

# **Chapter 2. Position Location Techniques**

## **2.1 Introduction**

This chapter is devoted to position location with an emphasis on air navigation. A discussion of basic pilotage and dead reckoning is followed by a presentation of radio navigation systems. These include terrestrial systems such as LORAN-C and VOR/DME as well as satellite systems such as the Global Positioning System (GPS) and GLONASS. The goal is not to present an all-inclusive review of position location methods; rather, the goal is to provide a basic understanding of navigation techniques and provide a background for a more detailed discussion of GPS in Chapter 3.

## **2.2 Pilotage and Dead Reckoning**

Pilotage is simply a visual navigation technique in which distinct landmarks are used as waypoints. These may include highways, railroads, power lines, towers, cities or towns, lakes, and other natural landmarks. One way to navigate between two points is to check an aeronautical chart for landmarks along the intended path, then go from one to the next until the destination is reached. Some landmarks are better than others, for instance one set of power lines looks very much like the next and can be easily confused.

Use of landmarks can be improved by using dead reckoning. This is a matter of maintaining heading and measuring distance according to ground speed and elapsed time. In the

air, one must account for wind and magnetic variation in choosing the appropriate compass heading [Glaeser et. al., 1989]. In general, navigation is possible using a combination of pilotage and dead reckoning. When visibility is poor, more sophisticated methods are required. This leads to a discussion of electronic aids to navigation.

## 2.3 Radio Navigation — Terrestrial Systems

### 2.3.1 Very High Frequency Omnidirectional Range (VOR)

A VOR provides the user with a bearing to the station. Using frequencies of 108-118 MHz, VOR stations require a direct line-of-sight (LOS) in order to be used. The received signal depends on which radial line from the station the user is located. The VOR signal is:

$$s(t) = m(t) \cos(2\pi f_c t) \quad (2.1)$$

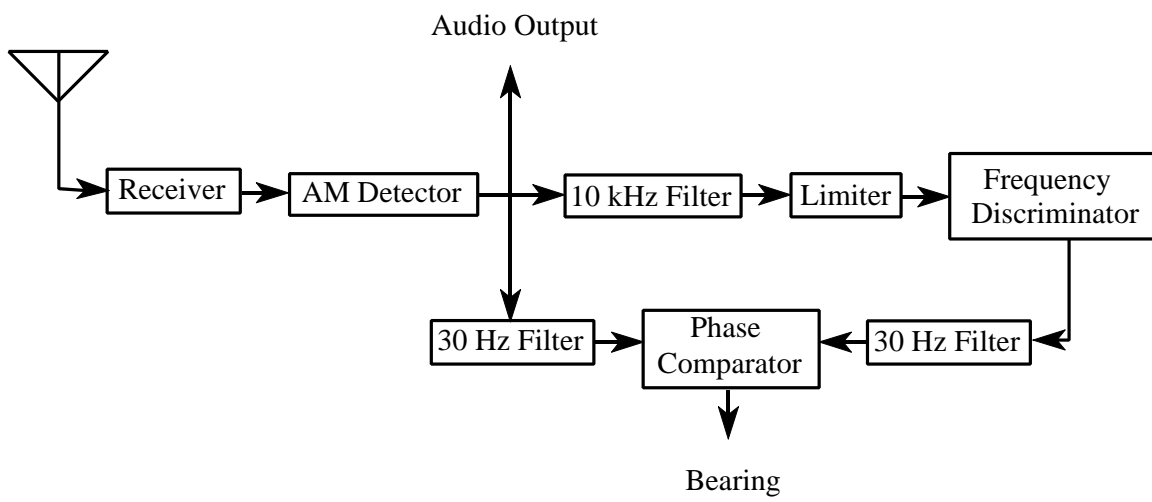
where:  $s(t)$  is the transmitted signal  
 $m(t)$  is the modulating signal  
 $f_c$  is the carrier frequency (108-118 MHz)

The modulating signal is:

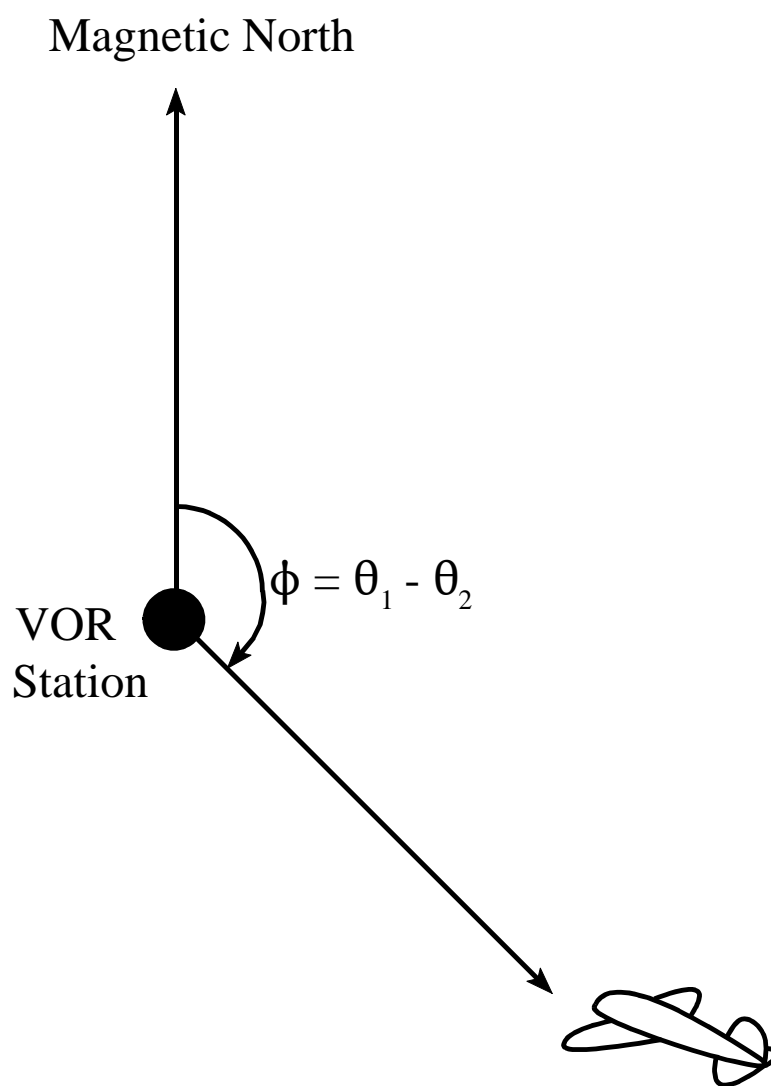
$$m(t) = 1 + 0.3 \cos(2\pi f_m t + \theta_1) + 0.3 \cos[2\pi f_{sc} t + 16 \cos(2\pi f_m t + \theta_2)] \quad (2.2)$$

where:  $f_m$  is 30 Hz  
 $f_{sc}$  is a subcarrier at 9960 Hz  
 $\phi = \theta_1 - \theta_2$  is the bearing angle

Equation 2.2 shows that the modulation index of the FM part of the signal is 16, yielding a peak-to-peak deviation of 960 Hz around the 9960 Hz subcarrier. A basic block diagram of a VOR receiver is shown in Fig. 2.1 [Kayton & Fried, 1969]. The output is bearing from the station, which yields a line of position (Fig. 2.2).



**Figure 2.1 Basic VOR Receiver Block Diagram**



**Figure 2.2 Illustration of Line of Position**

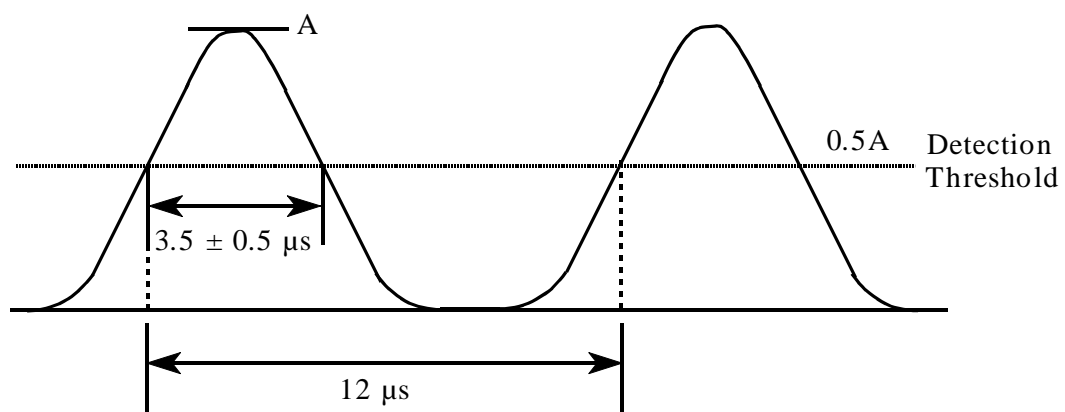
### 2.3.2 Distance Measuring Equipment (DME)

Using Distance Measuring Equipment (DME), a user can determine radial distance from the ground station. This is accomplished by a series of interrogations by the airborne receiver and replies by the ground station. The airborne receiver transmits pulse pairs that are approximately Gaussian in shape (see Fig. 2.3). The frequency range for the interrogations is 1025-1150 MHz [Forsell, 1991]. The ground station transmits a reply pulse pair after a delay of 50  $\mu$ s. The frequency ranges for the replies are 962-1024 MHz and 1151-1213 MHz. Upon receiving the reply pulse pair, the receiver determines the distance from the DME station as:

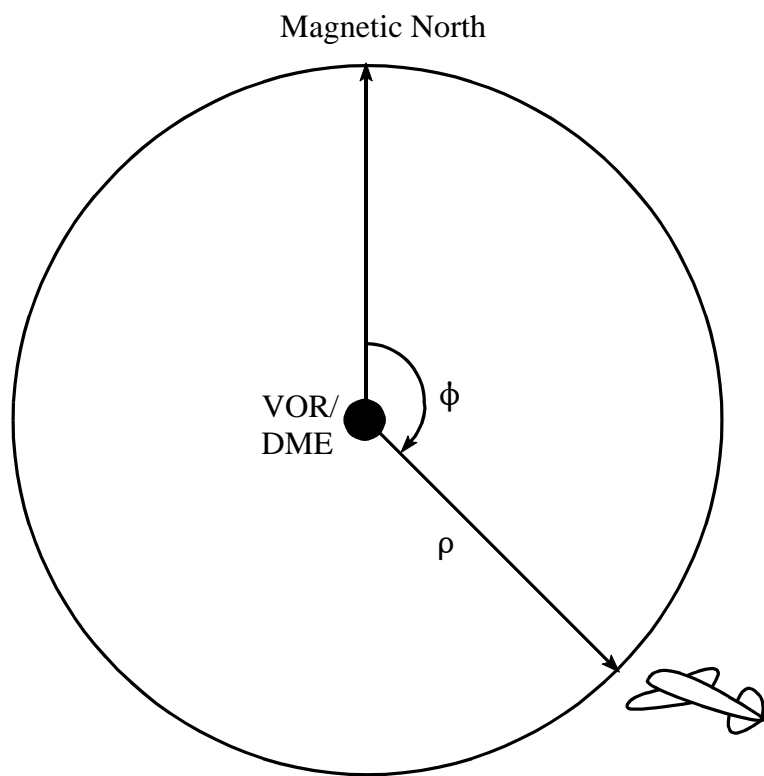
$$Distance = \frac{c}{2}(t_{roundtrip} - 50\mu s) \quad (2.3)$$

The airborne receiver transmits 120-150 pulse pairs per second in the search mode. Not all of the interrogations trigger replies because the ground station serves other DME users in the area — up to 100 total. After an interrogation is accepted, the ground station does not reply to other interrogations received within the next 60  $\mu$ s. However, each interrogator can select the correct reply pulses and eliminate all the others from consideration by averaging over time. Furthermore, by jittering the pulse pair rate, the airborne receiver can enhance the search for the appropriate reply pulses. After tracking has been established, the receiver slows to 24-30 pulse pairs per second.

In a two-dimensional application, a combination VOR/DME provides enough information to locate the aircraft (see Fig. 2.4). The distance from the DME places the user on a



**Figure 2.3 Illustration of DME Pulse Pair**



**Figure 2.4 Locating Position Using a VOR/DME**



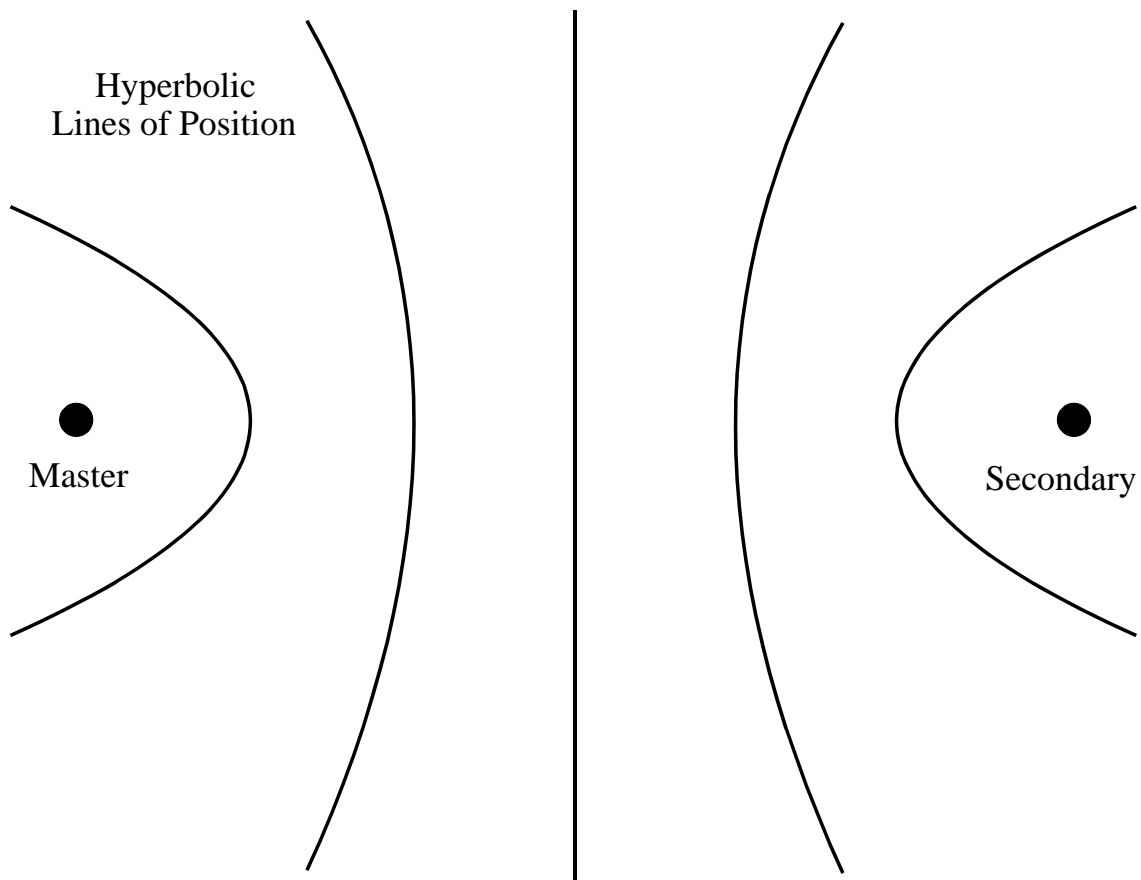
circle, and position is uniquely determined by the outbound radial from the VOR. Such a navigation aid is referred to as a  $\rho$ - $\theta$  system [Forssell, 1991].

The DME description herein relates to conventional DME use. A higher precision DME system (DME/P) has been designed for use with the microwave landing system (MLS) [Kelly, 1984]. The pulses exhibit quicker rise times in order to improve accuracy, but the system requires more bandwidth than conventional DME.

### **2.3.3 Long Range Navigation (LORAN-C)**

LORAN-C uses chains of up to five ground stations to provide the user with a position fix. A master station transmits a pulse, which is received by the user and the secondary stations in the chain. After receiving this pulse, the secondary stations transmit identifying pulses. The result is a unique time difference (TD) for each master-secondary pair that yields a hyperbolic line of position (Fig. 2.5).

LORAN-C is a low frequency system (~100 kHz) which does not require a line-of-sight for the signal to be received. The appropriate signal is a ground wave although unwanted sky waves may also be received. The transmitted pulse can be represented as [WGA Publication No. 1, 1976]:

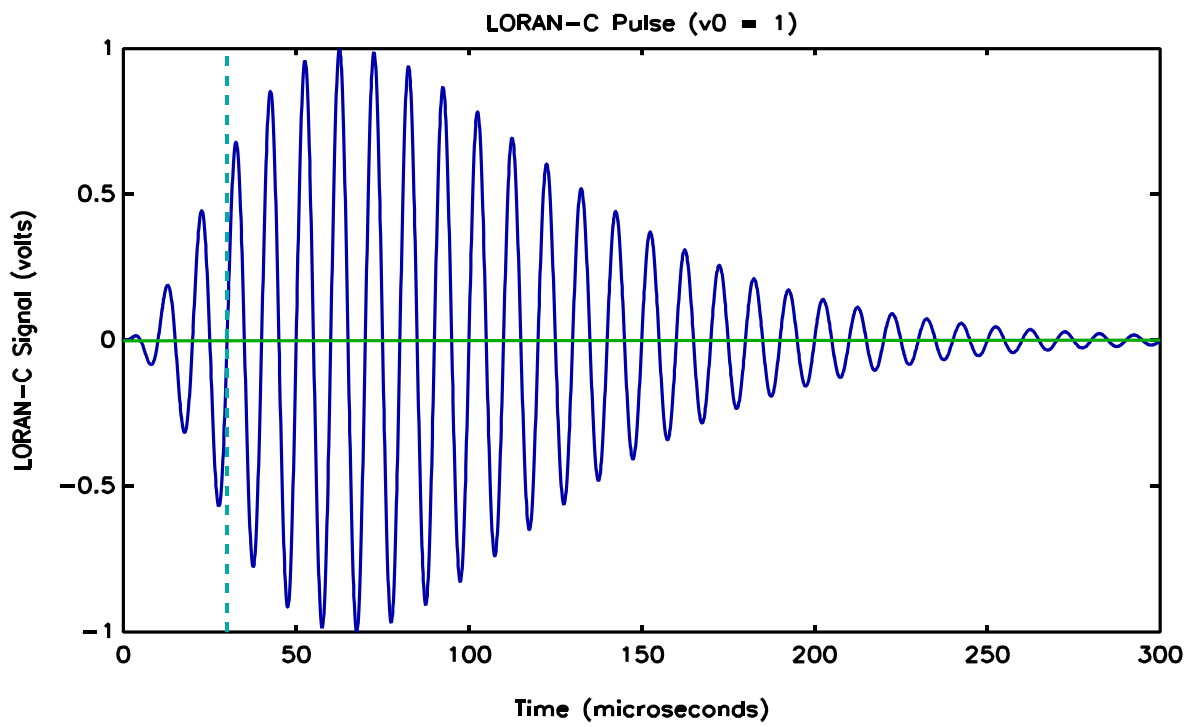


**Figure 2.5 LORAN-C Hyperbolic Lines of Position**

$$v(t) = v_0 \left( \frac{t}{t_p} \right)^2 \exp \left( 2 - 2 \frac{t}{t_p} \right) \sin(2\pi f_0 t + n\pi) \quad (2.4)$$

where:  $v(t)$  is the LORAN-C Pulse  
 $v_0$  is a constant  
 $t_p$  is the rise time of the pulse (65  $\mu$ s)  
 $f_0$  is the carrier ( $\sim$ 100 kHz)  
 $n = 0$  or  $1$

The nominal tracking point is the zero crossing marked in Fig. 2.6, which is 30  $\mu$ s after the beginning of the pulse. The master and secondary stations in a chain transmit a series of such pulses. The time difference measured by a user is the time between reception from the master station and the time of reception from a secondary station (accounting for a coding delay). This time difference places the user on a hyperbola that runs between the master station and the secondary station. The user can determine horizontal position by using measurements from the master station and at least two secondary stations.



**Figure 2.6 LORAN-C Pulse (n = 0)**

## 2.4 Satellite Navigation Systems

### 2.4.1 TRANSIT

TRANSIT is an early satellite navigation system developed by the United States Navy. In 1957, the Doppler frequency shift was used to determine the position of Sputnik I in orbit by researchers at the Applied Physics Laboratory at Johns Hopkins University [Black, 1981]. This was accomplished by using Doppler measurements from Sputnik I made by receivers at surveyed locations. By reversing the problem and using known satellite orbits, users can determine position by making repeated Doppler measurements to the same satellite.

The TRANSIT system typically has at least five satellites in polar orbits at an altitude of 1075 km above the surface of the Earth. The orbital velocity is about 7300 m/s (~1 hr 45 min orbital period), and the maximum Doppler shift of the 400 MHz signal is  $\pm 8.3$  kHz. Actually, the transmitted signal is at a frequency of 399.968 MHz to ensure that the frequency of the received signal will be below the locally generated 400 MHz reference signal [Forsell, 1991]. TRANSIT satellites also transmit a signal at 149.988 MHz, which allows for an ionospheric correction to be made.

The idea of Doppler positioning is to measure the change in distance to the satellite during a particular interval. Thus, TRANSIT is a range-rate system. Following [Forsell, 1991], the frequency of the received signal is shifted due to the Doppler effect:

$$f_{received} = f_{transmitted} \left( 1 + \frac{v_r}{c} \right) \quad (2.5)$$

where:  $f_{\text{received}}$  is the frequency of the received signal (Hz)  
 $f_{\text{transmitted}}$  is the frequency of the signal transmitted by the satellite (Hz)  
 $v_r$  is the component of satellite velocity directed toward the user (m/s)  
 $c$  is the speed of light (m/s)

The difference between the Doppler shifted signal and the reference signal is then integrated:

$$count = \int_{t_1 + \Delta t_1}^{t_2 + \Delta t_2} f_{\text{ref}} - f_{\text{transmitted}} \left( 1 + \frac{v_r}{c} \right) dt \quad (2.6)$$

where:  $count$  is the result of integrating over one epoch (carrier cycles)  
 $t_1$  is the time of transmission at the beginning of the epoch (s)  
 $t_2$  is the time of transmission at the end of the epoch (s)  
 $\Delta t_1$  is the travel time of the signal at the beginning of the epoch (s)  
 $\Delta t_2$  is the travel time of the signal at the end of the epoch (s)  
 $f_{\text{ref}}$  is the reference signal generated by the receiver (Hz)

Performing the integration yields:

$$count = (f_{\text{ref}} - f_{\text{transmitted}}) \cdot (t_2 - t_1 + \Delta t_2 - \Delta t_1) - \frac{f_{\text{transmitted}}}{c} \int_{t_1 + \Delta t_1}^{t_2 + \Delta t_2} v_r dt \quad (2.7)$$

The integral represents the change in range from the satellite to the user during the epoch:

$$\Delta R = \int_{t_1 + \Delta t_1}^{t_2 + \Delta t_2} v_r dt = c(\Delta t_1 - \Delta t_2) \quad (2.8)$$

where  $\Delta R$  is the change in range during the epoch. Substituting into Eq. 2.7 yields:

$$count = (f_{ref} - f_{transmitted}) \cdot (t_2 - t_1) + f_{ref}(\Delta t_2 - \Delta t_1) \quad (2.9)$$

The first term on the right hand side represents a constant offset which is determined by multiplying the differential frequency by the elapsed time during the epoch. The second term on the right hand side represents a time difference very similar to what is implemented in LORAN-C, a hyperbolic system. Three such time differences can be used to solve for the three-dimensional position of the user. In practice, a fourth measurement is made to correct for errors in the reference signal. Thus, position can be determined using four measurements from the same satellite, allowing time between measurements for the satellite to move to a new transmitting position.

#### 2.4.2 GPS and GLONASS

The U.S. Global Positioning System (GPS) and the Russian Global Navigation Satellite System (GLONASS) are considered together here because they are very similar systems. Each provides a constellation of satellites from which range measurements can be made. A detailed discussion of GPS is reserved for Chapter 3. For reference, Table 2.1 highlights the major characteristics of the two systems.

These are ranging systems similar in theory to the DME discussed in section 2.3.2, noting that GPS and GLONASS are one-way ranging systems while DME is a round trip system. Here we wish to solve for three-dimensional position by making range measurements to various

**Table 2.1 Comparison of GPS and GLONASS System Characteristics**

	<b>GPS</b>	<b>GLONASS</b>
Link Frequencies	L-Band (2)	L-Band (2)
Data Encoding	Spread Spectrum	Spread Spectrum
Satellite Identification	CDMA	FDMA
Constellation Size	24	24
Orbital Planes	6	3
Orbit Inclination	55°	64.8°
Orbital Altitude	20,200 km	19,100 km
Orbital Period	11 hr 58 min	11 hr 15 min
Data Rate	50 bps	50 bps



satellites. Each measurement places the user on a sphere centered at the satellite. The keys are knowing the position of the satellite at the time the signal is transmitted and having synchronized satellite clocks. Orbital parameters are transmitted to the user to allow for calculations of orbital position and satellite clock offset. The range measurements are offset by a bias in the receiver clock, which can be resolved by including a fourth measurement.

### 2.4.2.1 Satellite Pseudorange Measurements

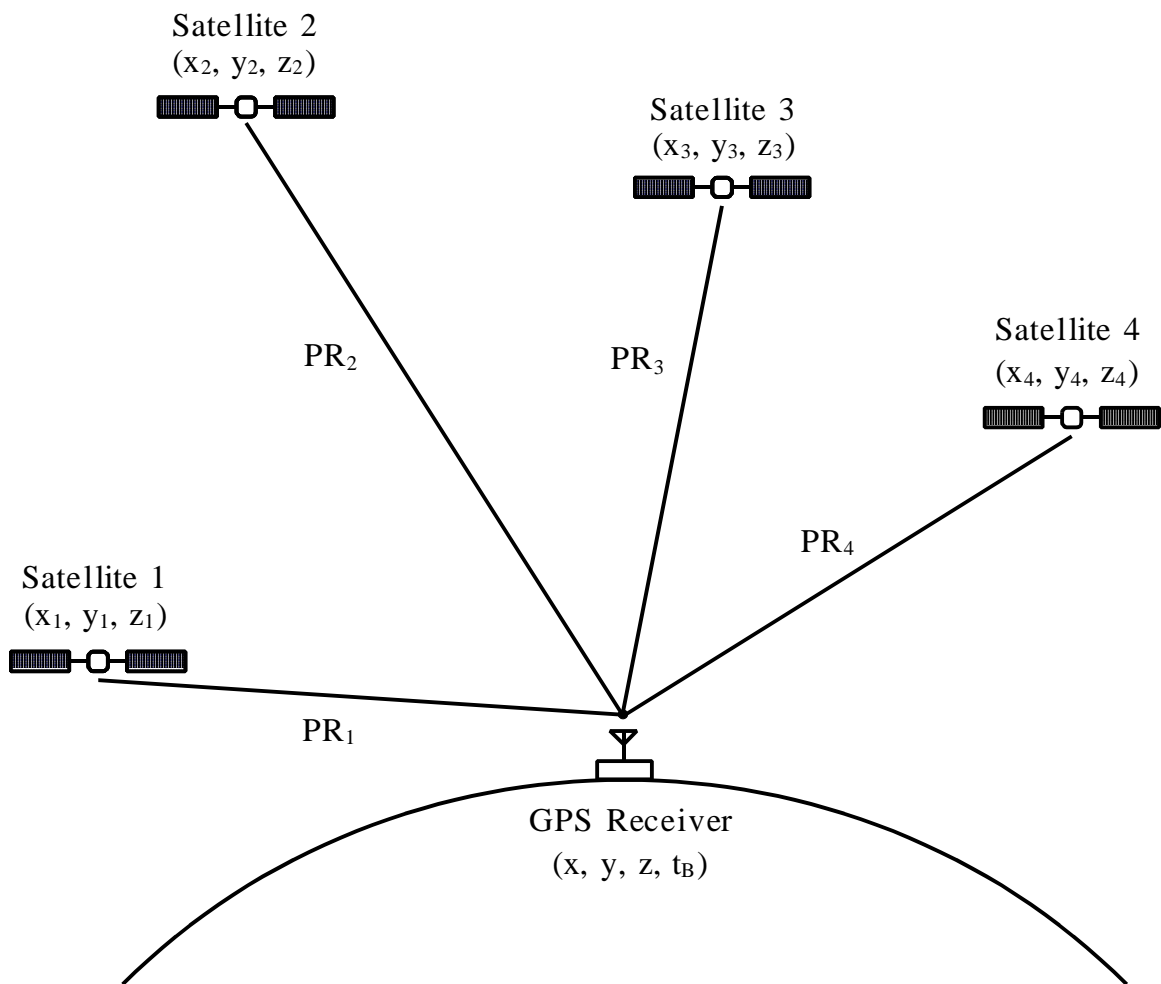
Satellite navigation depends on accurate range measurements in order to determine the position of the user. Because the receiver clock is generally not synchronized with GPS system time, the range measurements are erroneous and are therefore called pseudoranges [Milliken & Zoller, 1978]. There are other measurement errors as well, such as selective availability, but those will be discussed in Chapter 3. A pseudorange equation is thus:

$$PR_i = \sqrt{(X-X_i)^2 + (Y-Y_i)^2 + (Z-Z_i)^2} + ct_B \quad (2.10)$$

where:

- $PR_i$  is the pseudorange to the  $i^{\text{th}}$  satellite (m)
- $(X, Y, Z)$  is the unknown three dimensional user position (m)
- $(X_i, Y_i, Z_i)$  is the known three dimensional satellite position (m)
- $t_B$  is the unknown receiver clock offset (s)
- $c$  is the speed of light (m/s)

Because there are four unknowns, at least four measurements are required to solve for  $X, Y, Z$  and  $t_B$ , as illustrated in Fig. 2.7. Because the pseudorange equations are nonlinear, the system must be linearized in order to derive a linear relationship between pseudorange errors and



**Figure 2.7 Calculating Position Using GPS Pseudoranges**

position errors. Appendix A contains the procedure for linearizing the GPS pseudoranges using a first order Taylor series expansion. The result is a linear equation of the form:

$$\begin{pmatrix} \partial PR_1 \\ \partial PR_2 \\ \partial PR_3 \\ \partial PR_4 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & 1 \\ h_{21} & h_{22} & h_{23} & 1 \\ h_{31} & h_{32} & h_{33} & 1 \\ h_{41} & h_{42} & h_{43} & 1 \end{bmatrix} \begin{pmatrix} \partial X \\ \partial Y \\ \partial Z \\ ct_B \end{pmatrix} \quad (2.11)$$

where the  $h_{mn}$  elements represent direction cosines to each of the satellites. Eq. 2.11 can be used to iterate to a solution as described in Appendix A. Here we consider the minimum four required pseudorange measurements, but in practice all satellites in view could be included. In general, this equation can be written as:

$$\partial \underline{Y} = \mathbf{H} \partial \underline{\beta} \quad (2.12)$$

Thus, a receiver modifies an initial estimate of  $\underline{\beta}$  using  $\partial \underline{\beta}$  and iterates until convergence is achieved.

The column of ones in  $\mathbf{H}$  shows that the receiver clock offset ( $ct_B$ ) biases each pseudorange measurement by exactly the same amount. In practice, there may be interchannel biases that affect each measurement differently. However, great care is taken by receiver manufacturers to calibrate these effects.

## 2.5 Interoperability of Systems

Measurements from different navigation aids can be incorporated into a position solution [van Graas and Kuhl, 1991]. Range, bearing, and time difference measurements can all be used in a linearized equation to determine position. For these three measurements we have (two-dimensional case):

$$\text{Range:} \quad R_i = \sqrt{(X - X_i)^2 + (Y - Y_i)^2} \quad (2.13)$$

$$\text{Bearing:} \quad \theta_i = \tan^{-1} \left( \frac{X - X_i}{Y - Y_i} \right) \quad (2.14)$$

$$\text{Time Difference:} \quad TD_{ij} = \frac{b - R_i - R_j}{c} \quad (2.15)$$

where:

- (X,Y) is the user position
- (X<sub>i</sub>,Y<sub>i</sub>) is the position of transmitting station i
- R<sub>i</sub> is the distance between the user and station i
- θ<sub>i</sub> is the angle between the user and station i with respect to North
- TD<sub>ij</sub> is the elapsed time between signal arrival from stations i and j
- b is the straight line distance between the two stations

To determine position using Eqs. 2.13-15, an *a priori* estimate  $(\hat{X}, \hat{Y})$  is used as the center of a Taylor Series expansion. For example:

$$R_i = R_{i0} + \frac{\partial R_i}{\partial \hat{X}} \Big|_{(\hat{X}_0, \hat{Y}_0)} \partial X + \frac{\partial R_i}{\partial \hat{Y}} \Big|_{(\hat{X}_0, \hat{Y}_0)} \partial Y \quad (2.16)$$

where the higher order terms have been neglected. The initial position estimate  $(\hat{X}_0, \hat{Y}_0)$  is used to calculate the estimated distance to the station  $(R_{i0})$ . The terms  $\partial X$  and  $\partial Y$  are corrections to the estimated position. From the Taylor series we have:

$$\partial R_i = \begin{pmatrix} \frac{\hat{X} - X_i}{R_i} & \frac{\hat{Y} - Y_i}{R_i} \end{pmatrix} \begin{pmatrix} \partial X \\ \partial Y \end{pmatrix} \quad (2.17)$$

Similar linearizations are done in order to place the bearing and time difference equations into a form useful for updating the position estimate:

$$\partial \theta_i = \begin{pmatrix} \frac{\hat{Y} - Y_i}{R_i^2} & \frac{X_i - \hat{X}}{R_i^2} \end{pmatrix} \begin{pmatrix} \partial X \\ \partial Y \end{pmatrix} \quad (2.18)$$

$$\partial TD_{ij} = \left[ \frac{1}{c} \left( \frac{\hat{X} - X_j}{R_j} - \frac{\hat{X} - X_i}{R_i} \right) \quad \frac{1}{c} \left( \frac{\hat{Y} - Y_j}{R_j} - \frac{\hat{Y} - Y_i}{R_i} \right) \right] \begin{pmatrix} \partial X \\ \partial Y \end{pmatrix} \quad (2.19)$$

Any of Eqs. 2.17-2.19 can be expressed in the form:

$$\partial y_i = h_i \begin{pmatrix} \partial X \\ \partial Y \end{pmatrix} \quad (2.20)$$

where  $y_i$  is measurement  $i$  and  $h_i$  is the corresponding row vector for that measurement. Using multiple measurements yields a linear equation:

$$\partial \underline{Y} = H \partial \underline{\beta} \quad (2.21)$$

where  $\partial \underline{\beta}$  is the update to the user state vector. The geometry matrix  $H$  relates the measurements to the user position. Accordingly, measurement errors are translated to position errors using  $H$  (section 3.3 discusses this topic as it relates to the Global Positioning System). Eq. 2.21 can be used recursively to solve for the user state vector.

More states can be added to  $\underline{\beta}$  as necessary to solve for other parameters. For instance, an unknown clock offset in a range measurement would have to be solved for. The range equation becomes:

$$PR_i = R_i + ct_B \quad (2.22)$$

where  $PR_i$  is called a pseudorange and  $ct_B$  is the clock bias. Eq. 2.17 is altered to accommodate the additional unknown:

$$\partial PR_i = \begin{pmatrix} \frac{\hat{X} - X_i}{R_i} & \frac{\hat{Y} - Y_i}{R_i} & 1 \end{pmatrix} \begin{pmatrix} \partial X \\ \partial Y \\ ct_B \end{pmatrix} \quad (2.23)$$

For example, combining GPS and LORAN-C would require four measurements in order to solve for 2D position and two clock offsets, one for the GPS receiver and one for the LORAN-C receiver.