# Appendix A: Details of Deformation Analyses

This appendix presents details of theory and procedures for some aspects of the analyses presented in Chapter 4. Specifically this appendix consists of the following:

- A discussion on the theory and procedure for calculating the seepage and buoyant forces used in the deformation analyses.
- A discussion on the theory and procedure for calculating the tensile stress in the seepage barrier due to bending.

## Seepage and Buoyant Forces

The deformation analyses for this study were performed using the computer program Phase2 by Rocscience (2007). Phase2 is a finite element program for calculating deformations in soil and rock. The key to these analyses is the modeling of seepage forces. Because Phase2 does not have the capability of calculating seepage forces due to hydraulic gradients, the seepage forces needed to be calculated outside of the program and then inserted into the program as line loads (running normal to the two-dimensional model) at each node. The calculation of the seepage forces was performed using a procedure described by Clough and Duncan (1969) and is performed by manipulating the nodal water pressure results obtained from Phase2 finite element seepage analyses, the nodal coordinates, and node/element connectivities through Excel spreadsheet files and a MatLab m-file program written specifically for this purpose.

The Clough and Duncan procedure assumes that the pore pressures on the sides of the element vary linearly between the nodal pore pressures as shown on Figure A-1. The x- and y-components of the forces at each node are computed by summing the contribution of the water pressure distributions on each connected element boundary attached to that node. Equation 1 is used to calculate the x- and y- components of forces due to water pressures acting on the boundaries of a quadrilateral element.

$$\begin{Bmatrix} R_{xi} \\ R_{yi} \\ R_{xj} \\ R_{yj} \\ R_{xk} \\ R_{yk} \\ R_{xl} \\ R_{yl} \end{Bmatrix} = \frac{1}{6} \begin{bmatrix} 2(Y_{li}+Y_{ij}) & Y_{ij} & 0 & Y_{li} \\ 2(X_{il}+X_{ji}) & X_{ji} & 0 & X_{il} \\ Y_{ij} & 2(Y_{ij}+Y_{jk}) & Y_{jk} & 0 \\ X_{ji} & 2(X_{ji}+X_{kj}) & X_{kj} & 0 \\ 0 & Y_{jk} & 2(Y_{jk}+Y_{kl}) & Y_{kl} \\ 0 & X_{kj} & 2(X_{kj}+X_{lk}) & X_{lk} \\ Y_{li} & 0 & Y_{kl} & 2(Y_{kl}+Y_{li}) \\ X_{il} & 0 & X_{lk} & 2(X_{lk}+X_{il}) \end{bmatrix} \begin{Bmatrix} \Delta u_i \\ \Delta u_j \\ \Delta u_k \\ \Delta u_l \end{Bmatrix} \qquad \text{A-1}$$

Where $R_{xi}$ and $R_{yi}$ are the x- and y-components of the sum of the loads distributed to node i from the changes in pressures acting on the sides of the element, $Y_{li}$ is the y-coordinate of node l minus the y-coordinate of node i, $X_{il}$ is the x-coordinate of node i minus the x-coordinate of node l, and $\Delta u_i$ is the change in water pressure at node i for the stage in the analysis.

An application of Equation A-1 on a simple model consisting of four elements is presented in Figure A-2. Stage 1 of the model represents a hydrostatic condition with the phreatic surface located at an elevation of 5 feet. In Stage 2 the phreatic surface is raised to 15 feet on the left side and 10 feet on the right side, imposing a hydraulic gradient of 1.0 across the model. The first row of the table in Figure A-2 shows the changes in pore water pressure acting on the boundaries of the elements. The second row in the table shows the nodal body forces due to the pore water pressure changes calculated using Equation A-1 and the changes in pore pressure at each node of the element.

When Equation A-1 is used to calculate nodal pore water body forces, the full water pressure differential between opposite sides of the element acts over the entire width of the element. However, because the differential water pressure is assumed to decrease linearly from the maximum difference to zero across the element (with the average differential water pressure of one-half the total value), the full pore water pressure differential body forces will result in twice the actual amount of stress on the element and will result in more strain than appropriate (twice the amount of strain if a linear stress-strain relationship is assumed). To correct this, all of the forces on all of the nodes of an element are summed and redistributed evenly between the nodes. This results in the average stress being applied to the element and maintains the same sum of body forces for the element that would result from forces calculated using Equation A-1. The equation for calculating the distributed x- and y-components of the nodal forces due to the changes in pore water pressures acting on a quadrilateral element is as follows:

$$\begin{Bmatrix} R_x \\ R_y \end{Bmatrix} = \frac{1}{8}\begin{bmatrix} (Y_{li}+Y_{ij}) & (Y_{ij}+Y_{jk}) & (Y_{jk}+Y_{kl}) & (Y_{kl}+Y_{li}) \\ (X_{il}+X_{ji}) & (X_{ji}+X_{kj}) & (X_{kj}+X_{lk}) & (X_{lk}+X_{il}) \end{bmatrix}\begin{Bmatrix} \Delta u_i \\ \Delta u_j \\ \Delta u_k \\ \Delta u_l \end{Bmatrix} \qquad \text{A-2}$$

Where $R_x$ and $R_y$ are the x- and j-components of the loads distributed to each node from the changes in pressures acting on the element. The distributed body force loads due to the change in water pressure in the Figure A-2 example are shown in the third row of the table in Figure A-2.

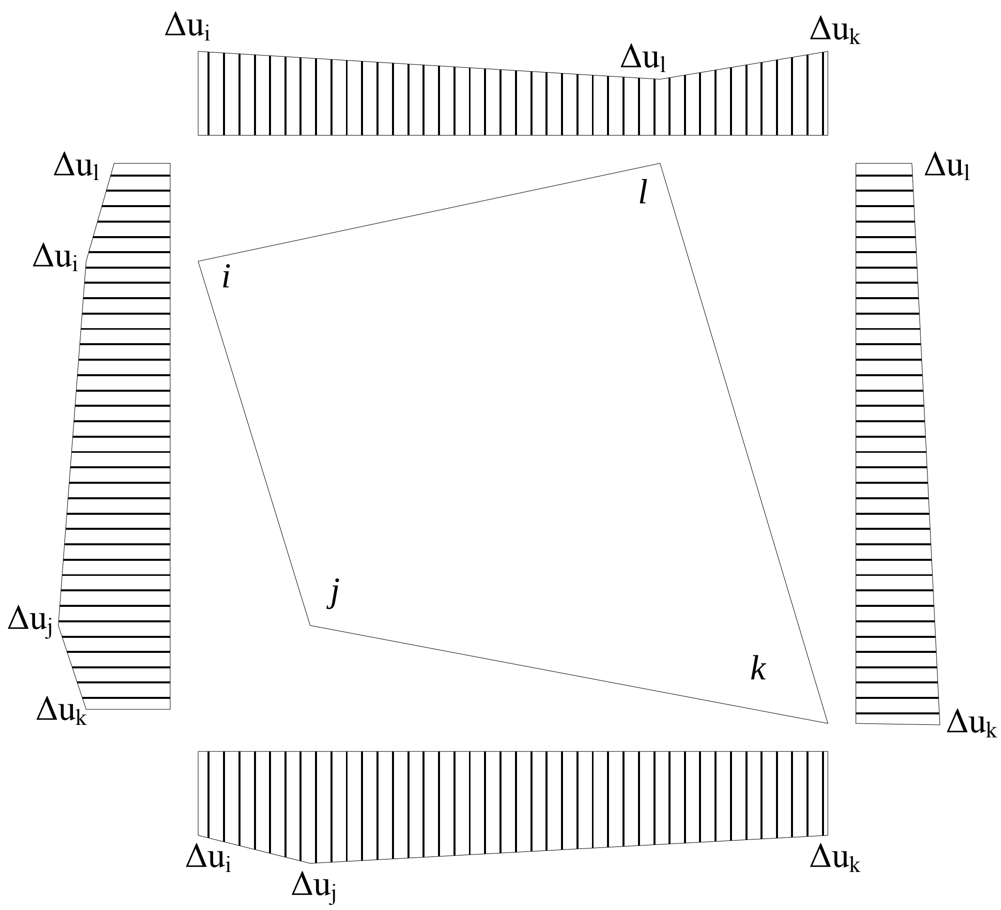


Figure A-1 Orthogonal components of water pressure acting on the boundaries of a quadrilateral element (after Clough and Duncan, 1969)

For the entire finite element mesh, the contributions from all of the elements connected to a node are summed to calculate the total water pressure body force acting on the node. The changes in water pressure body forces are calculated for each stage in the analyses where there is a change in the groundwater regime. Because the forces calculated by equation 1 also include the changes in buoyant force on an element, the model should be set up with total unit weights. When the forces are applied the buoyant component of the forces will result in an effective stress condition.

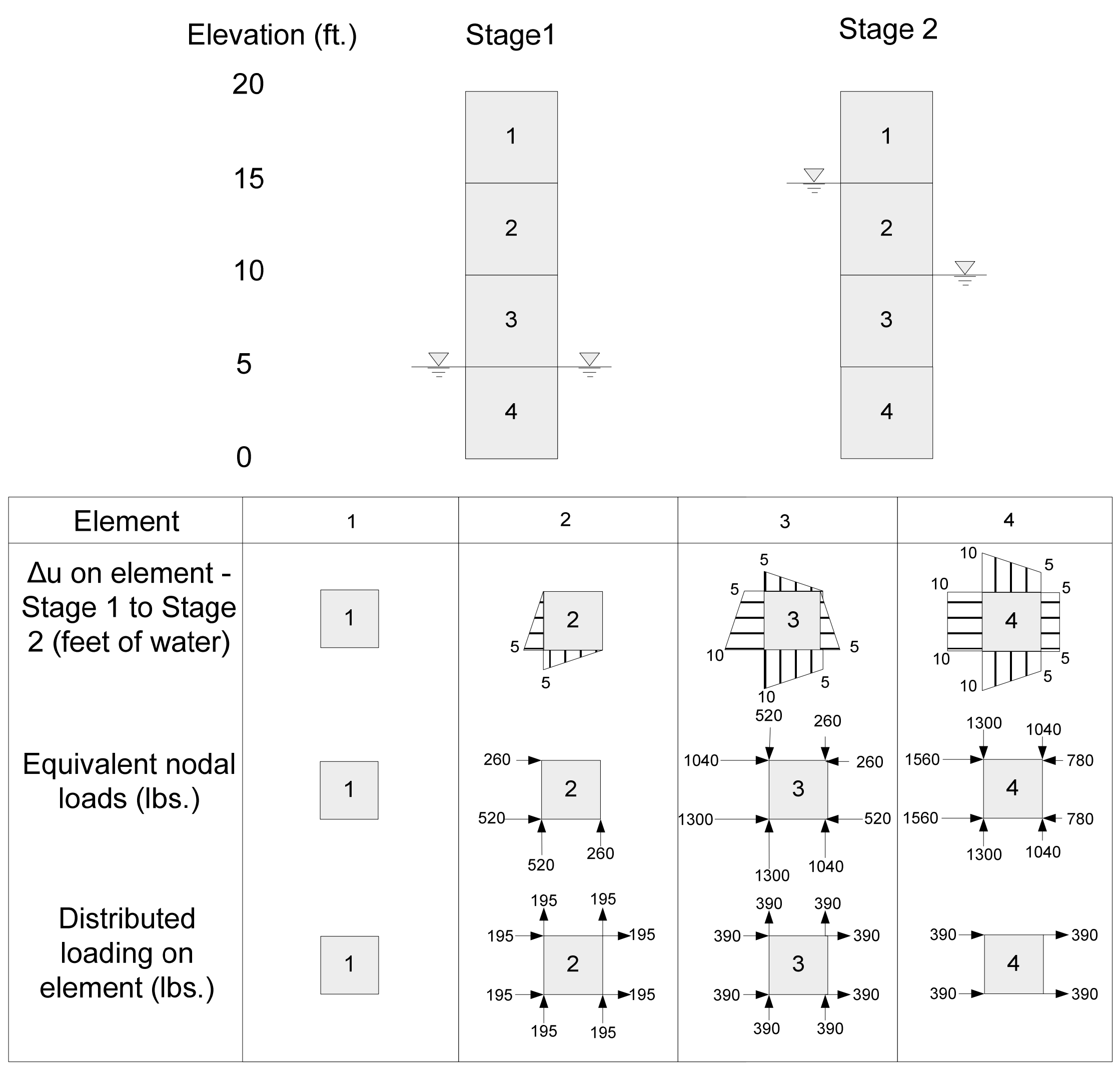


Figure A-2 Application of the pore pressure body force calculation procedure on a four-element, two-staged model (after Clough and Duncan, 1969)

The water pressure forces also account for surface water pressures acting on the outside of the model. To illustrate this principle, consider the two element model shown in Figure A-3(a). The top of the upper element is a distance, d, below the water surface. Figure A-3(b) shows the water pressures acting on the single elements. Summing the forces on each element results in a net force equal to the area of the element times the unit weight of water or the buoyant force. When the elements are joined back together, the water pressure at the base of element 1 and the top of element 2 are equal in magnitude but acting in opposite directions, and thus cancel each other out in terms of the external forces acting on the model. The remaining pressures are the external forces acting on the outer boundaries of the model. In this way, exterior water pressures will be applied to any submerged exterior surface in a finite element mesh. In the actual modeling, the pressures will be converted to nodal body forces and distributed evenly among the nodes connected to the elements along the boundaries in the manner illustrated in Figure A-2.

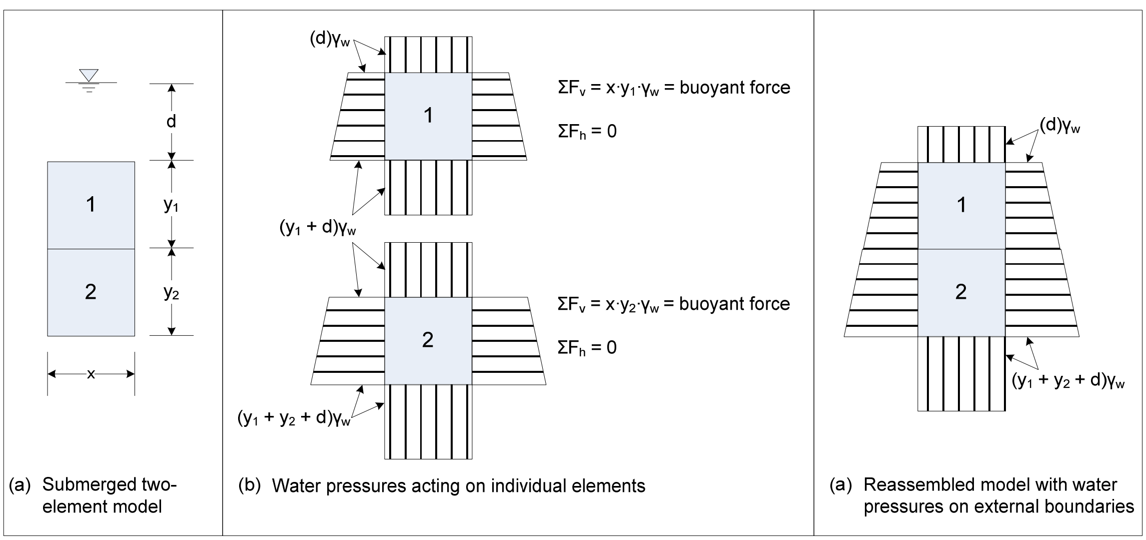


Figure A-3 Model illustrating how surface water pressures are included in the seepage and buoyant force calculations

**Tensile Stress and Cracking Potential**

The results of the deformation analyses provide deformations of the dam and seepage barrier and bending moments induced in the seepage barrier as a result of the deformation. The potential for the development of cracks in the seepage barrier was assessed by comparing the tensile stresses in the barrier with the tensile and shear strength of the barrier infill material. The maximum tensile and compressive stress in the barrier due to the imposed bending moment was assessed from the equation:

$$\sigma_m = \frac{M\ c}{I} \qquad \text{A-3}$$

where: $\sigma_m$ is the tensile or compressive stress, $M$ is the imposed bending moment per horizontal foot of seepage barrier, $c$ is the distance from the edge of the barrier to the center of the barrier, and $I$ is the moment of inertia per horizontal foot of barrier.

Due to the general lack of tensile strength tests performed on the backfill material for the seepage barriers analyzed, the tensile strength of the concrete, $f'_t$, was generally assessed based on the unconfined compressive strength, $f'_c$, using the following equation (Oluokun 1991):

$$f'_t = 4.5\sqrt{f'_c} \qquad \text{A-4}$$

Where $f'_c$ and $f'_t$ are the unconfined compressive strength and the tensile strength of the concrete in pounds per square inch.

The maximum tensile stress calculated using Equation A-3 and the maximum moment calculated in the finite element deformation analysis is compared to the estimated tensile strength of the seepage barrier backfill to assess the potential for the concrete tensile strength to be exceeded, resulting in the barrier cracking.