Chapter 6. Conclusions and Recommendations

6.1 Conclusions

Based on the results presented in this dissertation it is possible to conclude the following:

- An expression for an upper bound of the uncertainty of specific distribution factors between two surfaces of an enclosure has been developed. This expression relates the estimated distribution factors to the number of energy bundles traced in the MCRT algorithm by

$$\mathrm{Err}_{D'_{ij}} \equiv \left| D'^{t}_{ij} - D'^{e}_{ij} \right| \leq W_c \sqrt{\frac{D'^{e}_{ij}(1 - D'^{e}_{ij})}{N}} \tag{3.5}$$

- Numerical experiments show that as the number of energy bundles becomes large, the values obtained using Equation 3.5 approach zero, as indicated by the equation. They also show that in all cases, Equation 3.5 indeed gives an upper bound for the uncertainty in the distribution factors in a cavity. The bound provided by Equation 3.5 is conservative for the cavity studied, but not overly so.

- Equation 3.13,

$$\mathrm{Err}_{D'} \leq \omega_{D'_{ij}} \leq W_c \sqrt{\frac{n-1}{Nn^2}} \tag{3.13}$$

provides a way to estimate the upper bound on the global uncertainty of all the distribution factors in a cavity.

- The numerical experiments agree with the theoretical result represented by Equation 3.13, and in all the cases explored in Chapter 4 it provides a bound for the global uncertainty in the distribution factors.

- Figure 4.8 and Figure 4.29 show that the global uncertainty in the distribution factors increases when the number of surface elements decreases.

- Tables 4.3 and 4.4, derived from Equation 3.5, show the values of the global uncertainty in the distribution factor as a function of the number of energy bundles traced per surface element N, and the number of surface elements n in the cavity for a 90–percent and 95–percent confidence interval. Both tables are very useful for determining the minimum number of energy bundles required to be traced per surface element to obtain a desired global uncertainty of the distribution factors.

- Using the properties given in Table 4.1 a factor of 0.3 was found to transform Equation 4.3 into an approximation of the global uncertainty of the distribution factors of the cavity of Figure 4.1 as stated by Equation 4.3a,

$$\mathrm{Err}_{D'_{ij}} = 0.3\, W_c \sqrt{\frac{n-1}{Nn^2}} \qquad \text{(Cavity of Figure 4.1)} \quad (4.3a)$$

- For the new properties given in Table 4.5 the factor C of Equation 4.3 is 0.36 for the cavity of Figure 4.1.

- Equation 3.24, later written as

$$\omega_{q''_i} = \varepsilon_i \sigma \sqrt{\left[\left(\frac{\omega_\varepsilon}{\varepsilon}\right)^2 + \left(\frac{4\omega_T}{T}\right)^2\right]\left[T_i^4 - \sum_{j=1}^{n} T_j^4 D'_{ij}\right]^2 + \left[\sum_{j=1}^{n} T_j^4 \omega_{D'_{ij}}\right]^2} \qquad (4.4)$$

provides an approximation of the uncertainty of the net heat flux among surfaces and also provides bounds of the range where the error of the net heat flux should lie.

- It was shown in Section 4.4.1 that the seeds used in the random number generator to obtain the estimated temperatures and emissivity have little effect on the experimental uncertainty and the uncertainty from Kline and McClintock (right-hand side of Equation 4.4) of the net heat transfer. However, these seeds seem to have a large impact on the error of the net heat transfer which can be explained by the effect of the fourth power on the temperatures, which is more explicit in the calculation of the error than in the uncertainty.

- It was also shown in Section 4.4.3 that the fractional uncertainty has a big effect in Equation 4.4. Figure 4.19 shows that when the uncertainty in emissivity and temperature are much grater than the uncertainty of the distribution factor, the uncertainty predicted using the Kline and McClintock formalism converges quickly to a value that is much greater than the value of the experimental uncertainty. When the fractional uncertainty in emissivity and temperature decreases, the uncertainty predicted by the Kline and McClintock formalism seems to converge to a value approximately equal to the value of the uncertainty predicted by the large number of numerical experiments (experimental uncertainty).

- Equation 3.39,

$$\left\langle\left(\frac{\omega_{q_i''}}{\varepsilon_i\sigma T_i^4}\right)^2\right\rangle_i \le \left[\left(\frac{\omega_\varepsilon}{\varepsilon}\right)^2+\left(\frac{4\omega_T}{T}\right)^2\right]\left[1+n^2\frac{\left\langle T^4\right\rangle^2}{\left\langle\left(T^4\right)^2\right\rangle}\right]+\frac{W_c^2}{N}\left(1-\frac{1}{n}\right) \qquad (3.39)$$

is an expression for a conservative upper bound of the mean of the uncertainties of the net heat flux of all the surfaces in an enclosure. It also allows the estimation of the minimum number of energy bundles required to be traced per surface element to achieve a desired uncertainty of the heat transfer.

- Figures 4.20 through 4.22 show that the seeds of the random number generator used to generate the estimated temperature and emissivity has little effect on the global uncertainty of the net heat transfer obtained from the Kline and McClintock theorem as well as the experimental uncertainty. However, the seeds have more effect on the global error of the net heat transfer due to the fourth power of the temperatures, which produces considerable differences between "true" and estimated temperatures.

- In Section 4.5.2 it was shown that keeping the fractional uncertainty small (compared with the percentage difference between minimum and maximum temperatures in the temperature distribution), a better approximation of the global uncertainty of the net heat flux can be obtained using the right-hand side of Equation 3.39.

- Defining the cavity of Figures 4.1 and 4.2 by three surface elements, the coefficient C of Equation 4.3 is found to be 0.41.

- In Chapter 5 a model of a CERES-type cavity was used in the FELIX environment to show an application of the expressions developed in Chapter 3 and used in Chapter 4 with another geometry.

- Table 5.2 gives the values of the uncertainty of the distribution factor from the source to the detector for different values of energy bundles, estimated using Equation 3.3.

- Table 5.3 gives a bound of global uncertainty of the distribution factors for different number of energy bundles. This table can be used to estimate the number of energy bundles required to be traced to achieve a specific uncertainty of all the distribution factors for the CERES-type cavity.

- Figure 5.1 shows the optimum number of energy bundles that produce the best approximation for the uncertainty of the net heat flux of three surfaces of the CERES-type cavity for three different values of fractional uncertainty.

- Results from Chapter 4 and 5 indicate that mean uncertainty in the net heat flux is relatively insensitive to the number of energy bundles when the fractional uncertainty in temperature is larger than the uncertainty in the distribution factors. Therefore in order to use Equation 3.39 to predict the number of energy bundles required in a MCRT analysis for a specific uncertainty, it is recommended then to keep the fractional uncertainty on par or smaller than the global uncertainty of the distribution factor.

6.2 Recommendations

Based on the experience gained in carrying out the effort described in this dissertation it is recommended that:

- The impact of different geometries be investigated to study with different geometries to ensure that the methodology is valid independent of specific geometries.
- Further studies be performed to extend the methodology developed to non-gray radiation heat transfer problems.
- Efforts be made to incorporate bi-directional surface properties into the methodology to analyze non-diffuse-specular surfaces.
- Further efforts be made to integrate the methodology developed with high-level numerical models to help study the overall uncertainty associated with complex systems such as radiometric instruments.