APPENDIX ONE

THE KRONECKER PRODUCT OF TWO MATRICES

Let $A$ be a $p \times m$ matrix and $B$ a $q \times n$ matrix, let $a_{ij}$ denote the element in the $i^{th}$ row and $j^{th}$ column of $A$ and let $b_{rs}$ denote the element in the $i^{th}$ row and $j^{th}$ column of $B$. The $pq \times mn$ matrix with $a_{ij}b_{rs}$ as the element in the $(iq + r)^{th}$ row and the $(jn + s)^{th}$ column is called the Kronecker or direct product of $A$ and $B$ is denoted by $A \otimes B$; that is,

$$A \otimes B = \begin{bmatrix}
a_{11}B & a_{12}B & \cdots & a_{1m}B \\
a_{21}B & a_{22}B & \cdots & a_{2m}B \\
\vdots & \vdots & & \vdots \\
a_{p1}B & a_{p2}B & \cdots & a_{pm}B
\end{bmatrix}.$$