3. TWO-DIMENSIONAL BOUNDARY LAYER RESULTS

The two-dimensional boundary layers discussed in this chapter mainly serve as a baseline upon which to compare the more complex three-dimensional boundary layers discussed in subsequent chapters. They are of comparable Reynolds number to the wing-body junction flows discussed in chapter 4. An understanding of p beneath a two-dimensional boundary layer is necessary in order to appreciate the features of p beneath three-dimensional boundary layers. Relevant boundary layer flow parameters are given in table 3. The velocity field measurements of the lower Re_θ (= 7300) 2-D boundary layer are reported by Ölçmen and Simpson (1996). The velocity field measurements of the higher Re_θ (= 23400) 2-D boundary layer are reported by Ölçmen *et al*. (1998). The U^+ (figure 28) profiles exhibit law-of-the-wall similarity,

$$U^+ = \frac{1}{\kappa} ln\left(y^+\right) + C \tag{47}$$

where κ and C are constants. Ölçmen and Simpson (1996) calculated u_τ in the lower Reynolds number flow by fitting the U data to equation 47 using Coles' (1956) constants, $\kappa = 0.41$ and $C = 5$. Ölçmen *et al*. (1998) calculated u_τ in the higher Reynolds number flow by averaging the u_τ determined by fitting the U data to equation 47 using Coles' (1956) constants with the u_τ determined by fitting the U data to a near-wall approximation of Spalding's (1961) law-of-the-wall,

$$U^+ = y^+ - \frac{\kappa^4}{24\, e^{\kappa C}} \left(y^+\right)^4 \tag{48}$$

where κ and C are Coles' constants. The Reynolds normal stresses are shown in figure 29.

3.1. Spectral Scaling of Surface Pressure Fluctuations

There is not a universal scaling that collapses the p spectra of different Reynolds number flows at all frequencies. However, scaling characteristics of the power density spectrum of p show which turbulent structures are dominant for a given frequency range. The high frequency end ($\omega^+ > 0.15$) of the p spectra collapse to within measurement uncertainties when normalized using τ_W as the pressure scale and ν/u_τ^2 as the time scale and agree with the previous investigations (figure 30). The collapse of the *p* spectra when normalized on inner boundary layers indicates that the high frequency *p* is due to inner layer turbulent motions near the wall. Additionally, at $\omega^+ > 0.8$, the *p* spectra decay as ω^{-5} which is in agreement with the analytical analysis of Blake (1986). It should be noted that no spatial resolution correction (i.e. Corcos (1963) correction) has been applied to the *p* spectra presented here. The favorable comparison with other data (figure 30), particularly the low d^+ data of Schewe (1983) and Gravante *et al*. (1998), indicate that a correction is not required. The discrepancy between the inner-scaled spectra for $\omega^+ > 0.5$ can be attributed to transducer resolution limitations. For $\omega^+ > 0.5$ the lower spectral values are reported by Blake (1970; $d^+ > 43$) and the higher spectral values are the present data at $Re_\theta = 23400$ ($d^+ = 31$) and the data of Schewe (1983) ($d^+ = 19$). A lower value of d^+ indicates better transducer resolution of small-scale, high-frequency fluctuations. Contributions to *p* from sources that are smaller than the transducer sensing area are spatially integrated, and thereby attenuated (§2.3.2).

The spectra presented here are single-sided. The *p* spectra of McGrath and Simpson (1987), Farabee and Casarella (1991), and Blake (1970) shown here were multiplied by 2 in order to make them consistent with the definition of Φ used here. Some relevant boundary layer parameters for the comparison *p* spectra are given in table 3. The data of McGrath and Simpson (1987) presented here is an unpublished re-reduction of the original data by Shinpaugh and Simpson that corrected for the low frequency response (< 100 Hz) of their transducer.

There is general agreement in the literature on the proper pressure and time scales for the *p* spectrum at high frequencies. The same is not true for the *p* spectrum in the low and middle frequency ranges. Many researchers such as Blake (1970) and Keith *et al*. (1992) have shown the *p* spectrum to collapse at low frequencies using an outer boundary layer variable scaling of Q_e as the pressure scale and δ^*/U_e as the time scale in addition to a mixed inner-outer variable scaling which uses τ_W as the pressure scale and δ^*/U_e as the time scale. Farabee and Casarella (1991) reported that the former (outer variable) scaling only collapse *p* spectra at very low frequencies, $\omega\delta^*/U_e \leq 0.03$. Farabee and Casarella (1991) and Gravante *et al*. (1998) used τ_W as the pressure scale and δ/u_τ as the time scale to collapse *p* spectra at middle frequencies.

In the present study, the p spectra for various investigations which cover a wide range of Reynolds number ($1400 < Re_\theta < 23400$) were normalized using the time scales δ^*/U_e, δ^*/u_τ, δ/u_τ, and δ/U_e, and pressure scales τ_W and Q_e (figures 31 - 38). None of the eight possible scaling combinations successfully collapsed the p spectra at the lowest frequencies presented here, which do not extend into the very low frequency range of Farabee and Casarella (1991). In a middle frequency range the p spectra collapse when normalized using τ_W as the pressure scale independent of the time scale used. The p spectra collapse at $0.7 < \omega_{O1} < 2.5$ with δ^*/U_e as the time scale (figure 31), at $20 < \omega_{O3} < 70$ with δ^*/u_τ as the time scale (figure 33), at $100 < \omega_{O5} < 500$ with δ/u_τ as the time scale (figure 35), and at $4 < \omega_{O7} < 20$ with δ/U_e as the time scale (figure 37). Since the recent studies of Farabee and Casarella (1991) and Gravante *et al*. (1998) favor δ/u_τ as the time scale, the following discussion will illustrate the relationship between inner layer and outer layer scaling using figure 35.

It has been postulated (Bradshaw, 1967; Panton and Linebarger, 1974; Blake, 1986), using arguments relating the existence of an inner scaling and an outer scaling, that an overlap region exists in the p spectrum beneath 2-D boundary layers at high Reynolds number. Both inner and outer boundary layer scaling collapse the power spectrum in this overlap region. Using dimensional analysis, Bradshaw (1967) argued that the p spectrum in this region decreases as ω^{-1} and is due to "universal" turbulent motions within the log layer where the convection velocity approaches the local mean velocity. The size/existence of this region increases as Re_δ increases and is related to Kolmogorov's hypothesis (Batchelor, 1953) of an energy cascade.

The p spectrum for the higher Re_θ flows ($Re_\theta > 18800$) exhibit an overlap region. For the present data at $Re_\theta = 23400$ the frequency range $0.03 < \omega^+ < 0.06$ corresponds to $250 < \omega_{O3} < 500$. Examination of figures 30 and 35 reveals that both scalings collapse the p spectra and follow a power law decay within this range. An ω^{-1} decay is included in figures 30 - 38 since an ω^{-1} decay has a theoretical basis. However, the observed spectral decay is closer to $\omega^{-0.8}$. Blake (1970) observed an $\omega^{-0.75}$ decay and McGrath and Simpson (1987) observed an $\omega^{-0.7}$ decay within the overlap region. It should be noted that exact slopes are difficult to measure. The size of the middle frequency range in which the p spectra exhibit an $\omega^{-0.8}$ decay increases with Reynolds number. The low Re_θ (= 1400) p data of Schewe (1983) only tangentially approach a power law decay while the high Re_θ (=23400) p data of the present study decay as $\omega^{-0.8}$ for $30 < \omega_{O3} < 2000$ (figure 35).

3.2. Root Mean Square of Surface Pressure Fluctuations

Each of the p spectra were integrated to obtain $\overline{p^2}$ values. Figure 39 shows p'/τ_w as a function of Re_δ. Although there is scatter in p'/τ_w values due to transducer resolution limitations and accumulated experimental errors in individual frequency-spectral values, there is a general trend of increasing p'/τ_w with Re_δ, albeit with a moderate correlation coefficient (= 0.66). A trend of increasing $p'$with Re_δ is in agreement with previous investigations (Bradshaw, 1967; Panton and Linebarger, 1974; Farabee and Casarella, 1991; Bull, 1996). The source of the increasing trend in p' with Reynolds number is the overlap region of the p spectrum. The logarithmically spaced ordinate in figures 30 - 38 makes it difficult to judge what features of the p spectrum significantly affect the $\overline{p^2}$ integral. However, since

$$\frac{\overline{p^2}}{\tau_W^2} = \int_0^\infty \left[\frac{\omega\Phi(\omega)}{\tau_W^2}\right] d\big(ln(\omega)\big) \tag{49}$$

figure 40 shows $\omega\Phi/\tau_W^2$ so that contributions to the $\overline{p^2}/\tau_W^2$ integral are evenly spaced along the logarithmically spaced ω_{O3} axis. The Reynolds number trend is clearly visible at high frequencies. As Reynolds number increases so does the area under the $\omega\Phi/\tau_W^2$ curve due to increased high frequency p content. The increased high frequency p content follows directly from the overlap region extending to higher frequencies as Reynolds number increases.

Bradshaw (1967) and Panton and Linebarger (1974) analytically show that the energy within the overlap range of the p spectrum is proportional to $\ln(Re_\delta)$. Farabee and Cassarella (1991) propose an equation for $\overline{p^2}/\tau_W^2$ (solid line in figure 39),

$$\frac{\overline{p^2}}{\tau_W^2} = \begin{cases} 6.5 & (Re_\delta \le 333) \\ 6.5 + 1.86\, ln\left(\dfrac{Re_\delta}{333}\right) & (Re_\delta > 333) \end{cases} \tag{50}$$

by numerically integrating their measured spectra over the range $\omega_{O3} < 100$, integrating the equation proposed by Bull (1979)

$$\Phi^+ = \frac{2.5}{\omega^+}\, exp\left[-1.45\left(ln\left(\frac{\omega^+}{\omega_V}\right)\right)^2\right] \tag{51}$$

over the range $\omega^+ > \omega_V = 0.3$[¶¶], and assuming that Φ^+ decays as ω^{-1} in the range $100 \leq \omega_{O3} \leq 0.3Re_\delta$. Note that $\omega_{O3} = Re_\delta \, \omega^+$. Therefore, $\omega_{O3} = 0.3Re_\delta$ is equivalent to $\omega^+ = 0.3$. The trend shown in figure 39 is consistent with a logarithmic increase in p'/τ_w with Reynolds number, however, the level proposed by Farabee and Casarella (1991) is lower than most of the data shown in figure 39. Note the use of Re_δ to characterize the overlap region. Panton (1990) calls Re_δ the "preferred" Reynolds number in his general discussion of turbulent boundary layer scaling since Re_δ is the ratio of the outer length scale to inner length scale.

[¶¶] Bull (1979) proposed that $\omega_V = 0.375$.