5. PROLATE SPHEROID RESULTS

5.1. Mean Flow

The velocity components are presented here as U, V, W in the Body Surface coordinate system. This is different from the Body Axis coordinate system which uses x, r, ϕ to define a position. The difference is shown in figure 90. The transformation from the Body Axis coordinate system to the Body Surface coordinate system involves a rotation about the ϕ axis. The rotations required are 1.948° and 6.167° at $x/L = 0.600$ and $x/L = 0.772$, respectively. There is also some mention of the Wall-Shear-Stress coordinate system in the following sections. In the Wall-Shear-Stress coordinate system, the U component of velocity is aligned with the wall-shear-stress (nearest wall velocity). The V component of velocity is normal to the model surface and the W component of velocity is normal to U and W forming a right-handed coordinate system. The Wall-Shear-Stress coordinate system is used to scale the mean velocity profile near the wall (Chesnakas and Simpson, 1997). The axial locations $x/L = 0.600$ and $x/L = 0.772$ were chosen for the present study because detailed LDV measurements of the 3-D, crossflow separation are available at these x/L locations (Wetzel *et al*., 1998; Chesnakas and Simpson, 1993, 1994, 1996, 1997; Goody *et al*., 1998).

The mean flow and Reynolds-averaged turbulence stresses have been previously discussed (Chesnakas and Simpson, 1994, 1996, 1997). Some key features which are relevant to the present discussion are given here. Tables 8 - 11 give some boundary layer parameters of the present flows. The values of ρ, ν, u_τ, and U_∞ given in tables 8 - 11 were calculated using the pressure, temperature, and U_∞ during the measurements of p and the C_f measurements of Chesnakas and Simpson (1997). Mean velocity profiles from LDV measurements (Chesnakas and Simpson, 1997) and outer layer hot-wire anemometer data (Goody *et al*., 1998) were used to calculate δ^*, θ, and U_e/U_∞. Figure 91 shows distributions of u_τ, δ^*, and Re_θ. The nearest wall region ($y^+ < 50$) of the present flow follows the law-of-the-wall mean velocity profile when expressed in wall shearing stress coordinates (Chesnakas and Simpson, 1994, 1996, 1997).

Figures 92 - 107 show secondary flow streamlines interpolated from the V, W data for each station. Figures 92, 93, 96, 97, 100, 101, 104, and 105 also contain contours of the mean velocity magnitude and figures 94, 95, 98, 99, 102, 103, 106, and 107 contain contours of the turbulent kinetic energy (*TKE*) to show qualitatively how the mean flow field and the turbulence field are related. Additionally, figures 93 - 107 (even) have a logarithmic *r*-axis in order to highlight the near-wall flow.

5.1.1. 10° Angle of Attack

The secondary flow field at $\alpha = 10°$, $x/L = 0.600$, is shown in figures 92-95. Mean-flow separation, as indicated by a minimum in skin friction magnitude (Wetzel *et al.*, 1998), is at $\phi = 145°$, with a vortex center (zero secondary flow velocity) at $\phi = 162°$ approximately 0.375 cm above the surface. Figure 93 indicates that the weak separation at this measurement station thickens the boundary layer while figure 95 shows that near-wall *TKE* is suppressed at $120° < \phi < 160°$. Figures 96-99 show the flow field at $\alpha = 10°$, $x/L = 0.772$. Mean-flow separation (Wetzel *et al.*, 1998) is at $\phi = 137°$, with a vortex approximately 1.25 cm above the surface at $\phi = 165°$. Like at $x/L = 0.600$, the boundary layer is thickest (figure 93) and near-wall *TKE* is suppressed (figure 99) within the middle range of ϕ ($120° < \phi < 150°$). However, at $x/L = 0.772$ there is a local maximum in *TKE* (along lines of constant ϕ) away from the wall within the middle range of ϕ ($120° < \phi < 150°$). The locus of these local maxima are nearly aligned with the convergence of secondary streamlines associated with separation. It bears repeating that these maxima are only local maxima. For each of the $\alpha = 10°$ cases, the *TKE* near the wall is maximum at $\phi = 90°$ and $\phi = 180°$ because of large production due to large mean velocity gradients and turbulent shear stresses. The *TKE* decreases in the middle range of ϕ (120° - 150°). However, at any given ϕ location the maximum *TKE* remains in the near-wall region.

5.1.2. 20° Angle of Attack

The flow field at $\alpha = 20°$, $x/L = 0.600$, is shown in figures 100-103. Mean flow separation (Wetzel *et al.*, 1998) occurs at $\phi = 131°$. The separation sheet rolls into a vortex centered at $\phi = 158°$ approximately 1.8 cm from the surface. The boundary layer is thickest near separation and there is a local minimum in the velocity magnitude within the secondary vortex (figure 101).

There are kinks in the streamlines at $\phi = 145°$ and high *TKE* contours extend out into the flow with a local maximum near $r = 2.0$ cm (figure 103). Flow visualization shows a secondary separation to be incipient at $\phi = 140°$ (Wetzel *et al.*, 1998).

Figures 104-107 show the flow field at $\alpha = 20°$, $x/L = 0.772$. This case has the highest degree of three-dimensionality of those considered here. The primary separation (Wetzel *et al.*, 1998) location is at $\phi = 115°$ and the primary vortex is outside the LDV measurement region. Hot-wire data show this vortex center to be at $r = 3$ cm, $\phi = 155°$ (figure 100) (Goody *et al.*, 1998). In addition, there is a fully-formed secondary vortex at $\phi = 140°$ approximately 0.6 cm from the surface. Associated with the secondary vortex is separation ($\phi = 145°$), and reattachment at $\phi = 135°$. It should be noted that near-wall velocity measurements were not carried out at $\phi = 135°$ which is why the secondary streamlines in figures 104 - 107 do not converge at $\phi = 135°$ like they do at the primary and secondary separation, where near-wall velocity measurements were carried out. The boundary layer is thickest near the separations (figure 105) and the mean velocity magnitude exceeds the tunnel flow velocity beneath the primary vortex (figures 104 and 105). The vortices away from the wall have relatively low mean velocity gradients and bring fluid with relatively low *TKE* from the outer layer toward the wall at the most leeward (highest) ϕ locations (figures 106 and 107). Figures 104 and 105 show the low velocity trough, first reported by Chesnakas and Simpson (1997), between the primary separation location and the primary vortex. The trough extends out a significant distance from the wall. Diminished mean flow gradients and Reynolds shear stresses within the trough cause *TKE* to be lower there, due to diminished *TKE* production (figures 106 and 107). However, at the edges of the trough mean velocity gradients are higher. Higher mean velocity gradients combined with elevated Reynolds stresses increase *TKE* production. This is most evident at the top of the low velocity trough ($\phi = 140°$, $r = 2.75$ cm, $r^+ = 3990$) where the highest *TKE* ($= 0.022\rho U_\infty^2$) was measured. The main contribution to this high *TKE* is $\overline{w^2}$ ($= 0.022U_\infty^2$) as compared to $\overline{u^2}$ ($= 0.012U_\infty^2$) and $\overline{v^2}$ ($= 0.009U_\infty^2$). Also notable is that the Reynolds shear stresses are maximum at this location.

The mean flow at the $\alpha = 20°$ measurement stations has a stronger effect on the turbulence field than the mean flow at the $\alpha = 10°$ measurement stations. High *TKE* is not confined to the near-wall region. There are small regions of highly turbulent fluid away from the wall due to the separations. Additionally, each of the separations is followed by low *TKE* levels near the wall. The *TKE* levels are higher where reattachment is present.

The outward secondary flow streamlines near regions of separation in figures 92-107 do not appear to agree with the separation locations given above. The apparent discrepancy is due to the coordinate system used to represent velocity components and is discussed in depth by Wetzel *et al*. (1998). In a coordinate system locally aligned with the separation line, the LDV data show zero cross-flow velocity very close to the local minimum in skin friction magnitude (Wetzel *et al*., 1998). While this coordinate system is useful to determine the precise location of flow separation, the orientation of this coordinate system changes in space. Therefore, such a coordinate system is not appropriate for the global, field-type plots shown here.

5.2. Surface Pressure Spectra

Surface pressure measurements were carried out at 5° increments of ϕ. The p spectra are divided into two groups in this section- first, the p spectra at $\alpha = 10°$ and second, the p spectra at $\alpha = 20°$. The p spectra are presented in this manner because the p spectra at $\alpha = 10°$ are similar to the p spectra beneath equilibrium boundary layers, whereas the p spectra at $\alpha = 20°$ are not, as will be shown later. For each group, features of the dimensional p spectra are discussed, then several boundary layer scalings are presented.

5.2.1. 10° Angle of Attack

The p spectra at $\alpha = 10°$, $x/L = 0.600$ are shown in figure 108. In general, the spectral level within a given frequency range is highest at $\phi = 90°$ and lowest at $\phi = 145°$. Recall that at $\alpha = 10°$, $x/L = 0.600$ the boundary layer separates at $\phi = 145°$. The range of spectral levels among different ϕ locations is largest (~ 13 dB) at the highest frequencies. The range of spectral levels within 600 Hz < f < 1 kHz is nearly equal to the measurement uncertainty. The p spectra at $\alpha = 10°$, $x/L = 0.772$ are shown in figure 109. At the lowest frequencies ($f < 400$ Hz), the spectral level of p is highest at $\phi = 110°$ and lowest is at $170° < \phi < 180°$. At the highest frequencies, the spectral level of p is highest at $\phi = 90°$ and is lowest where the flow is separating, at $135° < \phi < 140°$. At $\alpha = 10°$, $x/L = 0.772$, boundary layer separation is at $\phi = 137°$. Like at

x/L = 0.600, the range of spectral levels of p at a given frequency is greatest (~ 19 dB) at the highest frequencies.

Scalings characteristics of the p spectrum show which turbulent structures are dominant for a given frequency range. Figure 110 shows the p spectra at $\alpha = 10°$, $x/L = 0.600$ using τ_W as the pressure scale and ν/u_τ^2 as the time scale. These inner layer scales are equivalent to using u_τ as a velocity scale and ν/u_τ as a length scale which are the scales used for the familiar law-of-the-wall mean velocity profile that holds nearest the wall in these flows when the velocity is expressed in wall-shear-stress coordinates (Chesnakas and Simpson, 1997). The spectra collapse at the highest frequencies, $\omega^+ > 0.2$, and approach the ω^{-5} decay that exists beneath equilibrium boundary layers (Blake, 1986; McGrath and Simpson, 1987; Keith *et al.*, 1992; Gravante *et al.*, 1998). The p spectra at $\alpha = 10°$, $x/L = 0.772$ also collapse to an ω^{-5} decay at high frequencies when non-dimensionalized using inner variables (figure 111). The ϕ locations with an ω^{-5} region are those at which τ_W is smallest and ν/u_τ^2 is largest. The source of the ω^{-5} region of the p spectrum is the smallest turbulent structures nearest the wall.

The present flows contain separations and are complex and 3-D — definitely non-equilibrium. Therefore, it is significant that the high frequency p spectra of the present flows ($\alpha = 10°$) compare well with the p spectrum beneath a 2-D, zero pressure gradient boundary layer (figures 112 and 113). The favorable comparison indicates that the near-wall structure of p in the present flows at $\alpha = 10°$ is similar to that in a 2-D, zero pressure gradient boundary layer.

The spectra presented here are single-sided. The p spectra of McGrath and Simpson (1987), Farabee and Casarella (1991), and Blake (1970) shown here were multiplied by 2 in order to make them consistent with the definition of Φ used here. Some relevant boundary layer parameters for the comparison p spectra are given in table 3. The data of McGrath and Simpson (1987) presented here is an unpublished re-reduction of the original data that corrected for the low frequency response (< 100 Hz) of their transducer.

Figure 114 shows the spectra at $\alpha = 10°$, $x/L = 0.600$ non-dimensionalized using mixed inner and outer variables. The pressure scale is τ_W and δ^*/U_e is the time scale. The p spectra leeward of separation ($\phi > 145°$) collapse for $0.8 < \omega_{O1} < 3$ and decay nearly as $\omega^{-0.6}$. The *p* spectra windward of separation ($\phi < 145°$) also collapse for $0.8 < \omega_{O1} < 2$, however, do not follow a discernable power law decay. The *p* spectra for all ϕ collapse for $\omega_{O1} > 4$ and decay nearly as $\omega^{-7/3}$, but note that only the *p* spectra at $\phi > 110°$ extend to $\omega_{O1} > 4$. The spectra at $\alpha = 10°$, $x/L = 0.772$, $\phi \leq 140°$ collapse at middle frequencies ($1 < \omega_{O1} < 6$) when normalized using mixed variables (figure 115), but they do not follow a discernable power law decay. The collapse of the *p* spectra at $\alpha = 10°$ normalized using mixed variables is limited to specific ranges of ϕ.

It has been postulated (Bradshaw, 1967; Panton and Linebarger, 1974; Keith *et al.*, 1992), using arguments relating the existence of an inner (viscous region) scaling and an outer (largest eddy) scaling within the boundary layer, that an overlap region exists in the *p* spectrum beneath 2-D boundary layers at high Reynolds number. Both inner layer and outer layer scaling hold in this overlap region. Bradshaw (1967) argued that the *p* spectrum in this region decreases as ω^{-1}. The low frequency *p* spectra at $\alpha = 10°$, $x/L = 0.600$ (figure 116) vary about as ω^{-1} for $0.07 < \omega_{O1} < 0.3$ when normalized using Q_e as a pressure scale and δ^*/U_e as a time scale. The *p* spectra in the same range at $x/L = 0.772$ do not scale as well using these variables (figure 117). Although this flow differs from a 2-D adverse-pressure-gradient separation, the spectral levels are comparable to those upstream of detachment reported by Simpson *et al*. (1987) (figures 116 and 117). In the mid-frequency range, around $\omega_{O1} = 2$, the $\omega^{-0.5}$ variation observed by Simpson *et al*. (1987) is also present.

The variation of the *p* spectra in the mid-frequency range may be a Reynolds number effect. The low frequency spectral contributions are from the largest shear layer structures. The power spectral contribution of these large structures increases with Reynolds number. The high frequency scaling is Reynolds number independent. This requires a greater decay in the power spectrum within the mid-frequency range as Reynolds number increases.

5.2.2. 20° Angle of Attack

The spectral level at low frequencies ($f < 300$ Hz) at $\alpha = 20°$, $x/L = 0.600$ is highest at $\phi = 150°$ and is lowest at $\phi = 105°$ (figure 118). The spectral value at 1 kHz has three local maxima: one at $\phi = 130°$ which is near primary separation; one at $\phi = 145°$ which is near incipient secondary separation; and one at $\phi = 160°$ which is near the ϕ location of the center of the shed vortex. The spectral value increases with increasing ϕ to primary separation, and decreases with increasing ϕ for $160° \leq \phi \leq 180°$. At the highest frequencies ($f \approx 25$ kHz) the range of spectral levels among different ϕ locations is 28 dB. The largest high frequency spectral level is at $\phi = 160°$ (under the shed vortex) and is over 75 dB — significant high frequency content. The smallest high frequency spectral level is at $\phi = 130°$ which is very near boundary layer separation ($\phi = 131°$). The p spectra at $\alpha = 20°$, $x/L = 0.772$ are shown in figure 119. The range of spectral levels among different ϕ locations at the lowest frequencies ($f < 300$ Hz) is nearly 10 dB. The highest spectral level within this frequency range is at $\phi = 150°$ and the lowest is at $\phi = 175°$. The spectral level at 1 kHz increases with ϕ to reattachment near $\phi = 135°$ and then decreases toward the leeward plane of symmetry ($\phi = 180°$). The range of spectral levels among different ϕ locations at the highest frequencies is nearly 25 dB. The highest spectral level within this frequency range is 76 dB at $\phi = 150°$. The lowest spectral level is at the primary boundary layer separation ($\phi = 115°$) like the other measurement stations.

None of the boundary layer variable scalings mentioned above collapse the p spectra at $\alpha = 20°$ as well as they roughly do for the p spectra at $\alpha = 10°$ (figures 120-127). However, around the primary separation and at windward ϕ locations, at $110° < \phi < 130°$, the p spectra collapse at $2 < \omega_{O1}$ with an ω^{-3} variation when normalized using τ_W as the pressure scale and δ^*/U_e as the time scale (figures 123 and 125). Simpson *et al*. (1987) also observed an ω^{-3} variation during 2-D detachment and downstream.

Even though the $\alpha = 20°$ case is a highly non-equilibrium flow, there are some p spectral features that can be consistently related to the flow above. As the flow moves from the windward to leeward sides ($\phi = 90°$), there is low level low-frequency large-scaled turbulence content due to the thin accelerating boundary layer with low mean velocity gradients in the outer layer. Substantial high frequency ($\omega \sim u_\tau^2 / \nu$) content is produced by the nearest wall-layer structure with a relatively large u_τ. The mid-frequencies (4 kHz $< f <$ 10 kHz) have nearly constant, or flat, spectral values.

At more leeward locations the low frequency content increases because of the thickening boundary layer and the separation with large-scale structures while the high frequency content is much lower because of much lower u_τ. Further leeward under the large vortex, the low frequency content decreases because there are low mean velocity gradients in the outer layer while much larger u_τ values increase the high frequency content. Again, the mid-frequencies have nearly constant spectral values.

5.3. Surface Pressure-Velocity Correlations

In order to examine the locations of turbulent flow that strongly influence p, simultaneous p and velocity fluctuation measurements were made for three of the stations; $\alpha = 10°$, $x/L = 0.772$; $\alpha = 20°$, $x/L = 0.600$ and $x/L = 0.772$. Although the Reynolds stresses are discussed by Chesnakas and Simpson (1997), they are presented here as they relate to the surface pressure-velocity correlation coefficient. The correlation coefficients and Reynolds stresses are presented in this section with a logarithmic radial (r) coordinate in order to emphasize the near-wall turbulent structures. The surface pressure-velocity covariances measured in the vicinity of $y^+ = 10$ are expected to be low because of attenuation of higher frequency surface pressure fluctuations by the pinhole that is much larger ($38 < d^+ < 92$) than the near wall coherent structures ($12\,\nu/u_\tau$). Two-point correlation coefficients are presented in this section. When evaluating the magnitude of the correlation coefficients presented here, it should be kept in mind that the single-point correlation coefficient of the uv-stress in a 2-D, zero pressure gradient boundary layer is -0.3 to -0.5.

5.3.1. $\alpha = 10°$, $x/L = 0.772$

At $\alpha = 10°$, $x/L = 0.772$, there is substantial R_{pu} ($\approx$ -0.07) at $r \approx 1$ cm ($r^+ \approx 1200$), $90° < \phi < 120°$ (figure 128). This r^+ corresponds to the wake region in which free-stream flow is entrained into the boundary layer. Relatively high correlation coefficient in the outer layer is also significant because pressure fluctuation sources that interact with the free-stream are radiated away as sound. There are two regions of significant positive R_{pu}. The first region is at $r \approx 0.13$ cm ($r^+ \approx 125$), $120° < \phi < 135°$ and $R_{pu} \approx 0.09$. This r^+ is the outer edge of the semi-logarithmic part of the mean velocity profile in wall-shear-stress coordinates (log layer). The second region is at $r \approx 0.2$ cm ($r^+ \approx 240$), $170° < \phi < 180°$ and $R_{pu} \approx 0.12$. None the regions of high R_{pu} correspond to regions of particularly high $\overline{u^2}$ (figure 129).

Figures 130 and 131 show R_{pv} and the fluctuating v-velocity component, respectively, at $\alpha = 10°$, $x/L = 0.772$. The maximum negative correlation is $-0.08 > R_{pv} > -0.11$ at $160° < \phi < 120°$, $0.1 \text{ cm} < r < 0.2 \text{ cm}$, which is roughly at $r^+ \approx 100 - 250$. Highly turbulent fluid in this case is present mainly near the wall. This maximum negative R_{pv} occurs at the outer edge of the mean velocity profile semi-logarithmic region. Bradshaw (1967) showed that in 2-D boundary layers, with both zero and equilibrium adverse pressure gradients, the semi-logarithmic region of the mean flow velocity profile is the source of the overlap region of the pressure spectrum. The correlation magnitude is high at this radial location because the overlap region of the spectrum is the main contribution to the $\overline{p^2}$ integral for high enough Re_θ (Bradshaw, 1967).

The maximum in R_{pv} in the semi-logarithmic region of the mean flow velocity profile can be seen through the solution to the Poisson equation relating surface pressure fluctuations to velocity fluctuations within the boundary layer (§1.2.2). Although the solution is an integral over all of space, the influence of any individual source decreases as $1/r_S$. In contrast, the magnitude of the v' source terms are small for small r_S, where they are constrained by the wall. They increase to a maximum toward the middle of the boundary layer. The combined effect is that the semi-logarithmic region is where the strength of source terms are high, and $1/r_S$ is still high enough that these sources influence the pressure at the wall.

Returning to figure 130, the high R_{pv} magnitude at the edge of the log layer is not evident at all ϕ positions. In the separation region, $120° < \phi < 160°$, R_{pv} remains low for all r. However, by considering the behavior of $\overline{v^2}$ (figure 131), this is to be expected. The band of maximum v' is farther away from the wall in the separation region. Pressure fluctuation sources located farther from the wall have less influence on p through the $1/r_S$ term in the Poisson integral (equation 25).

There are two regions of significant negative R_{pw} (≈ -0.08). One is centered about $r = 0.2$ cm ($r^+ = 180$), $\phi = 140°$, the other is at $r = 0.5$ cm ($r^+ = 600$), $\phi = 170°$ (figure 132). Values of $\overline{w^2}$ are low in both of these regions (figure 133). There are two regions with high positive R_{pw}. One is at 0.02 cm $< r <$ 0.04 cm ($20 < r^+ < 40$), 90° $< \phi <$ 120° where $0.08 < R_{pw} < 0.10$. The other region is at 0.02 cm $< r <$ 0.07 cm ($20 < r^+ < 80$), 160° $< \phi <$ 180° where $0.10 < R_{pw} < 0.16$. Values of $\overline{w^2}$ are highest in both regions of high R_{pw} (figure 133).

5.3.2. α = 20°, x/L = 0.600

There is significant R_{pu} at α = 20°, x/L = 0.600 (figure 134) in the same regions as at α = 10°, x/L = 0.772 (figure 128). At 1 cm $< r <$ 1.5 cm ($1000 < r^+ < 1500$), 90° $< \phi <$ 120°, R_{pu} reaches -0.1 and $0.10 < R_{pu} < 0.16$ at 0.15 cm $< r <$ 0.4 cm ($200 < r^+ < 700$), 170° $< \phi <$ 180°. Also there is high positive R_{pu} (= 0.1) at r = 0.1 cm (r^+ = 100), 120° $< \phi <$ 130°. However, these regions do not appear as prominent in figure 134 as they are in figure 128 due to other regions of higher correlation associated with a stronger shed vortex and the incipient formation of a secondary vortex. At ϕ = 140°, R_{pu} is high from r = 0.02 cm (r^+ = 20) where R_{pu} = 0.085 to r = 0.3 cm (r^+ = 340) where R_{pu} = 0.11 reaching a maximum of R_{pu} = 0.16 at r = 0.2 cm (r^+ = 230). It is also interesting that at ϕ = 140° R_{pu} goes from R_{pu} = +0.16 at r = 0.2 cm (r^+ = 230) to R_{pu} = -0.20 at r = 0.6 cm (r^+ = 680), and then positive again (R_{pu} = +0.09) at r = 1.5 cm (r^+ = 1700). There is a secondary vortex at r = 0.6 cm, ϕ = 140° downstream, at α = 20°, x/L = 0.772 (figure 105). There is also a region of high negative R_{pu} at 0.1 cm $< r <$ 1.0 cm, 150° $< \phi <$ 170° where R_{pu} goes from -0.1 to a maximum (magnitude) of -0.3 at r = 0.2 cm (r^+ = 550), ϕ = 160°. The only region of the flowfield in which both R_{pu} (figure 134) and $\overline{u^2}$ (figure 135) are high is at r = 1.5 cm ($1700 < r^+ < 2200$), 140° $< \phi <$ 150° where R_{pu} = 0.1.

Like at α = 10°, x/L = 0.772 (figure 130), there is significant negative R_{pv} at r = 0.125 cm ($100 < r^+ < 170$), 100° $< \phi <$ 125° where $-0.07 > R_{pv} > -0.11$ and at 0.125 cm $< r <$ 1 cm ($200 < r^+ < 1800$), 155° $< \phi <$ 180° where $-0.1 > R_{pv} > -0.18$ (figure 136). Unlike at α = 10°, x/L = 0.772, there is a region of small spatial extent centered at r = 0.5 cm (r^+ = 570), ϕ = 140° in which R_{pv} is highly negative ($R_{pv} \approx -0.18$). Again, the secondary vortex at α = 20°, x/L = 0.772, r = 0.6 cm, ϕ = 140° (figure 105) is probably incipient at this location. Also unlike α = 10°, x/L = 0.772 there is high positive R_{pv} in the outer layer at most ϕ locations. At 1.5 cm $< r <$ 3 cm, 110° $< \phi <$ 150° and 165° $< \phi <$ 180°, $0.07 < R_{pv} < 0.19$. Only regions of negative R_{pv} correspond to regions in which $\overline{v^2}$ is high (figure 137).

Figure 138 is especially interesting due to the topological structure present in R_{pw} that is not associated with the mean flow and only weakly associated with $\overline{w^2}$ (figure 139) at $x/L = 0.600$. There is an arch-shaped region of negative R_{pw} (figure 138). The legs of this arch are at $\phi = 130°$ and $\phi = 150°$ and extend from measurement locations closest to the wall ($r \approx 0.008$ cm, $r^+ \leq 11$) out to $r = 0.3$ cm ($r^+ = 235, 440$ at $\phi = 130°, 150°$, respectively). At $\phi = 130°$, R_{pw} ranges from -0.06 to -0.11 and at $\phi = 150°$, R_{pw} ranges from -0.05 to -0.10. The legs of the arch-like structure are connected at $130° < \phi < 150°$, 0.2 cm < r < 0.6 cm where $-0.06 > R_{pw} > -0.21$. Within this region, R_{pw} is maximum (magnitude) at $\phi = 140°$, $r = 0.6$ cm ($r^+ = 690$). However, closer to the wall at $\phi = 140°$ (0.007 cm < r < 0.125 cm, $8 < r^+ < 144$), R_{pw} is highly positive ($0.08 < R_{pw} < 0.20$). Between these regions of positive R_{pw} and negative R_{pw}, R_{pw} is necessarily zero. It is interesting that R_{pw} is near zero at $\phi = 135°$ and $\phi = 145°$ since downstream, at $x/L = 0.772$, there is a reattachment at $\phi = 135°$ and a secondary separation at $\phi = 147°$. At $\phi = 170°$, there is another region of high, negative R_{pw} ($-0.10 > R_{pw} > -0.18$) from $r = 0.08$ cm ($r^+ = 138$) to $r = 1.0$ cm ($r^+ = 1690$).

There are two region of significant positive R_{pw}. One region is at ϕ near 180°, 0.022 cm < r < 0.06 cm ($35 < r^+ < 95$) where $0.09 < R_{pw} < 0.11$. The other region is in the outer layer, 1 cm < r < 3 cm ($975 < r^+ < 3500$), at $120° < \phi < 140°$ where $0.14 < R_{pw} < 0.26$. None of the regions of significant R_{pw} correspond to any regions of high $\overline{w^2}$ (figure 139). In fact, R_{pw} is near zero at the ϕ location where $\overline{w^2}$ is highest (near $\phi = 160°$, figure 139)

5.3.3. $\alpha = 20°$, $x/L = 0.772$

Like at both $\alpha = 10°$, $x/L = 0.772$ and $\alpha = 20°$, $x/L = 0.600$, there is a region of high negative R_{pu} in the outer layer at $90° < \phi < 110°$ (figure 140). At 0.4 cm < r < 2.75 cm ($350 < r^+ < 3100$), $-0.1 > R_{pu} > -0.35$. Also like the other measurement stations, there is high positive R_{pu} in the middle ϕ range, $110° < \phi < 145°$. Between the primary separation ($\phi = 115°$) and reattachment ($\phi = 135°$), $0.07 < R_{pu} < 0.16$ at 0.1 cm < r < 0.4 cm ($10 < r^+ < 340$). Within the middle range of ϕ ($110° < \phi < 145°$), $\overline{u^2}$ is only relatively high near reattachment at $130° < \phi < 140°$ (figure 141).

Just windward of the secondary separation at $\phi = 145°$, $0.07 < R_{pu} < 0.19$ at 0.009 cm $< r < 0.3$ cm ($10 < r^+ < 300$). The value $R_{pu} = 0.19$ was measured at $r = 0.2$ cm ($r^+ = 200$). Just leeward of the secondary separation, $150° < \phi < 155°$, the magnitude of R_{pu} is exceptionally high ($-0.1 > R_{pu} > -0.5$) at 0.009 cm $< r < 0.3$ cm ($10 < r^+ < 1800$). The value $R_{pu} = 0.5$ was measured at $r = 0.3$ cm ($r^+ = 500$), $\phi = 155°$. The magnitude of R_{pu} remains high away from the wall (0.2 cm $< r < 1.5$ cm, $360 < r^+ < 2600$) in a region extending leeward to $\phi = 170°$ (figure 140). It is also notable that R_{pu} is near zero along the convergence of streamlines indicative of the secondary separation at $\phi = 147°$.

The behavior of R_{pu} is particularly interesting at $\phi = 140°$ along a line of constant ϕ. Near the wall at $r = 0.008$ cm ($r^+ = 12$), $R_{pu} = 0.1$. As r increases, R_{pu} becomes negative and reaches $R_{pu} = -0.1$ at $r = 0.124$ cm ($r^+ = 180$). As r increases more, R_{pu} becomes positive again and reaches $R_{pu} = 0.34$ at $r = 0.6$ cm ($r^+ = 870$) which is the approximate center of the secondary vortex (figure 140). There is similar behavior of R_{pu} near the center of the primary vortex ($r = 3$ cm, $\phi = 155°$). Measurements of R_{pu} were limited to $r < 2.75$ cm, however, at $r = 2.75$ cm, $\phi = 155°$, $R_{pu} = 0.22$ which is significant.

Like at both $\alpha = 10°$, $x/L = 0.772$ and $\alpha = 20°$, $x/L = 0.600$, there is significant negative R_{pv} at the middle r locations windward of $\phi = 115°$ and leeward of $\phi = 160°$ (figure 142). At 0.08 cm $< r < 0.12$ cm ($62 < r^+ < 132$), $105° < \phi < 115°$, $-0.09 > R_{pv} > -0.12$ with a local maximum R_{pv} at $r = 0.08$ cm ($r^+ = 90$), $\phi = 105°$. At 0.12 cm $< r < 0.6$ cm ($216 < r^+ < 1050$), $160° < \phi < 170°$, $-0.10 > R_{pv} > -0.17$ with a local maximum R_{pv} at $r = 0.3$ cm ($r^+ = 520$), $\phi = 105°$. However, these regions of high negative R_{pv} are overshadowed by the presence of a localized source of p. The localized source is associated with the secondary vortex present at $x/L = 0.772$, r = 0.6 cm, $\phi = 140°$. Here the term *localized source* is used to describe a small range of r and ϕ in which the correlation is high. This particular source is where the secondary flow streamlines have high curvature and the streamwise flow is rapidly decelerating. There is a small region of high $\overline{v^2}$ at $\alpha = 20°$, $x/L = 0.772$ near $r = 1.5$ cm, $\phi = 140°$ (figure 143) which is also a localized source of p. While this localized source is away from the wall ($r^+ \approx 2200$) the presence of reattachment causes pressure fluctuations associated with this source to be convected to the surface. High negative R_{pv} is observed near this region where high $\overline{v^2}$ and reattachment is present.

Also like at both $\alpha = 10°$, $x/L = 0.772$ and $\alpha = 20°$, $x/L = 0.600$, there is significant positive R_{pv} in the outer layer at $90° < \phi < 120°$ (figure 142). At $1 \text{ cm} < r < 2.75 \text{ cm}$ $(1000 < r^+ < 3000)$, $0.10 < R_{pv} < 0.33$ with a local maximum R_{pv} at $r = 2.0$ cm $(r^+ = 1700)$, $\phi = 120°$. There is another region of high R_{pv} in the outer layer between the primary and secondary vortex (figure 142). At $0.4 \text{ cm} < r < 2.75 \text{ cm}$ $(490 < r^+ < 4700)$, $145° < \phi < 155°$, $0.10 < R_{pv} < 0.30$ with a local maximum R_{pv} at $r = 2.0$ cm $(r^+ = 2000)$, $\phi = 145°$.

The correlation, R_{pw}, is positive at the windward ϕ locations, $\phi = 100°$, 105° (figure 144). At $0.008 \text{ cm} < r < 0.05 \text{ cm}$ $(9 < r^+ < 230)$, $0.09 < R_{pw} < 0.18$ with a local maximum R_{pw} at $r = 0.023$ cm $(r^+ = 25)$, $\phi = 100°$. Between the primary separation and reattachment, R_{pw} is negative and low. At $\phi = 140°$, R_{pw} is high from close to the wall ($r = 0.006$ cm, $r^+ = 9$) where $R_{pw} = 0.34$ out to $r = 0.2$ cm $(r^+ = 290)$ where $R_{pw} = 0.13$ reaching a local maximum of $R_{pw} = 0.45$ at $r = 0.02$ cm $(r^+ = 30)$. Similar to R_{pu} and R_{pv}, R_{pw} is nearly zero along the convergence of streamlines indicative of secondary separation ($\phi = 147°$, figure 144).

Leeward of the secondary separation, at $\phi = 150°$, R_{pw} is negative near the wall ($r = 0.006$ cm, $r^+ = 7$) where $R_{pw} = -0.15$. It remains high and negative well out into the flow ($r = 1$ cm, $r^+ = 1225$, $R_{pw} = -0.23$). At $\phi = 150°$, R_{pw} reaches a local maximum of $R_{pw} = -0.39$ at $r = 0.6$ cm $(r^+ = 730)$. This region of high negative R_{pw} away from the wall extends windward to $\phi = 140°$. At $0.2 \text{ cm} < r < 2 \text{ cm}$ $(200 < r^+ < 2900)$, $-0.15 > R_{pw} > -0.46$ with a maximum $R_{pw} = -0.46$ at $r = 1$ cm $(r^+ = 1450)$, $\phi = 140°$. This r, ϕ location is just above the secondary vortex (figure 144) where the secondary streamlines associated with the primary vortex show large curvature as they extend over the secondary vortex. There are also high levels of $\overline{w^2}$ at $\phi = 140°$, however, farther away from the wall than the high, negative R_{pw} (figure 145).

At the leeward ϕ locations, $160° < \phi < 170°$, R_{pw} is low and positive near the wall, however, away from the wall R_{pw} is high and negative. At $0.12 \text{ cm} < r < 1.5 \text{ cm}$ $(220 < r^+ < 2750)$, $160° < \phi < 170°$, $-0.10 > R_{pw} > -0.15$ (figure 144). Away from the wall, R_{pw} is high and positive at most ϕ locations. At 100° < ϕ < 130°, 0.4 cm < r < 2.75 cm ($300 < r^+ < 3500$), $0.09 < R_{pw} < 0.41$ and at 130° < ϕ < 155°, 2.0 cm < r < 2.75 cm ($2030 < r^+ < 4690$), $0.11 < R_{pw} < 0.27$.

5.4. Mean Square of Surface Pressure Fluctuations

Each of the p spectra were integrated to obtain $\overline{p^2}$ values. In order to use a more complete spectrum, a high frequency contribution to the mean square integral was added to the numerically integrated experimental spectral estimates. At $\alpha = 10°$, the p spectra collapse at the highest frequencies when scaled on inner variables. Therefore, the high frequency behavior can be described by a single curve. At $\alpha = 20°$, most of the measured p spectra do not extend to frequencies that are high enough to exhibit inner variable scaling, despite the high sampling rate and high frequency spectral correction used here. It is clear that an additional high frequency contribution to the $\overline{p^2}$ integral is required, especially at $\alpha = 20°$ (figures 120 and 121).

5.4.1. The Calculation Method

The procedure used here to calculate $\overline{p^2}$ was to first numerically integrate the experimental spectral estimates. Then, the last spectral estimate (highest frequency), was scaled on inner variables. This non-dimensional estimate served as the lower limit of an analytical integral contribution (AIC). For p spectra that extended to frequencies that are high enough to exhibit an ω^{-5} spectral decay, the integrand of the AIC was proportional to ω^{-5} (figure 146). For p spectra that did not extend to frequencies that are high enough to exhibit an ω^{-5} spectral decay, the integrand of the AIC consisted of two parts. The first part is an extrapolation of the observed spectral decay of p. The second part of the AIC was proportional to ω^{-5} (figure 147). The highest frequency AIC integrand was $\Phi^+ = 0.50(\omega^+)^{-5}$ (the ω^{-5} line in figures 110 and 111) for the p spectra at $\alpha = 10°$ and $\Phi^+ = 2.00(\omega^+)^{-5}$ (the ω^{-5} line in figures 120 and 121) for the p spectra at $\alpha = 20°$. Once made dimensional, the value of the AIC yields the high frequency contribution to $\overline{p^2}$.

For the data at $\alpha = 10°$, the AIC is generally within 10% of the total $\overline{p^2}$ (tables 12 and 13). However, it reaches 22% at $x/L = 0.600$, $\phi = 90°$ (table 12). For $\alpha = 20°$, the AIC varies from less than 1% of the total $\overline{p^2}$ near separation, where $\overline{p^2}$ is lowest, to 73% of the total

$\overline{p^2}$ at *x/L* = 0.600 (table 14) and 75% at *x/L* = 0.772 (table 15) at ϕ locations under the vortex, where $\overline{p^2}$ is large. While the AIC to $\overline{p^2}$ is large for some ϕ locations at $\alpha = 20°$, it must be noted that the values presented here are a lower bound on the true $\overline{p^2}$ values. Therefore, the variation of $\overline{p^2}$ with ϕ that is presented here is accurate. At ϕ locations where $\overline{p^2}$ is small, the AIC is small. At ϕ locations where $\overline{p^2}$ is large, the AIC is large, however, the true $\overline{p^2}$, if different, is higher than the values presented here. Additionally, the *p* spectrum at ϕ locations where the AIC is large are nearly flat in the middle to high frequency range. The large AIC at these ϕ locations emphasizes the large contribution of the *p* spectrum at high frequencies to the $\overline{p^2}$ integral.

5.4.2. Results

The *p*' estimates are presented here normalized using the far upstream dynamic pressure, Q_∞, and the wall shear stress, τ_w (figure 148). Since Q_∞ is nearly constant, p'/Q_∞ closely approximates the true variation of *p*' with ϕ. Recall that Re_L, rather than Q_∞, was kept constant during these measurements. The recent review of Bull (1995) concluded that p'/τ_W was the most appropriate parameter for consideration of *p*' beneath 2-D flows. Also, τ_w is a local minimum at separation (Wetzel *et al*., 1998).

At $\alpha = 10°$, the values of p'/Q_∞ and p'/τ_W are in the range observed beneath 2-D flows (Blake, 1970; Simpson *et al*., 1987). The value of p'/Q_∞ is largest at $\phi = 90°$ and $\phi = 180°$ and smallest very near separation (figure 148). The decrease in p'/Q_∞ at middle ϕ locations is almost entirely a result of the minimum in high frequency *p* very near separation (tables 12 and 13). The low frequency contribution to p'/Q_∞ is nearly constant with ϕ (tables 12 and 13). Values of p'/τ_W are largest at the middle ϕ locations near separation (figure 148). However, the maximum in p'/τ_W does not correlate with the separation location as well as the minimum in p'/Q_∞.

At $\alpha = 20°$, p'/Q_∞ is a local minimum very near separation due to decreased high frequency spectral values (figures 118 and 119 tables 14 and 15). The maximum p'/Q_∞ is near the ϕ location of the center of the primary vortex (figure 148) and is mostly due to the AIC (tables 14 and 15). The large AIC here is a result of the measured spectral values at high frequency which are nearly constant or even increasing at some ϕ locations (figures 110 and 111). Although the AIC is a model, it most likely follows *p*' accurately since the application of the AIC was consistent for each ϕ location. At $x/L = 0.772$, there is also local maximum in p'/Q_∞ at reattachment. This local maximum occurs in the contribution to $\overline{p^2}$ from all frequencies (tables 14 and 15). A local maximum in p'/τ_W occurs at both the primary and secondary separations (figure 148). The p'/τ_w value magnifies the effect of the separation (through u_τ) and follows the relative contribution of the outer layer and low frequency spectral contribution as compared to the viscous wall layer high frequency contribution (tables 14 and 15). Around the separation locations larger values of p'/τ_w occur because low frequency contributions are relatively large while τ_w is much lower with a lower contribution from the viscous region.