Numerical Simulation of Injection and Mixing in Supersonic Flow

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Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Aerospace Engineering

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November 20, 1997
Blacksburg, Virginia

Keywords: Scramjet, RNG, CFD, Ramp Injector

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Susan K. Cox-Stouffer

(ABSTRACT)

A numerical investigation of the performance of two candidate designs for injection into supersonic flow, including a comparison of two renormalized group theory (RNG) based K-ε turbulence models with a more conventional K-ε model. The chosen designs were an unswept ramp injector with four injection ports and a novel nine-hole injector array. The objectives of the investigation were to provide reliable computational solutions to the flowfields in question using both RNG and standard K-ε turbulence models and to compare the solutions to experiment, thereby to judge the relative performance of the turbulence models. A second objective of the investigation was to use the computed data to provide design insights for the nine-hole injector array.

This investigation made use of GASP^{TM} version 2.2, a commercial computational fluid dynamics code that was augmented by the addition of one RNG-based K-ε turbulence model derived by Zhou, et. al. and one variant of Zhou's model, which was derived by the author. Mesh sequencing studies were performed to measure solution quality, with the fine mesh for the injector array containing roughly one million grid nodes and the fine mesh for the ramp injector containing more than six million grid nodes. Results of these studies indicated that the injector-array solution was significantly under-resolved in the farfield, though the quality was better in the vicinity of the injector itself. The ramp-injector solution, while not perfectly grid-resolved, showed much better grid convergence in both the nearfield and farfield. Accordingly, comparison with experiment was better for the ramp injector than for the injector array. For both injectors, the differences between solutions generated with RNG-based K-ε and standard K-ε turbulence models were negligibly small.

Despite inadequate grid resolution in the farfield, the computational investigation of the nine-hole injector array did yield several important design insights. Particularly, the significance to mixing and losses of the placement of the outer injectors of the second and third rows was determined.

Acknowledgements

This work would not have been possible without the help and support of a number of individuals and organizations. Any list would be incomplete, but I will attempt to name as many as possible.

First I am compelled to express gratitude to my committee, Drs. Joseph Schetz, Robert Walters, Bernard Grossman, Charles Yates, and Wing Ng. I also appreciate the help Dr. Walter O'Brien provided with my defense.

This work was supported in part by a grant from NASA via the National Aerodynamic Simulator and by a grant of HPC time from the following DoD HPC Centers: Air Force Aeronautical Systems Center, U.S. Army Corp of Engineers Waterways Experiment Station, and the Naval Oceanographic Office. Computers used were their Cray C-90's hpc01, pk, and vlsc, respectively. I am very grateful for the kind assistance provided by the user support staff of the various computer centers. Additional computer resources were provided by Aerosoft, Incorporated.

My deepest gratitude goes to Lana Couch, Charles McClinton, and Sharon Stack of the Hypersonic Vehicles Office at NASA's Langley Research Center, and to Dr. A. S. Nejad, formerly of the Experimental Research Branch of the Propulsion and Power Division of Wright Laboratory for their support of my work. I extend thanks also to all those who have assisted me in these organizations.

I am also very grateful to Dr. Tzong Chen and the employees of Taitech, Incorporated, for all their support. I owe particular depts of gratitude to Jeffery White and Dr. Robert Baurle for their invaluable advice and assistance, and for all they have taught me about computational fluid dynamics.

I wish to thank the staff of the Department of Aerospace and Ocean Engineering for their kindness and support throughout my graduate experience.

I wish also to express my deepest gratitude to and respect for my special friends in the Department of Aerospace and Ocean Engineering, Drs. Antoni K. Jakubowski and James F. Marchman, III, and to my inspiration, Dr. Charles L. Yates.

I regret that I am unable to name and individually thank the many friends who have supported me throughout my graduate experience. Without them this achievement would not have been possible.

Special thanks and dearest love go to my parents, Jim and Bea Cox, my sister Becky Cox, for all their understanding, encouragement, and support.

Deepest appreciation and love are also extended to my husband, Scott Stouffer, for his understanding, patience, and support.

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List of Symbols

A	Any flow variable
$\bar{A}$	Reynolds-averaged variable
$\tilde{A}$	Fabre'-averaged variable
A'	Fluctuating component of Reynolds-decomposed variable
A^+	Van Driest damping constant
a	Sound speed
C_{R1} and C_{R2}	Local interaction contribution coefficients in Reynolds stress equation in RNG turbulence model
$C_{\tau 1}$, $C_{\tau 2}$, and $C_{\tau 3}$	Nonlocal interaction contribution coefficients in Reynolds stress equation in RNG turbulence model
$C_{\varepsilon 1}$ and $C_{\varepsilon 2}$	Modelling coefficients in dissipation equation of $K-\varepsilon$ turbulence model
C_μ	Modelling coefficient in eddy viscosity relation of $K-\varepsilon$ turbulence model
c_p	mixture specific heat at constant pressure
D_{l_i}	Diffusion coefficient for species i
D_{t_i}	Turbulent diffusion coefficient for species i
d	Diameter of each nozzle in the injector array
d_{eff}	Effective diameter of injector array
e	Specific internal energy
e_o	Stagnation specific internal energy, $e_o = e + \frac{1}{2}(U^2 + V^2 + W^2)$
$\bar{F}$	Inviscid flux vector for the x coordinate direction
F_{kleb}	Klebanoff intermittency factor
$\bar{F}_v$	Viscous flux vector for the x coordinate direction
$\bar{G}$	Inviscid flux vector for the y coordinate direction
$\bar{G}_v$	Viscous flux vector for the y coordinate direction

$\overline{H}$	Inviscid flux vector for the z coordinate direction
$\overline{H}_v$	Viscous flux vector for the z coordinate direction
h	$k_t = \mu c_p / \text{Pr}_t$ Specific enthalpy (h = e + P/ρ)
h_o	Stagnation specific enthalphy, $h_o = h + \frac{1}{2}(U^2 + V^2 + W^2)$
K	Turbulence kinetic energy
k	Thermal conductivity ($k = \mu c_p / \text{Pr}$)
k_t	Turbulent thermal conductivity ($k_t = \mu c_p / \text{Pr}_t$)
l_m	Mixing length
m_{f_i}	Mass fraction of species i
P	Static pressure
Pr_T	Turbulent Prandtl number
$\overline{Q}$	Vector of conservative flow variables
$\overline{q}$	Vector of primative flow variables
q_i	Heat-flux vector
$\overline{R}$	Residual
Sc_l	Schmidt number
Sc_t	Turbulent Schmidt number
s	Highest characteristic speed
T	Temperature
T_o	Time period of turbulent fluctuations used in Reynolds averaging
t	Time
t.i.	Turbulence intensity
U_i	Cartesian velocity vector in tensor notation
u	Flow speed
V	Magnitude of the velocity vector
V_{dif}	Velocity magnitude difference for use in wake function
$\hat{V}_{ij}$	Mass diffusion velocity of species i in direction j
$\overline{\hat{V}}_i$	Mass diffusion velocity vector of species i
x_i	Cartesian position vector in tensor notation
y	Normal distance above surface

y^+	Nondimensional law-of-the-wall coordinate

α	Transverse injection angle
$\bar{\delta}$	Central difference
Δ	Forward difference
∇	Backward difference
δ_{ij}	Kronecker delta
ε	Dissipation rate
κ	Karman constant
λ	Courant-Friedrichs-Levy (CFL) number
μ	Molecular viscosity
μ_T	Turbulent (eddy) viscosity
μ_T^*	Effective eddy viscosity used for heat and mass transfer in “mixing” RNG
ρ	Mass density
ρ_i	Mass density of species i
σ_K	Prandtl number for turbulence kinetic energy
σ_ε	Dissipation rate Prandtl number
τ_{ij}	Viscous shear stress
$\tau^{R}{}_{ij}$	Reynolds stress
τ_{ij}^{R*}	RNG contribution to Reynolds stress
ω	Vorticity

Subscripts

i, j, or k	Evaluated at plane i, j, or k
max	Maximum or at maximum
w	Wall value

Superscripts

n	Evaluated at time-level n