11. Conclusions and Recommendations for Future Work

We have explored different ways to improve the performance of adaptive filters. A novel adaptation algorithm, which we called the Normalized LMS algorithm with Orthogonal Correction Factors, was proposed. The proposed algorithm provides faster convergence than the widely used NLMS, especially for colored inputs. The well-known affine projection algorithm (and also a class of algorithms equivalent to it) is a special case of NLMS-OCF.

We derived the convergence and tracking properties of NLMS-OCF under certain simplifying assumptions, such as the independence assumption and the discrete orientation assumption. The derived results are applicable to APA and the class of algorithms equivalent to APA as well. The condition for convergence is derived. We showed that increasing the number of orthogonal correction factors M improves the convergence rate, while the improvement itself diminishes as M is increased. It was also shown that, for white input, NLMS-OCF with M OCFs converges at a rate of 20 dB for every $5N/(M+1)$ iterations, where N is the length of the adaptive filter. Thus, NLMS-OCF provides faster convergence than NLMS even for white inputs. We provided simulation results to corroborate the analytical results.

The effect of the user-selectable parameters such as step size $\bar{\mu}$, number of OCFs M, and input vector delay D on the tracking behavior of NLMS-OCF was analyzed. Expressions were derived for optimal (in the sense that the steady-state error is minimized) values of $\bar{\mu}$ and M. We also showed that the steady-state error increases linearly with D. The theoretical results were found to closely match the simulation results.

The NLMS-OCF algorithm based on direct Gram-Schmidt orthogonalization has a complexity of $\mathrm{O}(NM^2)$. A fast NLMS-OCF algorithm with a reduced complexity of $\mathrm{O}(NM)$ has been derived. The fast algorithm performs Gram-Schmidt orthogonalization recursively using a lattice structure with vector inputs.

Using simulations, it was shown that increasing the input vector delay accelerates the convergence of NLMS-OCF under most conditions. Hence, NLMS-OCF can converge faster than APA, which restricts the input vector delay to unity.

Simulation results comparing the performance of NLMS-OCF with existing multiple-input-vector based adaptation algorithms such as APA, NDR-LMS, and DA were presented. We find that NLMS-OCF converges faster than the other algorithms, if the input process is white, AR, or reasonably colored MA.

We demonstrated the advantages of using NLMS-OCF in a practical application, namely stereophonic acoustic echo cancellation. It was shown that NLMS-OCF provides faster convergence than APA in terms of both echo reduction and misadjustment reduction. Most importantly, the flexibility provided by NLMS-OCF in choosing input vectors used for adaptation can be exploited to achieve better steady-state echo rejection (equivalently, lower misadjustment).

A balanced realization is known to minimize the parameter sensitivity as well as the condition number for Grammians. Furthermore, a balanced realization is useful in model order reduction. These properties of the balanced realization make it an attractive candidate as a structure for adaptive filtering. We proposed an adaptive filtering algorithm based on balanced realizations. The proposed algorithm attempts to minimize the mean-squared output error. Hence, the resulting estimates are unbiased. The problem of the existence of local minimums in the output-error surface is surmounted by first using an equation-error based adaptation algorithm, which provides a good initial estimate for the parameters. The balanced parameterization used for adaptation inherently assures that the adaptive filter remains stable. Consequently, the usual complex stability checks are not needed while using our balanced-realization based adaptive filter.

We showed that minimizing equation error subject to the unit-norm constraint yields an unbiased estimate for the parameters of a system, if the measurement noise is white. We proposed an algorithm that uses the hyper-spherical transformation to convert this constrained optimization problem into an unconstrained optimization problem. It was shown that the hyper-spherical transformation does not introduce any new minima in the equation error surface. Hence, simple gradient-based algorithms converge to the global minimum. Simulation results indicate that the proposed algorithm provides an unbiased estimate of the system parameters.

Further research would be worthwhile on the following: The convergence and tracking analysis results presented in this dissertation are based on certain simplifying assumptions and hence are approximate. A possible future effort would be deriving exact results without using the independence assumption. Also, it would be worthwhile to perform a finite-word length analysis of the NLMS-OCF algorithm. We showed that increasing the input-vector delay D improves the convergence rate as well as the steady-state error of NLMS-OCF. Further research aimed at determining the optimal value for D, perhaps adaptively, would be useful. Fast NLMS-OCF uses a lattice structure, which is conducive to VLSI implementation. Future work on hardware implementation of the fast NLMS-OCF algorithm seems appropriate. The balanced realization based adaptive filtering algorithm proposed in this dissertation has a very high complexity. Developing a faster algorithm would be useful as a future endeavor.