

Chapter 5. Clock Offset Due to Antenna Rotation

5.1 Introduction

The goal of this experiment is to determine how the receiver clock offset from GPS time is affected by a rotating antenna. Because the GPS signal is right hand circularly polarized (RHCP), a rotating antenna causes the received phase to wrap up. This error can be removed by solving for receiver clock offset, but the benefits of having a stable clock would be lost. Thus, the clock is treated as an independent source of measurements along with carrier phase measurements from the satellites. In this experiment, a rubidium oscillator was used to experimentally characterize the antenna rotation effect.

The following is a mathematical description of antenna-wave interaction highlighting the phase of the received signal. It is shown that a rotating antenna causes this phase measurement to be offset by the amount of rotation. Results are presented for two experiments using different types of GPS receivers.

5.2 Polarization of the GPS Transmitted Wave

The development herein closely follows the text *Polarization in Electromagnetic Systems* by Dr. Warren L. Stutzman. GPS transmits using right hand circular polarization. The RHCP wave can be described in terms of a polarization-phase vector, which is used here because it includes signal phase. The polarization-phase vector uses x and y components to describe a

signal in space propagating along the z axis. The normalized complex vector is used here for simplicity because the field intensity is not needed for this discussion. In general, we describe a wave as:

$$\hat{e}_w = \cos(\gamma)\hat{x} + \sin(\gamma)e^{j\delta}\hat{y} \quad (5.1)$$

where: \hat{e}_w is the normalized polarization-phase vector of the wave
 γ, δ are parameters that describe a polarization state
 \hat{x}, \hat{y} are unit vectors in the x and y directions

The x and y directions could be thought of as crossed dipoles which are fed a voltage depending on the parameters (γ, δ) . For instance, if $\gamma = 0^\circ$ and $\delta = 90^\circ$ the wave is linearly polarized along the x axis, which is typically chosen as the horizontal (y represents vertical). For circular polarization, the x and y components are equal in magnitude but 90° out of phase with each other. Thus, a RHCP wave is given by $\gamma = 45^\circ$ and $\delta = -90^\circ$, which yields:

$$\hat{e}_w = \frac{1}{\sqrt{2}}(\hat{x} - j\hat{y}) \quad (5.2)$$

This vector represents the polarization of a RHCP wave in terms of x and y components at $z = 0$.

5.3 Polarization of the GPS Antenna

A RHCP antenna has the same complex vector representation given in Eq. 5.2. This result is now generalized for a rotating antenna. Let θ be the angle of rotation in degrees at a

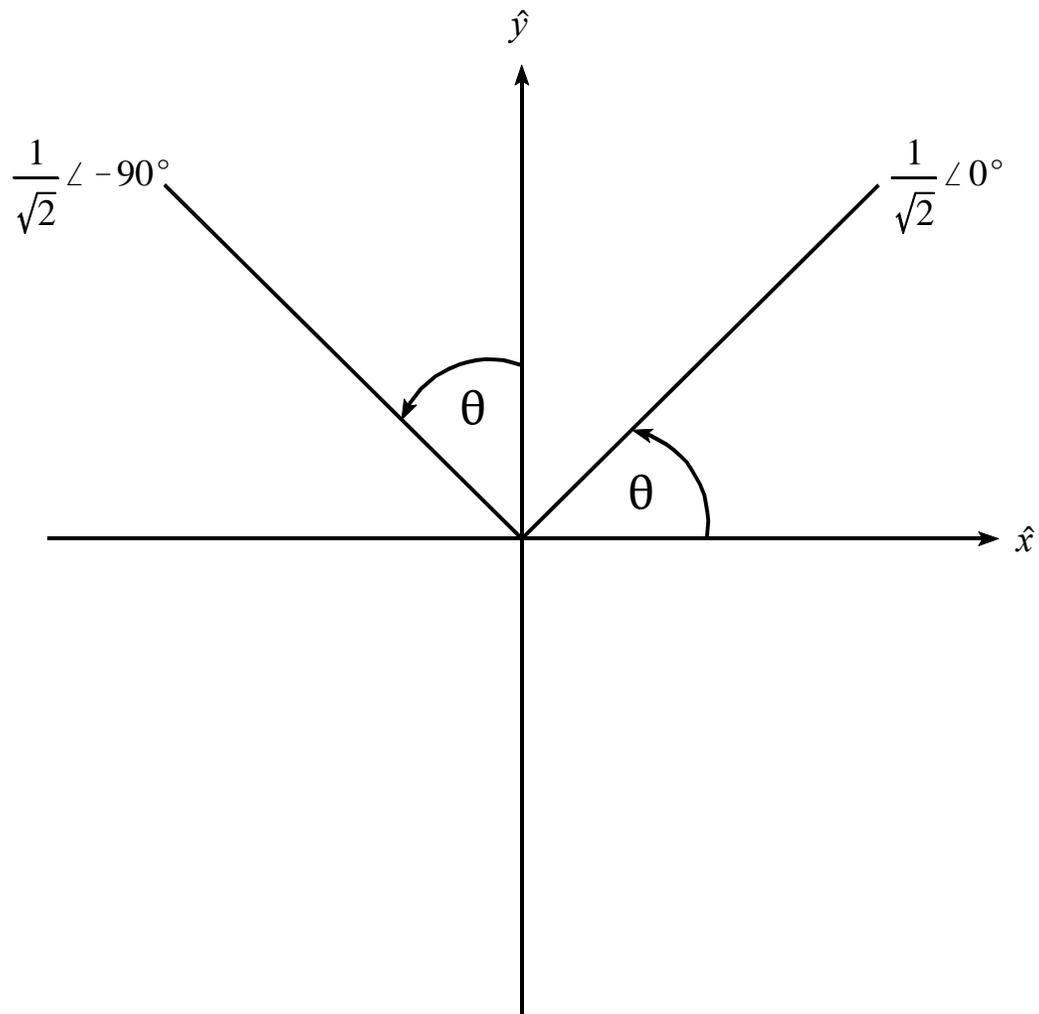


Figure 5.1 Polarization-Phase Representation of a Rotating RHCP Antenna

given point in time (Fig. 5.1). Note that θ is assumed to be in the counter-clockwise direction (CCW). The x and y components are rotated yielding a normalized complex vector that is a function of the rotation angle:

$$\hat{e}_a = \frac{1}{\sqrt{2}}(\cos\theta + j\sin\theta)\hat{x} + \frac{1}{\sqrt{2}}(\sin\theta - j\cos\theta)\hat{y} \quad (5.3)$$

The phase of the received signal can be determined from Eqs. 5.2 and 5.3.

5.4 Antenna-Wave Interaction

The normalized complex voltage of the received signal is determined from Eqs. 5.2 and 5.3:

$$\begin{aligned} \hat{e}_w \cdot \hat{e}_a^* &= \frac{1}{\sqrt{2}}[\hat{x} - j\hat{y}] \cdot \frac{1}{\sqrt{2}}[(\cos\theta - j\sin\theta)\hat{x} + (\sin\theta + j\cos\theta)\hat{y}] \\ &= \frac{1}{2}(\cos\theta - j\sin\theta - j\sin\theta + \cos\theta) \\ &= \cos\theta - j\sin\theta \end{aligned} \quad (5.4)$$

The complex conjugate is necessary because \hat{e}_a is defined for the antenna in transmitting mode. Consider two coordinate systems, one for the incoming wave and one for the antenna. The positive z axis for the wave is in the direction of propagation. For the antenna, the positive z axis is in the direction of a wave propagating from the antenna while transmitting. When a wave in space is traveling towards the antenna, the positive z axes of the two coordinate systems point at each other. We use the complex conjugate of \hat{e}_a to describe the antenna in receiving mode.

Thus, Eq. 5.4 yields the normalized complex voltage of the received signal, which can be represented by its magnitude and phase by Euler's formula for complex numbers:

$$\begin{aligned}v &= \cos\theta - j\sin\theta \\ &= 1 e^{-j\theta}\end{aligned}\tag{5.5}$$

This is the familiar polar form of a complex number. Equation 5.5 shows that the phase of the received signal is equal in magnitude to the rotation angle θ , but opposite in sign. This indicates that as the antenna is rotated in the counter-clockwise direction, an increasingly negative phase is induced in the received signal for a RHCP wave. The same phase offset would occur for a linearly polarized receiving antenna (see Appendix B).

This offset affects each phase measurement equally. A clock offset would also cause a common error in all the carrier phase measurements. Thus, the offset due to antenna rotation is virtually indistinguishable from an error in the GPS receiver clock. Also, note that as the antenna is rotated past 360° the phase error continues to wrap up. This is due to the cumulative nature of GPS carrier phase measurements (integrated Doppler).

As soon as the receiver achieves signal lock, it begins adding up carrier cycles based on the Doppler frequency shift. The advance in carrier phase during each epoch is determined by integrating the Doppler shift during the epoch. To get the total accumulated phase, the carrier phase advance for a given epoch is added to the value at the previous epoch [Diggle, 1994]:

$$\phi_k = \phi_{k-1} + \int_{t_{k-1}}^{t_k} f_D(\tau) d\tau + \phi_r \quad (5.6)$$

where: ϕ_k is the accumulated phase at epoch k
 f_D is the Doppler frequency as a function of time
 ϕ_r is the fractional carrier phase measured at the end of the epoch

This shows that as the antenna is rotated, carrier cycles are added to or lost from Eq. 5.6 depending on the direction of rotation. For instance, at the L_1 frequency of 1575.42 MHz each revolution causes an offset of 0.19 meters. This offset appears as a clock error because it is common to all of the carrier phase measurements.

To verify that Eq. 5.4 is correct, the polarization efficiency can be calculated. When the transmitted wave and receiving antenna are perfectly matched, the polarization efficiency is unity. This should hold even for a rotated antenna as long as the polarization is circular. The polarization efficiency is calculated as [Stutzman, 1993]:

$$\begin{aligned} \rho &= |\hat{e}_w \cdot \hat{e}_a^*|^2 \\ &= |\cos\theta - j\sin\theta|^2 \\ &= 1 \end{aligned} \quad (5.7)$$

for any θ .

5.5 Antenna Rotation — Experiment One

In the first experiment a RCHP antenna was used with a single channel receiver, the Stanford Telecommunications, Inc. Time Transfer System (TTS-502B). A rubidium oscillator was connected through an external frequency input. This receiver was modified so that up to ten measurements per second were possible.

The received carrier phase, or integrated Doppler, was the observable of interest in this experiment. The rotation of the antenna can be seen, but only if selective availability is turned off. It would be extremely difficult to distinguish antenna rotation from selective availability without differencing measurements from two receivers. Thus, a Block I satellite (SV PRN 12) was used for the experiment because it does not have SA capability. Another Block I satellite, SV PRN 3, had been declared unhealthy by the Department of Defense three months prior to this experiment and hence was not used. The remainder of the Block I satellites had all been declared unhealthy by May of 1993.

The experiment was conducted at Stocker Engineering and Technology Center on the campus of Ohio University. The TTS-502B receiver is located on the second floor in the Avionics Engineering Center. A cable connects the receiver to a RCHP antenna on the roof of Stocker Center. Figure 5.2 illustrates the setup for experiment one. In this experiment, the antenna was bolted to a ground plane which was mounted on a rotating stool. The stool was then placed on a rod to elevate the antenna a few feet. This minimized the effect of several masking features on the roof of Stocker Center and allowed for maximum satellite visibility. The

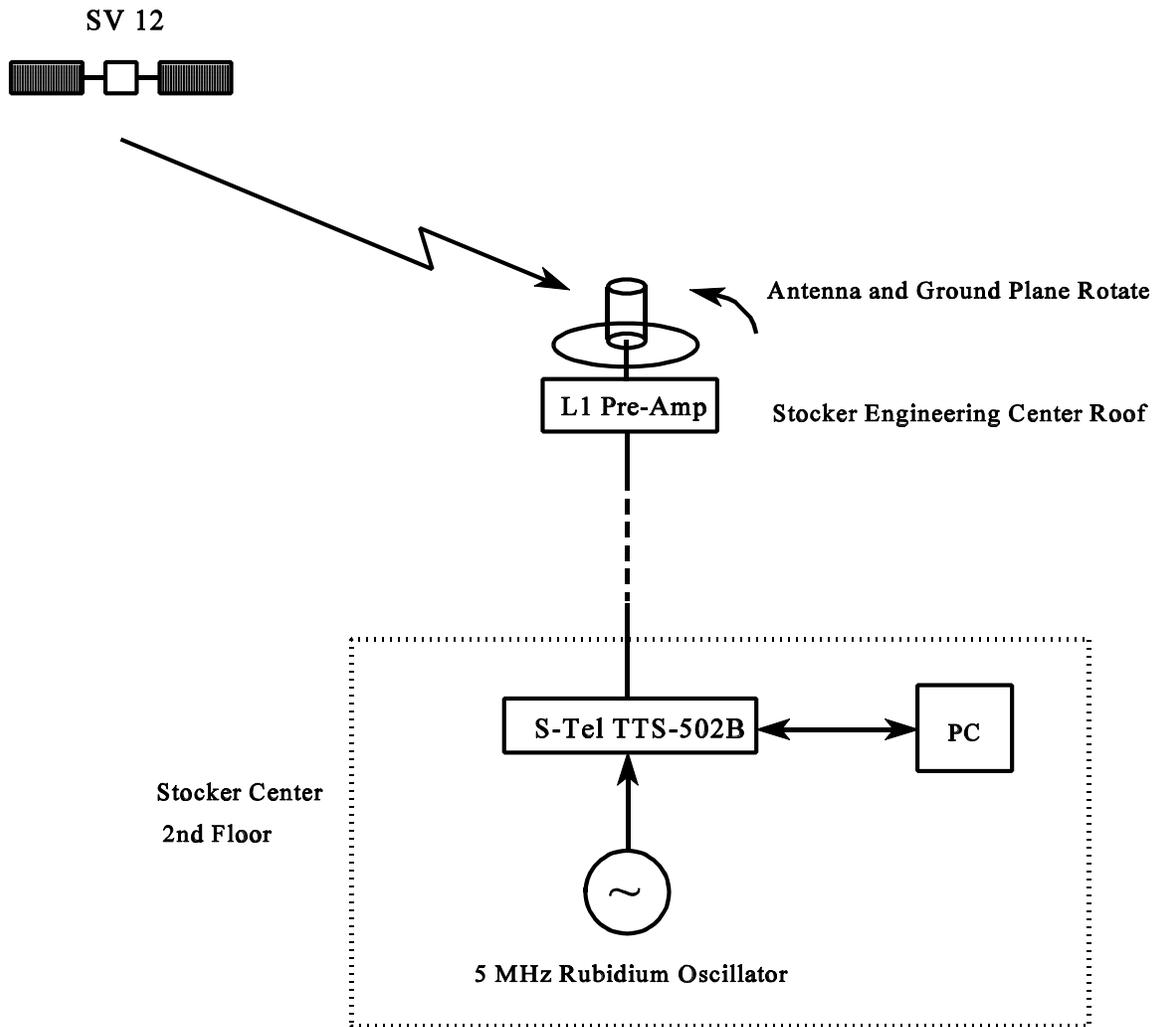


Figure 5.2 Setup for Antenna Rotation Experiment One

experiment began at approximately 3:40 pm on June 8, 1994. The elevation angle of SV 12 was about 30-35°. After the receiver achieved signal lock, the antenna was rotated ten times in the counter-clockwise direction (CCW as viewed from above). After a pause of ten to fifteen seconds the antenna was turned back to its original position. Following another pause of ten seconds, the antenna was rotated ten times in the clockwise direction and similarly turned back after about ten seconds.

The result is shown in Figs. 5.3 - 5.5. The integrated Doppler (Fig. 5.3) looks linear at first glance but the scale is large. Recall from Eq. 5.6 that the carrier phase measurement is an accumulated phase based on the Doppler frequency shift. In Fig. 5.3 a change of over 70 km is seen in about four minutes. Here the phase is scaled by the wavelength, i.e. 2π radians is equal to the L_1 wavelength of 0.19 meters. It is impossible to detect ten antenna rotations here because the offset is only 1.9 meters. Thus, a numerical fit to the data was used as a comparison so that higher order effects could be seen. A first order fit to the data proved to be useless because there is a substantial second order effect. Thus, a second order fit was used.

Fig. 5.4 depicts the higher order effects that remain after a second order fit was subtracted from fig. 5.3. There are two noticeable effects, one being the antenna rotation and the other being an oscillation. The oscillation could be a systematic error in the receiving system that introduces higher order harmonics. Another possibility is scintillation in the ionosphere or perhaps harmonics due to the aging clocks on this Block I satellite. This oscillation was removed

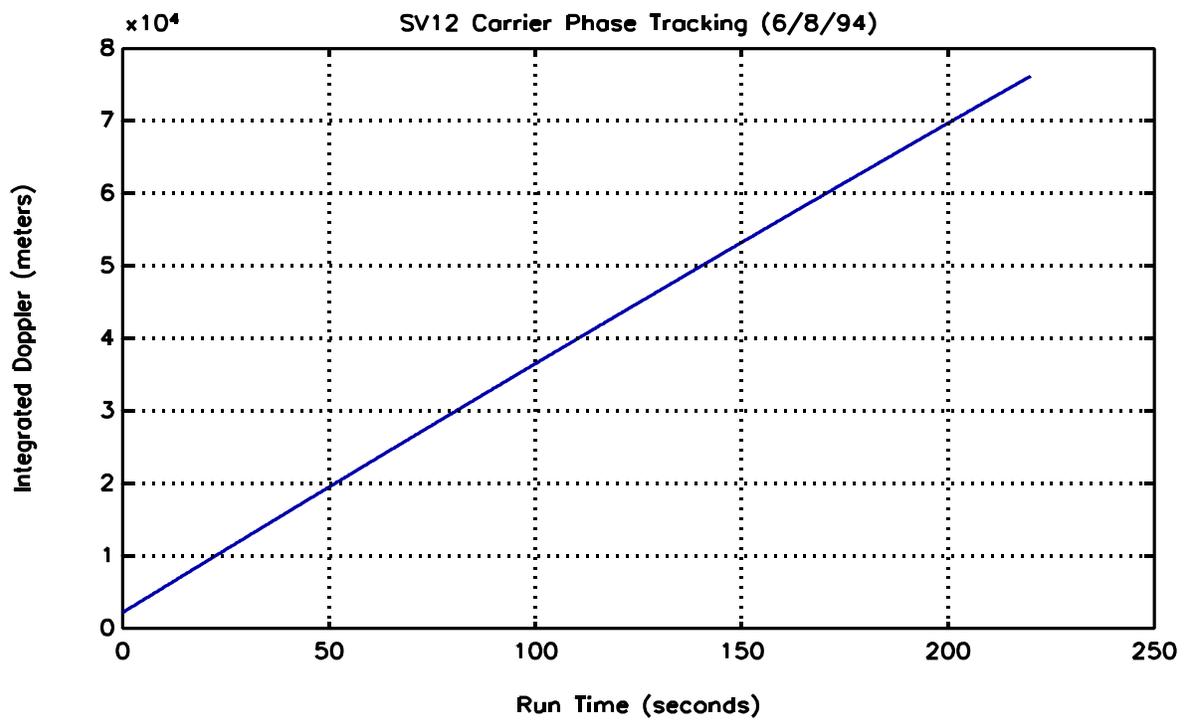


Figure 5.3 SV 12 Carrier Phase Measurement (Integrated Doppler) Using TTS-502B Receiver

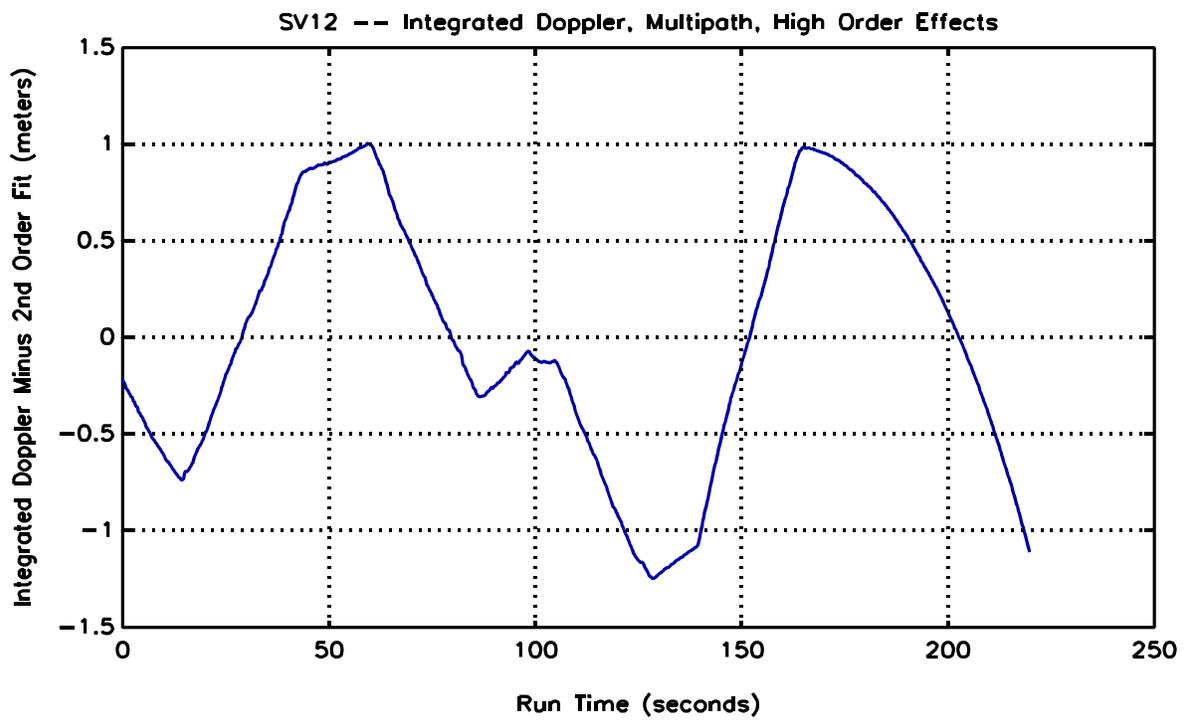


Figure 5.4 SV 12 Carrier Phase Measurement (Integrated Doppler) Minus a Second Order Fit

numerically to make the clock offset due to antenna rotation more visible. The result is seen in Fig. 5.5.

The first pulse in Fig. 5.5 was generated by ten CCW rotations followed by unwinding in the CW direction. As θ grows from 0 to 3600° , the offset grows from 0 to +1.9 m or about ten L_1 wavelengths. By Eq. 5.7, the phase offset from ten rotations is $-\theta$ or -3600° . If the actual range to the satellite contains N cycles, then the range after spinning appears to contain $N+10$ cycles. This offsets the range by +1.9 m as observed in Fig. 5.5. Likewise, when the antenna is rotated in the clockwise direction (θ goes from 0 to -3600°) the phase change is $+3600^\circ$. Thus, the range appears to contain $N-10$ cycles and the offset is -1.9 m as seen in the second pulse of Fig. 5.5.

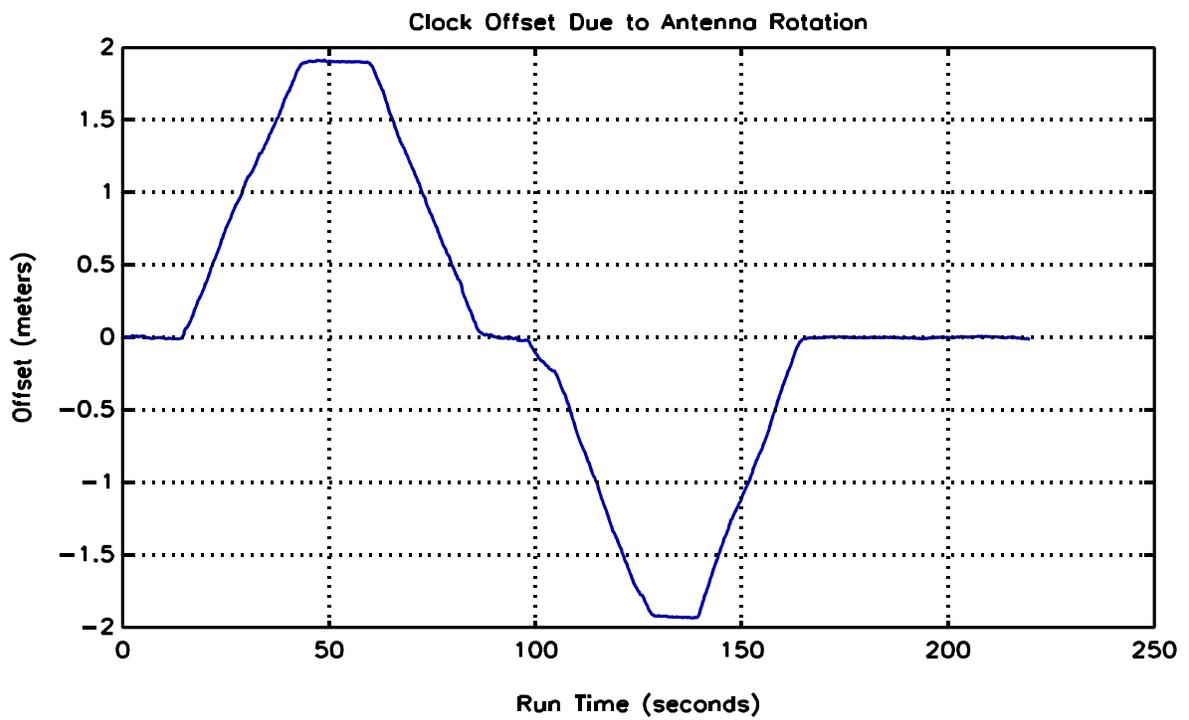


Figure 5.5 SV 12 Carrier Phase Measurement (Integrated Doppler) with Oscillation Numerically Removed

5.6 Antenna Rotation — Experiment Two

Because of the oscillations of undetermined origin in experiment one, a second experiment was conducted in which two Ashtech Z-12 receivers were used. Each is equipped with an external frequency input which can be requested as an add-on to the base model Z-12. Fig. 5.6 illustrates the setup for experiment two. Carrier phase measurements are made by each receiver, and the difference between the two — the single difference (SD) — is the observable of interest. Recall Eq. 3.14 which defines the single difference:

$$SD = \Delta\phi_{received} + \Delta N + \Delta S_n + f \cdot \Delta t_{bias} \quad (5.8)$$

Notice from Fig. 5.6 that the receivers share a common frequency input, but different antennas. Thus, spinning one antenna will cause an offset in received signal phase for one receiver only. The other receiver is a reference and the antenna it uses remains stationary. Thus, the effect of antenna rotation will be evident in the $\Delta\phi_{received}$ term of the single difference. Note that spinning both antennas in an identical fashion would have a very limited effect on the single difference, if any.

As the experiment was conducted, it became apparent that the Z-12 uses dual frequency processing to take advantage of both link frequencies. Specifically, the Z-12 appears to check L_1 and L_2 signals for inconsistencies. The L_1 wavelength is 19 cm and the L_2 wavelength is 24.5 cm. When the antenna is moved from one point to another, this causes a change in the received phase of the L_1 and L_2 signals that is proportional based on the different wavelengths. Spinning the antenna causes a different offset in the carrier phase of each signal. This can be

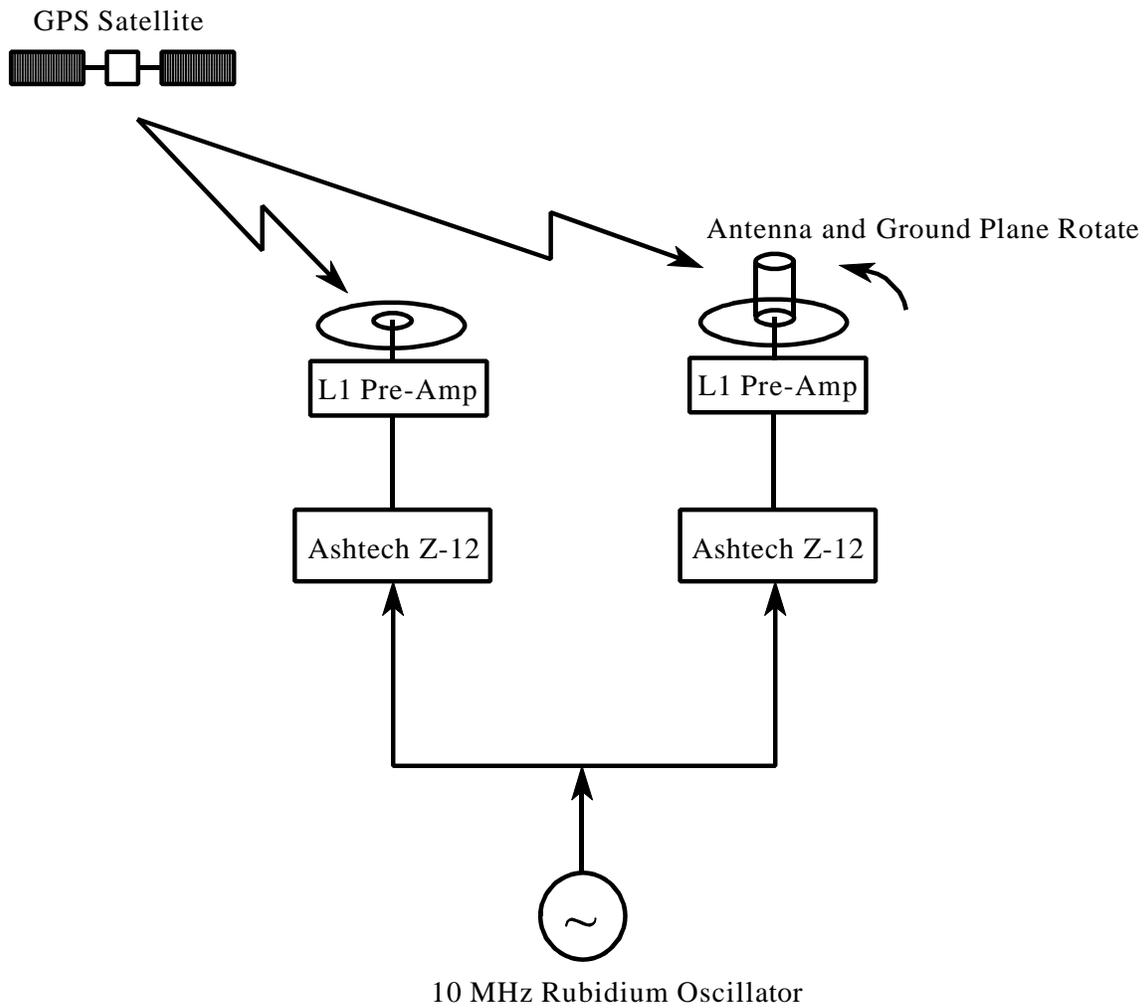


Figure 5.6 Setup for Antenna Rotation Experiment Two

understood by considering a rotation of one full turn. The Z-12 sees a change of 19 cm in the carrier phase of the L_1 signal, but a 24.5 cm change in the carrier phase of the L_2 signal. This discrepancy causes the Z-12 to discard the signals and reacquire the satellites in view. Because this experiment involved looking at the single difference over time, this periodic reset by the receiver disrupted the experiment.

The solution to the problem was to use an L_1 antenna and pre-amplifier so that the receiver could not lock on to the L_2 signal. This effectively disabled the dual frequency processing. For the reference receiver, the L_1 pre-amplifier was not absolutely necessary. However, it made sense to try to make everything the same for the two receivers, except for the rotating antenna. Thus, an L_1 pre-amplifier was used for the reference receiver as well.

The 10 MHz rubidium oscillator was connected by way of a splitter to the external frequency input to each receiver. The rubidium clock requires 22 - 32 volts DC and a power supply was used which was set to 28 volts DC. Each receiver recorded carrier phase measurements to internal memory once per second. This data was later downloaded to a personal computer for post processing. It should be noted that as an alternative, data can be stored real-time on a laptop computer by means of a serial port on the Z-12.

The simultaneous carrier phase measurements were differenced, yielding a single difference (Eq. 5.8) for each satellite in view. The combined integer ambiguity term, ΔN , is a constant but takes on a different value for each satellite. The initial integer ambiguities were

removed from each of the single differences which allows a direct comparison on the same scale. The result is plotted in Fig. 5.7.

During approximately twenty minutes of data collection, a variety of antenna rotations were performed. The initial uptick less than two minutes into the experiment was due to ten CCW rotations. The maximum number of rotations in either direction was ten, representing a maximum offset of 1.9 m (ten L_1 wavelengths). The direction, or polarity of the offset in Fig. 5.7 depends on which carrier phase measurement is used as the reference. In this case, the receiver connected to the fixed antenna was used as the reference. Note that the maximum deflection of ten rotations occurred five times, twice in the CCW direction and three times in the CW direction.

There is a drift of about eight meters over the period of data collection that is not related to antenna rotation. Although each receiver used the same 10 MHz frequency standard, the tracking loops in the two receivers behave differently. This effect is discussed in detail in Chapter 6.

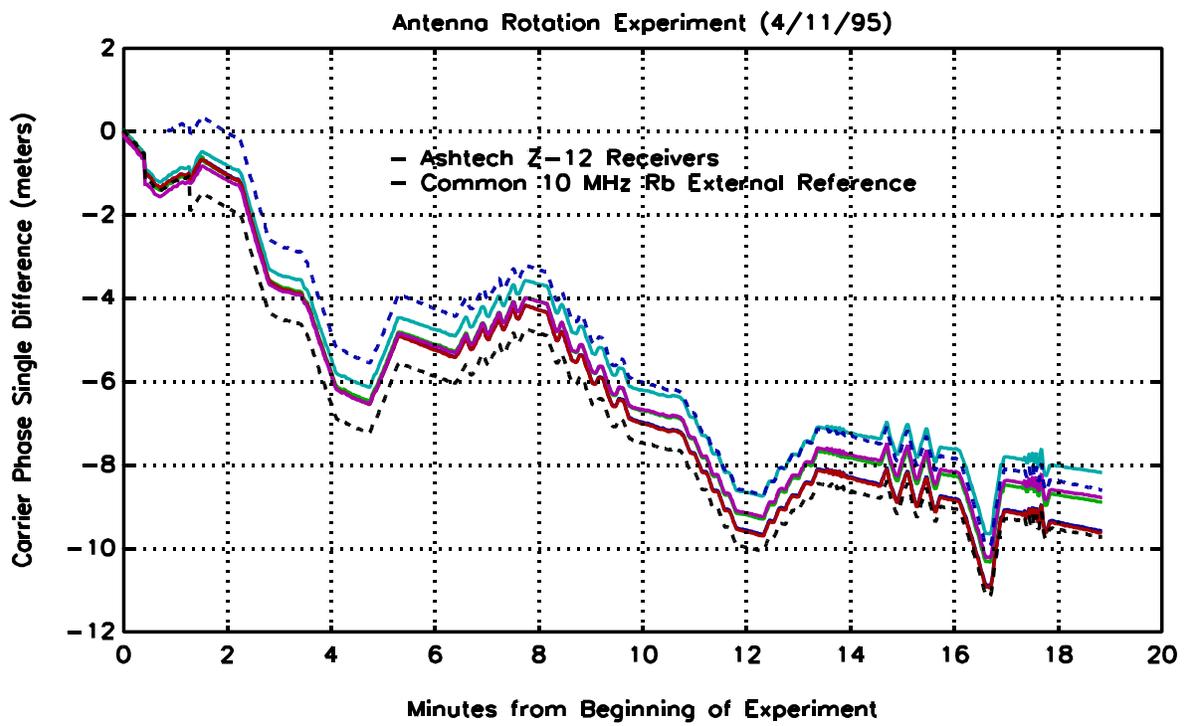


Figure 5.7 Carrier Phase Single Differences from Z-12 Antenna Rotation Experiment

5.7 Conclusions From Antenna Rotation Experiments

A significant clock error would occur if, for example, an aircraft coasted on the clock from the downwind leg to final approach. This 180° turn would cause an offset of close to 10 cm on L_1 , which could cause an alarm if not corrected. Similarly, if the clock is used as an independent measurement to improve system availability and integrity during the landing phase, the accuracy degradation due to turning could be unacceptable. Therefore, this problem needs to be accounted for in DGPS landing systems that rely in part on clock aiding.

One DGPS system that has been proposed utilizes a pseudolite on the ground for carrier cycle ambiguity resolution. If an aircraft is turning while receiving signals from below and above, the clock offset would be doubled. This could be verified experimentally by spinning the Z-12 antennas at the same rate in opposite directions.