Chapter 2

THE EFFECT OF THE T-STRESS ON CRACK PATH SELECTION IN ADHESIVELY BONDED JOINTS

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ABSTRACT

This paper investigates the effect of the T-stress on crack path selection in adhesively bonded joints. Fleck, Hutchinson, and Suo [1] concluded that, similar to the situation in homogeneous solid media, the directional stability of cracks in adhesive bonds is also significantly influenced by the magnitude of the T-stress. Cracks tend to be directionally stable when the T-stress is negative (compressive) and directionally unstable when the T-stress is positive (tensile). This T-stress dependence of crack path selection in adhesively bonded joints is demonstrated experimentally in this study using double cantilever beam (DCB) specimens with various levels of the T-stress. The technique reported to vary the T-stress involves a mechanical stretching procedure of the specimens and is able to continuously alter the T-stress level over a fairly wide range. Using the finite element analysis (FEA) method, the T-stress for DCB specimens is calculated and comparison is made with the analytical solution obtained by Fleck, Hutchinson, and Suo [1] for the bi-material sandwich geometry with semi-infinite adherends. The FEA results show that the T-stress increases as the thickness of the adherends decreases, indicating a mild effect of adherend thickness on the directional stability of cracks. This prediction is verified in this paper using DCB specimens with different thicknesses adherends.

KEYWORDS

Locus of failure, crack path selection, adhesively bonded joints, T-stress, interfacial failure, cohesive failure, failure mode, alternating failure, directional stability.

INTRODUCTION

Identifying and interpreting the locus of failure and crack propagation behavior are significant aspects in evaluating the mechanical properties of adhesively bonded joints, and have been of interest for many years. While conventional wisdom suggests that materials always fail at the weakest location, Dillard *et al.* [2] indicated that the locus of failure, while closely related to material properties such as tensile strength, quality of adhesion at the interface, and fracture toughness of the bonds, depends also on the stress state at the crack tip. Supportive information can also be found in Cao and Evans [3], and

Akisanya and Fleck [4] where both the locus of failure and the crack propagation behavior were shown to be dependent on the mode mixity of external loads. Although a number of important results have already been obtained [1-4] to interpret different crack path selection phenomena, further study in this area is still needed.

As shown in Fig. 1, four types of failure have been observed in mode I tests of double cantilever beam (DCB) specimens using aluminum adherends and an epoxy adhesive (details of material specifications and specimen fabrication are given in the next section). These modes are: a) cohesive failure, where the crack propagates along the middle of adhesive layer; b) interfacial failure (visually), where the failure occurs at the interface between adhesive and adherends; c) oscillatory failure, where the trajectory of the crack oscillates about the midplane of the bond but remains within the adhesive layer; and d) alternating failure, where the crack alternates between the two adhesive/adherend interfaces. Similar phenomena had also been observed by Wang and Suo [5], Cao and Evans [3], and Chai [6-8] in different systems such as 420 stainless steel/epoxy, plexiglass/epoxy and fiber-reinforced epoxy laminates.

Fig. 1 shows different failure locations, either within the adhesive layer (a and c) or at/near the interface (b and d), as well as different crack trajectories, either directionally stable (a and b) or directionally unstable (b and d). This information leads to two important issues closely related to the fundamental mechanism of crack path selection in adhesive bonds: locus of failure and directional stability of cracks.

The locus of failure is closely related to the direction of cracking. Over the years, several criteria have been developed to determine the direction of crack propagation for cracks in brittle homogeneous isotropic solids. Among these criteria, three primary ones have been widely discussed in the literature; namely,

1. **Maximum opening stress criterion** (by Ergodan and Sih, (1963) [9]). This criterion dictates that the direction of cracking is perpendicular to the direction of maximum opening stress at the crack tip.

2. **Maximum energy release rate criterion** (by Palaniswamy and Knauss, (1978) [10]). By applying this criterion, the direction of crack propagation can be obtained by maximizing the energy release rate as a function of the angle of crack kinking.

3. **Mode I fracture criterion** (by Goldstein and Salganik (1974) [11] and Cotterell and Rice (1980) [12]). According to this criterion, a crack will propagate along a path such that pure mode I fracture is maintained at the crack tip, i.e. $K_{II} = 0$ at the growing crack tip.

Although the three criteria specify different aspects, they all yield similar results and no experimentally distinguishable differences have been observed [4, 13, 14].

These criteria, although developed primarily for cracks in homogeneous materials, can be readily extended into bi-materials systems such as adhesively bonded joints. However, care should be used when applying these criteria to determine the direction of cracking for cracks located at a bi-material interface due to differences in fracture toughness in the vicinity of an interface. For adhesively bonded joints, Chen and Dillard [15] provided an example showing how to use these criteria to determine the direction of cracking when the crack is located at the interface.

According to these criteria, a crack in an adhesive bond can be steered to different locations if the local stress state at the crack tip is in mixed mode. Consequently, various failure locations can result and failure does not necessarily occur at the weakest site within the material, which conflicts with conventional wisdom.

The issue of directional stability of cracks in adhesively bonded joints was first discussed by Chai [6-8], who observed a unique form of crack trajectory in the mode I delamination failure tests of graphite reinforced epoxy composite laminates and aluminum/epoxy bonds. As the crack advanced, the crack propagated along one interface and then gradually deviated away with an increasing slope until the other interface was approached. An abrupt kink then occurred when the crack approached the opposite interface and stayed at the interface for a distance about 2-3 times the thickness of the adhesive layer before leaving the interface. As a result, the crack trajectory appeared to be periodically alternating between the two interfaces, which obviously reflected a very directionally unstable crack propagation.

Fleck, Hutchinson, and Suo [1] and Akisanya and Fleck [4, 16] later investigated this directional stability issue in adhesive bonds. In their analyses, for the geometry shown in Fig. 2, the adherends were assumed to be semi-infinite, the adhesive was assumed to be linear elastic, and a semi-infinite straight crack was present within the adhesive layer. According to the coordinate system in Fig. 2, the stress state at the crack tip can be expressed in the asymptotic form [17] as

$$\begin{aligned}\begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{xy} & \sigma_{yy}\end{bmatrix} &= \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\begin{bmatrix}1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right) & \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\\ \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right) & 1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\end{bmatrix}\\ &+\frac{K_{II}}{\sqrt{2\pi r}}\begin{bmatrix}-\sin\left(\frac{\theta}{2}\right)\left[2+\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right] & \cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]\\ \cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] & \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\end{bmatrix}\\ &+\begin{bmatrix}T & 0\\ 0 & 0\end{bmatrix}+O\left(\sqrt{r}\right)\end{aligned} \qquad \text{Equation 1}$$

where r and θ are the polar coordinates, and K_I and K_{II} are the stress intensity factors at the crack tip. The third term in Equation 1 with stress T is non-singular and acts in the direction parallel to the crack plane. By convention, this term is referred to as the "T-stress".

In investigating slightly curved or kinked cracks in homogeneous materials under mode I loading, Cotterell and Rice [12] concluded that the T-stress plays an important role in the directional stability of the crack propagation. The crack is directionally stable if the T-stress is negative, whereas is directionally unstable if the T-stress is positive. Through considering higher order terms in the William's asymptotic stress expansion for a crack in homogeneous materials [17], Chao *et al*. [18, 19] further investigated the effect of the T-stress on the crack propagation manner and indicated that the transition point between the directionally stable and unstable cracks is slightly influenced by the T-stress levels. For this study, however, the criteria developed by Cotterell and Rice [12] is still found to be satisfactory and therefore is used. Fleck, Hutchinson, and Suo [1], and Akisanya and Fleck [4, 16] indicated that, similar to the situations in homogeneous materials, the directional stability of cracks in adhesive bonds also depends on the magnitude of the T-stress, which in this case can be calculated as [1]

$$T = \frac{1-\alpha}{1+\alpha} T^{\infty} + \sigma_0 + C_I(c/t, \alpha, \beta)\frac{K_I^{\infty}}{\sqrt{t}} + C_{II}(c/t, \alpha, \beta)\frac{K_{II}^{\infty}}{\sqrt{t}} \qquad \text{Equation 2}$$

where σ_o is the residual stress in the adhesive and t is the thickness of the adhesive layer. K_I^{∞}, K_{II}^{∞}, and T^{∞} in Equation 2 are solutions for the homogeneous case obtained by neglecting the adhesive layer and here they are used as far field loading. The manner in which K_I^{∞} and K_{II}^{∞} are related to a specific applied loading can be found in Tada *el. al*. [20]. T^{∞} has been obtained by Larsson and Carlsson [21] for several commonly used testing geometries. $C_I(c/t, \alpha, \beta)$ and $C_{II}(c/t, \alpha, \beta)$ in Equation 2 are non-dimensional functions tabulated in Fleck, Hutchinson, and Suo [1], c is defined as shown in Fig. 2, and α and β are Dundurs' parameters reflecting the material mismatch, and are defined as

$$\alpha = \frac{\mu_1(\kappa_2+1) - \mu_2(\kappa_1+1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)}$$
$$\beta = \frac{\mu_1(\kappa_2-1) - \mu_2(\kappa_1-1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)} \qquad \text{Equation 3}$$

where the subscripts 1 and 2 refer to the materials for the adherends and adhesive, respectively; μ_i (i =1, 2) are shear moduli; $\kappa_i = 3 - 4\nu_i$ for plane strain and $\kappa_i = (3-\nu_i)/(1+\nu_i)$ for plane stress; and ν_i (i = 1, 2) are the Poisson's ratios. The value of α is between – 1 and 1, and according to Suga *et al.* [22] and Hutchinson and Suo [14], the value of β is between 0 and $\alpha/4$ for most material combinations.

Consistent with the results obtained by Cotterell and Rice [12] in homogeneous solids, Fleck, Hutchinson and Suo [1], and Akisanya and Fleck [4, 16] also concluded that under predominantly mode I loading, the crack propagation in an adhesive bond is directionally stable if the T-stress is negative and is directionally unstable if the T-stress is positive. This argument revealed the threshold of the transition of the directional stability of cracks in adhesively bonded joints and provided an important foundation for the experimental work in this paper. However, since the analyses were based on the assumption that the adherends were semi-infinite, the effect of adherend bending on the T-stress was excluded. In adhesively bonded DCB specimens, since the thickness of the adherends is finite, the T-stress is predicted to be higher than that predicted by Equation 2 and will increase as the thickness of the adherends decreases due to the effect of adherend bending. This adherend bending effect on the T-stress induces a mild influence of the thickness of the adherends on the directional stability of the cracks in the DCB specimens as discussed later in this study.

In this paper, the T-stress effect on the directional stability of cracks in adhesive bonds was demonstrated through achieving various levels of T-stress mechanically in DCB specimens. The technique reported herein to alter the T-stress level involves a mechanical stretching procedure of the specimens and has several advantages as compared to other techniques discussed in the literature. Through numerical analyses using the finite element method, the effect of adherend bending on the T-stress, and therefore on the directional stability of cracks, was investigated. The results show that the T-stress increases as the thickness of adherends decreases, and therefore the crack tends to be more directionally unstable. Experimental results of specimens with different adherends thicknesses were consistent with the numerical predictions.

EXPERIMENTAL SECTION

Specimen Fabrication

To understand the effect of the T-stress on the directional stability of crack propagation, standard double cantilever beam (DCB) specimens were made. The specimen dimensions are shown in Fig. 3 except that the thickness of the adherends varies. The adherends were aluminum 6061-T6 alloy cleaned with acetone, which was used simply to provide surface uniformity among specimens. The resin used was Dow Chemical epoxy resin D.E.R. 331, a low molecular weight, liquid bisphenol A-type resin. The curing agent used was dicyandiamide (“dicy”). A tertiary amine accelerator, 3-phenyl-1, 1 dimethyl urea (PDMU) was used to accelerate the curing. M-5 silica, a hydrophilic fumed silica manufactured by the Cabot Corporation, was used as a filler. The toughener used was Kelpoxy G272-100, a concentrate of an epoxy-terminated elastomeric copolymer designed by Reichhold Chemicals as an additive or modifier to toughen epoxies, epoxy novalacs, and PVC plastisols. The details of the adhesive formulation procedure can be found in Vrana *et al*. [23]. The final product contains about 69.3% D.E.R. 331, 4.1% DICY, 1.6%

PDMU, 4.7% Silica, and 20.3% Kelpoxy in weight. The final rubber concentration in this model system was approximately 8.1%. The specimens were cured at a temperature of 170°C for 90 minutes.

The material properties of the cured adhesive were characterized using differential scanning calorimetry (DSC), thermal mechanical analysis (TMA), and dogebone tensile tests. The results showed that the adhesive has a glass transition temperature T_g = 112 ^{o}C, coefficient of thermal expansion (CTE) $\alpha_2 = 62 \times 10^{-6} / {}^oC$, and Young's modulus E_2 = 2.97 GPa at room temperature. The Poisson's ratio of the adhesive is estimated as ν_2 = 0.33 at room temperature. The material properties for the aluminum 6061-T6 substrates are Young's modulus E_1 = 70 GPa, Poisson's ratio ν_1 = 0.33, and CTE $\alpha_1 = 26 \times 10^{-6} / {}^oC$ [24].

Due to the mismatch of the coefficients of thermal expansion, an equal bi-axial residual stress σ_0 is induced throughout the adhesive layer from the specimen fabrication. If the adherends are assumed to be thick and rigid as compared to the adhesive, the residual stress is then given by

$$\sigma_0 = \frac{E_2}{1-\nu_2}(\alpha_2 - \alpha_1)(T_{sft} - T_t) \qquad \text{Equation 4}$$

where T_{sft} (about 108 ^{o}C) is the stress free temperature, which is very close to the glass transition temperature for this particular material system, and T_t is the testing temperature, which is room temperature in this study. The residual stress estimated using Equation 4 is about 13 MPa, which is consistent with the experimental result obtained using a curvature measurement technique. Details of the residual stress measurement technique can be found in Dillard *et al.* [25].

According to Equation 2, for a straight crack within the adhesive layer of a DCB specimen, the T-stress depends on the residual stress σ_0. This relationship indicates that changing the residual stress in the adhesive layer can alter the T-stress level. In this study, to alter the residual stress and consequently the T-stress, DCB specimens were loaded uniaxially in an Instron machine until the substrates were plastically deformed as shown in Fig. 4. An MTS extensometer was attached to the specimens to monitor the strain. Upon unloading, the plastic deformation remaining in the substrate, ε_p, was recorded as shown in Fig. 5. According to the tensile test results for neat adhesive dogbone specimens shown in Fig. 5, the adhesive did not yield at the strain levels investigated. Due to the plastic deformation, the residual stress in the adhesive layer was increased (and so was the T-stress level). According to the coordinate system shown in Fig. 3, the total residual stress can be calculated as

$$\begin{aligned} \sigma_x &= \frac{E_2}{1-\nu_2}(\alpha_2 - \alpha_1)(T_{sft} - T_t) + \frac{E_2}{1-\nu_2^2}(1 - \nu_1\nu_2)\varepsilon_p \\ \sigma_z &= \frac{E_2}{1-\nu_2}(\alpha_2 - \alpha_1)(T_{sft} - T_t) + \frac{E_2}{1-\nu_2^2}(\nu_2 - \nu_1)\varepsilon_p \end{aligned} \qquad \text{Equation 5}$$

This result is consistent with what Dillard *et al.* [26] found in studying the stress state in polymer coatings subjected to similar combinations of biaxial residual plus uniaxial extension stresses.

Concerns have been raised whether any microcracks in the adhesive layer were induced during the stretching procedure. This issue is critical because the crack propagation behavior and resulting trajectory might be influenced by the microcracks. To further investigate the stretching processes, blue fountain pen ink was applied to both sides of the DCB specimens while they were being stretched. Due to the surface energy, the ink should wick into the microcracks as soon as they formed. The experiments showed no evidence of ink wicking before the maximum strain approached 2.1% and when the maximum strain was beyond 2.1%, ink wicking was observed in some specimens. This experimental result indicated that no microcrack should be induced during the stretching process as long as the total strain is controlled to be less than 2.1% for this particular material system, and the highest strain level reported herein is about 1.9%.

Besides the mechanical stretching method, the residual stress and consequently, the T-stress level can also be altered thermally. By bonding epoxy (GY260 + 2% rubber) adhesive to different substrates such as aluminum alloy and carbon fiber/epoxy composite, Daghyani, Ye, and Mai [27] achieved different residual stresses in DCB specimens due to different CTE mismatches, 11 MPa if the substrate was aluminum alloy and 26.9 MPa if the substrate was carbon fiber/epoxy composite. However, since the material system is completely different, the validity of comparing testing results between different specimens and concluding the T-stress effect based on the results must be carefully considered.

Different levels of residual stress and consequently, the T-stress, have also been achieved in our study by submerging the DCB specimens in liquid nitrogen or dry ice for one hour right before the tests were conducted. This is due to the fact that the coefficient of thermal expansion (CTE) of the epoxy adhesive ($62\times10^{-6}/^{o}C$) is greater than that of the aluminum alloy ($26\times10^{-6}/^{o}C$). Therefore, the residual stress in the adhesive layer (and consequently the T-stress) is increased when the specimen is cooled. However, since the properties of epoxy changes with temperature, care should be used when relating the experimental results obtained at low temperature with those obtained in room temperature.

The stretching method provides a convenient way to alter the residual stress and consequently, the T-stress level, in the adhesive bonds. First, with delicate control of the testing frame achieved through operating the GPIB interface using LabVIEW® [28] software, the expected plastic deformation level can be achieved within a 3% error; therefore, the expected T-stress level can be achieved rather precisely. Second, using this method, the T-stress can be continuously varied over a wide range (47 MPa for the material system studied). Third, since the stretching method is purely a mechanical method, no material properties have been altered after stretching, permitting direct comparison of the test results. Last, the availability of tensile test frames provides a relatively convenient way to alter the residual stress and the T-stress state.

DCB Testing

After stretching, the DCB specimens were then tested quasi-statically in a screw-driven Instron 4505 machine. The loading rate was controlled to be 1 mm/min and the load-displacement and load-time curves were recorded. A 10 power magnifying glass was used to measure the crack length from time to time as the crack arrested. From the load-displacement curve and the crack length information, the relation between the compliance of the specimen and the crack length could be obtained, from which, the effective stiffness of the DCB specimens could be calculated. The applied fracture energy or the strain energy release rate is given by

$$G = \frac{9\Delta^2 (EI_{eff})}{4B(a+x)^4} \qquad \text{Equation 6}$$

where, EI_{eff} is the effective stiffness of the DCB specimens, B is the width of the specimen, Δ is the specimen's opening displacement, a is the crack length, and the x is the apparent crack length offset which is obtained also from the compliance-crack length relation. Details of the DCB analysis and testing procedure can be found in Blackman *et al.* [29] and Parvatareddy *et al.* [30].

FRACTURE ANALYSIS

The T-Stress for Double Cantilever Beam Specimens

As discussed earlier, in the analysis of Fleck, Hutchinson, and Suo [1], the adherends were assumed to be semi-infinite. In the adhesively bonded DCB specimens, the T-stress is expected to be higher than that predicted by Equation 2 due to the effect of adherend bending. To quantify the T-stress in the DCB specimens, numerical methods must be used since the theoretical solutions are only available for a few geometries.

Over the years many numerical methods have been proposed [21, 31-35] to calculate the T-stress in general geometries and among them, finite element analysis (FEA) is the most direct one. According to Equation 1, the T-stress can be calculated by substituting the stress σ_{xx}, σ_{yy}, and the stress intensity factors obtained from finite element analysis into the equation [33, 35]. In this paper, to further simplify the calculation, the T-stress specimen was obtained as follows:

When the fracture is in pure mode I as with the mode I loaded DCB specimens, $K_{II} = 0$ in Equation 1. Then the T-stress along the crack plane ($\theta = 0$ and $\pm\pi$) can be obtained from Equation 1 as

$$T = \sigma_{xx} - \sigma_{yy} \qquad \text{Equation 7}$$

If the fracture is in mixed mode, as with asymmetric DCB specimens, in which one adherend is thicker than the other, K_{II} does not equal to zero. Then along the crack plane ahead of the crack tip ($\theta = 0$), the T-

stress can be still calculated using Equation 7; however, behind the crack tip ($\theta = \pm \pi$), the T-stress is now given by,

$$T = \sigma_{xx} + K_{II}\sqrt{\frac{2}{\pi r}} \qquad \text{Equation 8}$$

where σ_{xx} is obtained from the finite element analysis, and K_{II} can be calculated from fitting the σ_{xy} data, which is also obtained form the finite element analysis, as

$$K_{II} = \left(\sigma_{xy}\sqrt{2\pi r}\right)_{\theta=0} \qquad \text{Equation 9}$$

In this study, the finite element analysis was conducted using ABAQUS® [36]. The double cantilever beam (DCB) model analyzed is shown in Fig. 6. An adhesive layer (material #2) is sandwiched between two adherends (material #1). The thicknesses of both the adhesive and the adherends vary in the analysis. A straight crack is located in the middle of the adhesive layer, and the displacements of one end of the model are totally constrained. The finite element mesh around the crack tip is shown in Fig. 7; eight-node, plane-strain elements were used with reduced integration. Quarter point singular elements were constructed around the crack tip. Both the adherend and adhesive were modeled as linear elastic materials with material constants E_1 = 70 GPa, E_2 = 2.97 GPa, and $\nu_1 = \nu_2 = 0.33$. The coefficients of thermal expansion (CTE) used for adherend and adhesive, respectively, are $\alpha_1 = 26\times10^{-6}/^{o}C$ and $\alpha_2 = 62\times10^{-6}/^{o}C$. Since the measured fracture toughness of the adhesive bonds appeared to be independent of the crack propagation behavior for this material system as will be discussed later, the adhesive bonds were assumed to have an iso-fracture toughness value of 310 J/m^2, which was obtained from the quasi-static testing results of DCB specimens.

Analysis Results

Shown in Fig. 8 is the T-stress distribution along the x-axis (the coordinate is shown in Fig. 6 and Fig. 7) obtained from the FEA analysis for symmetric DCB specimens with zero residual stress. The adhesive thickness in the analysis was 0.5 mm and various adherends thicknesses (H = 20 mm, 6 mm, 4.8 mm, and 3.2 mm) were analyzed. The T-stress converges to the bending stress of the composite beam as x decreases behind the crack tip and converges to zero as x increases in front of the crack tip. Fig. 8 also shows that the T-stress is non-singular at the crack tip and increases as the adherend thickness decreases.

Shown in Fig. 9 is the relation between the T-stress and the thickness of the adhesive layer for the DCB specimens with zero residual stress. The figure shows that the T-stress decreases with the thickness of adhesive. The relationship between the adhesive thickness and the T-stress indicates that directionally unstable crack propagation is more unlikely to occur if the thickness of adhesive decreases. More details

about the adhesive thickness effect on the directional stability of crack propagation can be found in Chen *et al*. [37]. As for the effect of adherend bending, Fig. 9 shows that the T-stress obtained by Fleck, Hutchinson, and Suo [1] is the lower bound since the adherends were assumed to be semi-infinite in their analysis. As the adherend thickness decreases, the T-stress increases and the difference is not negligible if the adherend thickness is less than 6 mm for the configuration studied. Therefore, for this particular material system, when the adherend thickness is less than about ten times the thickness of the adhesive, the effect of adherend bending on the T-stress level is no longer negligible and the crack propagation is predicted to be more directionally unstable as the adherend thickness decreases.

If the residual stress in the adhesive is not zero, the T-stress is then given by

$$T = T_0 + \frac{E_2}{1-\nu_2}(\alpha_2 - \alpha_1)(T_{sft} - T_t) + \frac{E_2}{1-\nu_2^2}(1-\nu_1\nu_2)\varepsilon_p \qquad \text{Equation 10}$$

where T_0 is the T-stress under zero residual stress state. According to Equation 10, the T-stress increases linearly with the plastic deformation, ε_p, in the adherends. Consequently, cracks propagating in the DCB specimens are predicted to be more directionally unstable as the plastic deformation in adherends increases.

TEST RESULTS AND DISCUSSION

T-Stress and Directional Stability of Cracks

To demonstrate the T-stress effect on the directional stability of cracks, DCB specimens with adherend thicknesses 4.8 mm and various levels of the T-stress were tested quasi-statically. After failure, the failure surfaces of the specimens were carefully examined and three representative specimens were selected as shown in Fig. 10, from which, the effect of the T-stress on the directional stability of cracks could be inferred.

The initial residual stress based on thermal mismatch in all three specimens shown in Fig. 10 was 13 MPa. However, each specimen was stretched to a different level of plastic deformation in the adherends. As a result, the T-stress varied among these three specimens. Specimen a) was an as-produced specimen, no plastic deformation was introduced in the adherends, and the consequent T-stress was - 3.0 MPa. The failure surfaces of specimen a) appeared cohesive (except for a few spots along the edges where the debond had arrested) and the crack was directionally stable. On the other hand, specimen b) was stretched to about 1.1% plastic deformation. Consequently, the T-stress increased to 29 MPa and an oscillatory crack trajectory was observed on the failure surfaces, indicating a tendency toward directionally unstable crack propagation. When the plastic deformation reached the level $\varepsilon_p = 1.3\%$ as with specimen c), the corresponding T-stress was 35 MPa. The crack in this specimen alternated between the two

interfaces and failure occurred at or very close to the interfaces. This alternating crack trajectory is a very directionally unstable crack propagation.

To further investigate the characteristics of the directionally unstable cracks, a more careful surface examination including scanning electron microscopy was conducted on the representative areas of the failure surfaces of specimen b) and c). As shown in Fig. 11, the oscillatory features of the crack path were observed clearly in the magnified picture of specimen b) and the SEM micrograph of the failure surface further reveals that the failure occurred within the adhesive layer and the failure surfaces were relatively smooth. Fig. 12 shows the failure surfaces of specimen c), which shows that the crack propagation was directionally unstable. The crack trajectory alternates between the interfaces (sub-interfaces, to be exact) with a characteristic length, which was measured from the spacing of the adhesive blocks on the failure surfaces, to be 3 – 4 times the thickness of the adhesive layer. The SEM micrograph in Fig. 12 further reveals the characteristics of the morphology of the failure surfaces and the crack propagation behavior. First, the micrograph shows that the failure occurred at the sub-interface rather than at the interface, which was verified by the post-surface analysis results in reference 37. Second, this figure, along with Fig. 13, where the longitudinal cross-section of specimen c) is shown and from which, the crack trajectory is fully observed, suggested that as the crack advanced, the crack propagated along one interface and then gradually deviated away with an increasing slope until the other interface was approached. An abrupt kink then occurred when the crack approached the opposite interface and stayed at the interface for a distance about 2-3 times the thickness of the adhesive layer before deviated from the interface gradually again. These characteristics in the directionally unstable crack trajectory are very similar to what Chai [6-8] observed.

The experimental results demonstrate that the magnitude of the T-stress controls the directional stability of crack propagation in adhesively bonded joints and the magnitude of the oscillation of the crack trajectory appeared to increase with the T-stress in the tests. On the other hand, for this material system, the fracture toughness of the bonds appears to be independent of the crack propagation behavior. Fig. 14 shows the fracture toughness measured in the quasi-static tests for three sets of specimens with different plastic deformation level in the adherends. Since the crack is more directionally unstable as the plastic deformation increases, Fig. 14 indicates that the crack propagation behavior has an insignificant effect on the fracture toughness of the bonds. This result confirms the iso-fracture toughness assumption discussed earlier in the numerical model and the phenomenon can be explained through investigating the energy balance in the system. Details of the discussion can be found in Chen and Dillard [15].

The Effect of Adherend Bending on the Directional Stability of Cracks

As predicted by the finite element analysis results shown in Fig. 8 and Fig. 9, the T-stress level increases as the thickness of the adherends decreases, which indicated that crack propagation in specimens

with thin adherends tends to be more directionally unstable as compared to specimens with thick adherends. To verify the prediction, two groups of DCB specimens with adherend thickness of 4.8mm and 3.2mm, respectively, were prepared and stretched until 1.1% plastic deformation occurred in the adherends in order to alter the residual stress level (and the T-stress) as described in previous section. Then the quasi-static DCB tests were conducted on the specimens. After failure, as shown in Fig. 15, two typical specimens from each specimen group were selected based on visual examinations of the failure surfaces of the specimens, from which, the conclusions could be inferred.

The residual stress in all the four specimens was 13 MPa. However, due to the differences either in the adhered thicknesses or in the plastic deformation levels in the adherends, the T-stress among these four specimens varied. Specimens a) (H = 3.2 mm) and c) (H = 4.8 mm) were as-produced specimens and the T-stresses were relatively low (1 MPa for specimen a) and -3 MPa for specimen c)). Although failure in those two specimens was both cohesive, slight differences in crack trajectories were still apparent, indicating that crack propagation in specimen a) is more directionally unstable compared with specimen c). This adherend thickness effect on the directional stability of cracks became more pronounced in comparing stretched specimens b) (H = 3.2 mm) and d) (H = 4.8 mm), in which, the T-stress is relatively high (34 MPa for specimen b) and 29 MPa for specimen d)). The alternating crack trajectory in specimen b) indicates a much more directionally unstable crack as compared to specimen d) where the crack trajectory is characterized by a small magnitude oscillation.

SUMMARY AND CONCLUSIONS

This paper investigated the T-stress effect on the crack path selection in adhesively bonded joints. Through the study, the following conclusions are made:

1. The dependence of the directional stability of cracks on the T-stress level was demonstrated. A crack is directionally stable if the T-stress is compressive (negative), and tends to be more directionally unstable as the T-stress increases.

2. The residual stress and consequently, the T-stress, in a double cantilever beam specimen can be altered by stretching the specimen uniaxially until the adherends are plastically deformed. This method is convenient; the T-stress can be varied over a fairly wide range, and no material property alteration will be induced during the stretching procedure.

3. The T-stress in a DCB specimen can be calculated using the finite element method. Compared to the solution obtained by Fleck, Hutchinson, and Suo [1] for semi-infinite adherends, the results obtained using the finite element method are higher due to the effect of adherend bending.

4. The adherend bending effect on the directional stability of cracks was studied. The T-stress level increases as the adherend thickness decreases and consequently, a crack in specimens with thin adherends tends to be more directional unstable compared to specimens with thicker adherends.
5. For the particular material system studied, the T-stress level and the directional stability of cracks did not significantly affect the measured fracture toughness of the adhesive bonds.

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