

**UNCERTAINTY AND CONFIDENCE INTERVALS OF THE
MONTE CARLO RAY-TRACE METHOD IN RADIATION HEAT
TRANSFER**

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Abstract

The primary objective of the work reported here is to develop a methodology to predict the uncertainty associated with radiation heat transfer problems solved using the Monte Carlo ray-trace method (MCRT). Four equations are developed to predict the uncertainty of the distribution factor from one surface to another, the global uncertainty of all the distribution factors in an enclosure, the uncertainty of the net heat flux from a surface, and the global uncertainty of the net heat flux from all the surfaces in an enclosure, respectively. Numerical experiments are performed to successfully validate these equations and to study the impact of various parameters such as the number of surfaces in an enclosure, the number of energy bundles traced in the MCRT model, the fractional uncertainty of emissivity and temperature, and the temperature distribution in the enclosure. Finally, the methodology is successfully applied to a detailed MCRT model of a CERES-like radiometer.

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Nomenclature

Symbols:

A	Area (m^2)
D'_{ij}	Distribution factor from surface i to surface j
E	Estimated value
Err	Error
\vec{n}	Normal vector
n	Number of surfaces
N	Number of energy bundles
P	Power (W)
q''	Net heat flux (W/m^2)
Q	Radiative energy (W)
r_s	Specularity ratio
R	Random number between zero and unity
s	Standard deviation
\vec{t}	Tangential vector
T	Temperature (K)
W	W-statistic

Greek:

α	Absorptivity (—)
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ε	Emissivity (–)
ϕ	Circumferential angle (rad)
ρ	Reflectivity (–)
σ	Stefan-Boltzmann constant (W/m^4)
θ	Azimuth angle (rad)
ω	Uncertainty
$\omega_{\text{K\&M}}$	Uncertainty obtained using the Kline and McClintock formalism
ω_{exp}	Experimental uncertainty