

# Chapter 5

## Semiactive Adaptive Controller Theory

The primary purpose of this chapter is to present the theories leading to the development of the semiactive adaptive controller proposed here. The adaptive algorithm developed in this chapter is applied to design adaptive control for one class of nonlinear vibration systems, such as semiactive base-excited vibration isolation systems. The algorithm includes the on-line system identification and the adaptation of the control signal. Finally, the stability of the semiactive adaptive system is presented.

### 5.1 Rationale

As discussed in Section 2.2, the transmissibility ratio (TR) of the passive vibration isolators can be parameterized per the damping coefficient and therefore the damping tuning technique can be applied to obtain good isolation in a wide frequency range. If a semiactive damper is used, the damping coefficient can be changed to alter the dynamic system performance as desired.

As demonstrated in Chapters 2 and 4, the semiactive skyhook control is one method of adjusting damping for better vibration isolation. Even though we have developed two approaches to avoid the force discontinuities associated with skyhook control, there still exist higher harmonics, which can adversely affect the system performance. Another potential problem is that the gain of the skyhook has to be adjusted by trial and error in order to achieve a desirable performance. The skyhook control and its variations cannot easily achieve a compromise between minimizing the suspended body acceleration and the suspension travel. These practical problems motivate us to explore new methods for damping tuning with an adaptive control approach.

After reviewing some other adaptive control approaches, we will present the development of an adaptive control technique for the class of nonlinear systems that represent semiactive suspensions with non-stationary random input.

## 5.2 Adaptive Control Approaches

Some of the common adaptive control approaches are model reference adaptive control, LMS adaptive filter, self-tuning control, pole-placement adaptive control, gain scheduling, sliding mode control, and Lyapunov design. Their common element among these approaches is that they are able to achieve the adaptation of control signals based on the sensitivity of a specific performance index for the closed loop system. These methods cannot be directly applied to the semiactive vibration problems considered in this dissertation, due to the nonlinearity and un-measurable non-stationary excitation.

As mentioned in Chapter 1, indirect adaptive control refers to the adaptive algorithm that gives out the adaptive control signal based on the system identification of the plant model parameters. In contrast, for a direct adaptive control approach the plant parameters are incorporated into the controller parameters. Therefore, the estimation of control parameters implicitly represents the system identification of plant parameters. It may be difficult to apply such an adaptive approach into complicated nonlinear systems.

From the implementation point of view, the computational burden of on-line system identification and adaptation for DSP or microprocessors depends on the plant model. For example, if the dynamic system is nonlinear, Volterra series model, Wiener series model, Hermitian model, or their variations may be used for adaptive control systems design [7]. Their common difficulty, however, is that a long series of terms is required as FIR and IIR filters, which are the basis of LMS adaptive filters, even in order to capture the approximate dynamic responses of simple systems. Therefore, in order to reduce the computation, it is strongly recommended to use simple equations to model dynamic systems, and trade off between model simplification and dynamic performance.

Thus, in all, the proposed adaptive algorithm is a kind of indirect feedback approach for nonlinear non-stationary vibration systems. In the following section, a base-excited system with application of the magneto-rheological damper is used to represent one class

of nonlinear systems that can be described by continuous and differential modeling equations. Based on the system description, we will fully present the adaptive control algorithm.

### 5.3 System Formulation

The mechanical systems that are considered for this study can be described by the general second-order differential equation

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{f}_d(\dot{\mathbf{y}}, I) + \mathbf{f}_{sp}(\mathbf{y}) = \mathbf{u} \quad (5.1)$$

where  $\mathbf{M}$  is an  $n \times n$  mass matrix,  $I$  an input command,  $\mathbf{f}_d$  and  $\mathbf{f}_{sp}$  represent the damping and stiffness forces, respectively. The over dot “•” represents differentiation with respect to time. As such, the vectors  $\mathbf{y}$ ,  $\dot{\mathbf{y}}$ , and  $\ddot{\mathbf{y}}$  indicate the displacement, velocity, and acceleration vectors. The vector  $\mathbf{u}$  stands for un-measurable vibration sources, which can be periodic, stationary or non-stationary random.

As will soon become evident, the semiactive adaptive control results that we have developed are for base-excited systems such as those in Fig. 5.1. As such we will consider a one-degree-of-freedom form of Eq. (5.1), as given by

$$M\ddot{y}_1 + F_{mr}(\dot{y}_1, \dot{y}_2, I) + F_{sp}(y_1, y_2) = 0 \quad (5.2)$$

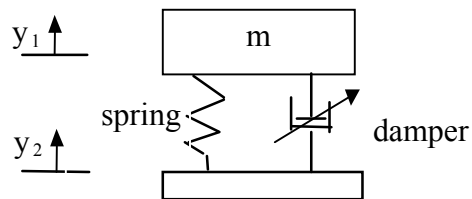


Figure 5.1 Base-Excited Systems Representing a Nonlinear Seat Suspension

The variables in Eq. (5.2) are shown in Fig. 5.1. We will further assume for the purpose of this study that the spring force is linear and can be expressed by

$$F_{sp} = K_{sp} (y_1 - y_2) \quad (5.3)$$

where  $K_{sp}$  is the spring constant and  $y_1 - y_2$  is the relative displacement across the spring. It is worth noting, however, that the adaptive control formulation that will be described next is not necessarily confined to linear systems or isolation systems with linear springs. The damping force  $F_{mr}$  is nonlinear, as expressed in details earlier in Eqs. (3.1)-(3.5).

#### **5.4 Semiactive Adaptive Controller Formulation**

The developed algorithm is composed of two parts: on-line system identification and adaptive controller. Both will be shown shortly.

##### **5.4.1 System Identification**

The purpose of system identification is to provide sufficient information for an adaptive algorithm to maintain an optimal control. Due to wear and tear, the mechanical components such as spring can vary with their characteristics in the long run. Considering the vehicle suspensions, the payload also often changes. Thus the resonant frequency of the vibration systems may not be constant. As is well known, the vibration responses are dependent on the relative distribution of excitation frequencies with respect to the resonant frequency. Obviously, it is important to estimate the parameters which are directly related to the resonant frequency in real time. Thus, for the proposed adaptive algorithm, it is necessary to identify the unknown parameters of the system on-line.

In order to apply the recursive least square (RLS) method to do system identification, Eqs. (5.2) and (5.3) are rewritten as

$$[-F_{mr}] = \varphi^T \theta \quad (5.4)$$

where  $F_{mr}$  is calculated by using a high fidelity MR damper model, which for example has been developed in Chapter 3 as shown in Eqs. (3.3)-(3.7), or measured through force sensors such as load cells. The vector  $\varphi$  is defined as

$$\varphi = [\ddot{y}_1(k) \ y_{12}(k)]^T$$

where  $\ddot{y}_1(k)$  is the measured acceleration and

$$y_{12}(k) = y_1(k) - y_2(k)$$

represents the measured relative displacement. Further, the vector

$$\theta = [M \ K]^T$$

includes unknown mass (M) and stiffness (K) of the vibration isolator, respectively.

The recursive least-square algorithm for the parameter estimation can be written as [7]

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \gamma(k)[(-F_{mr}) - \varphi^T \theta] \quad (5.5)$$

$$\gamma(k) = P(k)\varphi[\lambda + \varphi^T P(k)\varphi]^{-1} \quad (5.6)$$

$$P(k+1) = \{P(k) - P(k)\varphi[1 + \varphi^T P(k)\varphi]^{-1} \varphi^T P(k)\} \frac{1}{\lambda} \quad (5.7)$$

where  $\hat{\theta}$  is the estimated vector,  $\gamma$  is the update gain vector,  $\lambda$  is a weighting factor with a recommended range of [0.90, 0.995], and P is a weighting matrix.

The above algorithm is effective for system identification with ideal measurement signals. If there is noise in the measurement signals, the estimation accuracy can deteriorate. Introducing a weighting factor  $\lambda$  can improve the estimation accuracy for the

above algorithm, but at the expense of reducing the convergence speed [7]. In the practical system implementation, electrical noise and signal conditioning errors can cause significant errors in the adaptive algorithm. Therefore, an appropriate filter must be used for the system identification in order to improve the signal-to-noise ratio while maintaining the convergence speed of the estimation algorithm. The idea can be expressed in the frequency domain as

$$[-G(s)F_{mr}] = [G(s)\phi^T]\theta \quad (5.8)$$

where  $G(s)$  is a filter used to extract the most useful frequency components for the estimation algorithm. According to Eq. (5.4), the equality will not be changed if both sides are multiplied by an equal term, either a constant or transfer function. In the dynamic system, the signals used for system identification may be exposed to noises outside of the interested working frequency range. Those noises can cause estimation errors if they are not filtered out properly. Therefore,  $G(s)$  is designed to obtain the signals in the working frequency range and keep  $\lambda$  as large as possible to maintain a quick convergence speed for system identification.

#### 5.4.2 Adaptive Control

The configuration of the proposed nonlinear adaptive control is shown in Fig. 5.2. With application of magneto-rheological (MR) dampers, the damping level can be varied by adjusting the current  $I$  to MR dampers. The current  $I$  can be updated by minimizing the summation of the squared errors as the model parameters can be adjusted in the system identification.

The system identification can track the variations of mass and stiffness in the vibration control system. Through adaptation, the optimal control is maintained to minimize the desired cost function. For the base-excited system shown in Fig. 5.1, we can choose to minimize the acceleration of the mass and the relative displacement according to the performance index



If the mass and stiffness of this system are known (i.e., M and K are known constants in real time), then Eqs. (5.2) and (5.3) can be used to solve for the sensitivity which is used to calculate the adaptive update gradient. The derivative of Eq. (5.2) with respect to the current I is

$$M \frac{\partial \ddot{y}_1}{\partial I} + \frac{\partial}{\partial I} F_{mr}(\dot{y}_{12}, I) + K \frac{\partial}{\partial I} y_{12} = 0 \quad (5.12)$$

where  $\dot{y}_{12}$  can be obtained by differentiating  $y_{12}$  through either analog circuit or digital

filter. The gradient  $\frac{\partial}{\partial I} F_{mr}(\dot{y}_{12}, I)$  can be derived by using Eqs. (5.13)-(5.15),

corresponding to the MR damper model of Eqs. (3.3) to (3.7), as

$$\frac{\partial}{\partial I} F_s = A_{mr}(I) \frac{\partial}{\partial I} S_b(V) + S_b(V) \frac{\partial}{\partial I} A_{mr}(I) \quad (5.13)$$

$$\left\{ \begin{array}{l} \frac{\partial \dot{x}}{\partial I} = -(h_1 + 2h_2 I)x - (h_0 + h_1 I + h_2 I^2) \frac{\partial x}{\partial I} + \frac{\partial}{\partial I} (h_3 F_s) \\ \frac{\partial F_h}{\partial I} = (h_1 + 2h_2 I)x + (h_0 + h_1 I + h_2 I^2) \frac{\partial x}{\partial I} + \frac{\partial}{\partial I} (h_4 F_s) \end{array} \right. \quad (5.14)$$

$$\frac{\partial}{\partial I} F_{mr} = \frac{\partial}{\partial I} F_h + \frac{\partial}{\partial I} F_{bias} \quad (5.15)$$

The complete adaptive algorithm consists of Eqs. (5.5) to (5.7), which can identify the unknown parameters on-line, and Eqs. (5.10) to (5.15), which provide the adaptive control signal in real time.

It is worth noting that in our approach, the hysteretic nonlinearity is dealt with without using any linearization technique. Other nonlinearities such as backlash and dead zone can also be dealt with in a similar approach.



The adaptive control described above can be generalized for nonlinear controllers such as the skyhook control policy and its variations. For example, the gain  $K_s$  of the nonlinear control law

$$I = K_s |\dot{y}_1(k)| \quad (5.16)$$

which is developed from the skyhook control, can be adapted according to the following equations

$$K_s(k+1) = K_s(k) + \mu \left[ -\frac{\partial J}{\partial K_s(k)} \right] \quad (5.17)$$

$$\frac{\partial J}{\partial K_s(k)} = 2p\ddot{y}_1(k) \frac{\partial \dot{y}_1(k)}{\partial K_s(k)} + 2qy_{12}(k) \frac{\partial y_{12}(k)}{\partial K_s(k)} \quad (5.18)$$

$$\frac{\partial \dot{y}_1(k)}{\partial K_s} = \frac{\partial \dot{y}_1(k)}{\partial I(k)} \frac{\partial I(k)}{\partial K_s} \quad \text{and} \quad \frac{\partial y_{12}(k)}{\partial K_s} = \frac{\partial y_{12}(k)}{\partial I(k)} \frac{\partial I(k)}{\partial K_s} \quad (5.19)$$

where  $\frac{\partial I(k)}{\partial K_s}$  can be solved by differentiating Eq. (5.16) as follows

$$\frac{\partial I(k)}{\partial K_s} = \frac{|\dot{y}_1(k)|}{1 - \frac{\partial \dot{y}_1(k)}{\partial I(k)} \text{sgn}[\dot{y}_1(k)]} \quad (5.20)$$

In addition, Eqs. (5.12)-(5.15) and Eqs. (5.5)-(5.7) can be used with Eqs. (5.16)-(5.20) for adapting the gain for the skyhook control. This approach will be suggested as a future research topic in Chapter 8.

## 5.5 Stability Analysis

Although some methods, such as Lyapunov method, phase portrait and Popov criterion, are available for analyzing the stability of nonlinear dynamic systems [40], an analytic solution is generally not easy to achieve. For this control system, there are two aspects of stability which must be addressed: 1) stability of the closed-loop mechanical system, and 2) stability of the adaptive control algorithm. In this section, we present a qualitative approach to the first stability analysis and a numerical approach to the second stability analysis.

The dynamic mechanical system studied here uses a magneto-rheological damper as an actuator. Since a damper can not generate energy, it is intuitively obvious that the closed-loop dynamic system is passive. Furthermore, in such a system, the spring and mass transfer energy back and forth from kinetic to potential and thus do not create energy. In all, the dynamic closed-loop mechanical system dissipates energy only. Therefore, it is clear that the mechanical system is stable regardless of the damping tuning approach. We can further conclude that even if the damping tuning does not work properly, the passive vibration system will not become unstable per the bounded input-bounded output sense. This is a greater advantage over fully active systems.

The second form of stability concerns the adaptive control law itself. Passivity of the mechanical system certainly does not guarantee proper convergence of the adaptive controller. Remember that the adaptive control law is designed to minimize a quadratic cost function of acceleration as given by (5.9). Since the adaptive vibration control system is a forced system, the quadratic cost function will be dependent on the excitation frequency distribution with respect to the mechanical system resonant frequency. In Figure 5.3, the cost function, as represented by RMS (root mean square) acceleration, is plotted with respect to constant current to the MR damper for a variety of different excitation spectrums. The mechanical system resonance frequency of 1.4 Hz is described in Chapter 6, and the excitation spectrum designs are defined in Section 7.1.1.

Figure 5.3 clearly shows that, for these excitation spectrums, the cost function has a unique global minimum corresponding to a single optimal current command to the damper. Figure 5.3 also indicates that when the excitation spectrum is concentrated above the isolation frequency, for instance the 3.5 Hz pure tone and ISO4, the optimal current is zero A. When the excitation spectrum is concentrated below the isolation frequency, such as the 1.0 Hz pure tone and ISO1, the optimal current is one ampere.

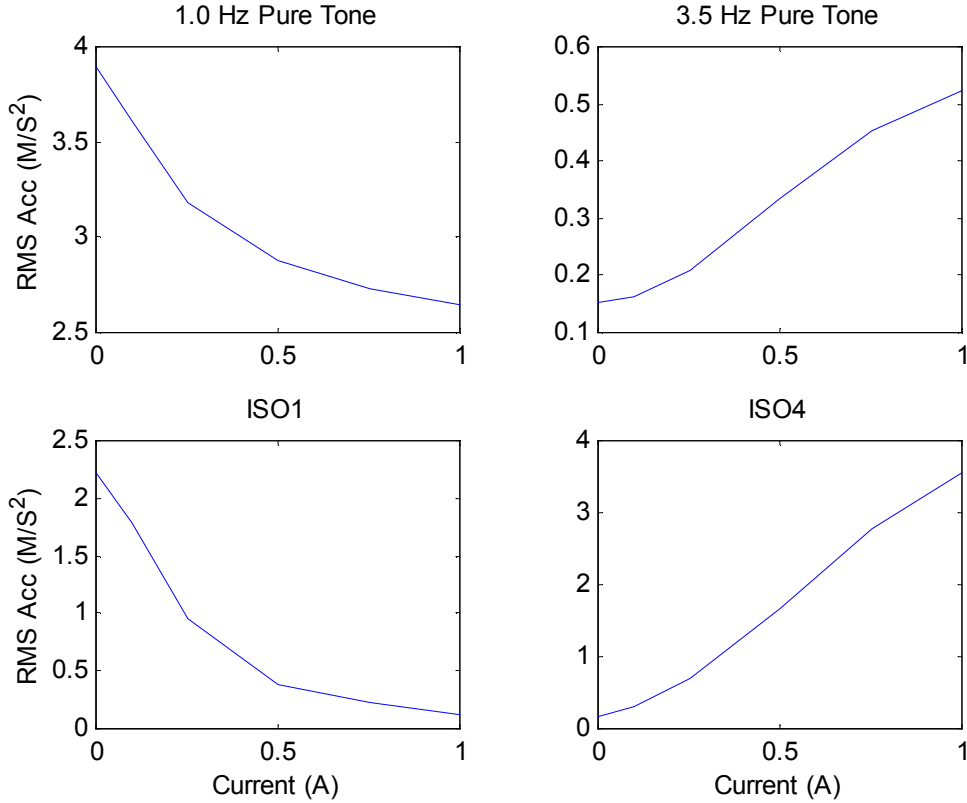


Figure 5.3 Acceleration RMS with Respect to the Tuning Current

For this study, the current is limited to a maximum of one ampere due to the saturation limit of the transconductance amplifier circuit used for the implementation of the adaptive controller as presented in Appendix A. As shown in Chapter 3, the larger current the more damping the MR damper produces. Thus, the current can represent the damping level. Furthermore, it can be observed that the optimal relationship between acceleration rms value and current in Figure 5.3 matches the prediction of Fig. 2.8.

From this analysis, we can at least conclude that unique global minima exist for certain classes of excitation spectra and therefore an associated unique optimal control will also exist. What remains is to show that the adaptive algorithm converges to the optimal control solutions. This result will be presented in Chapter 6 through a series of numerical simulation studies, and in Chapter 7 through experimental results. In addition, the dynamic accelerations obtained from the adaptive approach are compared with soft and hard damping cases in the time domain (i.e., peak by peak) in order to clearly show how the adaptive algorithm works.

## **5.6 Summary**

A complete adaptive algorithm that performs on-line system identification and semiactive control in real time was developed for a class of nonlinear systems, representing vibration isolation devices. The algorithm can deal with the complicated nonlinearities such as those arising from hysteresis, deadband, and backlash without the need for linearization. Furthermore, the stability of the proposed adaptive control was examined and it was determined that since the semiactive damper does not add any energy into the system, the system is stable.