Chapter 6

The Effects of Vibration on the Penetration Resistance of Light Castle Sand

6.1 Introduction

The purpose of this chapter is to synthesize the information presented in the previous chapters and to use this information to evaluate the effects of vibration on the penetration resistance of the soil. The chapter begins with a direct comparison of the static and vibratory penetration test data in saturated samples to quantify the influence of the vibration on the penetration resistance. Different stress and density conditions are considered in the analysis. The chapter includes a proposed method for estimating the relative density of the soil and the potential of the soil to undergo significant strength loss during dynamic loading based on direct comparisons between the static and the vibratory penetration test data. An analysis of the influence of vibration frequency on the penetration resistance and pore pressure measurements is also included. The chapter is concluded with a proposed method for evaluating the energy efficiency of the vibratory unit used in the investigation.

6.2 Influence of Vibration on Penetration Resistance Values

As noted in Chapter 5, static and vibratory penetration tests were performed in the calibration chamber on saturated Light Castle sand samples prepared at different densities and tested at different stress levels. The purpose of the investigation was to evaluate the effects of vibration on the penetration resistance and pore water pressure and to relate the measured values to the ability of the soil mass to resist significant strength loss when exposed to dynamic loading. One hundred and eighteen penetration tests were performed as part of the study, which allowed for the generation of representative penetration resistance and pore pressure trendlines for the different density and stress conditions. The representative trendlines for the penetration resistance and pore pressure ratios were presented in Chapter 5 in Figure 5.45 and Figures 5.46 to 5.47, respectively, for all of the test conditions.

The intent of the vibrating penetrometer was to induce cyclic pore water pressure within the zone of influence of the cone while simultaneously measuring the penetration resistance. As shown in the previous chapter, the penetration resistance measurement is a global measurement that takes into account the shear induced deformation adjacent to and laterally away from the cone penetrometer. Any reduction in the tip resistance due to the vibration is most likely related to a fluctuation in the soil strength within the zone of influence of the cone penetrometer. As noted in Section 5.9 and 5.11 of Chapter 5, however, the pore water pressure measured by the cone is a localized measurement related to the shear deformation in the soil adjacent to the cone penetrometer. This pore pressure measurement was much different from the pore pressure measured laterally away from the penetrometer during vibratory penetration of the cone, and was therefore not considered representative of the actual pore water pressure distribution laterally away from the cone.

The results of the calibration chamber tests indicate that the penetration resistance can be influenced by the vibration of the cone penetrometer, provided the magnitude of the force and frequency of vibration are large enough to add a significant dynamic load to the soil. This concept is illustrated through Figure 6.1, where the penetration resistance values from tests in saturated loose samples at the intermediate stress level are compared. The penetration resistance values included in the figure were obtained from a static penetration test and vibratory penetration tests using the pneumatic impact and rotary turbine vibrators.

A review of Figure 6.1 indicates that the static penetration resistance is approximately equal to the vibratory penetration resistance measured using the pneumatic impact vibrator. This suggests that the 4N dynamic load imparted into the soil at a 5 Hz frequency was not substantial enough to modify the static penetration resistance value. However, the penetration resistance of the soil is considerably less when the dynamic load of 1.2 kN at 125 Hz frequency is imparted into the soil by the rotary turbine vibrator. Based on this comparison, it appears that the penetration resistance value may be reduced during vibratory penetration of the penetrometer, provided the dynamic load imparted into the soil is substantial enough to modify the soil strength within the zone of influence of the cone penetrometer.

The influence of the vibration on the penetration resistance was quantitatively determined for a given test condition through a direct comparison of the vibratory and static penetration resistance. This direct comparison is referred to as the reduction ratio,

$$RR = 1 - \frac{q_{c(v)}}{q_{c(s)}} \tag{6.1}$$

where RR is the reduction ratio, $q_{c(v)}$ is the vibratory penetration resistance, and $q_{c(s)}$ is the static penetration resistance. A similar equation was proposed by Sasaki et al. (1985) during early work on vibrocone testing in Japan. A value of the reduction ratio near unity suggests that the vibratory penetration resistance was much less than that obtained from static penetration, while a value near zero suggests that the vibratory penetration resistance was approximately equal to the static.

A comparison of the representative penetration values for the different stress and density conditions was made using Equation 6.1 to quantify the influence of the vibration on the penetration resistance value. Plots showing this comparison are presented as Figure 6.2. A second abscissa was added to the plots to show the ratio of the vibratory to the static penetration resistance ($q_{c(v)}/q_{c(s)}$). The data from the representative trendlines presented in Figure 5.43 were used to generate these curves. As discussed in Chapter 5, the center of the sample (z = 0.75m) is chosen to represent the sample as a whole for the following analysis because the lateral and vertical boundary effects are minimized at this point.

As shown through the comparison presented in Figure 6.2a, the vibratory penetration resistance is approximately 60% less than the static penetration resistance for soil at the loose state and low stress conditions. In loose samples, the effect of the vibration on the penetration resistance is reduced as the mean stress in the soil is increased, as indicated by RR values of approximately 25% at the intermediate stress levels and 7% at the high stress levels. Thus, for the given input energy, any loss in soil strength and/or water pressure increase associated with the dynamic load was highly dependent on the mean stress in the soil for the range of stresses considered in the testing.

Presented in Figure 6.2b are the RR plots for penetration tests performed in the medium dense samples at different stress levels. It can be seen through the comparison that the vibratory penetration resistance is about 65% less than the static penetration resistance at the low stress level and is approximately equal to the static penetration resistance at the intermediate and high stress levels. Thus, for the given input energy, the fabric of the medium dense soil structure and/or the water pressure was only modified by the vibration at the low stress level, suggesting that effect of the vibration on the penetration resistance was not dependent on the mean stress for the entire range of stresses.

The influence of the vibration on the penetration resistance was identified by a reduction in the measured penetration resistance. It was previously shown through cavity expansion theory (e.g. Salgado et al. 1997) that the penetration resistance is controlled by plastic deformation of the soil adjacent to the penetrometer and by elastic deformation of the soil away from the penetrometer. Also, the zone of influence of the cone penetrometer increases as either the effective stress or density of the soil increases. In order to reduce the penetration resistance through vibration, the pore water pressure in this zone of influence of the penetrometer must be elevated through *cyclic softening* so that the effective stress is reduced. This cyclic softening must occur adjacent to the cone penetrometer, at finite distances laterally and vertically away from the probe, and must be present during the shear-induced deformation associated with the penetration of the cone. In order to reduce the penetration resistance at the higher confining stresses or density, the total volume of soil influenced by the vibration must increase. Consequently, for soils at the elevated stress levels or densities, the zone of influence of the vibration must increase in order to get a comparable decrease in penetration resistance. This implies that the vibratory unit used in the testing must impart different energies into the soil for the different test conditions in order to reduce the penetration resistance to low values that represent the *flow liquefaction* conditions defined by Robertson (1994).

As noted in Section 5.9 of Chapter 5, dummy cone tests in the loose samples at the low stress level revealed a water pressure approximately equal to 88% of the confining stress in the soil at a radial distance of 0.35m away from a vibrating penetrometer. It therefore appears that the zone of influence of the vibration has a minimum radius of 0.35m for the given densities and stress level. The increase in water pressure and subsequent reduction in the effective stress within this zone of influence of vibration overlaps the shearing zone of influence associated with the cone penetrometer, which results in a significant vibration-induced reduction in the penetration resistance. Both of these vibratory penetration resistance values are dramatically less than that observed during the static tests, with the vibratory penetration resistance value in the medium dense sample slightly greater than that measured in the loose sample. Since both sets of tests reveal water pressures laterally away from the cone penetrometer that place the soil near *flow liquefaction* conditions, the observed vibratory penetration resistance may be an approximate measure of the strength of the soil at or near the liquefied state.

The penetration resistance measured in the loose samples at the intermediate and high stress levels was only reduced by 25% and 5% through vibration, respectively. This suggests that the excess pore pressure induced by vibration was not substantial enough to induce flow liquefaction conditions throughout the entire zone of influence of the penetrometer. Consequently, the effective stress in the soil was not dramatically reduced throughout the entire zone of influence of the cone, resulting in small to moderate reductions in the penetration resistance. Similar conclusions were made for the tests performed in the medium dense samples at the medium and high stress levels.

The intent of the investigation was to evaluate the effect of vibration on the penetration resistance and to evaluate the soil response to dynamic loading. It was also expected that the vibratory penetration resistance would provide some insight into the shear strength of the soil in a liquefied state. As shown in the previous discussion, the energy imparted into the soil by the vibrator was large enough to increase the pore water pressure within the zone of influence of the cone to substantial levels at the low stress level. This caused a large reduction in the effective stress in the soil, which ultimately resulted in a substantial reduction in the penetration resistance. As such, the energy generated by the vibrator approached and/or exceeded the energy dissipation capacity of the soil (NDE at liquefaction), which may have moved the soil towards a *flow liquefaction* condition. Therefore, the penetration resistance value measured at this point may be indicative of the strength of the soil at or near a liquefied state. Tests performed at the elevated effective stress levels, however, did not exhibit similar dramatic reductions in penetration resistance. This limited response to dynamic loading suggests that the vibration did not induce a pore pressure increase that was substantial enough to modify the shear strength values within the zone of influence of the cone penetrometer. Although the lateral and vertical extent of this vibration-induced water pressure was not determined for the elevated stress levels, it is assumed that the reduced penetration resistance measured at these stress states was not indicative of a liquefied soil response within the entire zone of influence of the penetrometer. The penetration resistance value, therefore, should not be considered a direct measure of the liquefied strength of the soil at these test conditions. Instead, the response of the soil to the vibratory loading is a direct measure of the soil density and stress condition, and can possibly be used together with a static sounding to better determine the in-situ soil condition. A complete discussion of this principle is presented in the following section.

6.3 Estimation of the Relative Density Using Vibratory Penetration Resistance

The representative curves presented in Figure 5.45 provide information related to the relationship between the RR, the effective stress, and the relative density of the soil. A plot of the RR as a function of the effective stress is presented as Figure 6.3 for the different density conditions. The values of the RR and effective stress at the center of the sample are used in the plot. It can be seen through this comparison that the highest RR value occurs at the lowest effective stress level. It also appears that the RR decreases as the effective stress increases for a given relative density and decreases as the relative density increases for a given effective stress.

An expression was established for each of the density conditions that relates the value of RR to the vertical effective stress in the soil. The expression is based on a polynomial of the form:

$$RR = C_1 + C_2 \cdot \sigma_v' + C_3 \cdot (\sigma_v')^2 + C_4 \cdot (\sigma_v')^3 \qquad (6.2)$$

where C_1, C_2, C_3, and C_4 are unknown constants and σ_v’ is the vertical effective stress in the sample. The expression was established for each relative density using the established RR vs. σ_v’ relationships, resulting in 3 equations and 4 unknowns. The fourth equation was generated by assuming the slope of the curve was zero at the high stress level, which implied that the derivative of Equation 6.2 was zero at the high stress level. Based on this information, 4 equations were generated, which allowed for the definition of the constants C_1 through C_4 by simultaneously solving the system of linear equations. The procedure was performed for both relative densities, which resulted in two separate best-fit curves that extended through the range of stress levels used in the testing.

The best-fit curves generated for both the loose and medium dense conditions are presented as Figure 6.3. RR data obtained from penetration tests at the three target stress levels (σ_V' ~ 6.8 kPa, 55 kPa, and 110 kPa) are also included in the figure. As noted in

Section 5.4.1, a small variation in the testing stress level was present during each of the tests because of the procedure associated with the stress application using the air piston. The RR data obtained from three sets of penetration tests outside the target stress level are also superimposed on this figure. As shown through the comparison, the penetration test data measured in tests outside of the target stress levels agree fairly well with the empirical approximation generated using Equation 6.2.

The expression presented as Equation 6.2 includes the independent variable of vertical effective stress and the constants C_1 through C_4. The RR value for a given effective stress varies for the two different densities because the values of the constants are different. An empirical expression was generated using the information from the loose and medium dense curves to generate an overall expression for RR that included the independent variables of relative density and vertical effective stress. The procedure used to determine this expression involved plotting a given constant value determined from the parabolic curve fit versus the relative density (i.e. C_1 vs D_r). A linear regression line was used to fit the data. The slope and ordinate of this line were then substituted back into the RR expression for the constant, resulting in an expression for RR that included new constants, relative density, and vertical effective stress. Using this principle, a single empirical expression was generated that allowed for the computation of family of curves for the different density and stress conditions:

$$RR = [C_1 \cdot D_r][C_2 + \sigma_v']^2[C_3 + \sigma_v'] + C_4[C_5 + \sigma_v'] \cdot [C_6 + (C_7 + \sigma_v') \cdot \sigma_v'] \qquad (6.3)$$

where D_r is the relative density in units of percent, σ_v' is the vertical effective stress, and C_1 through C_7 are the empirical constants determined from the regression analysis. The empirical constants used to determine the family of curves are included in Figure 6.4.

The empirical family of curves generated using Equation 6.3 were compared to several data points obtained from the penetration testing in samples outside of the target range of densities identified in Chapter 5. The data points used in the figure are the actual data points obtained from the testing and should be considered independent of the normalized values associated with the target stress levels and densities. As shown through the comparison, the few data points available from the testing seem to agree well with the values estimated from the curves, suggesting that it may be possible to use a direct comparison of the static and vibratory penetration resistance to estimate the relative density of the soil. However, only a few data points outside of the target relative densities have verified the technique, which implies that further calibration chamber testing is needed to quantify the validity of the proposed relationships.

6.4 Effects of Frequency on the Penetration Resistance

Vibratory penetration tests were performed in the calibration chamber using the rotating mass vibrator to evaluate the effects of the frequency of vibration on the measured penetration resistance. The investigation was performed on samples prepared to loose and medium dense states and intermediate stress levels. Frequencies of 15, 25, 35, 45, and 55 Hz were used in the testing. The dynamic force measured at the tip location was adjusted for each of the tests to reach a fairly constant value of 1.3 kN, which was approximately the same as the force generated by the rotary turbine vibrator. A summary of the force and frequency combinations for each of the tests using the counter rotating mass vibrator was presented in Table 3.5 in Chapter 3.

The vibration used in the investigation was generated from a pair of unbalanced rotating masses illustrated in Figure 6.5. The phase relationship associated with the rotation is such that the masses reach their top and bottom positions at the same time so that the addition of the vector components results in the cancellation of the horizontal component of motion. The dynamic analysis of the system presented as Figure 6.5 leads to a force of $Q = 2me\omega^2 \sin(\omega t)$, which when introduced into Newton's Second Law of Motion yields the following equilibrium expression:

$$\sum F_y = m\ddot{x} = 2me\omega^2 \sin(\omega t) - kx - c\dot{x} \qquad (6.4)$$

where m is the mass, ω is the circular frequency of motion, k is the stiffness, c is the damping coefficient, and x is the displacement. As indicated by Equation 6.4, the force generated by the rotating mass has an amplitude proportional to the square of the frequency of oscillation. Using $\omega_n^2 = k/m$ as the natural frequency of the system and $D = c/c_c$, where c_c is the critical damping coefficient, Equation 6.4 is eventually reduced to:

$$M = \frac{A}{me} = \frac{(\omega/\omega_n)^2}{\left[1 - (\omega/\omega_n)^2 + (2D\omega/\omega_n)^2\right]^{\frac{1}{2}}} \quad \textit{and} \quad \tan\phi = \frac{2D\,\omega/\omega_n}{\left[1 - (\omega/\omega_n)^2\right]} \qquad (6.5)$$

The values of M and ϕ presented in Equation 6.5 represent the dynamic magnification factor and the phase angle between the force and the displacement during the vibration. Relationships of M and ϕ as a function of the applied versus natural frequency ratio are expressed graphically in Figure 6.6. As shown through this comparison, the magnitude of the amplified excitation is the largest where the applied versus natural frequency ratio (ω/ω_n) approaches a value of 1.0, which indicates a resonance condition.

The characteristic curves presented in Figure 6.6 also reveal magnification factors at ω/ω_n ratios above 1.0 that are greater than those below 1.0 for a defined distance away from ω/ω_n = 1.0. This suggests that a higher amplification of the motion should be observed for frequencies above the natural frequency of the material than for below, provided that the difference between the induced motion and the natural frequency of motion is the same in both cases. Evidence of this behavior was noted in the penetration testing, where a larger reduction in penetration resistance was noted in tests at a 55Hz frequency versus 15 and 25 Hz frequencies.

An estimation of the natural frequency of the soil sample was generated using an expression developed from the wave equation for the motion of uniform soil on rock:

$$f = \frac{(2n-1)v_s}{4H} \qquad n = 1,2...,\infty \tag{6.6}$$

where f is the fundamental frequency, n is the mode of vibration, v_s is the shear wave velocity, and H is the thickness of the soil layer. The value of the shear wave velocity of the soil deposit used in this equation was estimated using an empirical expression developed by Hardin and Richart (1963), which takes into account the void ratio and effective confining stress in the soil:

$$v_s = [170-(78.2)e]\cdot(\sigma_m')^{0.25} \tag{6.7}$$

where v_s is the shear wave velocity in ft/sec, e is the void ratio, and σ_m' is the mean effective stress in the soil in lb/ft^2.

The shear wave velocities using Equation 6.7 were determined to be about 184m/sec for the loose (i.e. Dr = 25%) and 195m/s for the medium dense (i.e. Dr = 55%) soils, which correspond to first fundamental frequencies of 31Hz and 33Hz, respectively, using Equation 6.6. Similar resonance frequency values were presented by Terzaghi et al. (1997) for both loose and dense fills. It should be noted that Equation 6.6 was developed based on a zero stress vertical boundary and a soil mass of infinite lateral extent. Since these boundary conditions are obviously different from those present in the calibration chamber, the estimation of the natural frequency of the calibration chamber sample using Equation 6.6 should be considered very approximate.

Included in Figure 6.7a are a series of vibratory penetration tests in loose saturated sand at the intermediate stress level. As shown through the comparison, the vibratory penetration resistance is highly dependent on the frequency of vibration. Vibratory penetration tests performed at frequencies of 15 and 25 Hz did not reveal penetration resistance values that were significantly different from those obtained from static penetration. Tests performed at vibration frequencies between 35 and 45 Hz, however, revealed penetration resistance values that were significantly lower than both the values obtained from static penetration and the vibratory penetration resistance value measured using the rotary turbine vibrator (Figure 6.7a and Figure 6.8). Similar results were noted for tests performed in the medium dense samples (Figure 6.7b).

The frequency of vibration associated with the largest reduction in penetration resistance was approximately equal to the natural frequency of the soil mass as determined through Equation 6.6. The ω/ω_n ratio at this frequency was approximately 1.0, and as noted through Figure 6.6, generated a large amplified signal and a phase lag of 180^o between the force and displacement. Evidence of this phase lag was noted in the force/displacement diagrams included in Chapter 3. Penetration tests performed with frequencies above the natural frequency of the soil (i.e. 125 Hz frequency using the rotary turbine vibrator) generated a larger reduction in the penetration resistance than those at the lower frequencies (i.e. 25 Hz). This behavior agrees with the amplification relationship presented in Figure 6.6, which suggests that the amplitude of the magnification factor is greater at the higher frequencies of motion. Similarly, the vibratory penetration resistance measured at the 55 Hz and 125Hz frequencies were approximately equal, which agrees with the relationship presented in Figure 6.6 that suggests that the ω/ω_n values are fairly constant above a value of about 2.0.

The above analysis suggests that the vibratory penetration resistance is directly dependent on the frequency of vibration for a given dynamic force. The largest reduction in penetration resistance was observed when the frequency of vibration approached the natural frequency of the soil deposit. As stated in Section 6.2, the magnitude of this reduction in penetration resistance is directly related to the ratio of the zone of influence of the vibration to the shearing zone of influence of the cone penetrometer. As this ratio approaches unity, the potential for elevated water pressure, reduced effective stress, and large strength loss increases.

6.5 Proposed Approach for Estimating the Energy Efficiency of the Vibratory Unit

6.5.1 Normalized Input Energy

The dynamic energy imparted into the soil was estimated by considering the work done during the vibration. Kovacs and Salomone (1982) and Yokel (1982) performed similar analyses during their investigation on the energy imparted into the soil during SPT testing, which resulted in the following equation:

$$E_r = \frac{c}{A \cdot E} \cdot CF \cdot \int_0^{\Delta t} \left[F(t) \right]^2 dt \tag{6.8}$$

where E_r is the energy generated by the compression wave, $F(t)$ is the force generated by the stress wave, CF is a correction factor to account for rod length effects and other geometrical parameters, c is the wave velocity, A is the cross sectional area of the rod, E is Young's Modulus, and Δt is the period of the compression portion of the wave. Equation 6.8 was derived by substituting the equation for work/energy into the 1-D wave equation presented as Equation 3.4 and then rearranging the relationships until an expression based on force and the properties of the rod are reached. The equation for work/energy is expressed by Equation 6.9:

$$E_r = \sum_{1cycle} F(t) \cdot u(t) \tag{6.9}$$

where E_r is the energy generated by the compression wave, $F(t)$ is the force generated by the wave, and $u(t)$ is the particle displacement.

The use of Equation 6.8 to estimate the energy imparted into the soil through the vibration involves the computation of material properties of the bar (c, E, and A). As noted in Chapter 3, the estimation of the force generated by the vibration using predefined material properties was approximately equal to that measured by the load cell and accelerometer. It should be noted, however, that this agreement between the two approaches was for wave propagation through a 1m section of rod. As the number of sections of rod increases, the wave propagation properties will most likely change, suggesting that the relationship between *c, E,* and *A* is dependent on the overall rod length and the number of rod connections. This suggests that the force imparted into the soil at the cone tip is a function of depth of penetration, the tightness of the rod connections, and the properties of the soil deposit, and further emphasizes the importance of direct measurement of the vibration at the penetrometer location. As such, Equation 6.9 was used to determine the actual energy inputted into the soil per cycle of vibration directly.

The input energy per cycle of vibration calculated through Equation 6.9 is not dependent on the confining stress present in the soil and the boundary conditions of the system. As noted through wave propagation theory, the system moves from a fixed-free boundary condition towards a fixed-fixed boundary condition during penetration, resulting in a change in the wave propagation abilities and the corresponding displacement and stress conditions at the end of the bar. This suggests that the boundary conditions are changing during the penetration test, which makes the particle displacement in the bar difficult to estimate using the theory. The importance of measuring the displacement at the boundary condition is therefore further emphasized. As the confining stress in the soil increases, the stiffness of the soil increases and the particle displacement at this lower boundary decreases. This suggests that energy imparted into the soil is dependent on the confining stress, which was recognized in the SPT correlation developed by Kovacs and Yokel (1982) through empirical correction factors (*CF*) (Equation 6.8). These correction factors were based on an impact-type motion and did not consider the full energy imparted into the soil by the propagating wave. To account for the influence of the confining pressure on the sinusoidal motion considered as part of this study, the input energy determined through Equation 6.9 was normalized by the effective confining stress at the vertical center of the sample:

$$NIE = \frac{\sum_{1cycle} F(t) \cdot u(t)}{\sigma'_{conf}} \tag{6.10}$$

where σ'_{conf} is the confining pressure in the soil. Similar normalization procedures were used during dissipated energy estimations from cyclic triaxial tests in the laboratory (e.g.

Simcock et al. 1983; Figueroa et al. 1994). The input energy value determined through Equation 6.10 is referred to as the normalized input energy per cycle (NIE) and has the units of volume (kN-m/kN/m^2 = m^3).

The NIE was reduced to a dimensionless quantity by dividing the NIE by the volume of soil exposed to the vibration. Given the nature of the testing program and the absence of measurement devices placed in the soil itself, the volume of soil exposed to the vibration is difficult to ascertain. In order to adequately define the dimensions of this vibratory zone of influence, accelerometers would have to be placed in numerous locations both vertically and laterally away from the cone penetrometer. A discussion of the estimate of the volume of soil used in the analysis is presented in Section 6.5.2

The value of the NIE imparted into the soil through a defined number of cycles of vibration was compared to the NDE at the point of where large strains were initiated. This comparison allowed for an evaluation of vibratory unit to determine whether or not it had the ability to generate a dynamic load that was substantial enough to influence the penetration resistance. A complete discussion of this NIE vs. NDE comparison is included in Section 6.5.3.

6.5.2 Zones of Influence of Cone Penetrometer and Vibratory Loading

As stated in Section 6.2, the zone of influence of the vibration was probably not the same for the different stress and density conditions. Given the minimal influence of the vibration on the penetration resistance at the elevated stress levels, it was likely that the vibration did not induce significant pore water pressures throughout the entire zone of influence of the cone penetrometer. In order to properly estimate the influence of the vibration on the penetration resistance, independent approximations of the zones of influence of both the cone penetrometer and the vibration were needed. The zone of influence of the cone penetrometer is defined as the lateral and vertical extent of the shear induced deformation associated with the penetration of the cone penetrometer. For the purpose of this investigation, the zone of influence of the vibration is considered as the lateral and vertical extent of the soil deformation associated with the input vibration.

6.5.2.1 Zone of Influence of Cone Penetrometer

An approximation of the zone of influence of the penetrometer was made using the penetration test results presented in Chapter 5. It was shown through a detailed analysis that the penetration resistance measurements in loose samples were similar for the center and off center testing locations. This suggests that the zone of influence of the cone penetrometer had a maximum radius of 0.35m, which was the distance from the off center testing location to both the cone penetrometer and the sample boundary. Tests performed in medium dense samples revealed a significant influence of the lateral boundary in tests performed at the off center location, which suggests that the radius of the zone of influence of the penetrometer was at least 0.35m.

An estimation of the plastic zone of deformation within the radius of influence of the cone penetrometer was estimated through cavity expansion theory using the computer program CONPOINT (Salgado et al. 1997). For loose specimens, the width of the zone of elastic deformation was estimated by subtracting the plastic radius of deformation obtained from CONPOINT from the distance between the two testing locations (0.35m). An average radius of the plastic zone of deformation for tests at the elevated stress levels was estimated at 0.17m and 0.20m for the loose and medium dense samples, respectively. This suggests that the elastic zone of deformation was about 0.18m for tests in the loose samples at the elevated stress levels. An estimation of the lateral extent of the zone of influence of the cone penetrometer and the subsequent width of the elastic zone of deformation was not accurately made for tests in medium dense samples because the zone of influence of the penetrometer was greater than 0.35m.

6.5.2.2 Zone of Influence of Vibration

As discussed in Chapter 5, substantial excess pore water pressures were measured in loose samples at the low stress levels at the off center testing location when a vibrating steel probe was penetrated at the center port. This water pressure rise was noted immediately upon entry of the probe into the sample, and assuming symmetry, revealed a minimum zone of influence of the vibration of the dimensions r = 0.35m and z = 0.75m. Given the relatively high values of the pore pressure ratio at this location ($\Delta u/\sigma_h' \sim 0.88$) and the distance from this point to the sample boundaries, it is assumed that the pore pressure ratio did not drop off dramatically beyond this point and that the zone of influence of the vibration extended to the lateral and vertical boundaries of the sample. It is therefore suggested that the zone of influence of the vibration overlapped the full volume of the soil sample for the low stress level. Thus, a volume of 2.65m^3 was used to reduce Equation 6.10 to a dimensionless quantity for the low stress conditions.

Nogami et al. (1997) performed a detailed study of the soil response during vibratory pile driving. They developed a set of pore pressure ratio ($\Delta u/\sigma_v'$) relationships for distance-to-pile diameter ratios (r/r_o) away from the vibrating unit. Based on these relationships, the vibratory unit used for this study induced a pore pressure ratio of approximately 1.0 adjacent to the vibrating cone, 0.2 at a lateral distance of 0.20m, and 0.05 at a distance of 0.25m. Thus, the bulk of the excess pore water pressure induced by vibration is contained within a 0.2m radius from the vibratory unit. As noted in Section 6.5.2.1, the soil deformation away from a penetrating cone penetrometer includes a 0.2m radius of plastic deformation and an elastic zone of deformation of a maximum thickness of 0.15m adjacent to the plastic zone. This suggests that the zone of influence of the vibration for tests at the elevated stress levels coincides with the plastic zone of deformation and does not involve significant portions of the elastic zone.

The investigation performed by Nogami et al. (1997) also revealed that the magnitude of the pore pressure generated laterally away from the vibrating probe is dependent on the magnitude of the shear strain at the soil/probe interface. They noted that the magnitude of this shear strain increased as the frequency of vibration increased, resulting in a smaller zone of influence of vibration for the higher frequencies of motion. As noted through the reduced sleeve friction values identified during vibratory penetration tests (versus static), a significant amount of soil deformation is induced at the soil/cone interface during the 125 Hz vibration. The presence of this soil deformation, along with the larger zones of plastic deformation of the cone penetrometer at the elevated stress level, may have prevented the vibration from propagating into the elastic zone of deformation of the cone penetrometer. Since the pore water pressure was not increased and the effective stress decreased from the vibration in this elastic zone, the penetration resistance did not deviate from that measured during static penetration. Based on these considerations, it was assumed that the zone of influence of vibration coincides with the plastic zone of deformation of the cone penetrometer. A cylindrical volume of $0.26m^3$ was therefore used to reduce Equation 6.10 to a dimensionless quantity for both densities at the elevated stress levels.

6.5.3 Relationship Between NDE and NIE

Presented as Figure 6.9a is the normalized dissipation energy (NDE) at the point of liquefaction, as determined through Equation 3.1, compared to the number of cycles to failure in the soil (N_f). The relationships for both the loose and medium dense conditions are included in this figure.

The NIE values per cycle of vibration generated from Equation 6.10 were determined to be 3.148E-05, 2.231E-05, and 1.116E-05 for the low, intermediate, and high stress levels, respectively. The total NIE imparted into the soil at any point during the penetration test can be determined by multiplying the NIE per cycle by the number of cycles of vibration. An example of this relationship is presented as Figure 6.9b for each of the stress levels.

The relationships presented in Figures 6.8 can be compared to evaluate the number of cycles needed to cause a significant pore pressure rise and subsequent strength loss in the soil. This phenomenon is suggested to occur once the input energy equals the energy capacity of the soil. An example showing this comparison is presented as Figure 6.10 for a loose soil at the intermediate stress level. According to the proposed relationship, the NIE generated through the vibration approaches the capacity of the soil after about 200 cycles. The exposure of the sample to cycles below this limit results in a moderate pore pressure rise because the capacity of the soil is greater than the NIE, where the reverse is true for points along the NIE curve above the NDE line.

The plots presented in Figure 6.10 have been superimposed over one another to show a direct comparison of NDE and NIE for soils at the different density and stress conditions (Figure 6.11). Also included in this figure are the number of cycles applied to the soil prior to the reduction in penetration resistance for 5 penetration tests where the vibration was started midway through the test. The number of cycles used for each of the tests were approximated by considering the vibration frequency and the time differential between the start of the vibration and the point where the penetration resistance began its reduction to the residual value. As shown through the comparison, the number of cycles generated by the unit agree well with the values approximated using the NIE curves, especially for the tests conducted at the low stress levels. The results from one of the tests in a medium dense sample seem to reveal an actual input energy that was well beyond that estimated by the intersection of the NDE and NIE curves. The added cycles associated with the NIE in this test are most likely attributed to testing errors induced by a faulty air line connection between the air compressor and the vibratory unit. This restricted the airflow into the vibrator and limited the force generated through the rotation of the mass. Consequently, a number of cycles of vibration were applied to the soil that had a lower NIE than the NIE generated per cycle during full operational mode of the vibrator. A test performed in a loose sample at the medium stress level falls slightly below the NDE curve, suggesting that the amount of energy imparted into the soil through the vibration was insufficient to cause large strains and significant strength loss in the soil. This absence of these phenomena is further identified by the 25% RR value for the test condition, which suggests that the penetration resistance was only moderately influenced by the vibration.

The comparison of the calibration test data to the estimation of pore pressure increase and strength loss using the NIE and NDE relationship seems to reveal a general agreement. In order to adequately use this approach for liquefaction evaluation purposes, the following issues must be addressed:

a) The cyclic triaxial testing revealed that the NDE at liquefaction was dependent on the CSR and the number of cycles to liquefaction. The relationship between NDE at liquefaction and the corresponding number of cycles was developed based on a limited number of tests at relatively high CSR's and low numbers of cycles to failure (Figure 6.9a). The magnitude of the NDE in undrained conditions at very low CSR values should be determined to verify the number of cycles extrapolated through the proposed curves beyond the actual data points.

b) The volume of soil used to compute the NIE per cycle of vibration at the low stress level was based on a limited number of dummy cone tests at the low stress levels. A proper identification of the actual zone of influence of the vibration should be made by placing pore pressures and accelerometers in the soil mass both laterally and vertically away from the penetrating penetrometer.

c) The volume of soil used to compute the dimensionless NIE at the elevated stress levels was based on estimations of the zone of influence of the cone penetrometer using cavity expansion theory and estimations of the vibration induced excess pore pressure field using relationships based on vibratory pile driving. As noted in (b) above, this zone of influence should be considered only approximate and should be properly quantified through instrumentation in the calibration chamber.

d) The influence of the level of stress in the soil mass on the input energy was addressed by normalizing the input energy by the effective confining stress. Since the particle displacement at the cone tip is a function of the stiffness of the soil, the wave propagation abilities of the rod, and the boundary conditions, it is imperative that a good measurement of the force and particle displacement be made to determine the energy imparted into the soil. A system comprised of both an accelerometer and a dynamic load cell at the tip would serve such a purpose.

e) The NIE and NDE relationship is based on an input motion of a defined force and frequency. The relationship should be considered unique to the rotary turbine vibratory unit and may not be applicable for other input motions.

f) The NIE and NDE relationship is based on a comparison of the input energy to the soil behavior during cyclic loading in the laboratory. The effect of resonance was not addressed in the relationship.

6.6 Summary and Conclusions

The information contained in the previous sections provides an evaluation of the effects of vibration on the penetration resistance and an analysis of the behavior of the soil under vibratory load. The following summarizes the results of the investigation:

a) Comparisons of the penetration resistance values measured during vibratory loading suggests that the penetration resistance is a function of both the magnitude and frequency of the dynamic load.

b) The RR values determined from the penetration tests indicate that the effects of the vibration on the penetration resistance substantially decrease as the mean effective stress in the soil increases.

c) The zone of influence of both the cone penetrometer and the vibration is dependent on the mean effective stress and relative density of the soil.

d) The increase in the zone of influence of the *cone* at higher relative densities and effective stresses suggests that a vibratory unit capable of generating a larger dynamic load would be needed to reduce the penetration resistance to values comparable to the lower stress levels.

e) The vibration seems to dramatically influence the penetration resistance at the low stress level because the zone of influence of the vibration overlaps the shearing zone of influence of the cone penetrometer. The pore water pressure is significantly increased in this zone, resulting in a reduction in effective stress and corresponding penetration resistance.

f) The vibration does not substantially decrease the penetration resistance at the elevated stress levels because the pore water pressure was most likely not elevated to high levels throughout the entire zone of influence of the cone penetometer. The extent of the effects of vibration and the pore water pressure was not measured away from the penetrometer due to difficulties in incorporating sensors into the calibration chamber.

g) Trendlines were generated to relate the RR to the vertical effective stress in the soil for both the loose and medium dense soils. Points that fall outside the target stress levels seem to agree well with the proposed relationship.

h) A family of curves was generated through a regression analysis that relates RR to vertical effective stress for different density soils. Several points outside of the target densities seem to agree with the proposed values. It may therefore be possible to use the proposed relationships to estimate the relative density of the soil using comparisons of the static and vibratory penetration resistance while at the same time obtaining a measurement of the collapse potential of the soil for a given input energy. Quantification of this proposed relationship is needed through additional testing at densities and stresses outside the target values.

i) An approximation of the first fundamental frequency of the soil deposit averaged 32 Hz for the two soil densities. The largest reduction in penetration resistance occurred at frequencies near this value, suggesting that resonance conditions possibly existed between the input motion and the soil.

j) A NIE based approach was developed to evaluate the energy imparted into the soil by the vibrator. The approach is similar to those developed for evaluating the energy imparted into the soil during SPT testing (e.g. Kovacs and Salomone 1982; Yokel 1982).

k) The NIE per cycle of vibration includes a volume of soil associated with elevated pore water pressures. This volume of soil was approximated at the low stress levels due to the measurement of pore water pressure away from the cone penetrometer. At the elevated stress levels, however, the pore water pressure was not measured away from the cone penetrometer, resulting in the assumption of the volume of soil influenced by the vibration.

l) The NIE per cycle of vibration imparted into the soil during the penetration of the cone is a function of the penetration rate of the cone penetrometer.

m) Test results indicate that NIE introduced into the soil mass prior to the reduction in penetration resistance agrees well with the NDE at the point of failure in laboratory samples.

n) The NIE and NDE relationship generated through the study is unique for a given vibratory input motion. Individual relationships should be established for motions outside of that generated by the vibratory unit used in the testing.

o) The NIE and NDE relationship involves a comparison of the input energy and the response of the soil to dynamic loading based on cyclic triaxial testing. The relationship ignores the effects of resonance and the harmonics of the system during dynamic loading.