Appendix 3a Proofs for Part 3a

Proof of Theorem 3.A.2: Observe that

$$\left\| \hat{l}^{*C} \hat{f} + (1-\hat{l}^{*C})\hat{g} - q \right\|^2 - \left\| l^* \hat{f} + (1-l^*)\hat{g} - q \right\|^2$$

$$= \frac{\sum_{i=1}^{n} (\hat{l}^{*C} \hat{f} + (1-\hat{l}^{*C})\hat{g} - q)^2}{n} - \frac{\sum_{i=1}^{n} (l^* \hat{f} + (1-l^*)\hat{g} - q)^2}{n}$$

$$= \frac{\sum_{i=1}^{n} (t_1 - q)^2}{n} - \frac{\sum_{i=1}^{n} (t_2 - q)^2}{n} \text{ (say)}$$

$$= \frac{\sum_{i=1}^{n} ((t_1^2 - t_2^2) - 2q(t_1 - t_2))}{n}$$

$$= \frac{\sum_{i=1}^{n} ((t_1 - t_2)(t_1 + t_2 - 2q))}{n}$$

$$= \frac{\sum_{i=1}^{n} ((t_1 - t_2)(t_1 - t_2 + 2t_2 - 2q))}{n}$$

$$= \frac{\sum_{i=1}^{n} (t_1 - t_2)^2}{n} + \frac{\sum_{i=1}^{n} (t_1 - t_2) 2 (t_2 - q)}{n}$$

$$= \frac{\sum_{i=1}^{n} (\hat{l}^{*C} - l^*)^2 (\hat{f} - \hat{g})^2}{n} + \frac{\sum_{i=1}^{n} (\hat{l}^{*C} - l^*)(\hat{f} - \hat{g})(l^* \hat{f} + (1-l^*)\hat{g} - q)}{n}$$

$$\le (\hat{l}^{*C} - l^*)^2 \left\| \hat{f} - \hat{g} \right\|^2 + 2\left| (\hat{l}^{*C} - l^*) \right| \left\| \hat{f} - \hat{g} \right\| \left\| (l^* \hat{f} + (1-l^*)\hat{g} - q) \right\|.$$

Then

$$\left\| \hat{l} *^{C} \hat{f} + (1 - \hat{l} *^{C}) \hat{g} - q \right\|^{2}$$

$$\leq (\hat{l} *^{C} - l*)^{2} \left\| \hat{f} - \hat{g} \right\|^{2} + 2 \left| (\hat{l} *^{C} - l*) \right| \left\| \hat{f} - \hat{g} \right\| \left\| (l * \hat{f} + (1 - l*) \hat{g} - q) \right\| + \left\| (l * \hat{f} + (1 - l*) \hat{g} - q) \right\|^{2}$$

and so

$$\left\| \hat{l} *^{C} \hat{f} + (1 - \hat{l} *^{C}) \hat{g} - q \right\| \leq$$

$$\left| (\hat{l} *^{C} - l*) \right| \left\| \hat{f} - \hat{g} \right\| + (2 \left| (\hat{l} *^{C} - l*) \right| \left\| \hat{f} - \hat{g} \right\| \left\| (l * \hat{f} + (1 - l*) \hat{g} - q) \right\|)^{.5}$$

$$+ \left\| (l * \hat{f} + (1 - l*) \hat{g} - q) \right\| \qquad \text{(A3.A.1)}$$

Using Lemmas 3.a.1 and 3.a.3, and Theorem 3.A.1, it is easy to see that the first two terms on the right side of A3.A.1 are asymptotically equal. Using only the first and third terms of A3.A.1, the desired results follow again from the aforementioned Lemmas and Theorem 3.A.1, as desired.//.

Whittle's Inequality: The following is a special case of Theorem 2 of Whittle (1960).

If $x_1, \ldots, x_n$ are independent random variables, with x_i ~ F(0 $unit$, s_i^2 $unit^2$) for i = 1, …, n, with

$$A = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{i,j} x_i x_j \,,\ for\ a_{i,j} \in \Re, i = 1, \ldots n,\ j = 1, \ldots, n \text{, and}$$

$$B = \sum_{i=1}^{n} b_i x_i \,,\ for\ b_i \in \Re, i = 1, \ldots, n \,,$$

then

$$V(A) \leq C_A (\sum_{i=1}^{n} \sum_{j=1}^{n} a_{i,j}^{2} s_i^{2} s_j^{2}) unit^{4} ,\ for\ some\ C_A \in \Re$$

$$V(B) \leq C_B (\sum_{i=1}^{n} b_i^{2} s_i^{2}) unit^{2} ,\ for\ some\ C_B \in \Re$$

provided $E(x_i^4) = s_i^4\ unit^4 < S\ unit^4 ,\ for\ S \in \Re, and\ i = 1, \ldots, n$.

This inequality will often be used in the dissertation with x_i replaced by e_i, for i = 1, …, n.