974). We consider the case of high-speed machining where the regenerative phase lag $\omega_c \tau$ is between $3\pi/2$ and $7\pi/2$, or the instability is constrained to occur within the first lobe. We presume that either passive or active control has been employed to increase the tool damping by an order of magnitude so that $\xi_{new} = 10\xi_{new}$. Thus, our hypothetical system is more representative of the coming state-of-the-art than the typical shop lathe that was employed in the experimental portion of
The equation of motion considered is then
\[ \ddot{x} + 2\xi \dot{x} + p^2 (x + \beta_2 x^2 + \beta_3 x^3) = -g_3 \dot{x}^3 - p^2 w \left[ x - x_\tau + \alpha_2 (x - x_\tau)^2 + \alpha_3 (x - x_\tau)^3 \right] \]  
(8.1)
where
\[ x_\tau = x(t - \tau), \]  
(8.2)
and \( g_3 \) is the nonlinear feedback gain.

**Normal Form**

The normal form can be determined in the same manner as shown in Chapter 6. Rather than repeat the entire analysis, we simply consider the secular terms generated by our nonlinear control and incorporate them into the solvability condition.

First, we note that our control will produce an additional term at \( Order \, e^3 \); that is
\[ \cdots = -g_3 (D_0 x_1)^3 + \cdots \]  
(8.3)
which gives rise to the secular term \(-3i g_3 \omega_c^3 A^2 \bar{A} \). This results in the new solvability condition
\[ 2(\xi + i\omega_c) A' + p^2 w_2 (1 - e^{-i\omega_c \tau}) A + \Lambda A^2 \bar{A} + 3i g_3 \omega_c^3 A^2 \bar{A} = 0 \]  
(8.4)
Introducing the polar transformation
\[ A = \frac{1}{2} a e^{i\beta} \]  
(8.5)
into the solvability condition and separating real and imaginary parts, we obtain the normal form as before
\[ a' = c_1 w_2 a + c_2 a^3 \]  
(8.6)
where now the coefficient $c_2$ is modified such that

$$c_2 = -\frac{1}{8(\xi^2 + \omega^2)}(\xi \Lambda_r + \omega_c \Lambda_i + 3g_3 \omega_c^4)$$

(8.8)

The qualitative behavior near the Hopf bifurcation can be determined by the sign of $c_2$. Here, we see that the choice of nonlinear velocity feedback has given us the ability to ensure that $c_2$ is negative, for the case when softening-type cutting forces might otherwise produce a subcritical Hopf bifurcation, and to increase the degree to which $c_2$ is negative in the case when hardening-type cutting force nonlinearities combined with structural nonlinearities produce a global subcritical instability. The former case is traditional bifurcation control, as discussed in Nayfeh and Balachandran (1995). The latter case is an extension to the concept of global instability.

**Stability Analysis**

It appeared in our earlier analysis of the linear chatter control that the application of a vibration-absorber-type control that enhances the tool damping could also affect the global stability. This is true in part. What occurs is that the global subcritical instability becomes substantially smaller in proportion to the tangential stability boundary, at least for the rotational speeds previously considered. However, at high rotational speeds we find that the effect once again becomes significant. This is important from a technological standpoint because the latest thrust in machine-tool dynamics research is towards high-speed machining. Spindles capable of tens of thousands of rpm’s are being brought into service by the major airframe manufacturers with little understanding yet of the dynamic implications.

In Fig. 8.1 we see the results of a global stability analysis for the case $1/\tau = 749.9$ and
This is approximately ten times the rotational speed considered previously. We see that the overall structure of the stability is the same and that there is a substantial region of globally subcritical behavior where both stable cutting and very large-amplitude chatter motions coexist.

Figure 8.1: Bifurcation diagram for the system of Hanna and Tobias (1974). The structural damping is ten times that originally considered and $1/\tau = 749.9$

Next we consider the effect of the nonlinear velocity feedback for two different levels of the
gain $g_3$. The results are shown in Fig. 8.2. It appears from this analysis that, for nonlinear feedback gains in excess of 10, the cyclic fold is eliminated and that the transition to chatter has been made a smooth function of the effective width of cut.

Figure 8.2: Comparison of the nonlinear tool response with and without bifurcation control for two different levels of cubic velocity feedback.
8.1.2 Simulation

In this section we consider the response of the system before and after application of nonlinear feedback control when \( g_3 = 10 \). We use MATLAB and Simulink (1993) to integrate the equation of motion and consider two cases with and without nonlinear feedback. The first case we consider is when the effective width of cut \( w = 2 \), which, according to a linear stability analysis, produces stable, chatter-free cutting. The second case we consider is in the post-bifurcation regime when \( w = 2.3 \).

We see in the phase portrait and time trace of Figs. 8.3 (a) and (b) that the uncontrolled tool response to a small initial condition decays to the trivial solution, while in Figs. 8.3 (c) and (d) we see that for a larger initial condition the tool motion grows to a stable limit cycle. We see that the controlled response in Fig. 8.3 (d) and (e) is stable for the initial condition that previously produced limit cycle behavior.

Finally, we show phase portraits in Figure 8.4 of the simulated response when \( w = 2.3 \), which is beyond the linearly established limit width of cut. The nonlinear feedback greatly reduces the severity of chatter, as evidenced by the considerable reduction in the size of the limit cycle.

8.2 Quench Control

We note that the previously considered bifurcation control has the desired effect of eliminating subcritical instability, while also attenuating the size of limit cycles that occur in the post-bifurcation region. In this section, we consider an open-loop control strategy known as quench control that in conjunction with the bifurcation control can further attenuate the size of the limit cycles.
Quench control is achieved by considering the forced response of the self-excited system. In essence, quench control seeks to replace a large-amplitude self-excited motion with a smaller-amplitude externally forced motion. Previously, the only systems for which the quench phenomenon has been explored are Van der Pol and Rayleigh oscillators (see Nayfeh and Mook, 1979). In this section we demonstrate that the limit-cycle behavior that occurs in the model of Hanna and Tobias can be quenched provided that the bifurcation control of the previous section is active.
Figure 8.4: Comparison of the phase portraits produced by the nonlinear response of the tool in the $x_1$ modal coordinate without bifurcation control (a) and with bifurcation control (b).

8.2.1 Theory

To illustrate the concept, we augment the previously considered model for high-speed machining as follows:

$$\ddot{x} + 2\xi \dot{x} + p^2 (x + \beta_2 x^2 + \beta_3 x^3) = -g_3 \dot{x}^3 - p^2 w \left[ x - x_\tau + \alpha_2 (x - x_\tau)^2 + \alpha_3 (x - x_\tau)^3 \right] + \epsilon_\omega \cos (\Omega t)$$

(8.9)

where $g_3$ is the nonlinear feedback gain for bifurcation control and $\epsilon_\omega \cos (\Omega t)$ is the forcing function that acts on the tool where the forcing frequency $\Omega$ is assumed to be incommensurate with the chatter frequency $\omega_c$ determined during the course of the analysis.

Due to the presence of both cubic and quadratic nonlinearities, the perturbation analysis must be performed using either the method of multiple scales or the generalized method of
averaging (Nayfeh, 1973, 1981). In this Dissertation, we choose a multiple-scales approach.

Using the method of multiple scales (Nayfeh, 1973, 1981), we introduce a fast time scale $T_0 = t$ and a slow time scale $T_2 = \epsilon^2 t$ and seek a third-order expansion in the form

$$x = \epsilon x_1(T_0, T_2) + \epsilon^2 x_2(T_0, T_2) + \epsilon^3 x_3(T_0, T_2) + \cdots$$

(8.10)

Moreover, we let

$$w = w_c + \epsilon^2 w_2 + \cdots$$

(8.11)

Substituting Eqs. (8.10) and (8.11) into Eq. (8.9) and equating coefficients of like powers of $\epsilon$, we obtain

**Order $\epsilon$:**

$$D_0^2 x_1 + 2\xi D_0 x_1 + p^2 x_1 + p^2 w_c(x_1 - x_{1\tau}) = f \cos(\Omega T_0)$$

(8.12)

**Order $\epsilon^2$:**

$$D_0^2 x_2 + 2\xi D_0 x_2 + p^2 x_2 + p^2 w_c(x_2 - x_{2\tau}) = -p^2 \beta_2 x_1^2 - p^2 w_c \alpha_2(x_1 - x_{1\tau})^2$$

(8.13)

**Order $\epsilon^3$:**

$$D_0^2 x_3 + 2\xi D_0 x_3 + p^2 x_3 + p^2 w_c(x_3 - x_{3\tau}) = -2D_0 D_2 x_1 - 2\xi D_2 x_1 - p^2 w_2(x_1 - x_{1\tau})$$

$$-2p^2 \beta_2 x_1 x_2 - 2p^2 w_c \alpha_2(x_1 - x_{1\tau})(x_2 - x_{2\tau})$$

$$-p^2 \beta_3 x_1^3 - p^2 w_c \alpha_3(x_1 - x_{1\tau})^3 - g_3(D_0 x_1)^3$$

(8.14)

The solution of Eq. (8.12) can be expressed as

$$x_1 = A(T_2)e^{i\omega_c T_0} + \lambda e^{i\Omega T_0} + cc, \quad \text{and} \quad x_{1\tau} = A(T_2)e^{i\omega_c (T_0 - \tau)} + \lambda e^{i\Omega (T_0 - \tau)} + cc$$

(8.15)

where

$$\lambda = \frac{f}{2(p^2 - \Omega^2 + 2i\xi \Omega + w_c p^2(1 - e^{-i\Omega \tau}))}$$

(8.16)
and \( \omega_c \) is the chatter frequency at the Hopf bifurcation point. Substituting Eq. (8.15) into Eq. (8.13) and solving for \( x_2 \), we obtain

\[
x_2 = -p^2 A^2 \Gamma_1 e^{2i\omega_c T_0} - \Gamma_2 A\bar{A} - p^2 \lambda^2 \Gamma_3 e^{2i\Omega T_0} - \Gamma_4 \bar{\lambda} \bar{A} \lambda \bar{A} \Gamma_5 e^{i(\omega_c + \Omega) T_0} - p^2 A \bar{\Lambda} \Gamma_6 e^{i(\omega_c - \Omega) T_0} + c c
\]

(8.17)

where

\[
\Gamma_1 = \frac{\beta_2 + w_c \alpha_2(1 - e^{-i\omega_c \tau})^2}{p^2 - 4\omega_c^2 + 4i\omega_c \xi + p^2 w_c(1 - e^{-2i\omega_c \tau})}
\]

(8.18)

\[
\Gamma_2 = \frac{\beta_2 + w_c \alpha_2(1 - e^{-i\omega_c \tau})(1 - e^{i\omega_c \tau})}{p^2 - 4\Omega^2 + 4i\Omega \xi + p^2 w_c(1 - e^{-2i\Omega \tau})}
\]

(8.19)

\[
\Gamma_3 = \frac{\beta_2 + w_c \alpha_2(1 - e^{-i\omega_c \tau})(1 - e^{-i\omega_c \tau})}{p^2 - 4(\omega_c + \Omega)^2 + 4i(\omega_c + \Omega) \xi + p^2 w_c(1 - e^{-i(\omega_c + \Omega) \tau})}
\]

(8.20)

\[
\Gamma_4 = \frac{\beta_2 + w_c \alpha_2(1 - e^{-i\omega_c \tau})(1 - e^{i\Omega \tau})}{p^2 - 4(\omega_c - \Omega)^2 + 4i(\omega_c - \Omega) \xi + p^2 w_c(1 - e^{-i(\omega_c - \Omega) \tau})}
\]

(8.21)

\[
\Gamma_5 = \frac{\beta_2 + w_c \alpha_2(1 - e^{-i\omega_c \tau})(1 - e^{-i\Omega \tau})}{p^2 - 4(\omega_c + \Omega)^2 + 4i(\omega_c + \Omega) \xi + p^2 w_c(1 - e^{-i(\omega_c + \Omega) \tau})}
\]

(8.22)

\[
\Gamma_6 = \frac{\beta_2 + w_c \alpha_2(1 - e^{-i\omega_c \tau})(1 - e^{i\Omega \tau})}{p^2 - 4(\omega_c - \Omega)^2 + 4i(\omega_c - \Omega) \xi + p^2 w_c(1 - e^{-i(\omega_c - \Omega) \tau})}
\]

(8.23)

Substituting Eqs. (8.15) and (8.17) into Eq. (8.14) and eliminating the terms that lead to secular terms, we obtain the solvability condition

\[
2(\xi + i\omega_c)A' + p^2 w_2(1 - e^{-i\omega_c \tau})A + \chi A\bar{\lambda} + \Lambda A^2 \bar{A} + i3g_3\omega_c^2 A^2 \bar{A} = 0
\]

(8.24)

where

\[
\chi = \chi_r + i\chi_i
\]

\[
= -2p^2 \beta_2 (2\Gamma_4 + p^2 \Gamma_5 + p^2 \Gamma_6) + 6p^2 \beta_3
\]

\[
-2p^4 w_c \alpha_2 \Gamma_5 (1 - e^{-i(\omega_c + \Omega) \tau})(1 - e^{i\Omega \tau}) + \Gamma_6 (1 - e^{-i(\omega_c - \Omega) \tau})(1 - e^{i\Omega \tau})
\]

\[
+ 6p^2 w_c \alpha_3 (1 - e^{-i\omega_c \tau})(1 - e^{-i\Omega \tau})(1 - e^{i\Omega \tau}) - 6i g_3 \omega_c \Omega^2,
\]

(8.25)

\[
\Lambda = \Lambda_r + i\Lambda_i
\]

\[
= -2p^2 \beta_2 (p^2 \Gamma_1 + 2\Gamma_2) + 3p^2 \beta_3 - 2p^4 w_c \alpha_2 \Gamma_1 (1 - e^{i\omega_c \tau})(1 - e^{-2i\omega_c \tau})
\]

\[
+3p^2 w_c \alpha_3 (1 - e^{-i\omega_c \tau})^2 (1 - e^{i\omega_c \tau}),
\]

(8.26)
and \( \Lambda_r, \Lambda_i, \chi_r, \) and \( \chi_i \) are real constants.

Introducing the polar transformation

\[
A = \frac{1}{2} a e^{i\beta}
\]

(8.27)

into Eq. (8.24) and separating real and imaginary parts, we obtain the normal form

\[
a' = c_1 w_a + c_2 a^3
\]

(8.28)

\[
a\beta' = c_3 w_a + c_4 a^3
\]

(8.29)

where

\[
c_1 = -\frac{1}{2(\xi^2 + \omega_c^2)}[p^2 \xi(1 - \cos \omega_c \tau) + p^2 \omega_c \sin \omega_c \tau + \lambda \bar{\lambda}(\xi \chi_r + \omega_c \chi_i)]
\]

(8.30)

\[
c_2 = -\frac{1}{8(\xi^2 + \omega_c^2)}(\xi \Lambda_r + \omega_c \Lambda_i + 3g_3 \omega_c^4)
\]

(8.31)

\[
c_3 = -\frac{1}{2(\xi^2 + \omega_c^2)}[p^2 \xi \sin \omega_c \tau - p^2 \omega_c(1 - \cos \omega_c \tau) + \lambda \bar{\lambda}(\xi \chi_i - \omega_c \chi_r)]
\]

(8.32)

\[
c_4 = -\frac{1}{8(\xi^2 + \omega_c^2)}(\xi \Lambda_i - \omega_c \Lambda_r)
\]

(8.33)

Machining instability only exists when \( c_1 > 0 \), otherwise the amplitude of the limit cycles decay exponentially to the trivial solution regardless of the nonlinearities. We see that the addition of quench control has produced terms in the coefficient \( c_1 \) that are proportional to the forcing level \( \lambda \). Thus, for forcing levels such that

\[
[p^2 \xi(1 - \cos \omega_c \tau) + p^2 \omega_c \sin \omega_c \tau + \lambda \bar{\lambda}(\xi \chi_r + \omega_c \chi_i)] > 0
\]

(8.34)

the limit cycles are quenched, leaving only the forced response.
8.2.2 Simulation

The model is programmed into a numerical simulation with the aid of MATLAB and Simulink (1993). The integration uses a 5th-order Runge-Kutta method and the maximum and minimum step sizes are both set for $\Delta t = 0.00001$ seconds to produce time series with a fixed sample rate of 100000 Hz. The model parameters are as before with $\tau = 749.9$ and $w = 2.3$. The results are plotted in Fig. 8.5.

Beginning at time $t = 0$, the simulation starts from an initial tool displacement of $x = 0.002$ that quickly grows to a large-amplitude limit cycle. At time $t = 0.5$ the bifurcation control is activated with a gain $g_3 = 10$. The limit cycle is seen to shrink considerably in amplitude.

Next, at time $t = 1.5$ the quench control is activated with $f = 0.0035/386\Omega^2$ and $\Omega = 10000$. The quenching force causes a further reduction in the tool response. In Fig. 8.6 we compare the steady-state tool responses before and after application of the quench control where a smaller time window has been employed to reveal the change in amplitude and frequency. Clearly, we have successfully exchanged a large-amplitude self-excited motion for a higher-frequency, lower-amplitude forced response.

8.3 Summary

In this chapter we have demonstrated two nonlinear chatter-control techniques on a model for high-speed machining. We show that by introducing nonlinear velocity feedback in the machine-tool structure it is possible to eliminate subcritical instabilities from the machine-tool operating envelope. Furthermore, such a control strategy has been shown to greatly attenuate the size of the post-bifurcation limit cycles.
A quench control technique has also been proposed as an open-loop method to enhance the performance of the nonlinear feedback in the post-bifurcation regime. Quench control requires high-frequency, large-amplitude, non-resonant forcing be applied to the tool to convert self-excited limit cycles to small-amplitude, forced oscillations. In other words, we convert the system from autonomous to nonautonomous behavior. At first glance, this seems an inefficient way to attack a vibration problem. However, from a technological standpoint it is fairly easy to conceive of a small “buzzer” embedded in the boring bar that is simply turned on if vibrations exceed a threshold. In fact, in cutting processes where nonlinear friction effects dominate, one can expect that quench control could be employed without resort to any feedback at all, making it a very simple and economical alternative to the more traditional, linear approach employed in the Smart Tool.

Regardless of the merits of the quench phenomenon as a control strategy, the theoretical
Figure 8.6: Comparison of the steady-state tool responses with and without quench control. The dashed line is the tool motion with the active quench control.

results obtained should prove applicable in the area of ultrasonic machining, where the cutting process is carried out using a tool that is forced to vibrate at frequencies corresponding to the wave speed of the tool material. The forced response analysis can also be employed to consider various resonant conditions that might exist due to external forcing. As noted at the very beginning of the Dissertation, machine-tool chatter can result from forced vibrations. There are many instances when rotating imbalance or tool run out contribute a periodic forcing to the tool that is detrimental to the part quality. One can reasonably assume that these forced vibration problems are excacerbated by conditions at high spindle speeds, and that the time when the problem of combined forced and self-excited tool motions must be considered is fast approaching.
Chapter 9

Summary

In this Dissertation we have explored the dynamics and control of machine-tool vibrations with particular attention to boring-bar chatter. We began with the classic models for machine-tool vibrations and laid the foundation for a simple linear control strategy. Using knowledge available from the smart structures research of the last decade, we devised a unique manufacturing research platform known as the Smart Tool to bring our control concepts to fruition. The design, integration, and modeling of this device constituted a major undertaking in itself, and is representative of the type of systems design, or mechatronic engineering, that is playing an ever increasing role in the advancement of engineering science and practice. The development of this platform constitutes a significant advancement in the fields of manufacturing design engineering, smart structures design and analysis, and applied controls.

We developed detailed models and simulations of the Smart Tool performance by adapting concepts from experimental modal analysis, classical controls, the mechanics of metal removal, signal processing, and nonlinear dynamics as needed to facilitate our efforts. The
resulting reduced-order models capture the observed experimental behaviors with remarkable accuracy, and the simulations, when combined with the experimental observations, bring the complex dynamic character of boring-bar chatter into much sharper focus. The experimental models of chatter greatly further our understanding of the complex machine-tool vibration problem, a problem that has been at the heart of a global research effort for the last forty years, and the experimental data base obtained can provide continued new insights into chatter behavior in coupled oscillatory systems as it is further explored.

The analytical treatments of nonlinear machine-tool stability define the state-of-the art and will be a valuable tool to other researchers in the years to come. In particular, the treatments of the nonlinear stability of the two-degree-of-freedom problem, the application of bifurcation control to affect both local and global change in the qualitative behavior of a retarded dynamical system, the application of these concepts to high-speed machining, and the treatment of the combination of time-delayed self-excitation and external forcing to yield a quenching phenomenon are firsts that represent significant achievements in the areas of manufacturing science, nonlinear dynamics, and controls.
References
Bibliography


References


[33] MATLAB (version 4.1) and SIMULINK Block Library (version 1.2d), copyright 1993, The MathWorks, Inc.


References


VITA

Jon Robert Pratt was born on November 17, 1961 in Biloxi, Mississippi. He earned his Master of Science degree in the Aerospace Engineering and Engineering Mechanics Department of Iowa State University in the Spring of 1993. He then worked briefly as a systems engineer for ETREMA Products, Inc. of Ames, Iowa. In the Spring of 1994, he enrolled at Virginia Tech where he completed his doctoral studies in November of 1997. Dr. Pratt won a Paul E. Torgersen award for research excellence from the President of Virginia Tech. He is currently a National Research Council Post-Doctoral Fellow working in the Manufacturing Engineering Laboratory of the National Institute of Standards and Technology.