# 1. Introduction

Adaptive filtering techniques are used in a wide range of applications, including echo cancellation, adaptive equalization, adaptive noise cancellation, and adaptive beamforming. These applications involve processing of signals that are generated by systems whose characteristics are not known *a priori*. Under this condition, a significant improvement in performance can be achieved by using adaptive rather than fixed filters. An adaptive filter is a self-designing filter that uses a recursive algorithm (known as adaptation algorithm or adaptive filtering algorithm) to “design itself.” The algorithm starts from an initial guess, chosen based on the *a priori* knowledge available to the system, then refines the guess in successive iterations, and converges, eventually, to the optimal Wiener solution in some statistical sense.

The performance of an adaptive filtering algorithm is evaluated based on one or more of the following factors [1]:

- *Rate of convergence*. This quantity describes the transient behavior of the algorithm. This is defined as the number of iterations required for the algorithm, under stationary conditions, to converge “close enough” to the optimum Wiener solution in the mean-square sense.
- *Misadjustment*. This quantity describes steady-state behavior of the algorithm. This is a quantitative measure of the amount by which the ensemble averaged final value of the mean-squared error exceeds the minimum mean-squared error produced by the optimal Wiener filter.
- *Computational Requirements*. This is an important parameter from a practical point of view. The parameters of interest include the number of operations required for one complete iteration of the algorithm and the amount of memory needed to store the required data and also the program. These quantities influence the price of the computer needed to implement the adaptive filter.

- *Numerical Robustness*. The implementation of adaptive filtering algorithms on a digital computer, which inevitably operates using finite word-lengths, results in quantization errors. These errors sometimes can cause numerical instability of the adaptation algorithm. An adaptive filtering algorithm is said to be numerically robust when its digital implementation using finite-word-length operations is stable.

Another practical measure of performance is the number of computations needed for the adaptive filter to reach steady state. This measure combines the rate of convergence and computational requirements and is the product of the number of iterations needed for the algorithm to converge close enough to the optimum solution and the number of computations needed per iteration.

Ideally, one would like to have a computationally-simple and numerically-robust adaptive filter with high rate of convergence and small misadjustment that can be implemented easily on a computer. As in any engineering problem, these desirable characteristics, in most cases, are incompatible with each other and some kind of trade-off is needed. For example, the widely used least mean squares (LMS) algorithm is computationally simple and numerically robust, but it has a drawback of converging slowly, especially when the input is colored. On the other hand, the well known recursive least squares (RLS) algorithm exhibits fast convergence, but it is very complex and is known to have serious numerical problems [1]. These algorithms are two popular examples of a vast repertoire of adaptation algorithms currently available to the user. These two algorithms may be loosely thought of as two extremes of the “trade-off spectrum.” Algorithms such as self-orthogonalizing adaptation algorithms attempt to reduce the complexity, by trading-off on convergence rate [1]. Steady research effort has been directed toward improving the performance of adaptive filters (achieving better trade-off). Our work also is aimed at that goal.

Part I of this dissertation presents a new adaptation algorithm, which we have termed the Normalized LMS algorithm with Orthogonal Correction Factors (NLMS-OCF). This algorithm, as will be shown later, occupies “user-selectable points” in the trade-off spectrum. The NLMS and rectangular-windowed RLS algorithms are shown to be two (extreme) special cases of our NLMS-OCF algorithm. The well-known Affine Projection Algorithm (APA) and algorithms equivalent to APA, such as the Partial Rank Algorithm (PRA) and the Generalized Optimal Block Algorithm (GOBA), are also special cases of NLMS-OCF [2, 3]. In this sense, we can consider NLMS-OCF to be a generalized version of APA.

The NLMS-OCF algorithm updates the adaptive filter coefficients (weights) on the basis of multiple input signal vectors, while NLMS updates the weights on the basis of a single input vector. This algorithm is derived as a better alternative to NLMS for colored input signals. The usual NLMS algorithm reduces the distance between the estimated and true system weights, where the correction is in the direction of the input vector. For colored inputs the correction is mostly in the direction of the largest eigenvector. We therefore generate additional, NLMS-like, corrections of the weight vector in directions orthogonal to the input vector and orthogonal to each other.

We also present an analysis of the convergence behavior of NLMS-OCF using a simple model for the input signal vector. In addition to the usual independence assumption, the angular orientation of the input vectors is assumed to be discrete [4]. This assumption renders the convergence analysis tractable. The convergence results that are presented here are applicable to APA and the entire class of algorithms equivalent to APA. Our analysis shows that the convergence rate of NLMS-OCF (and also APA) is exponential and that it improves with an increase in the number of input signal vectors used for adaptation. However, the *rate* of improvement in time-to-steady-state diminishes as the number of input vectors used for adaptation increases. While we show that, in theory, the misadjustment of the APA class is independent of the number of vectors used for adaptation, simulation results show a weak dependence. Thus APA provides a way to increase the convergence rate without compromising too much on misadjustment. For white input the mean squared error drops by 20 dB in about $5N/(M+1)$ iterations, where $N$ is the number of taps in the adaptive filter and $(M+1)$ is the number of vectors used for adaptation. Simulation results are provided to corroborate our findings.

We derive the tracking properties of the APA class of algorithms for a randomly time-varying system under independence and discrete-orientation assumptions. An expression is derived for the steady-state mean-squared error. The steady-state error consists of two distinct (decoupled) parts, namely the fluctuation error caused by measurement noise and the lag error caused by variations in the environment. The dependence of the steady-state error and of the tracking properties on the three user-selectable parameters, namely step size $\bar{\mu}$, number of vectors used for adaptation $(M+1)$, and input vector delay $D$ used for adaptation, is discussed. While the lag error depends on all of the above parameters, the fluctuation error depends only on

$\bar{\mu}$. Increasing $D$ results in a linear increase in the lag error and hence the total steady-state mean-squared error. The optimum choice for $\bar{\mu}$ and $M$ are derived. Simulation results support our theoretical deductions.

A straightforward way to compute the orthogonal correction factors is to use the Gram-Schmidt orthogonalization procedure. The resulting NLMS-OCF algorithm has a complexity of $O(NM^2)$. The square-law dependence of computational requirements on the number of orthogonal correction factors $M$ makes it too complex to be used in practical applications, when $M$ is larger than about two or three. Hence, we derive a fast version of our algorithm that has a complexity of $O(NM)$. The fast version of the algorithm performs orthogonalization using a forward-backward prediction lattice.

We use the stereophonic echo cancellation application to demonstrate the advantages of using NLMS-OCF over other algorithms. The flexibility provided by NLMS-OCF in choosing vectors used for adaptation can be exploited to achieve faster convergence than with the affine projection algorithm, in terms of faster improvement in echo-return loss as well as faster reduction in impulse response misalignment. The stereo echo cancellation scenario is simulated in MATLAB with speech signals as input. Simulation results corroborate the advantageous convergence behavior of stereo echo cancelers based on NLMS-OCF. It is also shown that NLMS-OCF can provide better echo rejection.

While the first part of this dissertation attempts to improve the adaptive filter performance by refining the adaptation algorithm, the second part of the work concerns an “unconventional” structure for adaptive IIR filters.

The input-output characteristic of linear systems is classically described by a ratio of polynomials in shift operator notation. However, there exist an infinite number of equivalent descriptions (also referred to as structures, realizations, representations, or parameterizations) with the same external (input-output) behavior, but different internal behaviors. These different realizations have different sensitivity measures and different numerical properties under finite-word-length conditions. An abundant literature comparing the performance advantages of different realizations is available in the fixed filtering context [6-9]. Also, a wide range of results is available regarding the “optimal” realizations for fixed digital filters [6-8]. However, sufficient attention has not been focussed on exploiting these structural properties in adaptive filtering. Some of the notable exceptions are the works by DeBrunner and Beex [11] and Gevers and Li

[15]. The adaptive filters discussed in most of the existing literature are based on “conventional” realizations such as the direct forms and lattice structures.

Researchers have proposed balanced realizations as a worthwhile candidate for adaptive filters due to many of its interesting properties. (It is worth pointing out here that a balanced realization is the optimal realization for fixed filters in the minimum sensitivity and minimum round-off noise senses [6-8].) For example, a balanced realization is known to have the least parameter sensitivity. This suggests that the balanced realization will have good noise rejection characteristics (robustness in the presence of noise), since the wrong parameter estimates, due to the misadjustment caused by noise, will describe a model that is still close to the true system [10]. The balanced realization minimizes the ratio of maximum-to-minimum eigenvalues of the Grammian matrices. DeBrunner heuristically argues that this property should lead to fast convergence of adaptive filters based on a balanced realization [10]. Furthermore, the balanced realization is very useful in model reduction [13]. Due to the absence of a parameterization that can guarantee that the realization stays balanced upon adaptation of the parameters, researchers could not develop a balanced-realization based adaptive filtering algorithm [10, 12]. Ober filled the void in balanced parameterization by developing a canonical representation for balanced realizations [14]. This dissertation presents an adaptive filtering algorithm based on Ober's parameterization.

The last part of the thesis proposes an unbiased equation-error based adaptive IIR filtering algorithm. The proposed algorithm aims to minimize equation error, recursively, under a unit-norm constraint on the characteristic polynomial instead of the usual monic constraint. The unit-norm constraint eliminates the bias associated with equation error based estimates, when the additive measurement noise is white. Adapting the parameters of the characteristic polynomial in hyper-spherical coordinates enforces the unit-norm constraint. Simulation results indicate that the proposed algorithm provides estimates that are unbiased.

Chapters 2 – 7 constitute Part I of this dissertation. The NLMS-OCF algorithm is developed in Chapter 2. Chapters 3 and 4 derive the convergence and tracking properties, respectively, of NLMS-OCF and algorithms equivalent to NLMS-OCF such as APA. A fast version of NLMS-OCF is presented in Chapter 5. Chapter 6 compares NLMS-OCF with existing algorithms. The performance advantages of an NLMS-OCF based stereophonic echo canceler are evaluated in Chapter 7. Part II consists of Chapters 8 and 9. Chapter 8 lists the desirable characteristics of "optimal" structures for adaptive filters. A balanced-realization based adaptive filtering algorithm is derived in Chapter 9. Chapter 10 develops an approach to unbiased equation-error based adaptive IIR filtering. Chapter 11 provides conclusions and also suggests some possible future work.