Appendix F Verification Problems

Four problems with plane or axisymmetric geometries were solved with JAM to test and verify that the material models, the eight-node quadrilateral element, and numerous other algorithms were correctly implemented in the FE program. For each of the four problems, selected output from JAM are compared with closed form or analytic solutions.

Verification Problem 1 exercises a single element under distributed normal and shear loads of 1000/length as shown in Figure F.1. The element simulates an isotropic linear elastic material having a Young's modulus of 30×10^{6} and a Poisson's ratio 0.3. The following boundary conditions were imposed:

$$u_x = u_y = 0 \text{ at point A and } u_y = 0 \text{ at points B and C}$$

Table F.1.
Results from Verification Problem 1

Stress or Strain	Plane Strain		Plane Stress	
	JAM	Analytic	JAM	Analytic
σ_x	-1000.	-1000.	-1000.	-1000.
σ_y	-1000.	-1000.	-1000.	-1000.
σ_{xy}	-1000.	-1000.	-1000.	-1000.
σ_z	-600.	-600.	0.	0.
ε_x	-1.733×10^{-5}	-1.733×10^{-5}	-2.333×10^{-5}	-2.333×10^{-5}
ε_y	-1.733×10^{-5}	-1.733×10^{-5}	-2.333×10^{-5}	-2.333×10^{-5}
γ_{xy}	-8.667×10^{-5}	-8.667×10^{-5}	-8.667×10^{-5}	-8.667×10^{-5}

Table F.1 compares stress and strain states calculated from the FE and analytic solutions (Hibbitt, Karlsson and Sorensen 1989) for this problem under plane strain and plane stress boundary conditions. This problem exercises the elastic constitutive model, verifies that the eight-node quadratic element accurately models constant strain states, and also checks that distributed loads are correctly simulated. The results from JAM match the analytic solution exactly. Verification Problem 1 was also successfully solved using the Cap model algorithm.

Verification Problem 2 exercises a single axisymmetric element under distributed normal loads of 1000/area as shown in Figure F.2. The element simulates an isotropic linear elastic material having a Young's modulus of 30×10^{6} and a Poisson's ratio 0.3. The following boundary conditions were imposed:

$$u_z = 0 \text{ at points A, B, and C}$$

Table F.2.
Results from Verification Problem 2

Stress or Strain	**Axisymmetric**	
	JAM	**Analytic**
σ_r	-1000.	-1000.
σ_z	-1000.	-1000.
σ_{rz}	0.	0.
σ_θ	-1000.	-1000.
ε_r	-1.333×10^{-5}	-1.333×10^{-5}
ε_z	-1.333×10^{-5}	-1.333×10^{-5}
γ_{rz}	0.	0.
ε_θ	-1.333×10^{-5}	-1.333×10^{-5}

Table F.2 compares stress and strain states calculated from the FE and analytic solutions (Hibbitt, Karlsson and Sorensen 1989) under the imposed axisymmetric boundary conditions. Like Verification Problem 1, problem 2 exercises the elastic constitutive model, verifies that the eight-node quadratic element accurately models constant strain states, and also checks that distributed loads are correctly simulated. The results from JAM match the analytic solution exactly.

Plane and axisymmetric patch tests were employed in Verification Problems 3 and 4. In the patch test, nodal point displacements are applied to a patch of elements such that a constant state of strain exists throughout the mesh. In Verification Problem 3, the elements simulate an isotropic linear elastic material having a Young's modulus of 30×10^6 and a Poisson's ratio 0.3. The imposed displacement boundary conditions were applied to the patch of elements shown in Figure F.3 and were calculated as:

$$u_x = \left(x + \frac{y}{2}\right) \times 10^{-3} \quad \text{and} \quad u_y = \left(y + \frac{x}{2}\right) \times 10^{-3}$$

Table F.3.
Results from Verification Problem 3

Stress or Strain	Plane Strain		Plane Stress	
	JAM	Analytic	JAM	Analytic
σ_x	1600.	1600.	1333.	1333.
σ_y	1600.	1600.	1333.	1333.
σ_{xy}	400.	400.	400.	400.
σ_z	800.	800.	0.	0.
ε_x	$1.\times10^{-3}$	$1.\times10^{-3}$	$1.\times10^{-3}$	$1.\times10^{-3}$
ε_y	$1.\times10^{-3}$	$1.\times10^{-3}$	$1.\times10^{-3}$	$1.\times10^{-3}$
γ_{xy}	$1.\times10^{-3}$	$1.\times10^{-3}$	$1.\times10^{-3}$	$1.\times10^{-3}$

Table F.3 compares stress and strain states calculated from the FE and analytic solutions (Hibbitt, Karlsson and Sorensen 1989) for this problem under plane strain and plane stress boundary conditions. The results from JAM match the analytic solution exactly. Verification Problem 3 was also successfully solved using the Cap model algorithm.

Verification Problem 4 consists of a patch of axisymmetric elements simulating an isotropic linear elastic material having a Young's modulus of 30×10^6 and a Poisson's ratio 0.3. The imposed displacement boundary conditions were applied to the patch of elements shown in Figure F.4 and were calculated as:

$$u_r = \left[(r - 1000) + \frac{z}{2}\right]\times 10^{-3} \quad \text{and} \quad u_z = \left[z + \frac{(r - 1000)}{2}\right]\times 10^{-3}$$

Table F.4.
Results from Verification Problem 4

Stress or Strain	Axisymmetric	
	JAM	Analytic
σ_r	5.769×10^4	5.769×10^4
σ_z	5.769×10^4	5.769×10^4
σ_{rz}	1.154×10^4	1.154×10^4
σ_θ	3.462×10^4	3.462×10^4
ε_r	$1.\times10^{-3}$	$1.\times10^{-3}$
ε_z	$1.\times10^{-3}$	$1.\times10^{-3}$
γ_{rz}	$1.\times10^{-3}$	$1.\times10^{-3}$
ε_θ	0.	0.

Table F.4 compares stress and strain states calculated from the FE and analytic solutions (Hibbitt, Karlsson and Sorensen 1989) for this problem. The results from JAM match the analytic solution exactly. Verification Problem 4 was also successfully solved using the Cap model algorithm.

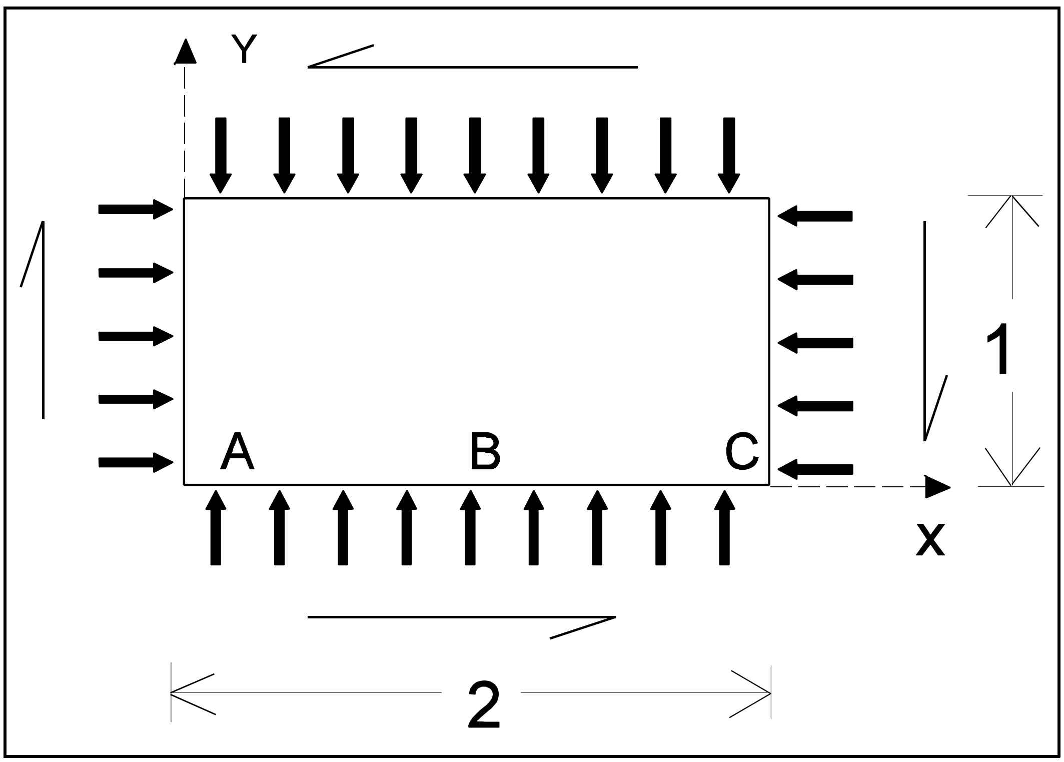

Figure F.1. Geometry and loading conditions for Verification Problem 1

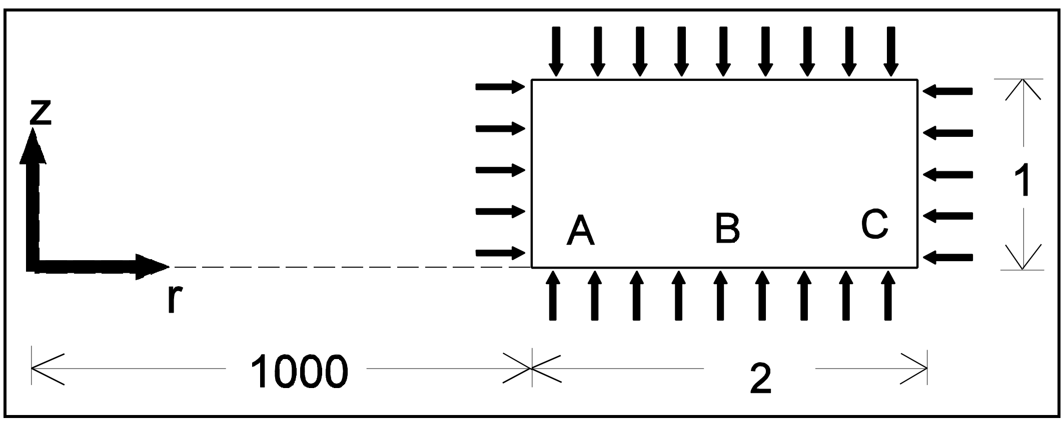

Figure F.2. Geometry and loading conditions for Verification Problem 2

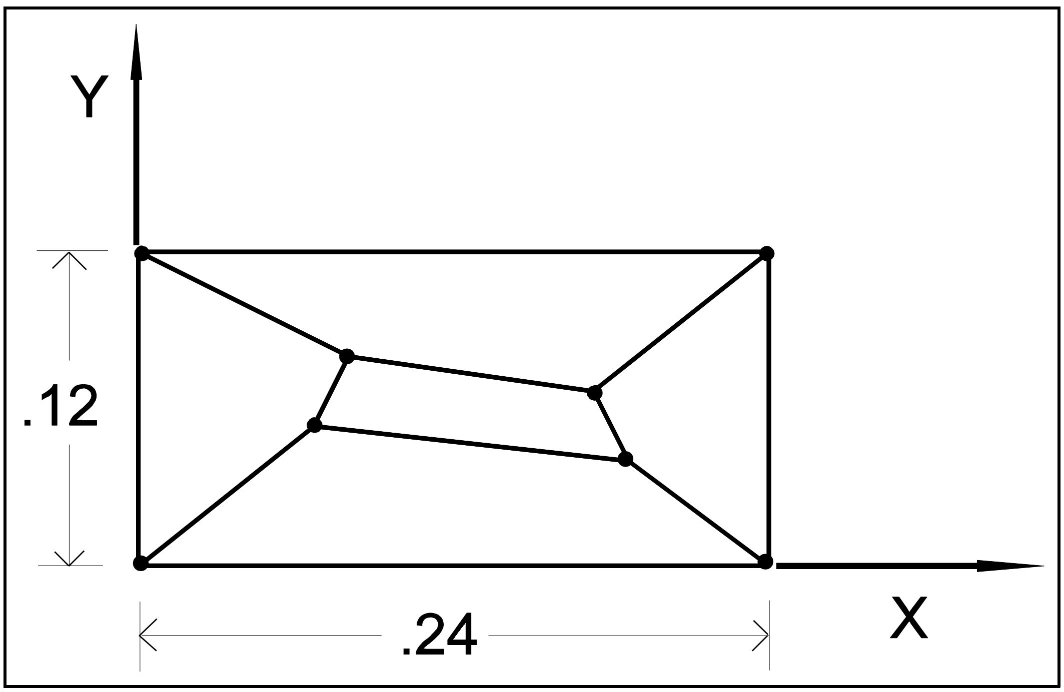

Figure F.3. Geometry for Verification Problem 3

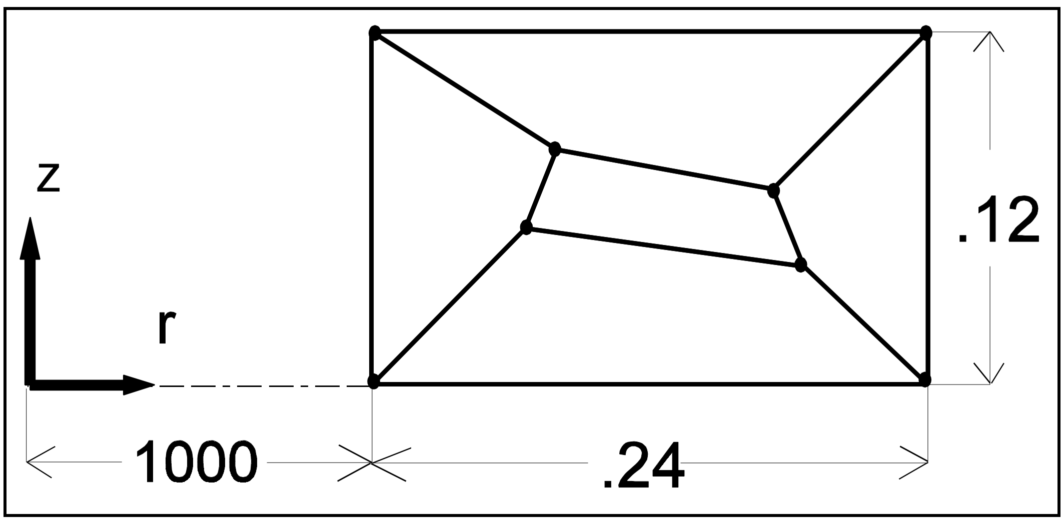

Figure F.4. Geometry for Verification Problem 4