Chapter 4
Review of Methods for Predicting Displacements in Lateral Spreads

4.1. Introduction to Lateral Spreading Models.

In assessing the seismic hazards at a site, the potential for soil liquefaction can be evaluated using in situ penetration data as discussed in Chapter 7. If liquefaction is likely to occur, the problem is to then anticipate the potential ground failure mechanisms. When the surface slope is mild, a common mode of failure is lateral spreading with surface displacements that can exceed several meters. Hence, to evaluate the impact of liquefaction, seismic hazard assessments often require estimates of ground deformations due to lateral spreading. Methods that have been proposed for predicting the magnitude of displacements in lateral spreads, described in Sections 4.2 through 4.4, can be grouped into three categories:

(1) Finite element models.
(2) Simplified analytical models.
(3) Empirical models.

Additionally, empirical methods for predicting liquefaction-induced settlements are discussed in Section 4.5.

In general, predicting permanent ground deformations due to soil liquefaction is a *"very difficult and complex nonlinear problem that is still far from being resolved"* (*Liquefaction...* 1985). As evident from the discussions in Chapter 3, lateral spreading involves complex mechanics and the interaction of a wide variety of factors. For example, deformations are driven by both static and dynamic loads during an earthquake, but movements can occur after seismic base motions have ended. Bartlett and Youd (1992b) contend that the magnitude of displacements in a lateral spread is controlled by four factors: (1) the degree of shear strength loss in the liquefied soil; (2) boundary conditions around the slide; (3) static and dynamic shear forces acting on the mass of moving soil; and, (4) the length of time for which the driving forces exceed the resisting forces. A rigorous model of this problem must consider dynamic and three dimensional effects as well as the anisotropic and heterogeneous nature of liquefiable soil deposits.

Conceptually, finite element or similar numerical models can address many important aspects of lateral spreading, particularly the influence of an irregular geometry. However, liquefiable sediments are often highly variable over short distances; developing a sufficiently accurate site model for a detailed numerical analysis can require extensive site characterization efforts. Moreover, accurate constitutive modeling of a liquefiable soil is a difficult problem, even when considerable laboratory testing is undertaken. Such efforts are hampered by the difficulty in obtaining representative, "undisturbed" testing samples from the in situ deposit. Consequently, in seismic hazard assessments of lifeline networks, where the risks due to lateral spreading must be evaluated at numerous sites over large geographic areas, comprehensive modeling of the geology in every potential lateral spread is simply not feasible. Numerical modeling techniques that require complete, three dimensional representations of the subsurface soils are viable options only for the less common analyses of critical structures subjected to lateral spreading.

In practice, deformations in a lateral spread are sometimes predicted with less-than-rigorous, simplified analytical models. These analytical models generally use both simple coefficients to characterize the behavior of liquefied soil and simple representations of the slide geometry. The success of these simplified analytical techniques depends on how well the relevant aspects of lateral spreading are captured, using parameters that can be reasonably defined for the soils and geometry of a given lateral spread.

The reliability and accuracy of any model for lateral spreading can be judged only on the basis of field case studies. Because knowledge of the subsurface soil conditions in a lateral spread is rarely complete, a simplified representation must be used in predicting deformations. For example, while the liquefied thickness may vary significantly over the area of a lateral spread, a uniform thickness might be assumed in an analysis. Any useful model for predicting displacements must be relatively insensitive to this simplified representation of the soil conditions. Scale model tests, which are nearly always constructed of uniform soil deposits, are thus of limited use in proving the adequacy of a model in predicting actual field behavior. Hence, the accuracy of a particular model for lateral spreading can be demonstrated only by comparison with actual field cases. Unfortunately, many of the analytical procedures described in the literature have been applied to only a very limited number of field cases and the reliability of these methods remain unproven.

Given the complexities of lateral spreading behavior and the typical lack of site information, it is not surprising that the educated guess of an experienced engineer may be as effective as more sophisticated analytical studies. Indeed, past engineering experience or expert opinion is frequently used to forecast ground deformation patterns and magnitudes (Honegger 1992). Moreover, the inherent limitations of the available numerical and analytical models point to the continued use of empirical methods for estimating deformations due to lateral spreading. In fact, the current state-of-the-practice for predicting liquefaction displacements relies on empirical methods (Glaser 1993; 1994). However, the best empirical models for lateral spreading can probably provide only bounding estimates of ground displacements.

4.2. Review of Finite Element Models.

For problems involving the deformation of a soil mass, finite element methods are often well suited for modeling the relevant mechanics and boundary conditions. However, a very sophisticated finite element model would be required to meticulously model all aspects of liquefaction and lateral spreading. A rigorous finite element model would need to simulate seismic excitation, soil softening with the accumulation of excess pore pressures, rapid loss of shear strength as the soil liquefies, large distortion of the liquefied deposit, redistribution of pore water, possible progressive failure, deformations continuing after dynamic loading ends, and reconsolidation as excess pore pressures drain. The resulting finite element formulation would be highly nonlinear and would need to consider both inertial loads and large strains. Paramount to the success of such a calculation would be the constitutive model used for the liquefied soil.

Simulating the behavior of liquefied soil is not a trivial matter, and any rigorous attempt would require fully-coupled, effective stress, nonlinear soil models. Much of this modeling challenge is a consequence of the transformations from solid soil to liquid and back to solid which is beyond the scope of conventional solid mechanics (Towhata et al. 1989). Soil may liquefy at fairly moderate strain levels and then undergo huge deformations. Predicting the post-liquefaction response of a soil is difficult and comparatively little research has been done in this area. Laboratory tests are generally unreliable at measuring the post-liquefaction response of a soil at large strains. In the field, partial drainage of excess pore pressures adds further complications. Also, given the limited scope of a typical site study, numerical simulations are hampered by inadequate knowledge of the soil conditions in a lateral spread. Seed (1979) observed that the *"level of analytical capability may have already out-stretched our engineering ability to provide significant details of soil profile stratification and soil property determinations with sufficient accuracy to take full advantage of these analyses."*

The general inadequacy of rigorous numerical models for liquefaction flow failures has been recently demonstrated (Manzari et al. 1994). As part of the VELACS project (see Section 3.3), numerical predictions were compared with the behavior of carefully constructed centrifuge models. While the onset of liquefaction could be accurately calculated, predictions of the ensuing ground deformations were much less reliable even though the soil conditions in the scale models were well known. Moreover, all of the available numerical procedures were deficient in simulating the behavior of dilative soils.

For lateral spreading due to liquefaction, rigorous numerical simulations are rarely attempted. Perhaps the most sophisticated finite element calculations that have been reported were performed by Finn and his associates (1990; 1991; 1994) using the TARA-3 and TARA-3FL codes. These codes feature an adaptive mesh procedure that permits modification of the finite element grid to track large deformations during the simulation. A ground slope is modeled in a two-dimensional, vertical cross section (plane strain model). Earthquakes can be simulated with an input seismic motion and triggering of liquefaction in specific soil elements can be identified. The stress-strain response of the liquefied soil is assumed to maintain a constant shape but is reduced to a residual strength upon liquefaction. The loss of strength in the liquefied elements creates an imbalance in the shear stresses that drives subsequent ground deformations; in the simulation, stresses are redistributed until equilibrium is re-established. Therefore, these finite element models are appropriate only for failures involving flow at a residual shear strength. Finn (1991) reports that the computed deformations are highly dependent on the residual strength specified, although determining reliable estimates is difficult (see Section 3.4). This procedure has been used to predict displacements in one lateral spread in Niigata, Japan (Finn 1990; 1991). Knowing that deformations were very large, a lower bound estimate of the residual strength was used to produce a reasonable estimate of the pattern and magnitude of displacements.

A somewhat less sophisticated finite element model has been used by Gu et al. (1993; 1994). These calculations also employ a plane strain model of a vertical cross section and are appropriate only for failures of liquefied soil at a residual shear strength. A static analysis is performed beginning with a calculation of the initial effective stresses. Excess pore pressures, independently estimated to result from an earthquake, are input to specific soil elements. Liquefaction of soil elements causes an imbalance of shear loads and the resulting deformations that end in an equilibrium condition are computed using an iterative procedure. Undrained conditions are assumed and a simplified model for the liquefied soils is used. When applied to the lateral spread at the Wildlife Site in California (Gu et al. 1994), this method correctly estimated the pattern of displacements. However, the magnitude of displacements was over-predicted by about 30%, despite the fact that inertial effects during the earthquake were not considered.

In a different approach, Kuwano et al. (1991) compute slope movements based on the potential deformation of unrestrained soil elements, as determined from laboratory tests. First, finite element calculations are used to determine the initial static stresses as well as the dynamic shear stresses generated by an earthquake. Cyclic shear tests are then conducted, at the computed dynamic stress levels, on soil samples consolidated to the computed static stresses. The *strain potential*, defined as the strain that would develop if a soil element could deform freely without constraint, is determined from these tests. Another finite element calculation is then performed to determine a compatible displacement pattern consistent with the strain potential of each soil element. While the authors report success in forecasting the observed deformations of actual embankments in earthquakes, they concede that this procedure can not adequately simulate flow type failures. Towhata et al. (1992) observe that this approach is especially hampered by difficulties in obtaining meaningful data on liquefied soils at large strains in laboratory tests.

Maybe the simplest finite element model for lateral spreading has been proposed by Yasuda et al. (1991a; 1991b; 1992a). Modeling a vertical cross section of the ground failure, two static finite element analyses are performed. In the first, static stresses are computed using elastic parameters for the soil prior to an earthquake. Then, in the second analysis, the static stresses are held constant while the stiffness of the soil is reduced to simulate softening due to liquefaction. The difference in the displacements computed in the two analyses is then taken to represent the liquefaction-induced movements of the slope. The key to the analysis is determining an appropriate stiffness reduction to represent the liquefied soil. This procedure has been used to analyze one lateral spread in Niigata, Japan, using cyclic torsional shear tests to estimate the stiffness reduction (Yasuda et al. 1991a; 1991b; 1992a). Using a stiffness reduction consistent with that measured in the laboratory, displacements of the correct magnitude were computed although the pattern of movements was predicted with less success.

Hamada et al. (1987) modeled a lateral spread in Noshiro, Japan, using a two dimensional finite element mesh representing a plan view of the slide area. Specifically, the unliquefied, surficial soil layer was modeled as an elastic plate using plane stress elements. Displacements were then computed in a static analysis by assuming the underlying liquefied soil provided zero resistance to movements of the surficial soil layer. Despite the very simple nature of this analysis, the computed displacements were in good agreement with the field observations at the Noshiro slide. However, this procedure is highly dependent on the assumed elastic properties of the surficial soil. Hamada and his colleagues estimated a representative elastic modulus based on an empirical correlation with in situ penetration test results.

An attempt to model a lateral spread in three dimensions has been published by Orense and Towhata (1992). Actually, these calculations are an extension of the analytical procedure developed by Towhata and his co-workers (1991; 1992), which are discussed in Section 4.3. The basic premise is that ground movements continue under gravity loads until a state of minimum potential energy is achieved. The total potential energy of the slide system is computed assuming no slip between the surficial layer and the liquefied soil beneath, a sinusoidal variation of lateral displacements with depth in the liquefied deposit, constant soil volume, no shear stiffness or strength in the liquefied soil, and the surficial soil layer acts as a linear elastic plate. A finite element formulation is then used to find the minimum energy state of the entire system, based on a two dimensional, plan view representation of the slide area. While this approach tends to produce upper bound estimates of displacements, Orense and Towhata (1992) were able to obtain a reasonable prediction of the displacements in a lateral spread in Noshiro, Japan.

In summary, a variety of simplified finite element schemes have been used to model liquefaction and lateral spreading. Some success has been shown in a very limited number of comparisons with observed field behavior. However, these simpler procedures are dependent upon material parameters that are fairly difficult to estimate, usually due to the limitations in laboratory soil tests. More rigorous numerical procedures have not been demonstrated to be more reliable at modeling lateral spreads, even when soil conditions are well defined in laboratory-scale model tests. Efforts aimed at modeling the behavior of liquefied soils, using constitutive laws and first principles of physics, often can not be applied in practice to solve real world problems (Glaser

1993). Given the expense of site characterization and constitutive modeling of liquefied soil, rigorous numerical models will probably never be widely used to predict lateral spreading displacements.

4.3. Review of Simplified Analytical Models.

Two simplified analytical procedures have been used to estimate ground deformations in lateral spreads and are discussed in this section. Calculations based on Newmark's sliding block model assume that slope deformations are driven by seismic base accelerations. On the other hand, Towhata's method assumes that all deformations result from static loads after liquefaction has been triggered.

Newmark's Sliding Block Model.

Newmark (1965) proposed a practical means of predicting slope movements in earthquakes, based on the analogy of a sliding block. Newmark's approach has been used extensively and described by numerous authors including Makdisi and Seed (1978), Wilson and Keefer (1985), National Research Council (*Liquefaction...* 1985), and Jibson (1993). Conventional Newmark-type analyses assume that the shear strength of a soil is unchanged during an earthquake and, therefore, are not valid for modeling ground deformations resulting from soil liquefaction. On the other hand, Baziar et al. (1992) considered the use of undrained residual strengths, and Byrne et al. (1992) proposed modifications, to model liquefaction failures with Newmark's analogy as described below.

Newmark's analogy of a sliding block on a frictional plane, subjected to horizontal base motions, is portrayed in Figure 4.1. Because of inertia, the block will tend to move in a direction opposite to the acceleration of the base. However, the block will begin to move relative to the base only when the sum of the static and dynamic driving forces exceed the resisting forces. This equilibrium condition, when the block begins to slide, is expressed in terms of a yield acceleration (a_y). Given the yield acceleration, the total accumulated displacement is computed by double integration of the base acceleration record as indicated in Figure 4.1. When the base acceleration exceeds the yield value, the block slides and the first integration gives the velocity of the block. The velocity reaches a peak after the base acceleration reverses direction and eventually the velocity declines to zero. The second integration, of the velocity record, produces the net displacement during each pulse where the yield acceleration is surpassed. Following this procedure for the entire base motion record yields the total, accumulated displacement.

As given at the top of Figure 4.1, the yield acceleration can be expressed in terms of the slope angle and the conventional, static factor of safety against sliding. If the slip surface is inclined, a_y for movement up the slope is greater than for down-slope movements. In Figure 4.1, the up-slope yield acceleration is never exceeded and all displacements are down-slope. When soil slopes are considered, the yield acceleration is computed as a function of the shear strength of the soil and the failure mechanism or slip surface. In a conventional Newmark analysis, sliding is assumed to exhibit a rigid, perfectly-plastic response implying a yield acceleration that does not change with displacement.

Using a measured or predicted seismic acceleration history, the net displacement of a soil slope can be computed using numerical integration techniques (see Chang et al. 1984; Jibson 1993). However, accurately forecasting the time history of accelerations, for a future seismic event at a particular site, is a significant problem. Hence, Jibson (1993) proposed a simpler approach where the ground motion record is represented by a single, quantitative measure of the total shaking intensity. Using the Arias intensity ($I_a=[\pi/2g]\int a^2 dt$) to represent shaking intensity, and analyzing several strong motion records, Jibson developed a relationship for displacements predicted by a Newmark-type analysis:

$$\log D = 1.460 \log I_a - 6.642\, a_y + 1.546 \qquad (4.1)$$

where D is the predicted displacement (cm), I_a is the Arias intensity (m/s), and a_y is the yield acceleration (g). Thus, displacements for a given yield acceleration can be estimated directly from Arias intensity, which can be estimated from other empirical attenuation equations.

An alternative approach to simplifying Newmark analyses, which also avoids the need to integrate strong motion records, relies on "equivalent" base acceleration histories. If the base acceleration can be expressed as a mathematical function, the necessary integrations can be performed closed-form and the solution expressed as (Yegian et al. 1991):

$$D = N_{eq}\, T^2\, a_p \;\; f(a_y/a_p) \qquad (4.2)$$

where D is the predicted displacement, a_y is the yield acceleration, and N_{eq}, T, and a_p are the number of cycles, period, and peak acceleration, respectively, of the equivalent, uniform base motion. The dimensionless function $f(a_y/a_p)$ depends on the assumed shape of the base motion record; solutions for rectangular, triangular, and sinusoidal motions are presented in Yegian et al. (1991). In addition, Yegian and his associates give a polynomial function for $f(a_y/a_p)$ that better represents the integration of actual strong motion records. Assuming an equivalent, sinusoidal base acceleration record, Baziar et al. (1992) proposed that the displacement predicted by a Newmark analysis could be computed with:

$$D = N_{eq}\, (v_{max}^2/a_{max}) \;\; f(a_y/a_{max}) \qquad (4.3)$$

where $f(a_y/a_{max})$ is a function based on a sinusoidal motion. In this equation, a_{max} and v_{max} are the peak horizontal surface acceleration and velocity of an earthquake record. For earthquakes in the western United States, Baziar et al. (1992) recommend N_{eq}=2. Using Equation 4.3, displacements predicted with a Newmark sliding block model can be easily calculated from the anticipated peak motions at a given site. Using a_y=0.0477g to represent a typical lateral spread, Baziar et al. (1992) found that Equation 4.3 agrees favorably with the Youd and Perkins' empirical LSI equation (see Section 4.4).

Regardless of whether the acceleration record is integrated or some type of simplified method is used, a Newmark sliding block analysis relies on a suitable yield acceleration. For conventional analyses of slope deformation, equations for the yield acceleration have been given by Newmark (1965), Wilson and Keefer (1985), and others. In general, these relationships for a_y are developed assuming (Jibson 1993; Bray et al. 1995):

- a well defined failure mechanism or slip surface,
- the soil exhibits a well defined yield stress that does not change with dynamic loading, and
- the soil undergoes plastic deformations at constant shear resistance (perfectly plastic behavior).

As discussed in Chapter 3, all of these assumptions are probably violated, to some degree, in a lateral spread. For instance, shearing generally occurs across the full depth of the liquefied deposit with no discernible slip surface. A more significant problem with liquefaction failures is the dramatic change in shear strength of the underlying soils during dynamic loading and subsequent deformations. Indeed, the yield acceleration must change significantly as the foundation soils liquefy. Consequently, the yield accelerations defined for most conventional analyses are simply inappropriate for liquefaction-induced slope deformations.

Conceivably, to model liquefaction failures, the yield acceleration could be expressed in terms of effective stress on the failure surface. However, the difficulty in then predicting the changes in pore pressures seems to preclude this approach. This problem is also faced in many conventional, static slope stability analyses, and Baziar et al. (1992) have proposed a similar solution based on total stress and undrained strengths. In their model, Baziar and his colleagues express the yield acceleration in terms of the normalized, undrained shear strength (S_u/σ'_v) of the liquefied soil. Note that S_u is defined here as the maximum shear stress measured during failure in an undrained test (and not the shear stress on the actual failure plane) and is assumed to remain fairly constant at large strains. A Newmark-type analysis based on (S_u/σ'_v) is valid only for soils that exhibit a "plateau" in the undrained shear resistance and should not be applied to dilative soils (Baziar et al. 1992). Moreover, as with all "undrained" analyses of this type, S_u is valid only if the laboratory test results are representative of the field conditions to be analyzed, particularly with respect to the generation of excess pore pressures. Consequently, the laboratory test procedures, including sample preparation techniques, are especially critical when the (S_u/σ'_v) approach is used to model liquefaction failures. A constant S_u implicitly neglects soil softening leading up to the liquefaction failure and means the yield acceleration is assumed to be constant for the entire analysis.

Expressions for yield acceleration, suitable for modeling liquefaction-induced lateral spreading in terms of (S_u/σ'_v), are given by Baziar et al. (1992) and presented here in Figure 4.2. Assuming that liquefied soil will fill tension cracks that form in a lateral spread, no up-slope movements are computed in this type of analysis. Hence, only the yield acceleration for down-slope displacements are given here. The two expressions given in Figure 4.2a and 4.2b represent an infinite slope with and without groundwater seepage. In the model given in Figure 4.2c, movement toward a free face is considered while the affect of a non-yielding toe is modeled in Figure 4.2d. In these last two models, numerical values for the angles β_1 and β_2 are chosen to give the minimum or critical value of a_y. Unfortunately, only one of these models has been evaluated in a single analysis of lateral spreading in the field. Using a model for an infinite slope, even though the actual site had a free face, Baziar et al. (1992) modeled the Wildlife Site in California which experienced liquefaction and lateral spreading in 1987. Using assumed values of (S_u/σ'_v) and α to compute bounding estimates of a_y, and integrating the strong motions recorded at the site, displacements ranging from 1.9 to 31.7 cm were computed. This compares with the average measured displacement of about 18 cm. Baziar and his co-authors conclude their analysis is very sensitive to the assumed inclination of the slip plane (α).

An alternative model, also based on Newmark's sliding block analogy, has been proposed by Byrne et al. (1992). Instead of considering equilibrium of forces, this formulation is derived from the conservation of energy during slope deformation; that is, the change in kinetic energy of the sliding mass is equal to the difference in the input energy of the seismic motions and the work done in overcoming internal soil resistance. Significantly, this approach can be used to construct a multiple degree-of-freedom, finite element model. Also, where conventional Newmark models are limited to a rigid-plastic behavior, the model developed by Byrne and his co-workers can encompass changes in shearing resistance as a function of displacement. When a rigid, perfect-plastic response is assumed, their solution reverts to a simple Newmark model. However, modeling the changing shear resistance associated with soil liquefaction requires knowledge of the nonlinear stress-strain response of the liquefied soil and adds complexity to the analysis.

To summarize, the application of a Newmark sliding block analysis to liquefaction-induced lateral spreading is inhibited by several shortcomings:

(1) An obvious problem is defining an appropriate shear strength for a liquefiable soil. In an approximate manner, this problem can be addressed with undrained shear strengths, although this approach requires special soil testing and is not applicable to all soil conditions.

(2) The lack of a well defined slip surface in a lateral spread confounds the definition of a simple yield acceleration. In reality, the "yield acceleration" in a lateral spread will change dramatically with the occurrence of liquefaction and subsequent deformations.

(3) As discussed in Section 3.8, lateral spreading can continue after earthquake motions stop and a Newmark-type analysis is incapable of modeling these deformations.

Nevertheless, somewhat simplified Newmark models have been proposed for lateral spreading.

Unfortunately, these models have not been adequately validated against field observations and their reliability is unknown.

Towhata's Minimum Potential Energy Model.
Based on observations from scale model tests, conducted on laboratory shake tables (see Section 3.3), Towhata and his various collaborators (1991; 1992) have developed an analytical model for lateral spreading. Towhata's model predicts maximum possible movements in a lateral spread, corresponding to the ultimate displacements that will occur if the foundation soil remains liquefied for a sufficient length of time. That is, lateral spreading is assumed to continue until complete flow and a minimum energy state is achieved. In laboratory scale model tests, ultimate displacements are produced by shaking the model until down-slope displacements stop while the underlying soil remains liquefied. In the field, Towhata's model corresponds to a long duration seismic event that results in the maximum possible movement of a lateral spread. Notably, Towhata's model assumes a flow failure under static loads and neglects inertial effects during dynamic loading.

Towhata's minimum potential energy model was developed from two earlier, simpler analytical models for lateral spreading (Towhata et al. 1992). In the simplest model, the unliquefied surficial soil was treated as an inclined, linear elastic column subjected to axial compression. Assuming the underlying, liquefied soil provided no resistance to down-slope movements, deformations of the surficial soil "column" were computed. Later, a "flow model" was proposed which predicted the deformation of a sloping, liquefied soil without any overburden soil. This model assumed displacements continued until a level surface was achieved, just as in the flow of a liquid. Neither of these two simple models were satisfactory, but the basic concepts were combined in developing Towhata's minimum potential energy model.

Details on the derivation of Towhata's model for lateral spreading, derived from the principle of minimum potential energy, are given by Towhata et al. (1991; 1992). This derivation assumes a simple slope geometry comprised of planar layer interfaces, as depicted in Figure 4.3. In addition, four key assumptions, based on observations from scale model tests, were used in Towhata's derivation:

(1) horizontal deformations of a vertical cross section in the liquefied deposit can be represented with a sinusoidal equation;
(2) the volume of liquefied soil is unchanged during deformation;
(3) the liquefied soil exhibits a linear-elastic, rigid-plastic response, although the shear stiffness and strength are generally set to zero when applying the model; and,
(4) the unsaturated, surficial soil layer does not liquefy and behaves like an elastic solid.

Using these assumptions, the potential energy is computed as the sum of the strain energy and the gravitational energy (associated with vertical displacements) of the liquefied and unliquefied soil layers. The net potential energy of the system is found by integrating over the volume of soil in the lateral spread, with corrections to account for boundary conditions at the head and toe of the slide. After developing an expression for the net potential energy at any deformation state, the minimum energy state is found mathematically using the variational principle. The closed-form solution for horizontal displacements is given by Towhata et al. (1991; 1992) and is not repeated here; although quite lengthy, the solution is an algebraic expression in terms of the parameters defined in Figure 4.3.

Only simple slide geometries, composed of planar interfaces as depicted in Figure 4.3, can be considered in Towhata's model. While laboratory scale models may be built to uniform conditions, lateral spreads in the field typically encompass more complicated geometries, especially vertical free faces such as stream banks. For these more complicated conditions, an approximate representation must be defined to apply Towhata's solution. Even more troublesome is the definition of "elastic modulus" (E) for the unliquefied surface soil layer. Contrary to the definition given by Towhata et al. (1991), where "E" is inappropriately identified as the Young's modulus, "E" must be a secant-type modulus representing the plane-strain compression of the soil layer over the full range of displacement. Since the surficial soil usually breaks into blocks of intact material, the elastic modulus used in Towhata's model is clearly not a fundamental soil parameter and can not be determined from laboratory or in situ testing. In a sense, this "material property" is really treated as an arbitrary constant (Glaser 1993; 1994). In addition to a poor understanding of the parameter "E", and a lack of guidance on how to estimate representative values, the sensitivity of the model to the stiffness of the surficial soil has not been addressed. Nevertheless, assuming E=10780 kPa, Towhata et al. (1991; 1992) analyzed three lateral spreads in Japan and reported a reasonable prediction of the observed deformations.

A simplified version of Towhata's minimum potential energy model has been proposed by Tokida et al. (1993). Using Towhata's model, simplified equations were developed from a regression analysis on the results of a suite of parametric calculations. For the maximum displacement in the center of the slide:

- for 10 m ≤ L ≤ 100 m:

$$D = 1.73\times10^{-5} \cdot L^{1.94} \cdot H^{0.298} \cdot T^{-0.275} \cdot \theta^{0.963} \qquad (4.4)$$

- for 100 m ≤ L ≤ 1000 m:

$$D = 1.29\times10^{-5} \cdot L^{1.99} \cdot H^{0.280} \cdot T^{-0.243} \cdot \theta^{0.995} \qquad (4.5)$$

where D is the horizontal displacement (m), L is the length (m) of the slide, H is the average thickness (m) of the liquefied layer, T is the average thickness (m) of the unliquefied surface layer, and θ is the slope (%) of the ground surface (assumed equal to the slope of each layer interface). Tokida et al. (1993) also give equations for computing somewhat greater displacements in the upper part of a slide. Apparently, in the parametric calculations that form the basis of these equations, E was assumed to be 10780 kPa.

In a more sophisticated application, Orense and Towhata (1992) extended the minimum potential energy model to three dimensions. They reduced the total energy equation to a two-dimensional surface integral and solved for the minimum energy state using finite element methods. This finite element model, which is based on the same assumptions described here, is discussed earlier in Section 4.2.

Towhata's minimum potential energy method can not model lateral spreads that do not reach an ultimate deformation state, as might occur in less severe seismic events. With this in mind, Towhata and Matsumoto (1992) have considered possible modifications to forecast the time rate of deformation in a lateral spread. Their approach is based on the ultimate displacements predicted by Towhata's basic model, but the dissipation of energy with time is also considered. Towhata and Matsumoto suggested a "dilatant flow model" which represents, in a simplistic manner, periodic increases in the shear resistance of the liquefied soil. They also considered the inclusion of an imaginary viscosity term to represent energy dissipation, but this seemed to require an unrealistically high viscosity coefficient. Towhata and Toyota (1994) have undertaken further development of a model utilizing a viscosity term for the liquefied soil and obtained reasonable predictions. Several researchers in Japan have recently attempted to measure the viscosity of liquefied soil including Towhata and Toyota (1994), Sato et al. (1994), Miyajima and Kitaura (1994), and Kawakima et al. (1994). While promising, this work is still preliminary and has yet to be demonstrated for predicting the deformation of lateral spreads in the field.

To conclude, Towhata's analytical model is fairly simple to use and shows promise for providing useful predictions of lateral spreading. However, this model appears to have four fundamental weaknesses:

(1) The model is developed from observations of shake table model tests which, as discussed in Section 3.3, may not accurately reflect all relevant aspects of field behavior. For example, low overburden stresses in a conventional shake table model results in a lower shear resistance during steady-state deformations than occur in the field. Not insignificantly, Towhata et al. (1991; 1992) assume the liquefied soil has no shear resistance.

(2) Only relatively simple slide geometries can be considered. Most importantly, steep free faces commonly associated with lateral spreading are difficult to consider in the basic model.

(3) Towhata's model relies on a parameter representing the elastic stiffness of the unliquefied, surface soil layer. Unfortunately, this parameter is poorly defined, not well understood, and difficult or impossible to measure. Moreover, sensitivity of the model to this parameter has not been established.

(4) The model is only capable of predicting the maximum or ultimate displacements; many lateral spreads experience more limited deformations due to the short duration of shaking. Proposed modifications that model the time rate of movements have not been fully developed and verified.

4.4. Review of Empirical Models.

The numerical and analytical procedures reviewed in Sections 4.2 and 4.3 are mechanistic methods which attempt to model, with varying degrees of simplification, the physical behavior of liquefaction-induced lateral spreading. Empirical models, on the other hand, make no attempt to explicitly model the system mechanics; rather, empirical models simply represent the observed relationship between displacements and various site parameters. Three empirical models for lateral spreading are described in this section. Given the unsatisfactory aspects of the available mechanistic models, an empirical approach shows considerable promise in terms of both simplicity and reliability. If formulated correctly, an empirical model has the advantage of a direct basis on observed field behavior. Also, empirical methods can be used to validate more complicated mechanistic models or inexpensively identify sites that warrant further study. In developing empirical models, however, relevant parameters must be chosen and accurately compiled into a sufficiently large database. For liquefaction failures, this is a difficult but critical task (Glaser 1993; 1994).

Youd and Perkins' LSI Model.

A fairly simple empirical method for predicting displacements due to liquefaction and lateral spreading has been proposed by Youd and Perkins (1987). They formulated a rudimentary relationship, between earthquake source parameters and severity of ground movements, suitable for use in developing regional maps of liquefaction hazards. Youd and Perkins introduced a single parameter, the *Liquefaction Severity Index* (LSI), to represent the severity of ground deformations due to lateral spreading. LSI is defined as the general maximum magnitude of ground failure displacement, measured in millimeters divided by 25 (i.e., inches). LSI is specifically defined for deformations of gently sloping, geologically recent, fluvial deposits of shallow, continuous, cohesionless, liquefiable soils. The "general maximum" displacement excludes single, anomalously large movements associated with unusual local conditions. Furthermore, LSI is assigned an arbitrary upper limit of 100, corresponding to a displacement of 2.5 m, which is considered to sufficiently indicate severe failures. Hence, LSI represents the maximum displacement due to lateral spreading in a typical geological setting.

Based on case studies from historical earthquakes in the western United States and Alaska, Youd and Perkins (1987) correlated LSI to earthquake magnitude and distance:

$$\log LSI = -3.49 - 1.86 \log R + 0.98 M_w \qquad (4.6)$$

where R is the horizontal distance (km) to the seismic energy source and M_w is the moment magnitude. This equation reflects the seismic attenuation characteristics of western North America and may not be valid for other regions of the world. Youd, Perkins, and Turner (1989) have investigated LSI attenuation in eastern North America.

The empirical LSI model provides a conservative, upper-bound estimate of deformations due to liquefaction and lateral spreading. That is, the displacements in any given lateral spread are likely to be less than predicted by Equation 4.6. As intended, the LSI model is an effective tool for mapping liquefaction hazards, but the model is poorly suited to the study of individual lateral spreads since site-specific factors are not considered.

Hamada's Empirical Model.

Hamada and his colleagues (1986; 1987) developed a simple empirical model for horizontal displacements from studies of lateral spreading in Niigata and Noshiro, Japan, and the San Fernando Valley, California. Using site cross sections, mean values of relevant parameters were compiled for segments with similar displacement patterns, slopes, and soil conditions. Based on 60 cases, mostly from Noshiro, a simple regression equation was obtained (Hamada et al. 1986; 1987):

$$D = 0.75 \sqrt[2]{H} \sqrt[3]{\theta} \tag{4.7}$$

where D is the horizontal displacement (m) and H is the thickness (m) of liquefied soil. When more than one soil layer liquefies, H is measured as the distance from the top-most to the bottom-most liquefied soil including all intermediate soil layers. θ is the slope (%) of either the ground surface or the base of the liquefied soil, whichever is greater. When present, the surface slope is measured to the toe of a free face.

Equation 4.7 is a fair fit to the data compiled by Hamada and his co-workers. However, this database is heavily biased towards the lateral spreads in Noshiro and thus represents a narrow range of seismic and site conditions. The accuracy of the fitted equation outside these confines is unknown and, consequently, the usefulness of Hamada's empirical model is limited.

Bartlett and Youd's MLR Model.

Recently, Bartlett and Youd (1992a; 1992b; 1995) have developed a more sophisticated empirical model for lateral spreading. They fit equations to a database of 467 horizontal displacement vectors compiled from lateral spreads in eight earthquakes in Japan and the western United States, including Alaska. However, 337 of these data points (72% of the database) are from slides in two Japanese cities, Niigata and Noshiro, resulting from earthquakes in 1964 and 1983, respectively. Soil parameters were compiled from 267 soil borings. When more than one boring was available in a given locale, parameters were interpolated to the location of the displacement vector using a weighted-average scheme. However, data from soil borings were often interpolated over such large distances (several hundred meters) that the accuracy of some compiled soil parameters is questionable.

Bartlett and Youd developed two model equations: a free face component for lateral spreading toward a steep vertical face and a ground slope component for lateral spreads without a free face. These equations were fit to the compiled database using a multiple linear regression (MLR) analysis in which the model variables were selected with a modified stepwise procedure. For the final regression equations, the coefficient of determination (R^2) is 0.826 (indicating that 82.6% of the variability in the observed displacements is explained by the fitted model) and all coefficients are significant to a 99.9% significance level. The MLR equations given by Bartlett and Youd (1995), which are slightly modified from those published in 1992, are:

• Ground Slope component:

$$\log D = -15.787 + 1.178 M_w - 0.927 \log R - 0.013 R + 0.429 \log S + 0.348 \log T_{15} + 4.527 \log(100 - F_{15}) - 0.922 D50_{15} \tag{4.8}$$

• Free Face component:

$$\log D = -16.366 + 1.178 M_w - 0.927 \log R - 0.013 R + 0.657 \log W + 0.348 \log T_{15} + 4.527 \log(100 - F_{15}) - 0.922 D50_{15} \tag{4.9}$$

where D is the horizontal displacement (m), M_w is the moment magnitude of the earthquake, and R is the nearest horizontal distance (km) to the seismic energy source or fault rupture. T_{15} is the thickness (m) of saturated, cohesionless soils (excluding soils deeper than 20 m or with ≥15% clay content) with $N_{1,60} \leq 15$, where $N_{1,60}$ is the standardized SPT blowcount. In addition, F_{15} is the average fines content (% finer than 0.075 mm) in T_{15}, and $D50_{15}$ is the average D_{50} grain size (mm) in T_{15}.

For the ground slope equation, S is the gradient of the ground surface (%) defined as the change in elevation over horizontal distance for long, uniform slopes. When the surface topography is not uniform, a special definition of S is used to account for small, local benches (ridges, banks, etc.) as illustrated by Bartlett and Youd (1992b). For a location within a distance of 100h (where h is the toe-to-crest height of the bench) below a bench, S is defined as the change in elevation over the horizontal distance from the point of interest to the crest of the bench. Similarly, for points within 100h above a bench, S is computed using the vertical and horizonal lengths to the toe of the bench. In the free face equation, W is the free face ratio (%), defined as the toe-to-crest height of the face divided by the horizontal distance to the face toe.

Before employing Bartlett and Youd's MLR model to forecast displacements due to lateral spreading, liquefaction and lateral spreading must be independently predicted for the design earthquake. Moreover, the model is valid only when liquefaction occurs over a widespread area, and not just in isolated pockets. Bartlett and Youd (1995) recommend the use of the free face component (Equation 4.9) when $5 < W \le 20\%$, and the ground slope component (Equation 4.8) when $W < 1\%$. In addition, the MLR model is expressly valid only for $6 \le M_w \le 8$, $1 \le W \le 20\%$, $0.1 \le S \le 6\%$, $1 \le T_{15} \le 15$ m, and a depth to the top of the liquefied soil from 1 to 10 m. The model is not generally appropriate for gravelly or very silty soils and should be used only if $F_{15} \le 50\%$ and $D50_{15} \le 1$ mm; other applicable bounds for the gradation of the liquefied soil are given by Bartlett and Youd (1995). The model is generally limited to Japan, western North America, and other areas with similar strong motion attenuation rates. Also, the MLR model is invalid at source distances closer than:

for M_w equal to:	6.0	6.5	7.0	7.5	8.0
minimum R (km):	0.5	1	5	10	20-30

Further details on the applicability of Equations 4.8 and 4.9, including a flowchart, are given by Bartlett and Youd (1995). In addition, a procedure is outlined to adjust the model for other regions of the world or to account for ground motions amplified by deep, soft sediments.

Bartlett and Youd's MLR model fits about 90% of the observed data to within a factor of two. While there is a lot of scatter in the fit of the model to the database, Bartlett and Youd's MLR model seems to provide a reasonable empirical model for lateral spreading. However, accuracy of the MLR model may be limited because:

(1) The model is heavily biased toward the very damaging slides in Niigata and Noshiro, Japan, which involved large displacements over widespread areas. This may explain why the MLR model greatly over-predicted displacements in a lateral spread in California, as reported by Holzer et al. (1994).

(2) The mathematical scheme used to interpolate between soil borings in the database may not produce representative soil parameters at the location of each displacement vector, especially when the distance to the nearest soil boring is great.

(3) No consideration is given to the relationship between local displacement vectors and the overall slide mass in a lateral spread. That is, no attempt is made to account for the possible effects of slide size, nearness to a boundary margin, average slope across the slide mass, etc.

The possible impact of these factors on the accuracy of Bartlett and Youd's empirical model is difficult to judge. Future comparisons between predictions from this model and additional field data are needed to verify the reliability of this empirical model.

4.5. Methods for Predicting Settlements due to Liquefaction.

While most damage in a lateral spread results from horizontal deformations, significant vertical displacements, including differential settlements, also occur. The simplified analytical and empirical models described in Sections 4.3 and 4.4 can only predict horizontal movements in a lateral spread. In this section, relatively simple, one-dimensional models for predicting settlements due to soil liquefaction are described. More complete reviews of methods for estimating settlements are given by Tokimatsu and Seed (1987) and Glaser (1994).

Scott (1986) has presented a simple analytical model of settlement resulting from soil liquefaction. Scott's model treats the liquefied soil column as a suspension of soil grains that settle toward the bottom. After initial liquefaction, a solidified soil layer forms at the bottom and increases in thickness with time. The top of the solidified soil, called the "solidification front", is assumed to move upward with a constant velocity. A liquefied state is said to continue until the solidification front meets the interface between clear water and water with suspended soil, which moves downward from the top of the deposit. Assuming that each soil grain reaches a terminal velocity (approximately equal to the permeability of the deposit) instantaneously upon liquefaction, Scott computes the net settlement due to solidification of the suspended soil grains. Additional settlement resulting from the consolidation of the re-solidified soil deposit is also computed. Hence, the density, permeability, and coefficient of consolidation of the liquefied soil are required to estimate the total surface settlement. Unfortunately, Scott's analytical model has not been adequately evaluated against field settlement data.

Assuming one-dimensional compression, settlements can also be calculated from estimates of the volumetric strain in the liquefied soil. Empirical charts for estimating volumetric strains in liquefied, clean sands have been proposed by Tokimatsu and Seed (1987) and Ishihara and Yoshimine (1992). In both cases, these charts were developed from laboratory cyclic shear tests on sands at different relative densities. The volumetric strains observed in these tests were correlated to the maximum shear strain imparted to the sample. Both charts also rely on field correlations between relative density and in situ penetration resistance. The chart given by Tokimatsu and Seed (1987) indicates the volumetric strain expected for a soil deposit with a given Standard Penetration Test blowcount when subjected to a particular cyclic shear stress. To estimate volumetric strain using the chart given by Ishihara and Yoshimine (1992), the factor of safety against liquefaction is used with the relative density (estimated from standard penetration or cone penetration data). Both of these methods were shown to give reasonable estimates of the settlements observed at a few sites in past earthquakes.

Unfortunately, the methods available for predicting settlements due to liquefaction were not developed directly from observed field performance data and, in general, have not been adequately evaluated with field data. Moreover, settlement predictions of this type are inherently difficult to make, as pointed out by Tokimatsu and Seed (1987):

> *"It should be recognized that, even under static loading conditions, the error associated with the estimation of settlements in sands is on the order of 25-50%. It is therefore reasonable to expect less accuracy in predicting settlements for the more complicated conditions associated with earthquake loading."*

On a lateral spread, the vertical displacement of a specific point is further complicated by the tilting of surficial soil blocks as well as the lateral movement of the overall slide mass.

Settlement on a lateral spread is not a simple one-dimensional problem involving volumetric strains in the liquefied soil, although the methods described here probably give reasonable settlement estimates.

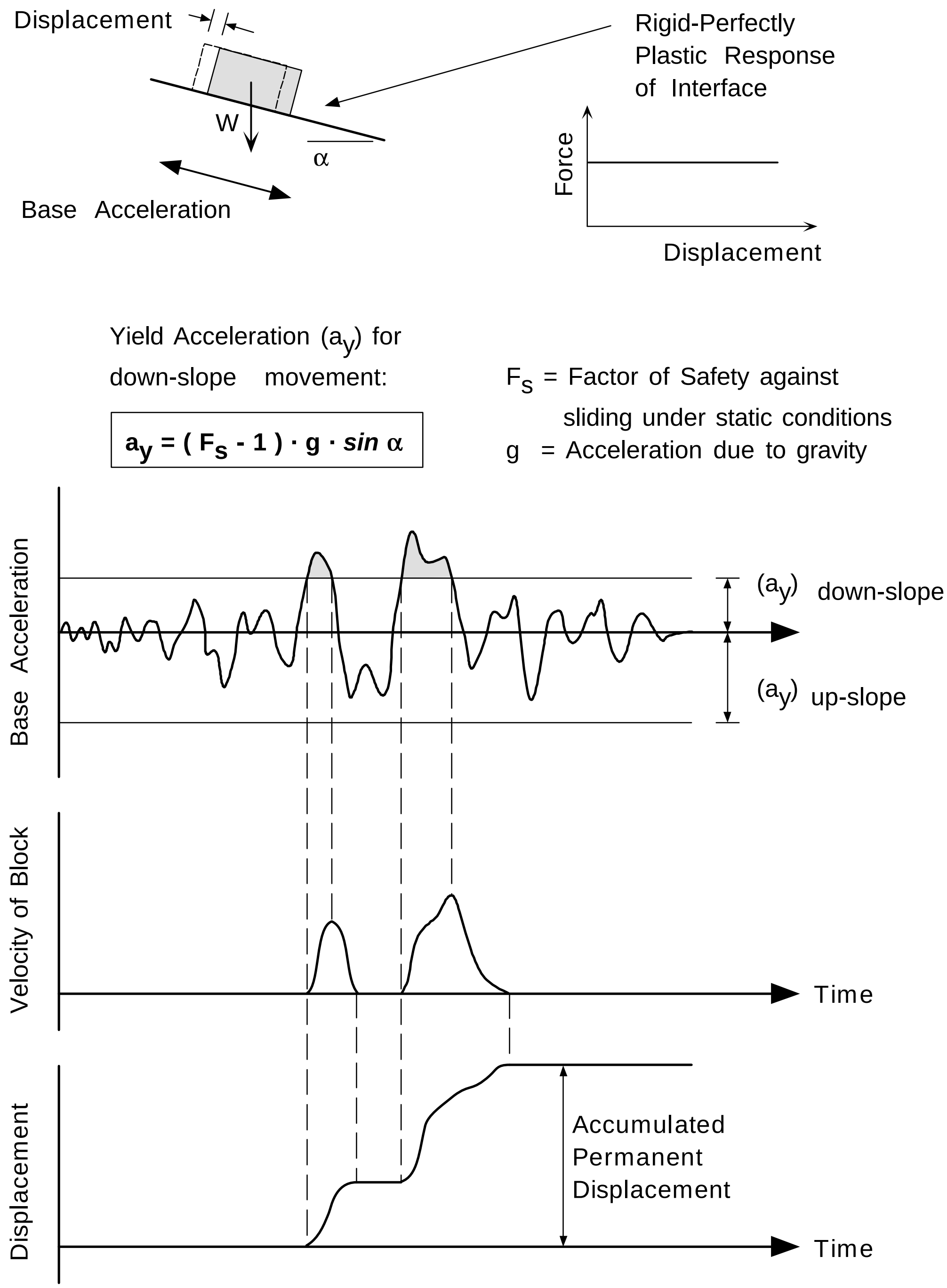


Figure 4.1. Calculation of displacements using Newmark's sliding block model (after Wilson and Keefer 1985).

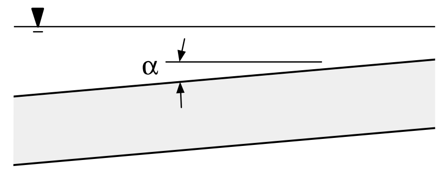
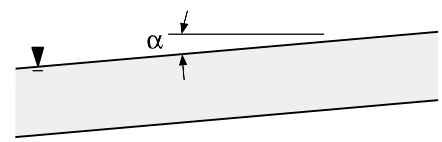
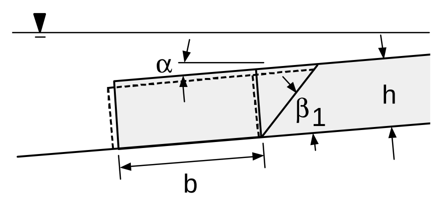
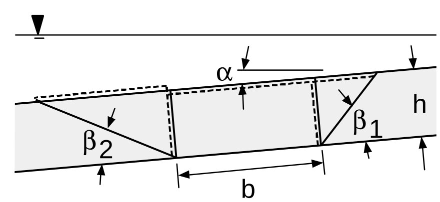
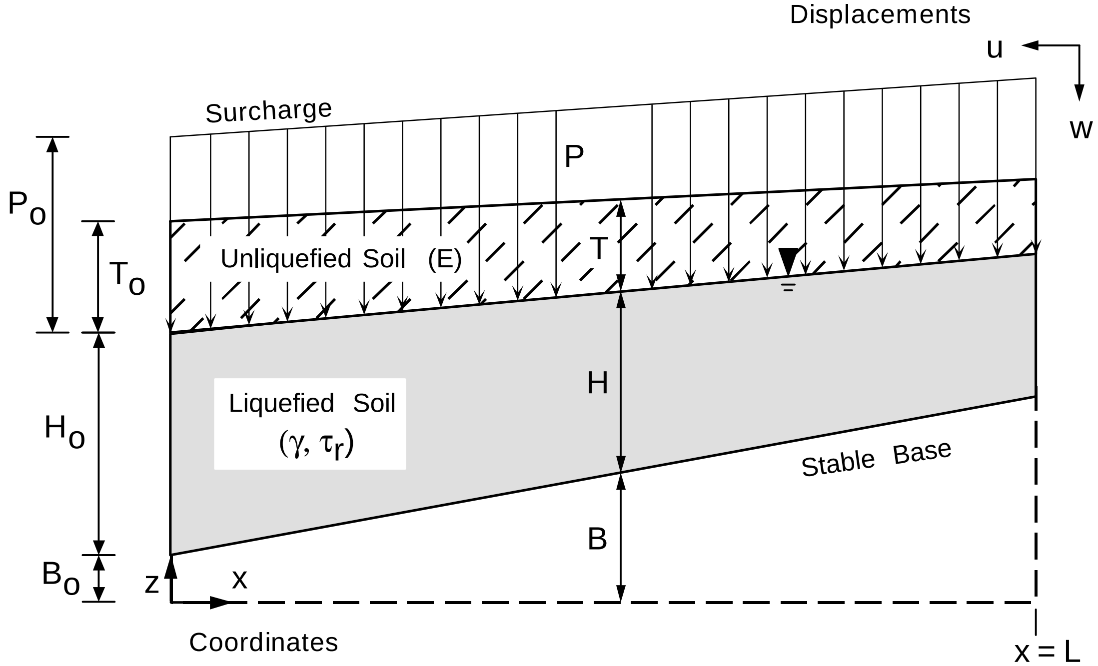
Yield Acceleration (a_y) for Down-slope Movement	Model
(a) *Submerged, infinite slope with no seepage* $$\frac{a_y}{g} = \frac{\sigma'_v}{\sigma_v}\left(\frac{S_u}{\sigma'_v} - \sin\alpha\right)$$	
(b) *Submerged, infinite slope with seepage* $$\frac{a_y}{g} = \frac{\sigma'_v}{\sigma_v}\left(\frac{S_u}{\sigma'_v} - \frac{\sigma_v}{\sigma'_v}\sin\alpha\right)$$	
(c) *Submerged block movement toward free face* $$\frac{a_y}{g} = \frac{\sigma'_v}{\sigma_v}\left(\frac{S_u}{\sigma'_v} - \sin\alpha + \frac{\frac{S_u}{\sigma'_v}\tan\beta_1 - \frac{\cos\alpha}{2}}{\frac{b}{h} + \frac{1}{2\tan\beta_1}}\right)$$	
(d) *Submerged block movement with no free face* $$\frac{a_y}{g} = \frac{\sigma'_v}{\sigma_v}\left(\frac{S_u}{\sigma'_v} - \sin\alpha + \frac{\frac{S_u}{\sigma'_v}(\tan\beta_1 + \tan\beta_2)}{\frac{b}{h} + \frac{1}{2\tan\beta_1} + \frac{1}{2\tan\beta_2}}\right)$$	

Figure 4.2. Determination of yield acceleration for Newmark-type analyses of liquefaction-induced lateral spreading (Baziar et al. 1992).



B = B_o + ax = thickness of stable base
H = H_o + bx = thickness of liquefied soil
T = T_o + cx = thickness of unliquefied, surficial soil
P = P_o + ex = surcharge pressure, including weight of surficial soil
a, b, c, and e = parameters defining linear variation of B, H, T, and P

x, y = coordinates
u, w = ultimate displacements

γ = unit weight of liquefied soil
τ_r = residual shear strength of liquefied soil (assume τ_r = 0)
E = elastic modulus of the unliquefied, surficial soil

Figure 4.3. Simplified geometry used in derivation of Towhata's minimum potential energy model for lateral spreading (after Towhata et al. 1992).