Chapter 5 Features and Verification of FE Program

5.1 Introduction

The objectives of this chapter are to (1) describe features in the FE program that have not already been introduced and (2) present solutions from several verification problems as proof that the program works correctly. Several features of the FE program have already been introduced. In Chapter 3, the benefits gained from effective stress simulations of multi-kilobar material behavior and the available material models were described. The equations of state of air, water, and solids were also described in Chapter 3. The cap model was documented in Chapter 4.

The following features will be presented in this chapter. A restart feature was implemented into the FE program to permit the simulation of certain laboratory tests. For example, a consolidated undrained triaxial compression test wherein the consolidation phase has drained boundary conditions and the shear phase has undrained boundary conditions requires changing fluid flux boundary conditions. A brief summary of the postprocessing procedures will also be presented. These features are described in the next section.

The final sections of this chapter document solutions from several verification problems. For each problem, the FE solution is compared to available closed form or analytic solutions. These verification problems establish the FE program's ability to correctly solve a variety of initial and boundary value problems.

5.2 Additional Features of FE Program

5.2.1 Restart feature

The restart feature was implemented for the purpose of allowing the user to change the boundary conditions at a preselected time in the calculation. A K_o/BX/STX test (acronyms defined subsequently) is an example of a laboratory test with changing boundary conditions. This test is conducted by loading a cylindrical specimen to a desired mean normal stress level under K_o or uniaxial strain boundary conditions, unloading to a desired mean normal stress level under constant axial strain (BX) boundary conditions, and then conducting a constant radial stress triaxial compression (STX) test at yet another mean normal stress level. The K_o loading and the BX unloading phases may be numerically simulated with displacement controlled boundary conditions. However, to realistically attempt to simulate the STX phase, the user should apply stress controlled boundary conditions. The restart feature implemented into JAM allows the user to perform this calculation in a simple manner.

5.2.2 Postprocessing

In many instances, one would like to plot FE results at a single location, e.g., at the nodal points. Many postprocessing FE software packages require stress and strain values at the nodes rather than at the Gauss integration points. A procedure was implemented in the FE program JAM to extrapolate and smooth Gauss point data to the element vertices, i.e., corner nodes. Values at the midside nodes were then calculated from the values at the appropriate corner nodes.

The implemented smoothing procedure was developed and described by Hinton, Scott, and Ricketts (1975) and Hinton and Campbell (1974). The procedure is simple and straightforward. The smoothed stresses at the nodes may be calculated from the expression

$$\begin{Bmatrix} \tilde{\sigma}_1 \\ \tilde{\sigma}_2 \\ \tilde{\sigma}_3 \\ \tilde{\sigma}_4 \end{Bmatrix} = \begin{bmatrix} a & b & c & b \\ b & a & b & c \\ c & b & a & b \\ b & c & b & a \end{bmatrix} \begin{Bmatrix} \sigma_I \\ \sigma_{II} \\ \sigma_{III} \\ \sigma_{IV} \end{Bmatrix} \qquad 5.1$$

where the $\tilde{\sigma}_i$ are the smoothed stresses, $\sigma_I, \sigma_{II}, \sigma_{III}$ and σ_{IV} are the stresses at the integration points, and $a = 1 + \frac{\sqrt{3}}{2}$, $b = -\frac{1}{2}$ and $c = 1 - \frac{\sqrt{3}}{2}$. At a given corner node, smoothed values from adjacent elements are averaged to yield a single value.

5.3 Verification Problems

Several problems with plane or axisymmetric geometries were solved with JAM to test and verify that the material models, the eight-node quadrilateral element, and numerous other algorithms were correctly implemented in the FE program. For each of the five problems, selected output from JAM are compared with closed form or analytic solutions. Verification problems 1-4 are documented in Appendix F.

Verification Problem 5 is a plane strain simulation of a thick wall cylinder subjected to an increasing internal pressure. Due to the symmetry of the problem, a quarter grid was used in the calculation; the problem geometry and FE mesh are shown in Figure 5.1. The material was modeled with the following properties, a Young's modulus of 21000, a Poisson's ratio of 0.3, a yield stress of 24 and a linear hardening modulus of 0. When a cylinder with these properties is subjected to an increasing internal pressure above 10.4, an elastic-plastic boundary moves through the cylinder; on the external side of the boundary, all of the strains are elastic, and on the interior side, the strains are elastic-plastic. Table 5.1 compares stresses calculated from the FE

Table 5.1.
Results from Verification Problem 5

Stress	Radius	Plane Strain	
		JAM	Analytic
Max. Principal Stress or σ_θ	163.64	22.12	22.09
	176.32	20.32	20.25
	193.64	18.37	18.31
σ_z	> 160	5.33	5.31
J_1	> 160	23.11	23.03

and analytic solutions (Hodge and White 1950; Prager and Hodge 1951) at several radii within the elastic region for an applied internal pressure of 18, which places the elastic-plastic boundary at a radius of 160. The computed results are well within the imposed convergence tolerance of 1 percent. The results from JAM also agree with the results calculated by Owen and Hinton (1980) for this problem. This validates the plasticity formulation in JAM.

5.4 Consolidation Problems

To verify that the FE program could solve consolidation problems, output from JAM was compared to the results calculated from closed form solutions. Boundary conditions and material properties were altered to fully exercise different features within the FE program. In Figure 5.2, depth versus calculated pore fluid pressures are plotted in a normalized format at two different time increments for a one-dimensional consolidation problem in which the soil column was idealized as an elastic porous skeleton with an incompressible pore fluid. Good agreement is shown between the FE results and the closed form solution. A similar one-dimensional problem was solved with two materials having compressible pore fluids, where the ratios of pore fluid modulus to skeletal modulus (N) were 2000 (nearly incompressible pore fluid) and 5 (highly compressible pore fluid). Results from the FE program and the closed form solution (Chang and Duncan 1983) are plotted in Figure 5.3 as normalized displacements versus time factor, i.e., normalized time. Again, the results show reasonable agreement between the FE results and the closed form solution.

A two-dimensional axisymmetric consolidation problem consisting of a circular foundation on a finite soil layer (Figure 5.4) was also calculated. The mesh is *A* units high by *10A* units wide, and a uniform vertical load of radius *A* was applied to the top surfaces of three elements to simulate the foundation loads. The following boundary conditions were invoked for this problem. The vertical edges of the mesh (A-D and B-C) were constrained in the radial direction, the bottom edge of the mesh (C-D) was constrained in the vertical direction, the top surface (A-B) was free draining, and no flow conditions were applied to the three remaining surfaces (B-C, C-D, and A-D). The calculated settlements (in dimensionless format) from JAM (solid circles) are compared in Figure 5.5 to settlements calculated from the analytical solution (solid line). Again, there is excellent agreement between the calculation and the analytical solution.

5.5 Cryer Problem

A numerical simulation of Cryer's problem (Cryer 1963) was conducted as an additional verification test of the FE program. Cryer developed a closed form solution to the problem of a sphere of elastic porous material loaded on the surface by a constant uniform pressure and having drained boundary conditions. For values of Poisson's ratio less than 0.5, pore pressure at the center of the sphere increases to stress levels greater than the externally applied pressure and then dissipates. The greatest increase in pore pressure occurs for values of Poisson's ratio equal to 0. This response is called the Mandel-Cryer effect after the two mathematicians who discovered the phenomena. Gibson et al. (1963) conducted laboratory experiments on clay spheres and were able to reproduce the Mandel-Cryer effect. They demonstrated that the total stress within a consolidating sphere is not time invariant as predicted by Terzaghi-Rendulic consolidation theory. Dimensionless total stress at the center of the sphere increases above unity and approaches unity at late time.

Utilizing the symmetry of the problem, the soil sphere was represented by the mesh depicted in Figure 5.6 and calculated as an elastic axisymmetric problem. A unit pressure was placed on the boundary during the first increment of loading and held constant thereafter. Figure 5.7 and Figure 5.8 compare the results from JAM with the closed form solution for a value of Poisson's ratio equal to 0. Figure 5.7 is a plot of dimensionless pore pressure at the center of the sphere versus the square root of dimensionless time; Figure 5.8 is a plot of dimensionless displacement of the outer surface of the sphere versus the square root of dimensionless time. The comparison between the FE calculation and the closed form solution is very good. Pore pressure contours at a dimensionless time of approximately 0.04 are plotted in Figure 5.9. At this early time, a significant portion of the sphere has pore pressures greater than unity.

5.6 Summary

In this chapter, the restart and post-processing features of the FE program were described. The documented verification problems indicate that the FE program correctly calculates one- and two-dimensional consolidation problems and elastic and elastic-plastic boundary value problems. Although the successful calculation of these verification problems does not certify the FE program is error free, they should increase the confidence of the end user.

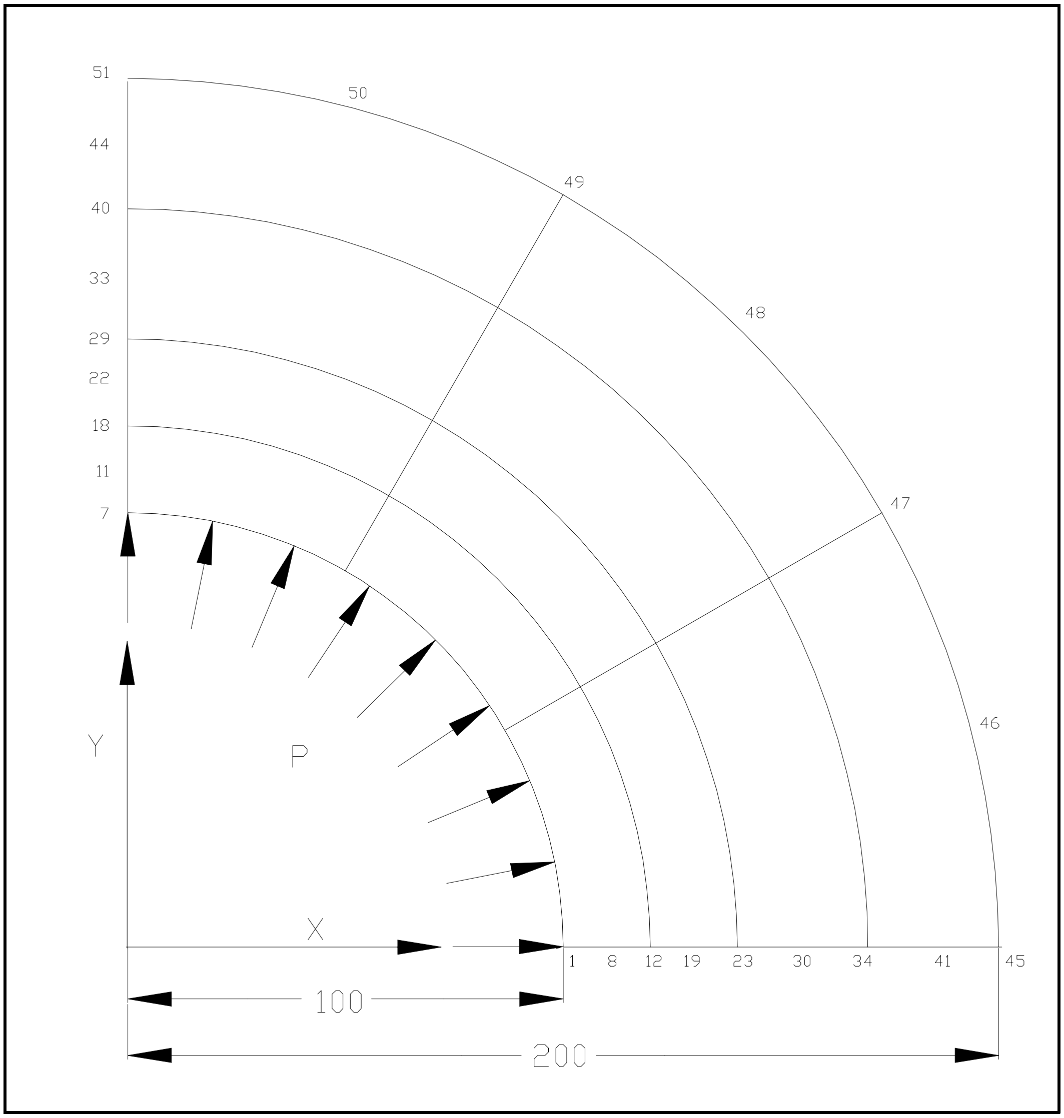

Figure 5.1. Mesh geometry for Verification Problem 5

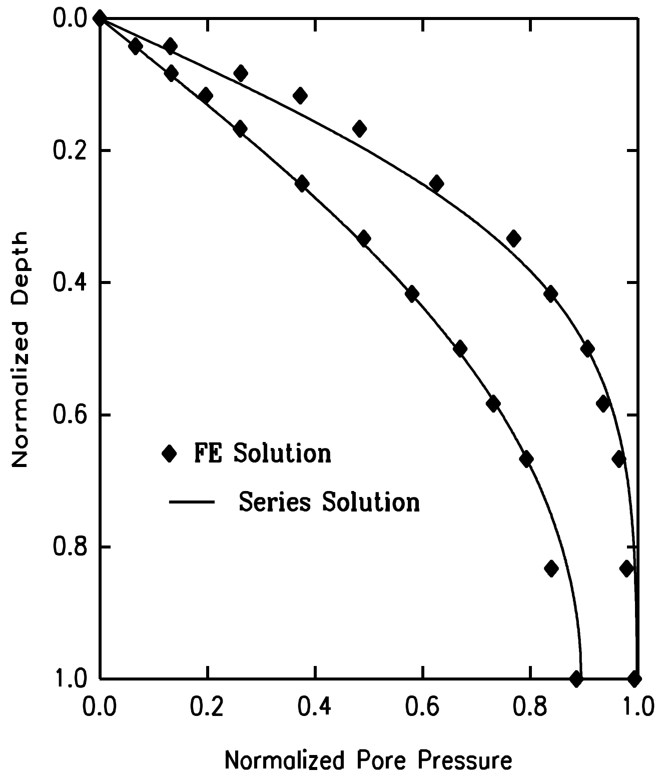

Figure 5.2. Pore pressure versus depth at two time increments

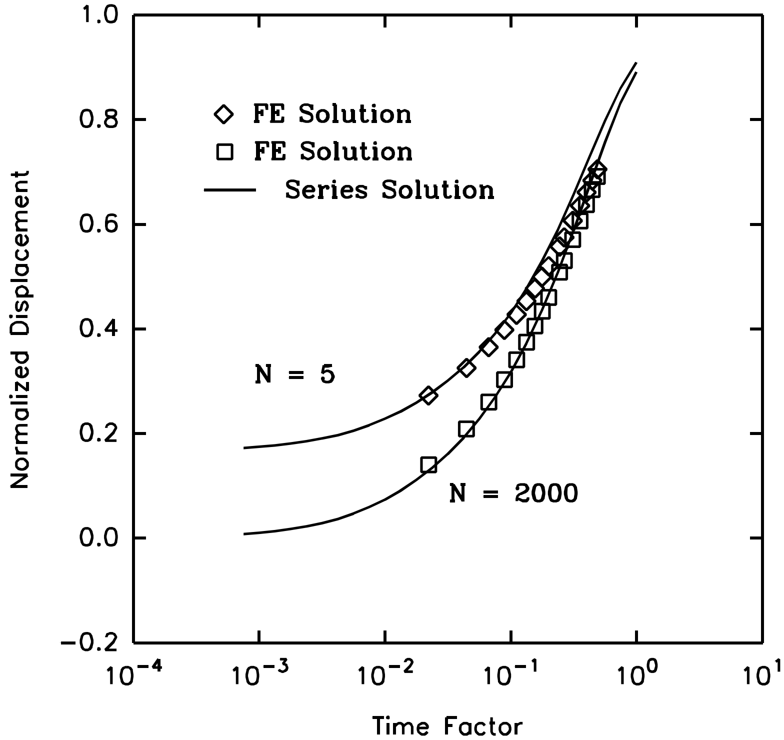

Figure 5.3. Displacement versus time for one-dimensional consolidation of an idealized elastic material

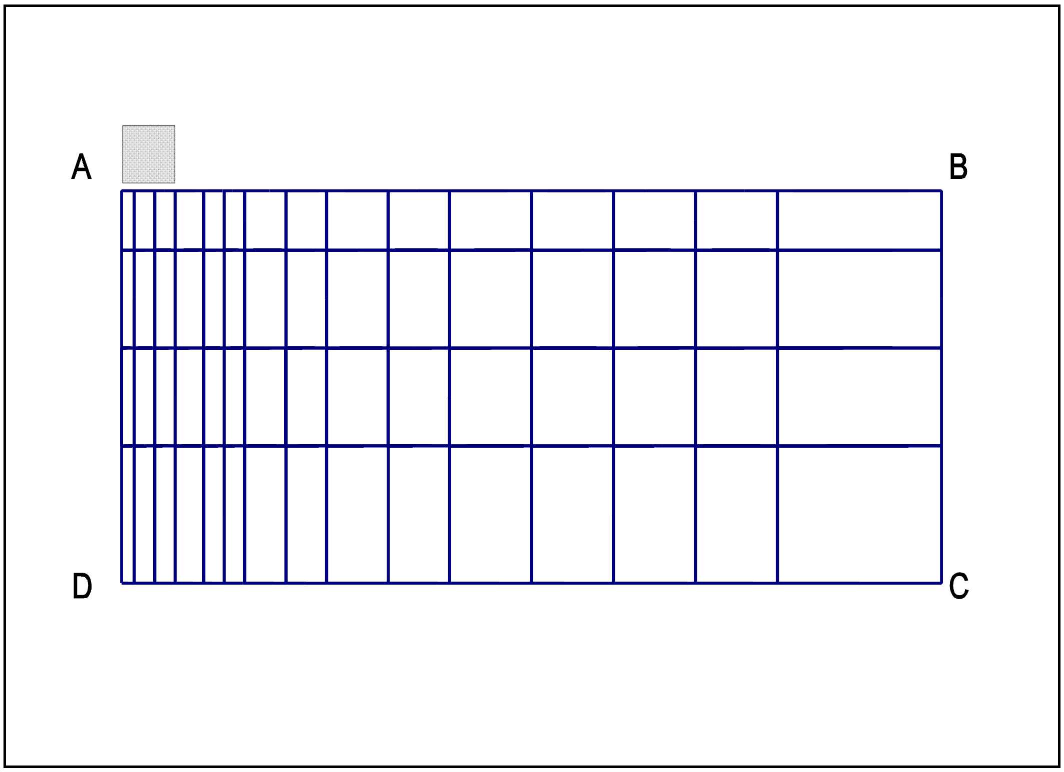

Figure 5.4. Mesh geometry for Axisymmetric Consolidation Problem

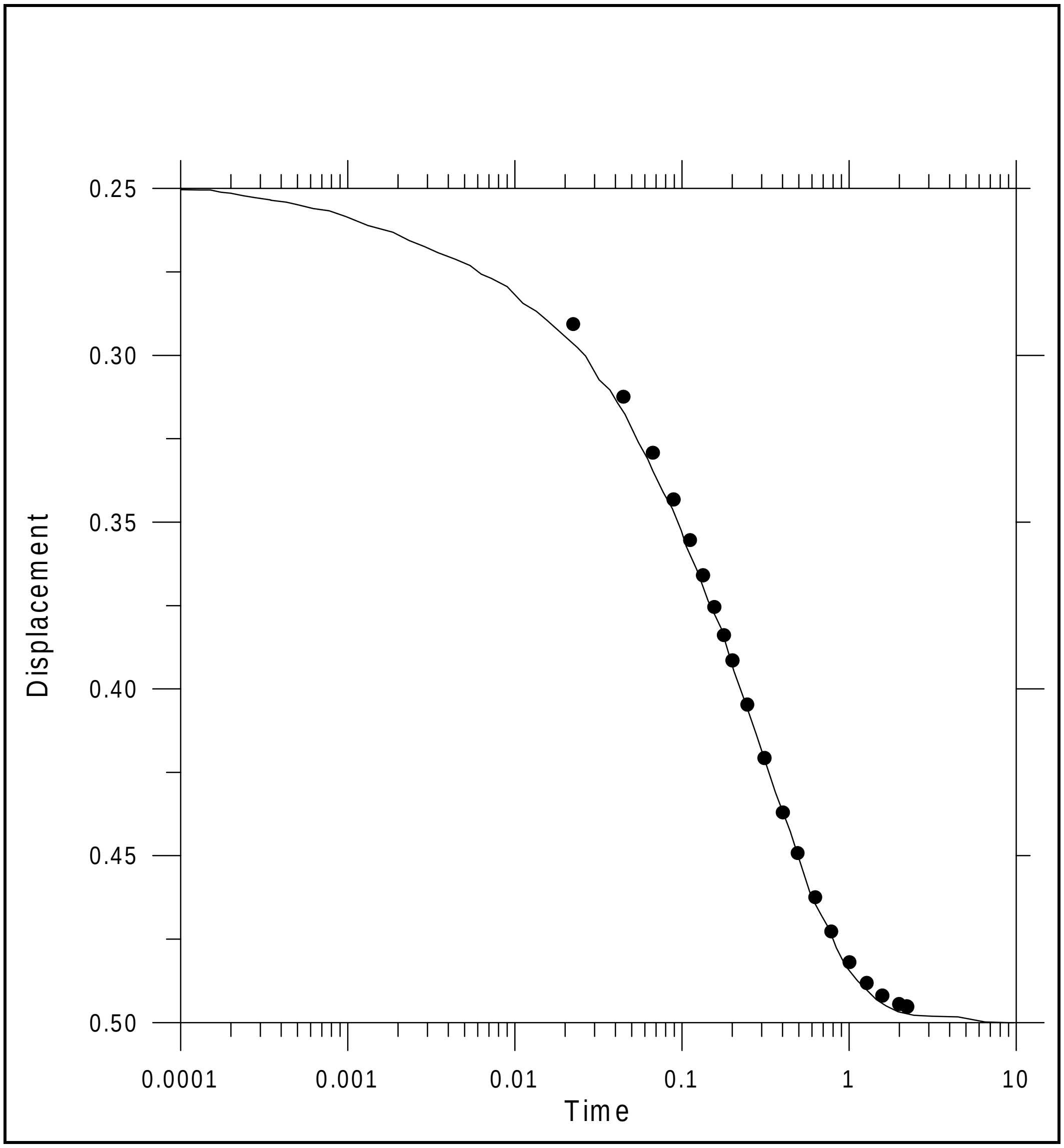

Figure 5.5 Displacement vs Time from 2D Consolidation Problem

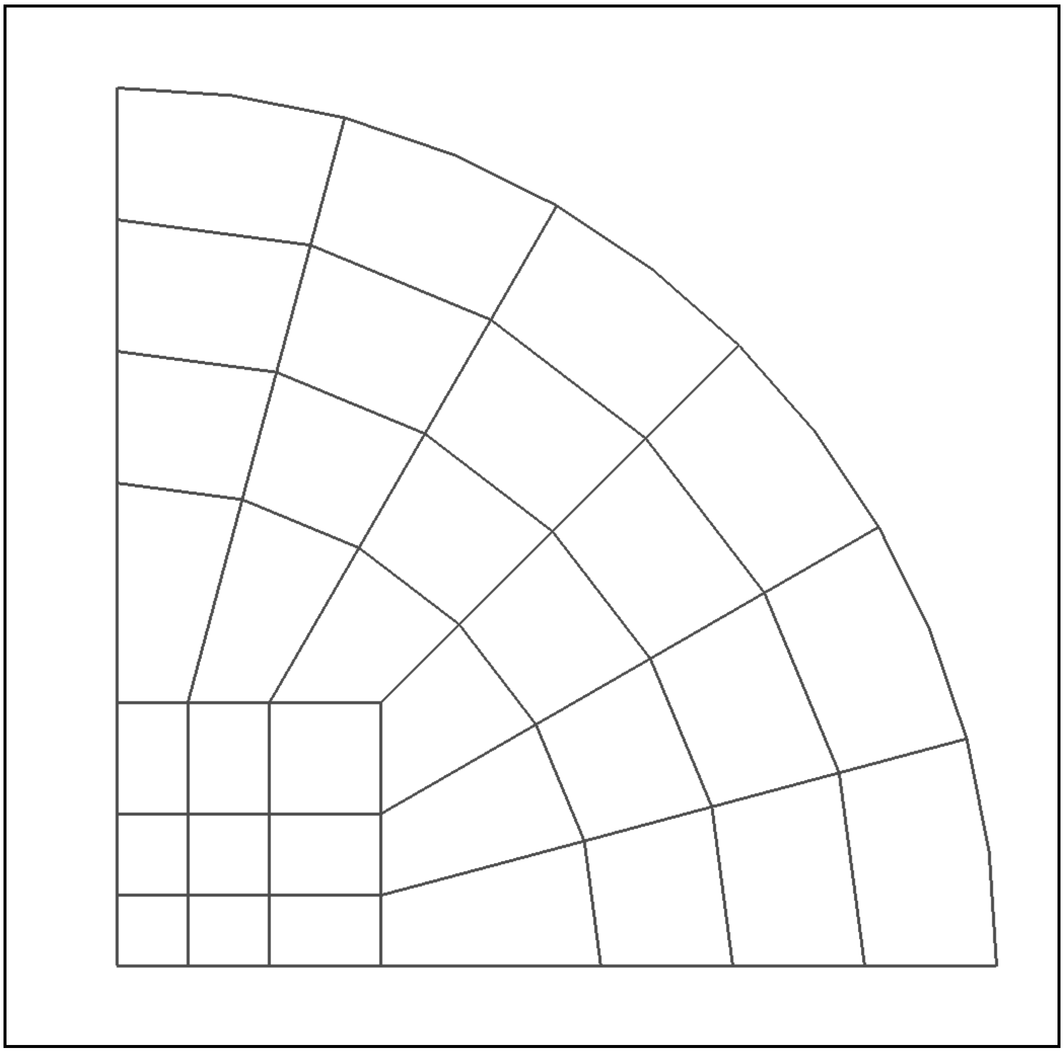

Figure 5.6. Mesh geometry for Cryer Problem

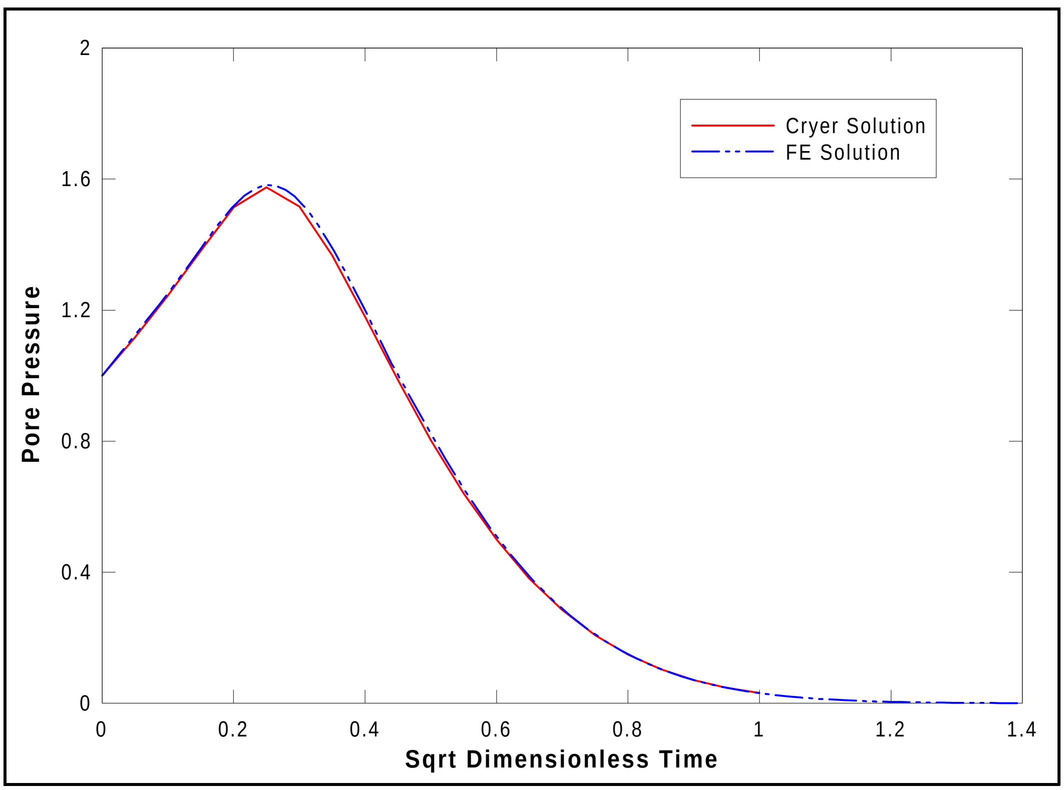

Figure 5.7. Dimensionless pore pressure response for Cryer's problem

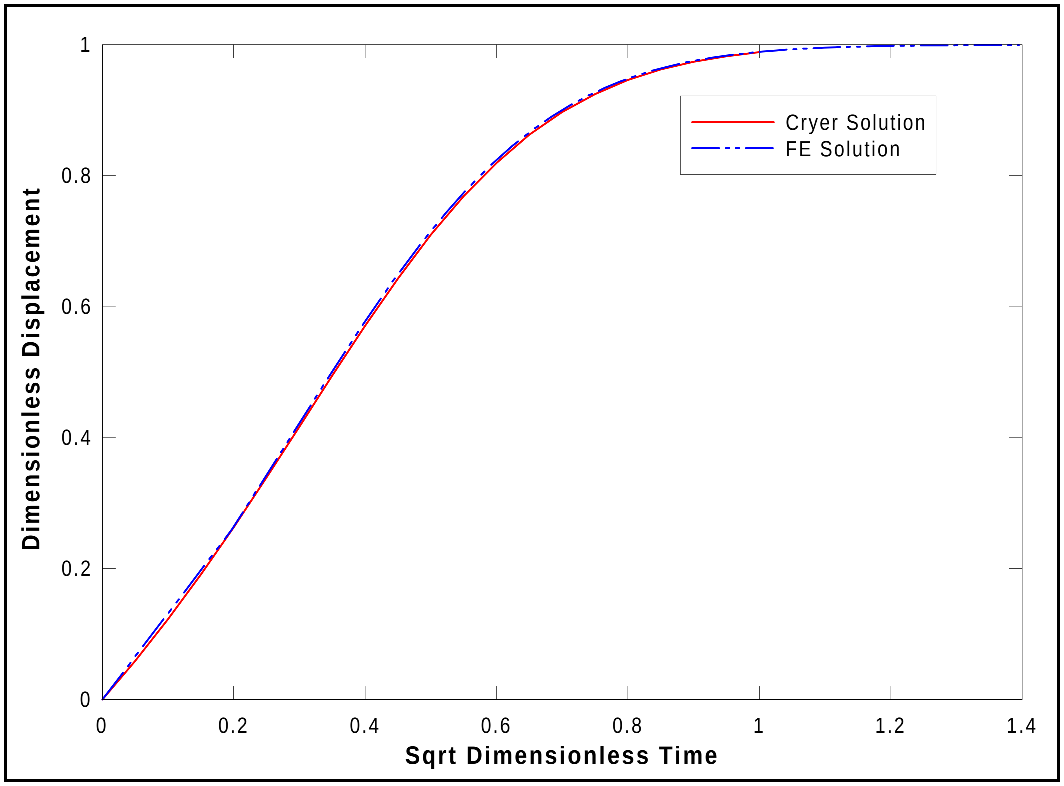

Figure 5.8. Dimensionless displacement response for Cryer's problem

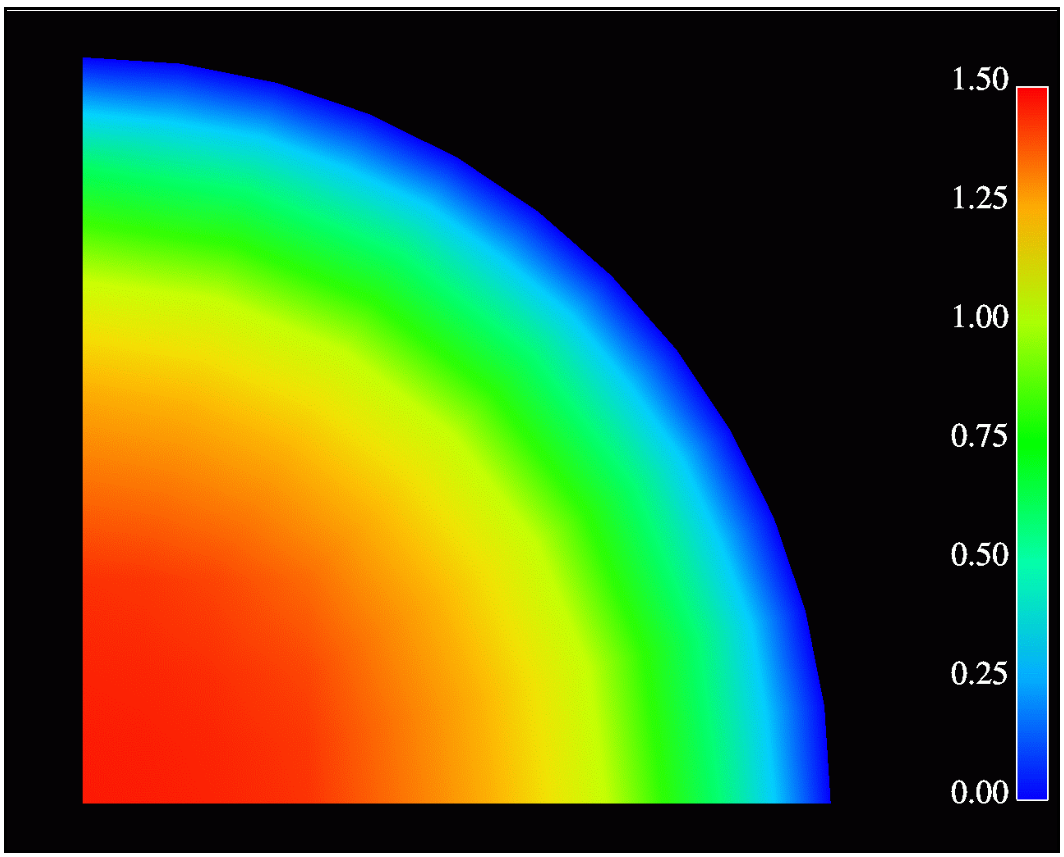

Figure 5.9. Pore pressure contours for Cryer Problem