

Appendix A Corrected Equations

Lewis and Schrefler (1987) describe in detail the components that contribute to the rate of fluid accumulation. However, one of the terms in their formulation is misleading if not incorrect. The following discussion expands on their analysis and then shows why their formulation requires modification. For the material and conditions of interest, Lewis and Schrefler specify that four volumetric strain components must be evaluated; they are the volume strain of the porous matrix ε_v^m , the volume strains of the grain solids ε_v^g and the pore fluid ε_v^f , and a component of solid volume strain ε_v^{ge} due to the applied effective stresses. To simplify the discussion of fluid accumulation, consider the following test conditions. A fully saturated porous material is contained in a sample chamber, which has frictionless sides, and is loaded by a frictionless piston of area A . A flow meter attached to the piston indicates the direction of fluid flow into (positive flow) or out of (negative flow) the sample. The top surface of the piston is loaded by a chamber pressure P_1 and a second fluid pressure P_2 is applied through the flow meter to the pore fluid within the sample. The fluid pressures P_1 and P_2 control the total and effective stresses within the sample. For a porous material with a volume of unity, the volume of the voids is n , and the volume of the solids is $1-n$.

If P_1 is increased and P_2 held constant, the effective stress in the specimen will increase, the material will compress, and pore fluid will flow out of the specimen. Since the specimen is fully saturated with fluid, the rate of change in fluid accumulation is equal to the volumetric strain of the porous matrix and may be written as:

$$\frac{\partial \varepsilon_v^m}{\partial t} = \frac{\partial \varepsilon_{kk}^m}{\partial t} = \frac{\partial \varepsilon_{ij}^m}{\partial t} \delta_{ij} \quad \text{A.1}$$

If P_1 and P_2 are increased at the same rate, the effective stress within the specimen will not change. However, both the grain solids and the pore fluid will compress due to the change in pore fluid pressure. For these conditions, fluid will flow into the specimen. Let us evaluate the solid and fluid components separately. The volume strain in the solids due to an applied pressure is expressed as:

$$d\varepsilon_s = \frac{dV}{V_s} = \frac{-dP}{K_g} \quad \text{A.2}$$

where K_g is the bulk modulus of the grains and V_s is the volume of the solids. From this equation, we can express the volume change within the unit volume due to the compression of the grains as:

$$d\varepsilon_v^g = (1 - n) d\varepsilon_s = (1 - n) \frac{-d\pi}{K_g} \quad \text{A.3}$$

where dP has been replaced by the pore fluid pressure $d\pi$.

In the same manner, the volume change within the unit volume due to the compression of the pore fluid is written as:

$$d\varepsilon_v^f = -n \frac{d\pi}{K_f} \quad \text{A.4}$$

where K_f is the bulk modulus of the pore fluid.

Lewis and Schrefler also evaluate the component of volume strain due to the compression of the grains caused by the increase in effective stress. A similar analysis was developed by Bishop (1973). Refer again to the test configuration and the example in which P_1 was increased

and P_2 held constant. For a statistically random distribution of pore space within the specimen, the area of solids (A_s) on any plane through the specimen will be

$$A_s = (1 - n)A \quad \text{A.5}$$

where n is the porosity of the specimen as previously defined. If $d\sigma_{ij}'$ is the average normal effective stress on any surface, then $d\sigma_{ij}' / (1 - n)$ is the average normal stress in the solids.

Using the concept implied by Equation A.2, the pressure applied to the solids can be expressed as:

$$dP = \frac{-d\sigma_{ij}'}{3(1-n)} \delta_{ij} = \frac{-d\sigma_{kk}'}{3(1-n)} \quad \text{A.6}$$

which when substituted into Equation A.3 gives:

$$d\varepsilon_s = \frac{dV}{V_s} = \frac{dV}{(1-n)V} = \frac{d\sigma_{kk}'}{3K_g(1-n)} \quad \text{A.7}$$

The component of volume strain due to the applied effective stresses for the unit volume is then:

$$d\varepsilon_v^{ge} = \frac{dV}{V} = \frac{d\sigma_{kk}'}{3K_g} \quad \text{A.8}$$

The components of the volume strain are now:

$$\frac{\partial \varepsilon_v^{\text{total}}}{\partial t} = \frac{\partial \varepsilon_v^m}{\partial t} - \frac{\partial \varepsilon_v^{ge}}{\partial t} - \frac{\partial \varepsilon_v^g}{\partial t} - \frac{\partial \varepsilon_v^f}{\partial t} \quad \text{A.9}$$

and the expressions for each component when substituted gives:

$$\frac{\partial \varepsilon_{kk}^{\text{total}}}{\partial t} = \frac{\partial \varepsilon_{kk}^{\text{matrix}}}{\partial t} - \frac{1}{3K_g} \frac{\partial \sigma_{kk}'}{\partial t} + \left(\frac{1-n}{K_g} + \frac{n}{K_f} \right) \frac{\partial \pi}{\partial t} \quad \text{A.10}$$

The term in dispute is the component of volume strain due to the applied effective stresses. Grain compression due to changes in effective pressure is already incorporated into all skeletal

constitutive models by default. In an analysis of drained test data, the skeletal strains are not decoupled from the grain strains, and the sum of the two is always measured by strain gauges or deformeters. Therefore, both components are included when a constitutive model is fit to drained hydrostatic loading data. A similar argument can be made by examining Equation A.10. During a drained test, Equation A.10 would reduce to the following

$$\frac{\partial \varepsilon_{kk}^{\text{total}}}{\partial t} = \frac{\partial \varepsilon_{ij}^{\text{matrix}}}{\partial t} \delta_{ij} - \frac{1}{3K_g} \frac{\partial \sigma_{kk}}{\partial t} \quad \text{A.11}$$

Clearly, this is in error; the only strains included in the above expression should be the skeletal strains. Therefore, one should only include grain compression due to changing pore pressures in the final formulation. The necessary modifications were made to Equations 3.8 and 3.11, which are rewritten below

$$L = \int_{\Omega} B^T \mathbf{m} \bar{N} d\Omega \quad \text{A.12}$$

and

$$s = \frac{1 - \phi}{K_g} + \frac{\phi}{K_f} \quad \text{A.13}$$