

# Control of Dynamic Response of Thin-Walled Composite Beams using Structural Tailoring and Piezoelectric Actuation

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## (ABSTRACT)

A dual approach integrating structural tailoring and adaptive materials technology and designed to control the dynamic response of cantilever beams subjected to external excitations is addressed. The cantilevered structure is modeled as a thin-walled beam of arbitrary cross-section and incorporates a number of non-classical effects such as transverse shear, warping restraint, anisotropy of constituent materials and heterogeneity of the construction.

Whereas structural tailoring uses the anisotropy properties of advanced composite materials, adaptive materials technology exploits the actuating/sensing capabilities of piezoelectric materials bonded or embedded into the host structure. Various control laws relating the piezoelectrically-induced bending moment with combined kinematical variables characterizing the response at given points of the structure are implemented and their effects on the closed-loop frequencies and dynamic response to external excitations are investigated. The combination of structural tailoring and control by means of adaptive materials proves very effective in damping out vibration.

In addition, the influence of a number of non-classical effects characterizing the structural model on the open and closed-loop dynamic responses have been considered and their role assessed.

*Dedicated*

*to my Father and Mother*

## ACKNOWLEDGEMENTS

It is my privilege and honor to present Dr. Liviu Librescu and Dr. Leonard Meirovitch, University Distinguished Professor, as my committee chairmen. They were always resourceful and encouraging with personal generosity, to whom I express all my gratitude. Dr. Librescu's enthusiasm was contagious to me, and from the discussions and interactions with him, new ideas necessary for the progress and achievement of this work have emerged. I also thank my committee members: Dr. Inman, director of Center for Intelligent Materials, Systems and Structures, Dr. Hyer, director of The NASA-Virginia Tech Composite Program, and Dr. Renardy, for showing a sincere interest in my work and giving me valuable advices.

Upon the completion of my doctorate work, the final destination of my education, I wish to thank most of all my father, Jong-Taek Na, and my mother, Sang-Hyun Kim. They have been always supporting and sacrificing for me at all levels; they made my education accomplished and allowed me to achieve this degree. They surely deserve all the credit.

My wife, Dong-Eun has been a caring, loving, brilliant and patient helper who provided endless encouragement to finish this research work, putting my dreams before her own. Because of her love, friendship and faith in me, I should have added my wife's name as a co-author on my dissertation. She shared all the difficulties and disappointments, and kept the hope of a happy-ending to this trip alive.

My wife and I witnessed two adorable daughters' birth and growth in Blacksburg. Hosanne(Sangwon) and Joanne(Sang-Yoon) have been always there for me to cheer up and constant reminders of what is truly important in my life.

Appreciation is also expressed to my brother Kyungsoo Na, father and mother in-law, and sisters and brothers in-law who have always provided me encouragement, love and prayer. I hope they realize how much I appreciate their thoughtful care and concerns for me.

I surely had unforgettable memories in Blacksburg with my friends and would like to deliver my gratitude to: Pastor H. Chung's, Dr. B. K. Ahn's, Dr. S. G. Kim's, Dr. D. Y. Ohm's, Dr. S. H. Park, T. I. Hyun's, T. H. Park's, J. H. Park's and M. H. Kim. They have been good friends with me and my family, and showed true care and friendship. I have to admit that with their prayers, we move on to the next stage with pleasant memories, hopes and visions.

I also thank Dr. O. Song who didn't mind "knock-at-the-door" at midnight in the beginning of the research and lots of long-distance calls afterward for technical discussions. He willingly helped me to accomplish this research.

Finally, I wish to thank God for sustaining me throughout this work. He has served as a constant source of strength and wisdom ,and reinforced my belief that through God all things are possible.

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