APPENDIX B

CLOCK OFFSET FOR A LINEARLY POLARIZED RECEIVING ANTENNA

This appendix addresses the clock error due to antenna rotation for a linearly polarized receiving antenna. It is shown that the same clock offset is expected whether a circularly polarized or linearly polarized antenna is used. The polarization-phase vector for a linearly polarized antenna is:

$$\hat{e}_a = \hat{x} \tag{B.1}$$

This choice is somewhat arbitrary, and represents horizontal linear polarization. Assume the antenna is rotated in the x-y plane while the incoming signal travels along the z axis. Fig. B.1 shows the polarization-phase representation of the antenna after a rotation of θ degrees in the CCW direction. The polarization-phase vector is now given by:

$$\hat{e}_a = \cos\theta\hat{x} + \sin\theta\hat{y} \tag{B.2}$$

Again, as in Chapter 5, we calculate the normalized complex voltage of the received signal:

$$\begin{aligned} v = \hat{e}_w \cdot \hat{e}_a^* &= \frac{1}{\sqrt{2}}(\hat{x} - j\hat{y}) \cdot (\cos\theta\hat{x} + \sin\theta\hat{y}) \\ &= \frac{1}{\sqrt{2}}(\cos\theta - j\sin\theta) \end{aligned} \tag{B.3}$$

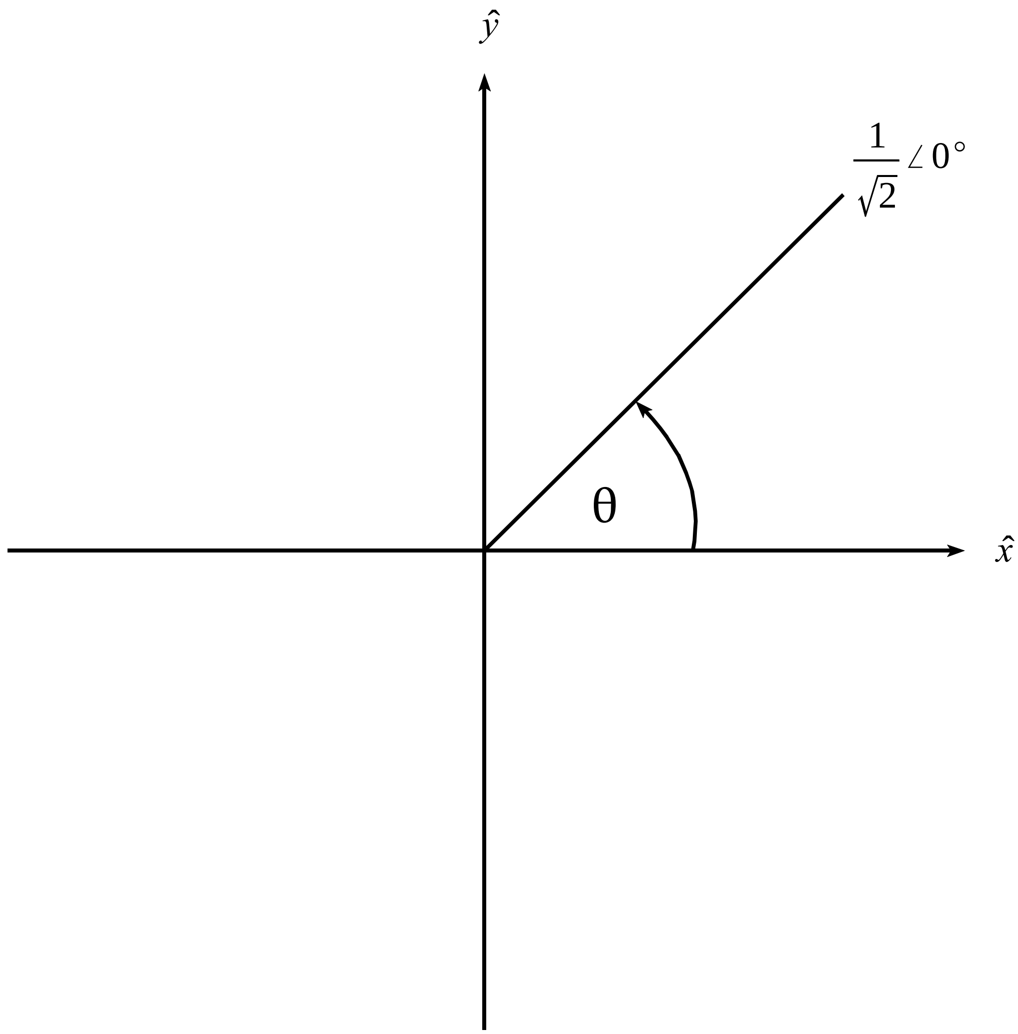

Figure B.1 Polarization-Phase Representation of a Rotating LP Antenna

where $\hat{e}_w$ is the polarization-phase vector for a RHCP wave. Expressing v in polar form yields the magnitude and phase:

$$v = \frac{1}{\sqrt{2}} e^{-j\theta} \tag{B.4}$$

where again Euler's formula has been applied. The phase of the received signal is $-\theta$, which is identical to the result obtained for a RHCP antenna in Eq. 5.5. Thus, the same clock offset would be observed if a linearly polarized antenna were used instead of a RHCP antenna. The clock offset occurs because of the RHCP wave and the way the carrier phase measurement is made — by integrating the Doppler frequency shift.

To help verify this result, the polarization efficiency for this case can be calculated. Circular polarization can be produced by a crossed pair of dipole antennas fed 90° out of phase. This is sometimes referred to as a turnstile antenna [Stutzman, 1993]. Thus, it is intuitive that the polarization efficiency for a CP wave and a linearly polarized receiving antenna should be 0.5. By the definition of polarization efficiency, we have:

$$\begin{aligned} \rho &= |\hat{e}_w \cdot \hat{e}_a^*|^2 \\ &= |v|^2 \\ &= \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= \frac{1}{2} \end{aligned} \tag{B.5}$$

for all θ, which verifies the result of Eq. B.4.