Chapter 8

Distribution of Displacement Magnitudes on Lateral Spreads

8.1. Variation in Horizontal and Vertical Displacements.

Horizontal and vertical displacements vary in magnitude across the area of a lateral spread. Movements tend to be larger in the central area of a slide and in the vicinity of a free face, but also change from place to place due to variations in surface topography and subsurface soil conditions. As discussed in Section 5.4, the EPOLLS model is designed to predict the average and standard deviation of the horizontal and vertical displacements. Statistical distributions (probability density functions) are employed to represent the variation in displacement magnitudes across the surface of a slide. Using the EPOLLS model to first predict the average and standard deviation of the horizontal or vertical displacements, these distributions can then be used to estimate maximum displacements.

In this chapter, well-documented EPOLLS case studies are examined to find suitable probability density functions to represent the variation in movements on a lateral spread. As illustrated in Figure 8.1, the measured displacements are plotted in histograms and candidate statistical distributions are considered for representing the observed variation. Based on statistical goodness-of-fit tests, the gamma distribution is found to give the best representation of horizontal displacements in a lateral spread, while the normal distribution is chosen for the vertical displacements. Reasonable estimates of maximum displacements can be made using percentile values of the gamma and normal distributions as discussed in Section 8.5.

The use of statistical distributions with the EPOLLS model is not intended to model the pattern of displacements along section lines or to predict movements at specific points on the surface of a lateral spread. Instead, statistical distributions represent the relative frequency with which a certain displacement magnitude might occur. For example, the EPOLLS model might be used to predict that 90% of the measurable horizontal displacements on a lateral spread will be less than 1.5 m. On the other hand, the EPOLLS model could not predict where the 10% of larger displacements (exceeding 1.5 m) will occur on the slide area. The EPOLLS model is designed only to predict the severity of potential lateral spreading deformations (average and maximum displacements) as opposed to predicting the movement of specific points on the ground surface.

The use of probability density functions in the EPOLLS is not meant for modeling the pattern of displacements imparted to a pipeline crossing a lateral spread. When a buried pipeline crosses a lateral spread perpendicular to the direction of movement, the pattern of ground displacements are needed to compute the forces imparted to the pipe. Along any line traversing the slide area, displacements vary from zero at the slide boundary to a maximum value somewhere in the middle. Mathematical equations have been used to represent these patterns of horizontal displacement, including cosine functions (Kobayashi et al. 1989; O'Rourke 1989) and a modified beta distribution (O'Rourke and Lane 1989). However, these equations represent the pattern of displacements along a given traverse of the slide area. This is fundamentally different from the use of stochastic distributions in the EPOLLS model to represent the relative frequency of displacements across the entire surface area of the lateral spread.

To identify areas subject to the largest movements on a lateral spread, knowledge of the site topography and geology can be used. For example, maximum settlements can be expected in areas where the liquefied deposit is thickest. When a free face is present, larger horizontal movements can be expected in vicinity of the face. In the empirical model developed by Bartlett and Youd (1992a; 1992b; 1995), horizontal displacements in a lateral spread are seen to decrease with increasing distance behind a free face (see Equation 4.9 in Section 4.4). In Kobe, Japan, a consistent pattern of horizontal displacements were observed that decreased with distance behind large quay walls damaged by liquefaction in the 1995 earthquake (Hamada et al. 1996; Ishihara et al. 1996). Although failure of the massive walls in Kobe do not conform to the EPOLLS definition of a lateral spread, this data suggests that the distance from a free face might be used to model the pattern of deformations on a lateral spread. Obviously, this approach could not be used to model displacement patterns on a lateral spread without a free face. While deformations over a substantial portion of a slide might be affected by a free face, this influence probably does not extend across the entire slide area in a typical lateral spread, because the face height is much smaller than the length of the slide area. For the forty-one EPOLLS case studies where both values are known, the face height is, on average, just 3% of the slide length.

To investigate the distribution of displacement magnitudes in a lateral spread, the EPOLLS case studies listed in Tables 8.1 and 8.2 were examined. In these selected case studies, the displacement field is believed to be represented adequately by the available displacement measurements. The case studies in Tables 8.1 and 8.2, used in the analyses that follow, meet the following criteria:

- At least ten horizontal or vertical displacements were measured on the surface of the lateral spread. However, in the evaluation of candidate statistical distributions (Sections 8.3 and 8.4), only those lateral spreads with at least twenty measurements were used. Twenty measurements are assumed to be the minimum necessary to sufficiently define the distribution of displacements.
- The displacement measurements are reasonably well dispersed across the surface of each lateral spread. While this may not be rigorously true for every case study in Tables 8.1

and 8.2, the available displacement vectors at these sites tend to be located across the entire slide surface including the center, head, toe, and sides. Case studies with displacements measured in only a small part of the slide, or measured along only one or two cross sections, were not used.

For these lateral spreads, the measured horizontal and vertical displacements are treated as random variables that follow some probability density function.

Twenty-nine lateral spreads, listed in Table 8.1, were analyzed for the distribution of horizontal displacements. Twenty-six of these case studies are from Japan where horizontal displacements were determined mainly from comparisons of aerial photographs taken before and after the earthquake. At three sites in California, displacements were determined mostly from the offsets of street curbs and other reference points. The number of measurements as well as the average, standard deviation, and maximum reported horizontal displacements for each lateral spread are given in Table 8.1. For those case studies with more than twenty measured horizontal displacements, histograms are plotted in Figures 8.2a-c. Similarly, nineteen case studies (thirteen from Niigata, Japan), listed in Table 8.2, were used to study the distribution of vertical displacements. Histograms of the vertical displacements, for those case studies with more than twenty measurements, are plotted in Figures 8.3a-b.

8.2. Statistical Distributions and Tests for Goodness-of-Fit.

Three statistical distributions were considered for representing the histograms of the measured displacements: normal, lognormal, and gamma distributions. The *probability density functions* for these three distributions are specified in Table 8.3. The probability density function, f(x), indicates the relative frequency with which a value of x occurs in a data population. Note that the lognormal distribution is, in essence, the normal distribution fit to the natural logarithms of the data (Scheaffer and McClave 1990). The general shape of the probability density functions for the normal, lognormal, and gamma distributions are shown at the bottom of Figure 8.1.

An infinite number of displacement vectors could be measured in a lateral spread, but only a fairly small sample of the displacement field is available for the EPOLLS case studies. The mathematical equations in Table 8.3 are thus fit to a specific site using the mean and standard deviation of the available, finite sample of displacements. The relationship between the sample statistics and the distribution parameters are given in the last column of Table 8.3. For the gamma distribution, simple "modified moment estimators" are used for λ and β instead of "maximum likelihood estimators" (Johnson et al. 1994). In theory, this simpler approach is a less accurate means of fitting the gamma distribution, but the effort required for the more complicated estimators is not justified for this analysis. Moreover, the "modified moment estimators" for the gamma distribution can be computed directly from the mean and standard deviation of the displacements, consistent with predicted values from the EPOLLS model.

The fitted distributions are superimposed on the histograms in Figure 8.2 to give a rough indication of how well each function represents the observed displacement patterns. A class of statistical tests known as *goodness-of-fit tests* give a more objective measure of how well a particular distribution fits this data. Good reviews of goodness-of-fit testing are given by Scheaffer and McClave (1990) and Conover (1971), and more detailed discussions are given in D'Agostino and Stephens (1986). Goodness-of-fit tests are formally based on a null hypothesis that the sample data is taken from a larger population that follows a given mathematical distribution. If the null hypothesis is accepted at a given level of significance (α), then we can believe the statistical distribution fits the sample data. The higher the significance level at which the hypothesis is accepted, the more confident we can be that the distribution fits the data. In analyzing the EPOLLS displacements, a level of significance of α=2.5% was deemed appropriate for judging the fit of each statistical distribution. Viewed another way, this implies a 97.5% confidence that the fit of a given distribution has not been erroneously rejected. In a strict sense, accepting the fit in a goodness-of-fit test indicates only that the hypothesized distribution is a reasonable approximation of the population from which the sample data was taken.

The chi-squared test is probably the most familiar goodness-of-fit statistical test. Calculated from histograms of the sample data, the chi-squared test is based on the difference between the histogram and a given probability density function. The chi-squared test is easily employed with data having only discrete values (for example, the number of faulty components produced in a factory, which can only be whole number values). For non-discrete or continuous data (like measured displacements), the chi-squared tests requires grouping the sample data into arbitrary histogram cells. The selection of these cell limits has a direct impact on the results of the chi-squared test. Consequently, the chi-squared test is not preferred for testing goodness-of-fit with continuous data (D'Agostino and Stephens 1986).

More powerful goodness-of-fit tests for continuous data are based on cumulative frequency plots like those in Figure 8.4. Eight of the histograms in Figure 8.2 are re-plotted in Figure 8.4 as *empirical density functions* (EDFs) that represent the cumulative frequency of the measured displacement magnitudes. The EDFs of the sample data are overlain with *cumulative density functions* (CDFs) for each of the candidate statistical distributions. The CDF, denoted mathematically as F(x), indicates the frequency of occurrence of values less than or equal to x in the data population and is computed as the integral of the probability density function up to the value of x. Small vertical departures between the EDF and CDF in Figure 8.4 indicate a good fit between the data and a particular distribution. Goodness-of-fit tests based on EDF statistics are discussed in Conover (1971) and D'Agostino and Stephens (1986). Two EDF goodness-of-fit tests were chosen for this analysis, the Kolmogorov-Smirnov "*D*" test and the Cramér-von Mises "W^2" test.

The Kolmogorov-Smirnov "*D*" test statistic is based on the single, maximum vertical offset between the EDF and CDF over the range of the sample data. The maximum offset will always occur just to the left or right of an observation point on the EDF. Thus, the value of *D* can be computed with the equation:

$$D = \max\left(\left| F(x_i) - \frac{i-1}{n} \right| , \left| F(x_i) - \frac{i}{n} \right| \right) \tag{8.1}$$

where n is the sample size, x_i is the sample data arranged in ascending order, and $F(x_i)$ is the cumulative density function at x_i for the statistical distribution under consideration. The first term in Equation 8.1 is the vertical offset between the EDF and CDF to the left of x_i, while the second term is the offset to the right of x_i. The value of *D* is the maximum of all offsets computed for the entire sample. On the other hand, the Cramér-von Mises "W^2" test statistic is computed from all of the departures between the EDF and CDF over the full range of the sample data. Consequently, W^2 is usually considered to yield a more powerful goodness-of-fit test than *D* (D'Agostino and Stephens 1986). The W^2 statistic is computed with:

$$W^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left(F(x_i) - \frac{2i-1}{2n} \right)^2 \tag{8.2}$$

where the variables are defined the same as for Equation 8.1. For both *D* and W^2, smaller values indicate a closer fit of the hypothesized distribution to the sample data.

To test the hypothesis that a certain distribution fits the sample data, critical values of the test statistic are needed for a given level of significance (α). When the parameters for the population distribution are estimated from sample data, as done in this analysis, these critical values depend on the distribution tested. Stephens (1974) developed critical values of *D* and W^2, which depend on the size of the data sample, for testing the normal distribution with estimated parameters. Because the CDF of the lognormal distribution at "x" is equal to the CDF of the normal distribution at "ln(x)", critical values of the test statistics for the normal distribution can be used to test the fit of the lognormal distribution. For testing the fit of the gamma distribution with estimated parameters, D'Agostino and Stephens (1986) give critical W^2 values; however, critical values of the *D* statistic are not available for testing the fit of the gamma distribution. The available critical values of *D* and W^2 for a significance level of 1 to 10% are reproduced here in Table 8.4.

Testing the fit of a hypothesized distribution to the horizontal or vertical displacements in a lateral spread is done in five steps:

(1) Compute the mean and standard deviation of the measured displacements and fit the chosen distribution using the estimated parameters as indicated in Table 8.3.
(2) Arrange the measured displacements in ascending order.
(3) Compute the cumulative density function for the hypothesized distribution at each data point.

(4) Compute the test statistic, D or W^2, using Equations 8.1 or 8.2.
(5) If the computed test statistic is less than the critical value for α=2.5% from Table 8.4, conclude that the hypothesized distribution fits the data to a 2.5% level of significance.

Goodness-of-fit tests for the candidate statistical distributions, based on the D and W^2 statistics, are discussed in Sections 8.3 and 8.4 for the horizontal and vertical displacements, respectively, measured in the EPOLLS case studies.

8.3. Distribution of Horizontal Displacements.

Histograms of the observed horizontal displacements, for twenty-three case studies with more than twenty measurements, are shown in Figures 8.2a through 8.2c. Inspection of these histograms reveals that many are skewed toward the smaller magnitudes. This tendency is strongly evident in Figure 8.2a for Slide Nos. 9 and 26, and less so for Slide Nos. 6 and 29. Although this trend could result from occasional, anomalously large displacements, the larger vectors are often found in groups at these sites. Consistent with the discussion above, the skew of the displacement distributions seem to result from larger displacements occurring in smaller, central areas of a lateral spread, or in zones closer to a free face. In addition, the displacement histograms from lateral spreads with or without a free face do not appear to be significantly different.

In Figures 8.2a-c, the fitted normal, lognormal, and gamma distributions are shown on the histograms of the measured horizontal displacements. While the normal distribution is symmetric about the mean value, both the lognormal and gamma distributions are non-symmetric. Hence, the apparent skew of the histograms is better represented with the lognormal or gamma distributions. More significantly, both the lognormal and gamma distributions are defined only for positive values of displacement whereas the normal distribution extends to values less than zero. Since all horizontal displacements are positive by definition, the normal distribution is not a good choice for modeling this data.

Goodness-of-fit tests, based on the D and W^2 statistics, were performed on the EPOLLS case studies with more than twenty measured horizontal displacement vectors (considered the minimum sample size needed for this analysis). The fit of the normal, lognormal, and gamma distributions were tested. The Kolmogorov-Smirnov D test results are given in Table 8.5, with the Cramér-von Mises W^2 results presented in Table 8.6. For each case study, computed values of D and W^2 are paired with the appropriate critical values for a level of significance (α) of 2.5%. In the adjacent columns labeled "Fit?", "yes" entries signify that the computed test statistic is less than the critical value, which is interpreted to mean that the distribution fits the data reasonably well. In general, the results of these tests are mixed and none of the three distributions tested are accepted for all cases in Tables 8.5 and 8.6. However, the W^2 test for the gamma distribution gives a positive result, at α=2.5%, in nineteen of twenty-three (83%) cases. By the same criteria, the lognormal and normal distributions are accepted in only fifteen (65%) and ten (43%) of the case studies, respectively. Therefore, the Cramér-von Mises W^2 tests suggest that, for the majority of the lateral spreads investigated, the horizontal displacements follow a gamma distribution.

Moreover, the fit of the three candidate distributions to the sample data can be ranked using the D and W^2 statistics. Smaller values of the D or W^2 test statistics indicate a closer match between the EDF of the data and the CDF of a given distribution. For each case study, the statistical distribution that best fits the observed displacements yields the lowest value of D or W^2. Hence, as indicated in the last two columns of Tables 8.5 and 8.6, the distributions yielding the best and second-best fits to the data can be identified from the numerical values of D and W^2. However, no single distribution emerges as the best-fit for the majority of the lateral spreads studied. On the other hand, the gamma distribution is the first or second choice (based on either D or W^2) in the greatest number of cases. Also, the normal distribution yields the best or second-best fit in only eight cases; that is, the normal distribution gives the worst match in two-thirds of the cases. This clearly indicates that the normal distribution is not a good choice for representing the pattern of horizontal displacements in a lateral spread.

The goodness-of-fit tests yield mixed results for the three distributions considered, which may be related to the small number of measured displacements in the available case studies. However, the Cramér-von Mises W^2 tests indicate that the gamma distribution fits the measured horizontal displacements (to a 2.5% significance level) in the majority of the case studies. In addition, the gamma distribution gives the best or second best fit to the data in the greatest number of cases. The normal distribution is clearly a poor choice for modeling the observed horizontal displacements because the normal distribution is symmetric, extends to values less than zero, and gives the worst match to the data in the majority of the case studies.

8.4. Distribution of Vertical Displacements.

Histograms of the measured vertical displacements are shown in Figure 8.3a-b. Unlike the horizontal displacements, the observed vertical displacements follow a more symmetric distribution with no pronounced skew. Recall also that settlement is defined as a positive vertical displacement while heaving or uplift is measured as a negative vertical displacement. Unlike the horizontal displacements, the distribution of vertical displacements extends to values less than zero.

These observations suggest that the normal distribution would be a good choice for modeling the vertical displacements on a lateral spread. The two-parameter lognormal and gamma distributions, as defined in Table 8.3, are incapable of modeling both positive and negative vertical displacements. On the other hand, three-parameter lognormal or gamma distributions could be used to overcome this deficiency (the third parameter shifts the minimum value of the distribution). However, employing a third parameter to model the distribution of vertical displacements would add complexity to the EPOLLS model and is probably not warranted. Hence, only the normal distribution was considered for representing the observed vertical displacements on lateral spreads.

Results of the goodness-of-fit tests for the normal distribution and the measured vertical displacements, based on the Kolmogorov-Smirnov D and the Cramér-von Mises W^2 statistics, are presented in Table 8.7. As done for the evaluation of horizontal displacements, only those case studies with more than twenty measured displacements were considered in this analysis. Again using a level of significance of 2.5%, the Kolmogorov-Smirnov D test indicates that the normal distribution fits the data in nine of thirteen cases as shown in Table 8.7. The Cramér-von Mises W^2 test consistently indicates a positive fit of the normal distribution in eight of thirteen cases. Therefore, based on the available data, it appears that the normal distribution is well-suited for representing the distribution of vertical displacements on the surface of a lateral spread.

8.5. Prediction of Maximum Displacements.

In the EPOLLS model, probabilistic distributions are used to forecast maximum likely displacements from the predicted mean and standard deviation. The statistical distributions considered above were thus evaluated in predicting the maximum observed displacements. For each of the lateral spreads in Tables 8.1 and 8.2, maximum horizontal and vertical displacement were computed using the statistical distributions fit with the mean and standard deviation of the measured displacements. The predicted maximum displacement was then compared to the maximum measured displacement. Note that the true maximum deformation may not have been measured on each lateral spread; hence, somewhat conservative over-predictions of the maximum measured displacement are desired.

Predictions of the maximum horizontal displacement were made at the 99.0, 99.5, and 99.9 percentiles of the normal, lognormal, and gamma distributions. In Figure 8.5, histograms of the resulting "error" (difference between the predicted and observed maximum displacement) are shown for each distribution and percentile level. From Figure 8.5, it appears that 99.5 percentile predictions from the normal and gamma distributions yield reasonable, conservative estimates of maximum horizontal displacement. That is, the maximum displacement is over-predicted by less than 2 meters for most of the cases and is under-predicted in only a few cases. More significantly, the lognormal distribution tends to produce several excessively large predictions of the maximum displacement (>4 m error even at the 99.0 percentile). This indicates that the lognormal distribution is a poor choice for estimating maximum horizontal displacements in a lateral spread.

In Figure 8.6, errors in the predicted maximum settlement are plotted as histograms for the 99.0, 99.5, and 99.9 percentiles of the normal distribution. Here, it appears that the 99.5 percentile of the normal distribution gives a reasonably conservative estimate of the maximum settlement. To predict the maximum uplift, the 0.5, 1.0, and 1.5 percentiles of the normal distribution were evaluated with histograms of the resulting errors plotted in Figure 8.7. Because uplift is defined as a negative displacement, an over-prediction of the maximum uplift produces a negative "error" (predicted minus observed uplift) in Figure 8.7. From these histograms, it appears that the 1.0 percentile of the normal distribution gives a reasonable estimate of the maximum uplift.

8.6. Summary.

The normal, lognormal, and gamma distributions were evaluated for representing the pattern of deformations in the EPOLLS lateral spreads. The gamma distribution was found to be the best choice for representing the distribution of horizontal displacements on a lateral spread. This conclusion is based on the following:

(1) By definition, displacements are non-negative and the gamma distribution is defined only for positive values. The normal distribution, which is defined for negative values, is a poor choice for modeling horizontal displacements.

(2) According to the W^2 goodness-of-fit test, the gamma distribution fits the sample data in 83% of the cases to a 2.5% level of significance.

(3) Based on both the Kolmogorov-Smirnov D and Cramér-von Mises W^2 statistics, the gamma distribution yields the best and second-best fit to the sample data in the greatest number of cases. The normal distribution gives the worst fit in the majority of cases.

(4) The lognormal distribution produces excessively large over-predictions of the maximum horizontal displacement and is thus a poor choice for modeling horizontal displacements.

Conservative, yet reasonable, estimates of the maximum horizontal displacement were obtained at the 99.5 percentile of the gamma distribution.

The normal distribution was found to provide a satisfactory representation of the measured vertical displacements. The lognormal and gamma distributions were rejected because neither could model both positive settlement and negative uplift. Reasonable predictions of the maximum settlement were obtained at the 99.5 percentile, while good estimates of the maximum uplift were obtained at the 1.0 percentile, of the normal distribution.

To estimate maximum movements on a lateral spread using statistical distributions, predictions of the mean and standard deviation of the displacements are made first. The gamma distribution is then fit by computing the parameters λ and β, as shown in Table 8.3, using the predicted mean and standard deviation of the horizontal displacements. For the vertical displacements, the normal distribution is fit directly with the predicted mean and standard deviation. Maximum displacements are then forecast using the cumulative density function (CDF) of the fitted gamma or normal distribution.

Values of the cumulative density functions, for either the normal or gamma distributions, can not be computed from simple equations. Instead, good approximations of the normal and gamma CDF values can be obtained from statistical tables or from a variety of computer software packages, including many spreadsheet programs. In Tables 8.8 through 8.10, values of the CDF for the 99.5 percentile of the gamma and normal distribution, as well as the 1.0 percentile of the normal distribution, are given in ranges suitable for use in predicting maximum displacements on a lateral spread. Satisfactory estimates of maximum displacement can be obtained by interpolating the appropriate values in Tables 8.8 through 8.10.

Table 8.1. EPOLLS case studies used to investigate the distribution of horizontal displacements.

Slide No.	Location	Horizontal Displacements (m)			
		Number Measured	Average	Standard Deviation	Maximum
6	Fukui, Japan	24	1.96	0.84	4.00
7	Fukui, Japan	25	1.89	0.99	4.30
8	Fukui, Japan	36	1.69	0.71	3.40
9	Fukui, Japan	24	1.56	0.65	3.69
*25	Niigata, Japan	14	3.75	2.45	9.25
26	Niigata, Japan	75	3.94	2.97	11.81
27	Niigata, Japan	24	3.76	1.94	8.72
28	Niigata, Japan	38	2.08	1.21	6.49
29	Niigata, Japan	46	4.21	1.98	8.82
30	Niigata, Japan	26	4.78	2.64	10.15
31	Niigata, Japan	37	1.22	0.41	2.07
32	Niigata, Japan	72	2.34	1.01	4.65
*34	Niigata, Japan	16	0.98	0.64	2.16
35	Niigata, Japan	22	4.59	2.66	10.55
37	Niigata, Japan	63	3.23	1.55	6.46
38	Niigata, Japan	66	4.74	2.10	8.34
39	Niigata, Japan	84	2.76	1.43	7.64
40	San Fernando, California	26	1.02	1.19	3.69
41	San Fernando, California	79	0.90	0.58	1.82
43	Imperial Valley, California	33	1.40	1.19	4.24
45	Noshiro, Japan	28	1.47	0.66	2.92
46	Noshiro, Japan	34	1.46	0.43	2.72
47	Noshiro, Japan	59	1.58	0.83	4.01
48	Noshiro, Japan	57	1.26	0.42	2.65
49	Noshiro, Japan	187	1.55	0.58	3.25
*109	Shiribeshi-toshibetsu River, Japan	11	1.38	1.06	3.91
*110	Shiribeshi-toshibetsu River, Japan	17	0.68	0.35	1.40
*116	Shiribeshi-toshibetsu River, Japan	11	0.67	0.27	1.10
*117	Shiribeshi-toshibetsu River, Japan	13	1.36	0.90	3.39

* Case studies with < 20 measured displacements not used to evaluate fit of statistical distributions.

Table 8.2. EPOLLS case studies used to investigate the distribution of vertical displacements.

Slide No.	Location	Vertical Displacements (m)				
		Number Measured	Average	Standard Deviation	Maximum Settlement	Maximum Uplift
*25	Niigata, Japan	14	0.27	0.44	1.07	-0.54
26	Niigata, Japan	71	0.65	0.64	3.45	-0.68
27	Niigata, Japan	23	1.23	0.59	2.24	0.00
28	Niigata, Japan	37	0.14	0.61	1.82	-0.78
29	Niigata, Japan	45	1.13	0.87	2.87	-0.81
30	Niigata, Japan	26	1.14	1.03	3.78	-0.87
31	Niigata, Japan	37	0.04	0.47	1.11	-0.98
32	Niigata, Japan	64	1.04	0.61	2.96	-0.37
*34	Niigata, Japan	16	1.40	1.00	3.16	0.00
35	Niigata, Japan	22	0.61	0.62	2.03	-0.80
37	Niigata, Japan	63	0.08	0.59	1.39	-1.05
38	Niigata, Japan	63	0.52	0.78	1.85	-0.92
39	Niigata, Japan	82	0.29	0.50	2.10	-0.76
41	San Fernando, California	84	0.25	0.24	0.86	-0.44
56	Watsonville, California	52	0.08	0.07	0.24	-0.02
*109	Shiribeshi-toshibetsu River, Japan	11	0.55	0.18	0.84	0.00
*110	Shiribeshi-toshibetsu River, Japan	18	0.43	0.29	0.97	-0.27
*116	Shiribeshi-toshibetsu River, Japan	12	0.27	0.28	0.54	-0.25
*117	Shiribeshi-toshibetsu River, Japan	13	0.14	0.23	0.45	-0.25

* Case studies with < 20 measured displacements not used to evaluate fit of statistical distributions.

Table 8.3. Candidate statistical distributions for representing observed displacements on a lateral spread.

	Probability Density Function, *f(x)*	Distribution Parameters *estimated from a finite sample of the total population*
Normal Distribution	$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2}$	μ_x = mean of sample x σ_x = standard deviation of sample x
Lognormal Distribution (for x > 0)	$f(x) = \frac{1}{x \, \sigma_{\ln x} \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu_{\ln x}}{\sigma_{\ln x}}\right)^2}$	$\mu_{ln x}$ = mean of sample ln(x) $\sigma_{ln x}$ = standard deviation of sample ln(x)
Gamma Distribution (for x ≥ 0)	$f(x) = \frac{x^{\lambda-1}}{\beta^{\lambda} \, \Gamma(\lambda)} e^{-\frac{x}{\beta}}$ $\Gamma(\lambda) = \int_0^{\infty} u^{\lambda-1} e^{-u} du$ $\Gamma(\lambda)$ = gamma function	$\lambda = \mu_x^2/\sigma_x^2$ $\beta = \sigma_x^2/\mu_x$ μ_x = mean of sample x σ_x = standard deviation of sample x

See Figure 8.1 for a comparison of the probability density function shapes for these distributions.

Table 8.4. Critical values of D and W^2 statistics for goodness-of-fit tests.

Distribution and EDF Test Statistic	Critical Value (n = sample size)	Level of Significance, α (%)			
		1.0	2.5	5.0	10.0
Normal Distribution: Kolmogorov-Smirnov "D" Statistic	$D_{critical} = \frac{C^*}{\sqrt{n} - 0.01 + (0.85/\sqrt{n})}$ C^* = (Stephens 1974)	1.035	0.955	0.895	0.819
Normal Distribution: Cramér-von Mises "W^2" Statistic	$W^2_{critical} = \frac{C^*}{1 + (0.5/n)}$ C^* = (D'Agostino and Stephens 1986)	0.179	0.148	0.126	0.104
Gamma Distribution: Kolmogorov-Smirnov "D" Statistic	[critical values not available]				
Gamma Distribution: Cramér-von Mises "W^2" Statistic	$W^2_{critical}$ for: λ = 1	0.196	0.162	0.136	0.111
	λ = 2	0.187	0.155	0.131	0.107
	λ = 3	0.184	0.153	0.129	0.106
	4	0.183	0.152	0.128	0.105
	λ = estimated shape parameter for gamma distribution 5	0.182	0.151	0.128	0.105
	6	0.181	0.151	0.128	0.105
	8	0.181	0.150	0.127	0.104
	10	0.180	0.150	0.127	0.104
	(D'Agostino and Stephens 1986) 12	0.180	0.150	0.127	0.104
	15	0.180	0.149	0.127	0.104
	20	0.180	0.149	0.126	0.104

Table 8.5. Results of Kolmogorov-Smirnov goodness-of-fit tests for three distributions and measured horizontal displacements.

Slide No.	Normal distribution			Lognormal distribution			Gamma distribution			Best fit	Second best fit
	D	*Crit D*	Fit?	*D*	*Crit D*	Fit?	*D*	*Crit D*	Fit?		
6	0.126	0.189	yes	0.121	0.189	yes	0.104	--	--	Gamma	Lognorm
7	0.177	0.185	yes	0.101	0.185	yes	0.109	--	--	Lognorm	Gamma
8	0.162	0.156	no	0.084	0.156	yes	0.107	--	--	Lognorm	Gamma
9	0.186	0.189	yes	0.122	0.189	yes	0.133	--	--	Lognorm	Gamma
26	0.218	0.109	no	0.102	0.109	yes	0.120	--	--	Lognorm	Gamma
27	0.156	0.189	yes	0.194	0.189	no	0.187	--	--	Normal	Gamma
28	0.190	0.152	no	0.093	0.152	yes	0.155	--	--	Lognorm	Gamma
29	0.131	0.138	yes	0.098	0.138	yes	0.080	--	--	Gamma	Lognorm
30	0.115	0.182	yes	0.153	0.182	yes	0.131	--	--	Normal	Gamma
31	0.099	0.154	yes	0.170	0.154	no	0.125	--	--	Normal	Gamma
32	0.144	0.111	no	0.173	0.111	no	0.171	--	--	Normal	Gamma
35	0.135	0.196	yes	0.130	0.196	yes	0.101	--	--	Gamma	Lognorm
37	0.094	0.119	yes	0.085	0.119	yes	0.087	--	--	Lognorm	Gamma
38	0.127	0.116	no	0.153	0.116	no	0.161	--	--	Normal	Lognorm
39	0.061	0.103	yes	0.116	0.103	no	0.097	--	--	Normal	Gamma
40	0.266	0.182	no	0.133	0.182	yes	0.153	--	--	Lognorm	Gamma
41	0.161	0.106	no	0.178	0.106	no	0.173	--	--	Normal	Gamma
43	0.156	0.162	yes	0.148	0.162	yes	0.121	--	--	Gamma	Lognorm
45	0.141	0.175	yes	0.184	0.175	no	0.167	--	--	Normal	Gamma
46	0.148	0.160	yes	0.125	0.160	yes	0.114	--	--	Gamma	Lognorm
47	0.148	0.123	no	0.123	0.123	yes	0.112	--	--	Gamma	Lognorm
48	0.177	0.125	no	0.125	0.125	yes	0.136	--	--	Lognorm	Gamma
49	0.094	0.070	no	0.081	0.070	no	0.057	--	--	Gamma	Lognorm

D = Kolmogorov-Smirnov test statistic
Crit D = critical value of *D* for 2.5% level of significance

Table 8.6. Results of Cramér-von Mises goodness-of-fit tests for three distributions and measured horizontal displacements.

Slide No.	Normal distribution			Lognormal distribution			Gamma distribution			Best fit	Second best fit
	W^2	*Crit* W^2	Fit?	W^2	*Crit* W^2	Fit?	W^2	*Crit* W^2	Fit?		
6	0.064	0.145	yes	0.046	0.145	yes	0.040	0.151	yes	Gamma	Lognorm
7	0.161	0.145	no	0.031	0.145	yes	0.048	0.152	yes	Lognorm	Gamma
8	0.160	0.146	no	0.033	0.146	yes	0.050	0.151	yes	Lognorm	Gamma
9	0.237	0.145	no	0.074	0.145	yes	0.129	0.151	yes	Lognorm	Gamma
26	1.009	0.147	no	0.240	0.147	no	0.339	0.157	no	Lognorm	Gamma
27	0.082	0.145	yes	0.153	0.145	no	0.130	0.152	yes	Normal	Gamma
28	0.367	0.146	no	0.073	0.146	yes	0.141	0.153	yes	Lognorm	Gamma
29	0.128	0.146	yes	0.072	0.146	yes	0.057	0.151	yes	Gamma	Lognorm
30	0.048	0.145	yes	0.110	0.145	yes	0.070	0.153	yes	Normal	Gamma
31	0.048	0.146	yes	0.165	0.146	no	0.090	0.150	yes	Normal	Gamma
32	0.350	0.147	no	0.421	0.147	no	0.472	0.151	no	Normal	Lognorm
35	0.071	0.145	yes	0.049	0.145	yes	0.031	0.153	yes	Gamma	Lognorm
37	0.124	0.147	yes	0.114	0.147	yes	0.094	0.152	yes	Gamma	Lognorm
38	0.236	0.147	no	0.387	0.147	no	0.380	0.151	no	Normal	Gamma
39	0.051	0.147	yes	0.302	0.147	no	0.135	0.152	yes	Normal	Gamma
40	0.451	0.145	no	0.101	0.145	yes	0.135	0.162	yes	Lognorm	Gamma
41	0.586	0.147	no	0.535	0.147	no	0.731	0.154	no	Lognorm	Normal
43	0.130	0.146	yes	0.111	0.146	yes	0.053	0.159	yes	Gamma	Lognorm
45	0.117	0.145	yes	0.118	0.145	yes	0.125	0.151	yes	Normal	Lognorm
46	0.149	0.146	no	0.086	0.146	yes	0.089	0.150	yes	Lognorm	Gamma
47	0.380	0.147	no	0.101	0.147	yes	0.125	0.152	yes	Lognorm	Gamma
48	0.276	0.147	no	0.104	0.147	yes	0.136	0.150	yes	Lognorm	Gamma
49	0.282	0.148	no	0.172	0.148	no	0.125	0.150	yes	Gamma	Lognorm

W^2 = Cramér-von Mises test statistic
Crit W^2 = critical value of W^2 for 2.5% level of significance

Table 8.7. Results of goodness-of-fit tests for the normal distribution and measured vertical displacements.

Slide	Kolmogorov-Smirnov Test			Cramér-von Mises Test		
No.	*D*	*Crit D*	Fit?	W^2	*Crit* W^2	Fit?
26	0.173	0.112	no	0.414	0.147	no
27	0.112	0.192	yes	0.038	0.145	yes
28	0.108	0.154	yes	0.079	0.146	yes
29	0.065	0.140	yes	0.023	0.146	yes
30	0.119	0.182	yes	0.061	0.145	yes
31	0.078	0.154	yes	0.031	0.146	yes
32	0.081	0.118	yes	0.058	0.147	yes
35	0.159	0.196	yes	0.117	0.145	yes
37	0.073	0.119	yes	0.056	0.147	yes
38	0.107	0.119	yes	0.201	0.147	no
39	0.138	0.104	no	0.430	0.147	no
41	0.130	0.103	no	0.248	0.147	no
56	0.233	0.130	no	0.673	0.147	no

D = Kolmogorov-Smirnov test statistic
Crit D = critical value of *D* for 2.5% level of significance
W^2 = Cramér-von Mises test statistic
Crit W^2 = critical value of W^2 for 2.5% level of significance

Table 8.8. Values of the gamma distribution at the 99.5 percentile.

λ	β									
	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
0.20	0.55	1.10	1.65	2.20	2.75	3.31	3.86	4.41	4.96	5.51
0.40	0.72	1.44	2.16	2.88	3.60	4.32	5.05	5.77	6.49	7.21
0.60	0.85	1.70	2.55	3.40	4.25	5.09	5.94	6.79	7.64	8.49
0.80	0.96	1.92	2.88	3.84	4.80	5.76	6.72	7.68	8.64	9.60
1.00	1.06	2.12	3.18	4.24	5.30	6.36	7.42	8.48	9.54	10.60
1.20	1.15	2.31	3.46	4.61	5.76	6.92	8.07	9.22	10.38	11.53
1.40	1.24	2.48	3.72	4.96	6.21	7.45	8.69	9.93	11.17	12.41
1.60	1.33	2.65	3.98	5.30	6.63	7.95	9.28	10.60	11.93	13.26
1.80	1.41	2.81	4.22	5.63	7.04	8.44	9.85	11.26	12.66	14.07
2.00	1.49	2.97	4.46	5.94	7.43	8.92	10.40	11.89	13.37	14.86
2.20	1.56	3.13	4.69	6.25	7.81	9.38	10.94	12.50	14.07	15.63
2.40	1.64	3.28	4.91	6.55	8.19	9.83	11.47	13.10	14.74	16.38
2.60	1.71	3.42	5.13	6.85	8.56	10.27	11.98	13.69	15.40	17.12
2.80	1.78	3.57	5.35	7.14	8.92	10.70	12.49	14.27	16.05	17.84
3.00	1.85	3.71	5.56	7.42	9.27	11.13	12.98	14.84	16.69	18.55
3.20	1.92	3.85	5.77	7.70	9.62	11.55	13.47	15.40	17.32	19.25
3.40	1.99	3.99	5.98	7.97	9.97	11.96	13.96	15.95	17.94	19.94
3.60	2.06	4.12	6.19	8.25	10.31	12.37	14.43	16.49	18.56	20.62
3.80	2.13	4.26	6.39	8.52	10.64	12.77	14.90	17.03	19.16	21.29
4.00	2.20	4.39	6.59	8.78	10.98	13.17	15.37	17.56	19.76	21.96

Note: λ = (mean)^2 / (standard deviation)^2 ; β = (standard deviation)^2 / (mean)

Table 8.9. Values of the normal distribution at the 99.5 percentile.

Mean	Standard Deviation									
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.10	0.36	0.62	0.87	1.13	1.39	1.65	1.90	2.16	2.42	2.68
0.20	0.46	0.72	0.97	1.23	1.49	1.75	2.00	2.26	2.52	2.78
0.30	0.56	0.82	1.07	1.33	1.59	1.85	2.10	2.36	2.62	2.88
0.40	0.66	0.92	1.17	1.43	1.69	1.95	2.20	2.46	2.72	2.98
0.50	0.76	1.02	1.27	1.53	1.79	2.05	2.30	2.56	2.82	3.08
0.60	0.86	1.12	1.37	1.63	1.89	2.15	2.40	2.66	2.92	3.18
0.70	0.96	1.22	1.47	1.73	1.99	2.25	2.50	2.76	3.02	3.28
0.80	1.06	1.32	1.57	1.83	2.09	2.35	2.60	2.86	3.12	3.38
0.90	1.16	1.42	1.67	1.93	2.19	2.45	2.70	2.96	3.22	3.48
1.00	1.26	1.52	1.77	2.03	2.29	2.55	2.80	3.06	3.32	3.58
1.10	1.36	1.62	1.87	2.13	2.39	2.65	2.90	3.16	3.42	3.68
1.20	1.46	1.72	1.97	2.23	2.49	2.75	3.00	3.26	3.52	3.78
1.30	1.56	1.82	2.07	2.33	2.59	2.85	3.10	3.36	3.62	3.88
1.40	1.66	1.92	2.17	2.43	2.69	2.95	3.20	3.46	3.72	3.98
1.50	1.76	2.02	2.27	2.53	2.79	3.05	3.30	3.56	3.82	4.08
1.60	1.86	2.12	2.37	2.63	2.89	3.15	3.40	3.66	3.92	4.18
1.70	1.96	2.22	2.47	2.73	2.99	3.25	3.50	3.76	4.02	4.28
1.80	2.06	2.32	2.57	2.83	3.09	3.35	3.60	3.86	4.12	4.38
1.90	2.16	2.42	2.67	2.93	3.19	3.45	3.70	3.96	4.22	4.48
2.00	2.26	2.52	2.77	3.03	3.29	3.55	3.80	4.06	4.32	4.58

Table 8.10. Values of the normal distribution at the 1.0 percentile.

Mean	Standard Deviation									
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0.10	-0.13	-0.37	-0.60	-0.83	-1.06	-1.30	-1.53	-1.76	-1.99	-2.23
0.20	-0.03	-0.27	-0.50	-0.73	-0.96	-1.20	-1.43	-1.66	-1.89	-2.13
0.30	0.07	-0.17	-0.40	-0.63	-0.86	-1.10	-1.33	-1.56	-1.79	-2.03
0.40	0.17	-0.07	-0.30	-0.53	-0.76	-1.00	-1.23	-1.46	-1.69	-1.93
0.50	0.27	0.03	-0.20	-0.43	-0.66	-0.90	-1.13	-1.36	-1.59	-1.83
0.60	0.37	0.13	-0.10	-0.33	-0.56	-0.80	-1.03	-1.26	-1.49	-1.73
0.70	0.47	0.23	0.00	-0.23	-0.46	-0.70	-0.93	-1.16	-1.39	-1.63
0.80	0.57	0.33	0.10	-0.13	-0.36	-0.60	-0.83	-1.06	-1.29	-1.53
0.90	0.67	0.43	0.20	-0.03	-0.26	-0.50	-0.73	-0.96	-1.19	-1.43
1.00	0.77	0.53	0.30	0.07	-0.16	-0.40	-0.63	-0.86	-1.09	-1.33
1.10	0.87	0.63	0.40	0.17	-0.06	-0.30	-0.53	-0.76	-0.99	-1.23
1.20	0.97	0.73	0.50	0.27	0.04	-0.20	-0.43	-0.66	-0.89	-1.13
1.30	1.07	0.83	0.60	0.37	0.14	-0.10	-0.33	-0.56	-0.79	-1.03
1.40	1.17	0.93	0.70	0.47	0.24	0.00	-0.23	-0.46	-0.69	-0.93
1.50	1.27	1.03	0.80	0.57	0.34	0.10	-0.13	-0.36	-0.59	-0.83
1.60	1.37	1.13	0.90	0.67	0.44	0.20	-0.03	-0.26	-0.49	-0.73
1.70	1.47	1.23	1.00	0.77	0.54	0.30	0.07	-0.16	-0.39	-0.63
1.80	1.57	1.33	1.10	0.87	0.64	0.40	0.17	-0.06	-0.29	-0.53
1.90	1.67	1.43	1.20	0.97	0.74	0.50	0.27	0.04	-0.19	-0.43
2.00	1.77	1.53	1.30	1.07	0.84	0.60	0.37	0.14	-0.09	-0.33

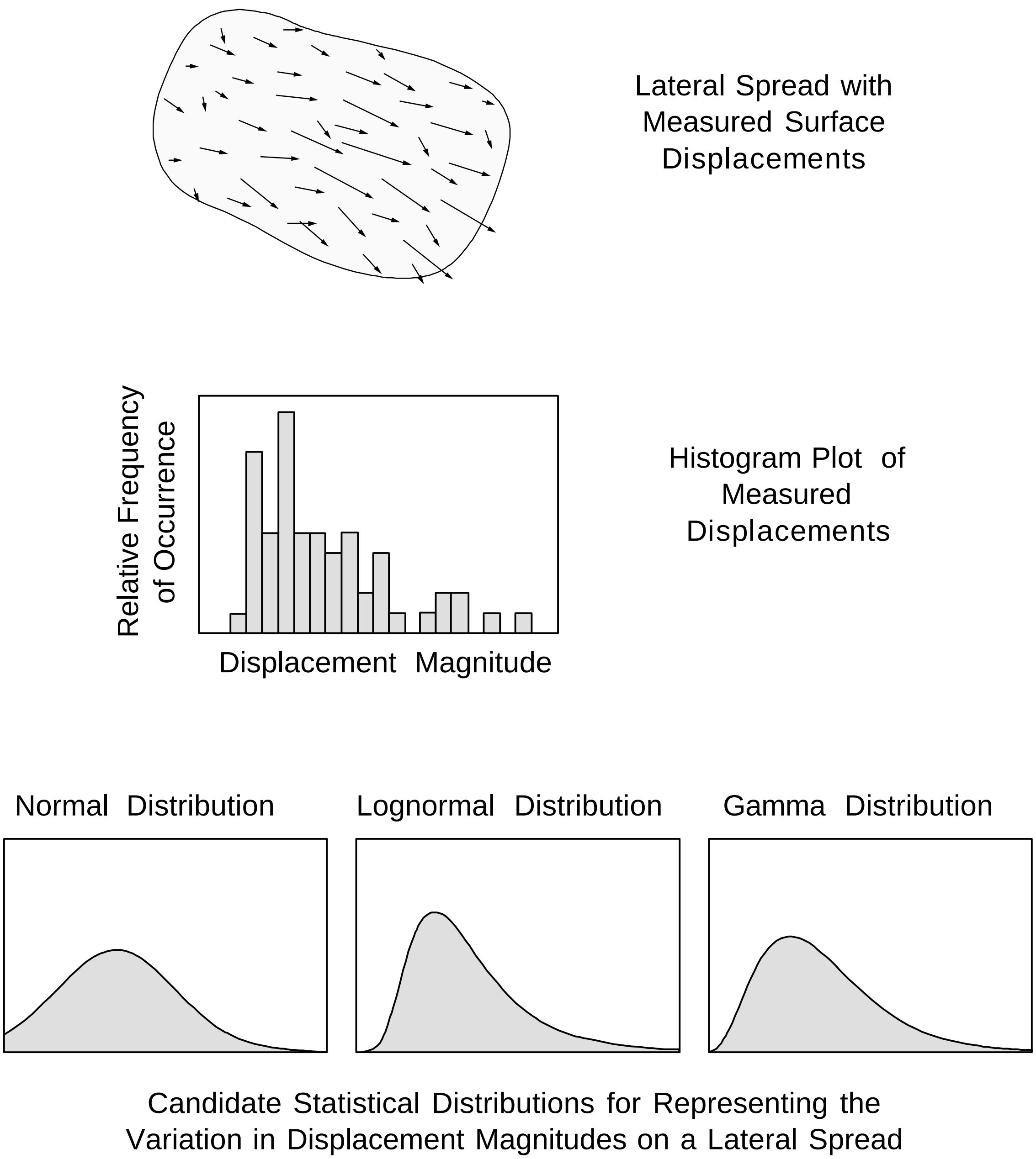

Figure 8.1. Modeling the variation in displacement magnitudes on a lateral spread using statistical distributions.

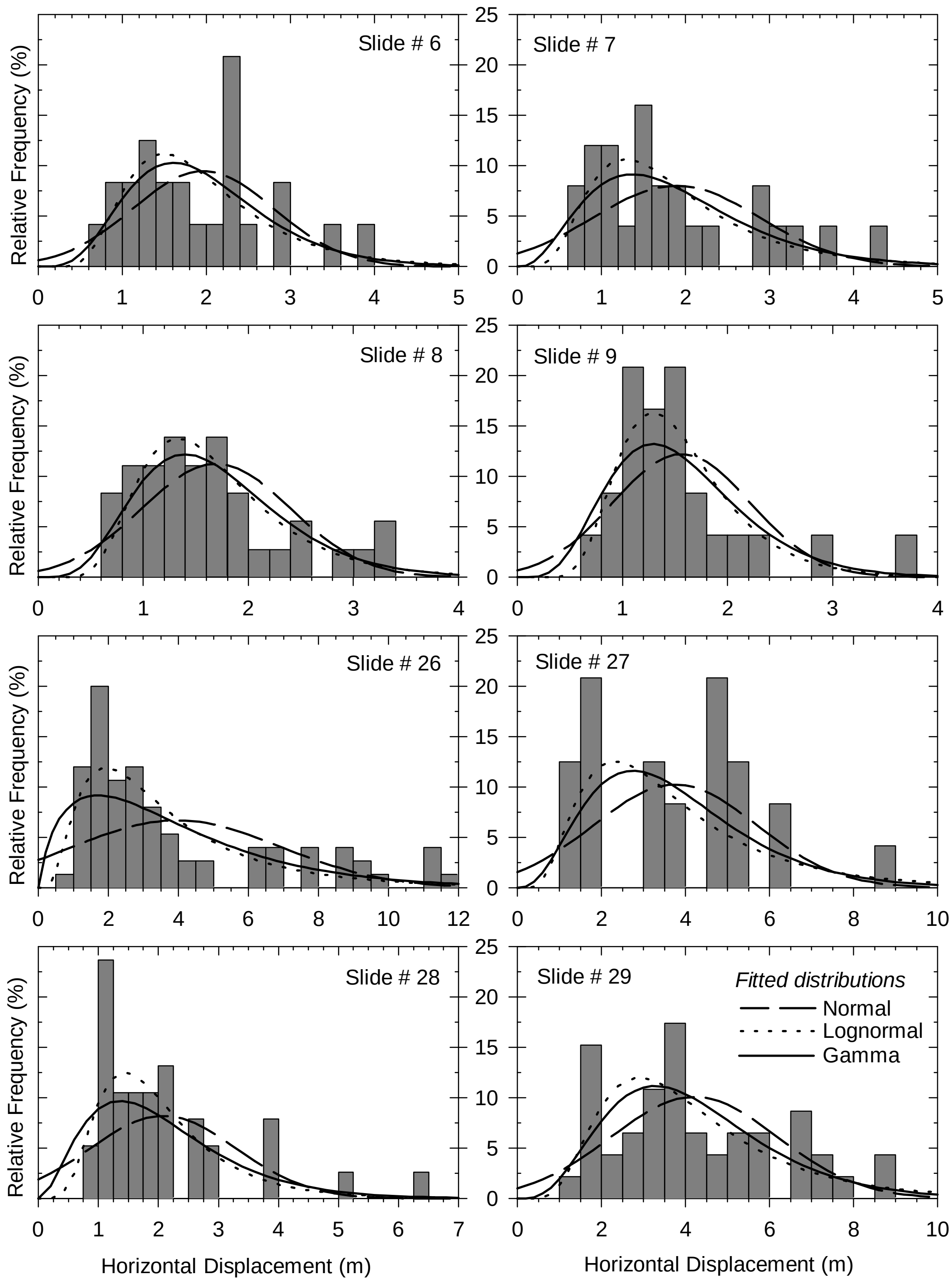

Figure 8.2a. Histograms of measured horizontal displacements with fitted statistical distributions.

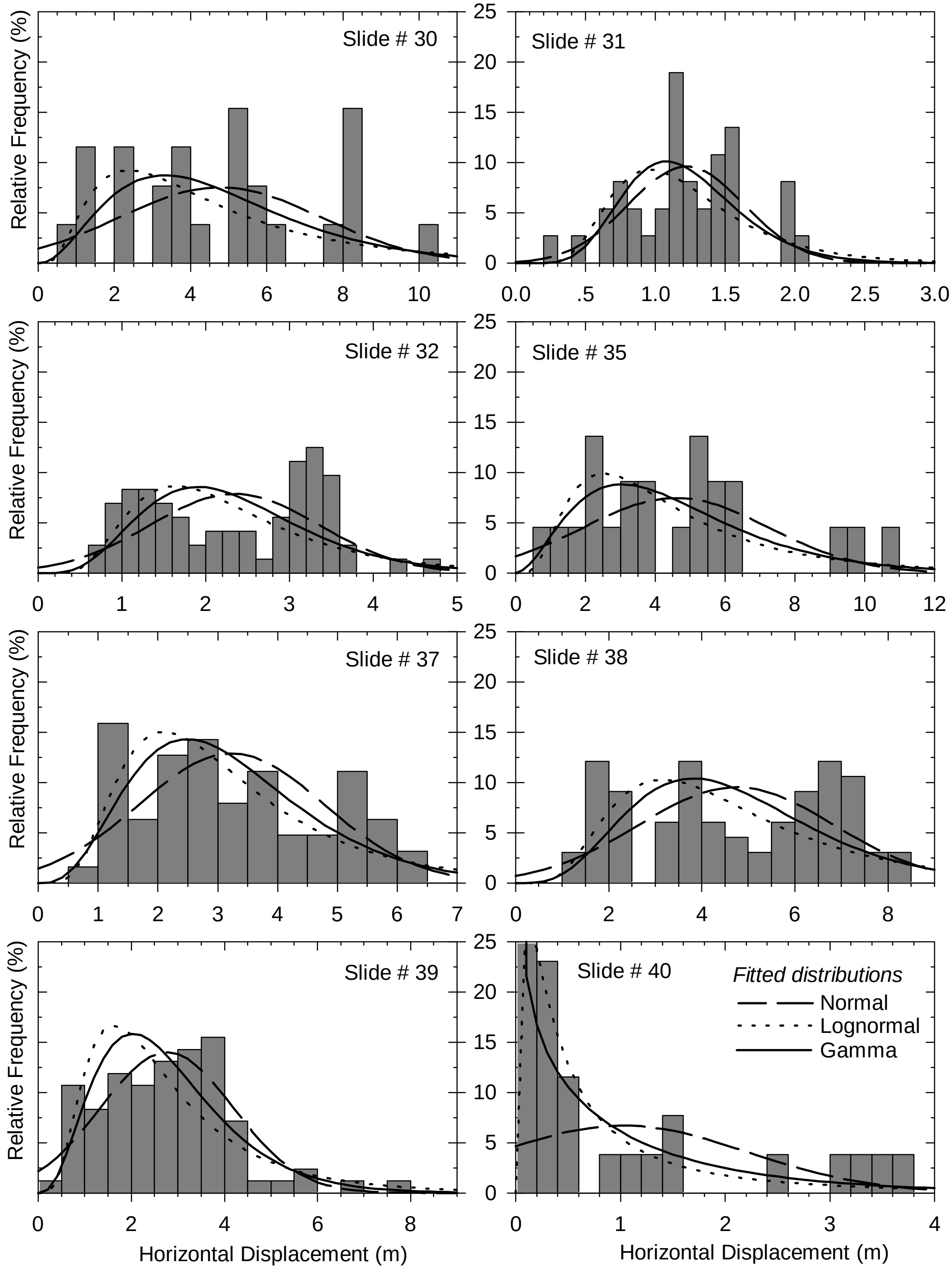

Figure 8.2b. Histograms of measured horizontal displacements with fitted statistical distributions.

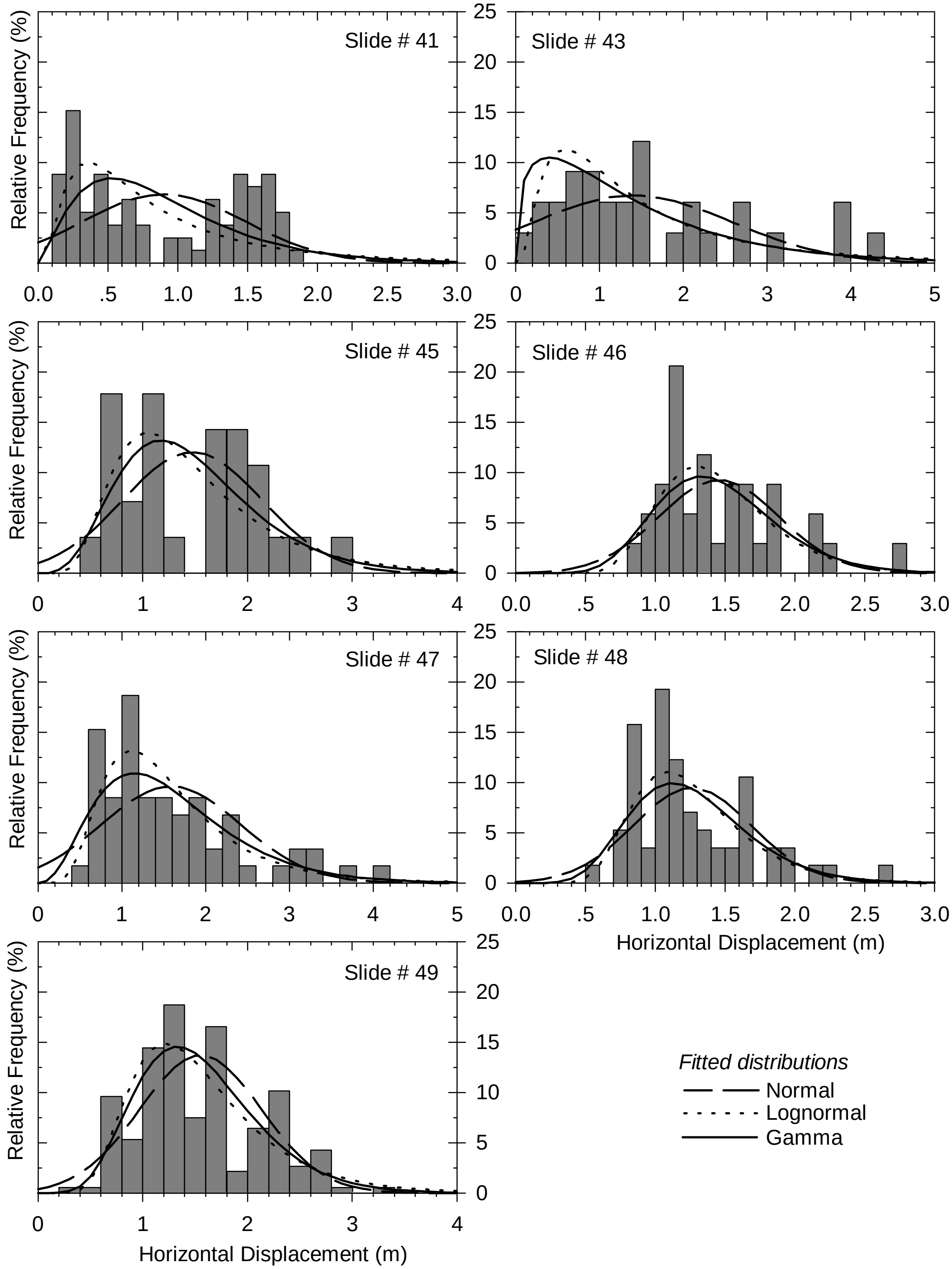

Figure 8.2c. Histograms of measured horizontal displacements with fitted statistical distributions.

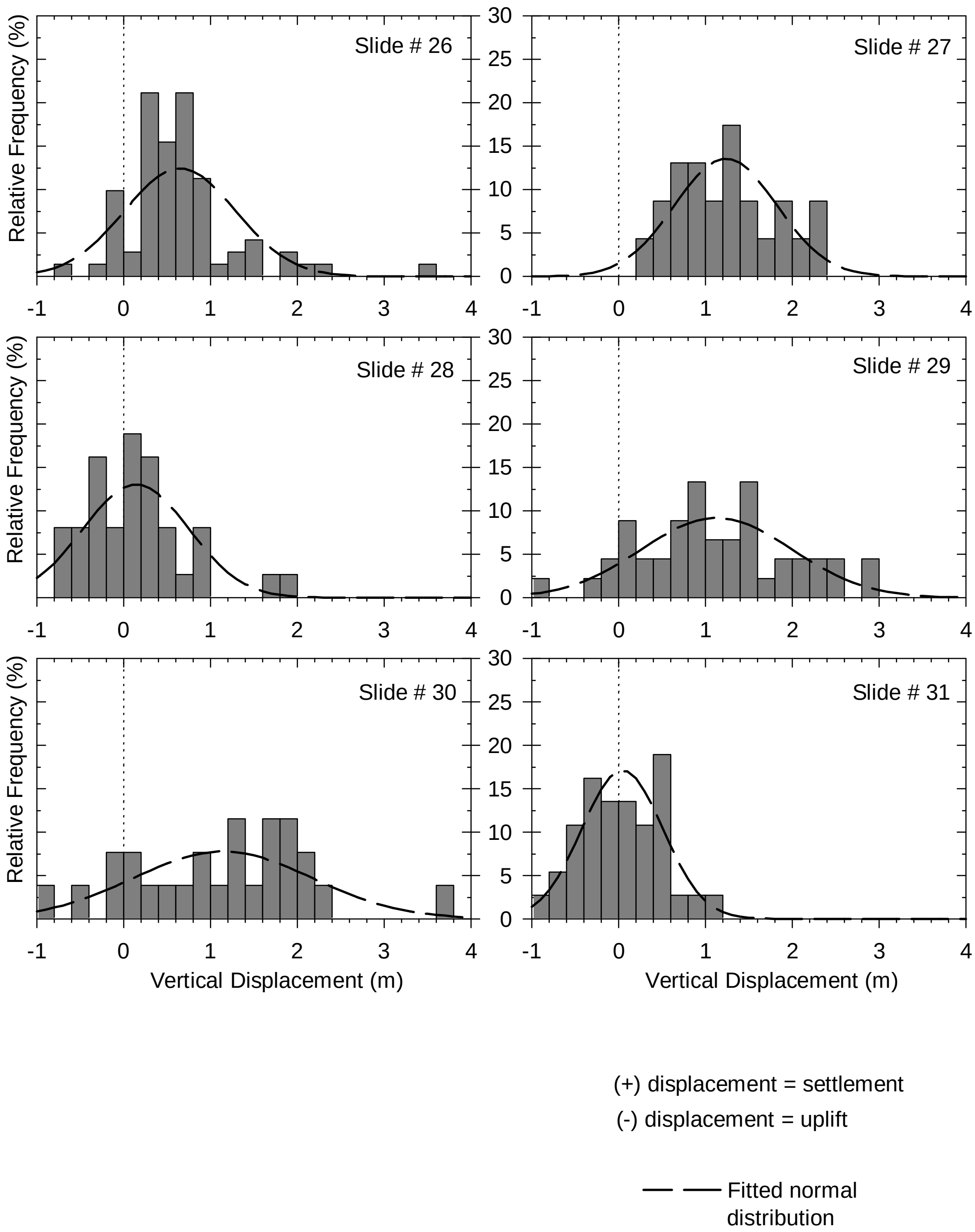

Figure 8.3a. Histograms of measured vertical displacements with fitted statistical distribution.

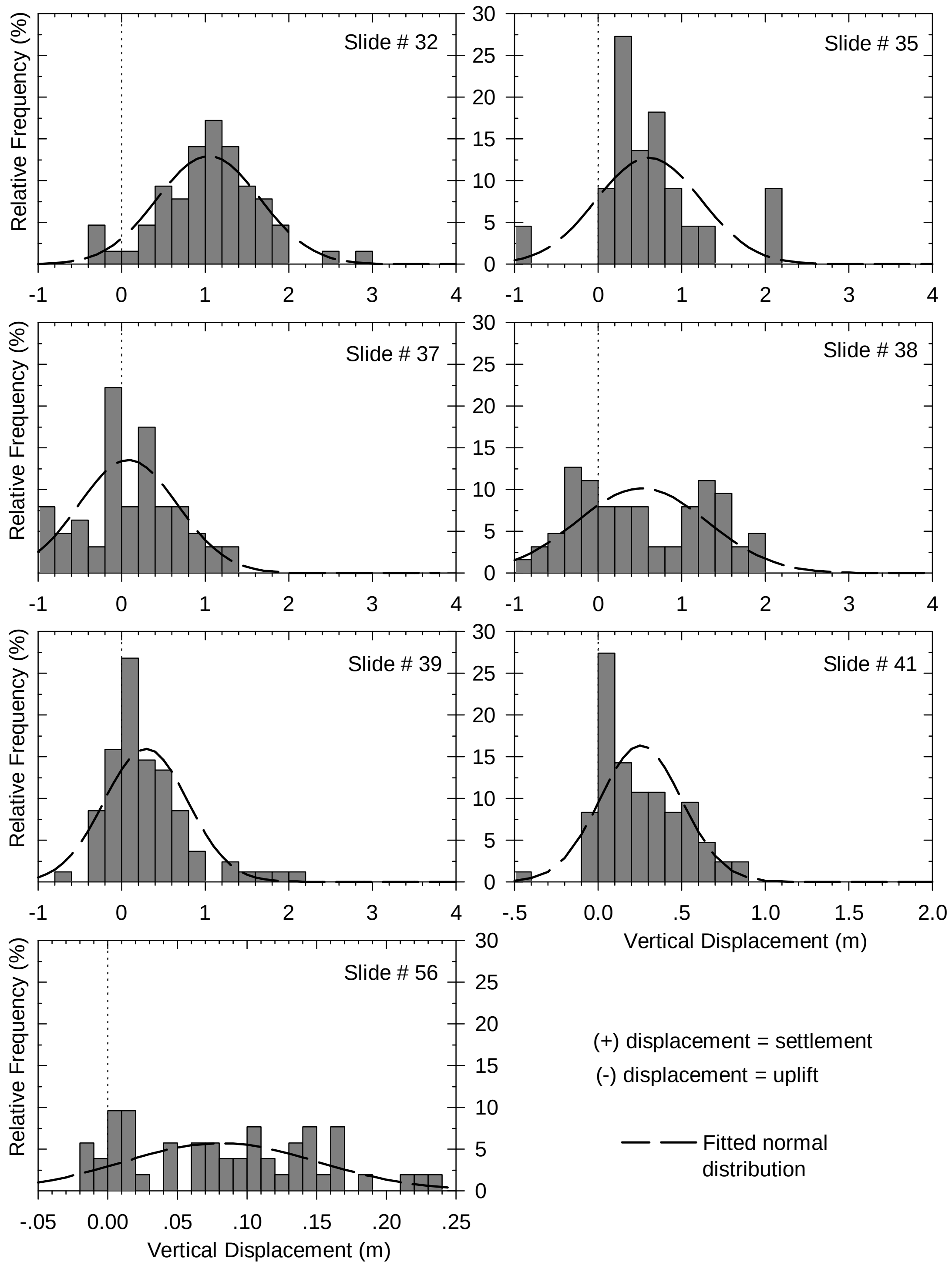

Figure 8.3b. Histograms of measured vertical displacements with fitted statistical distribution.

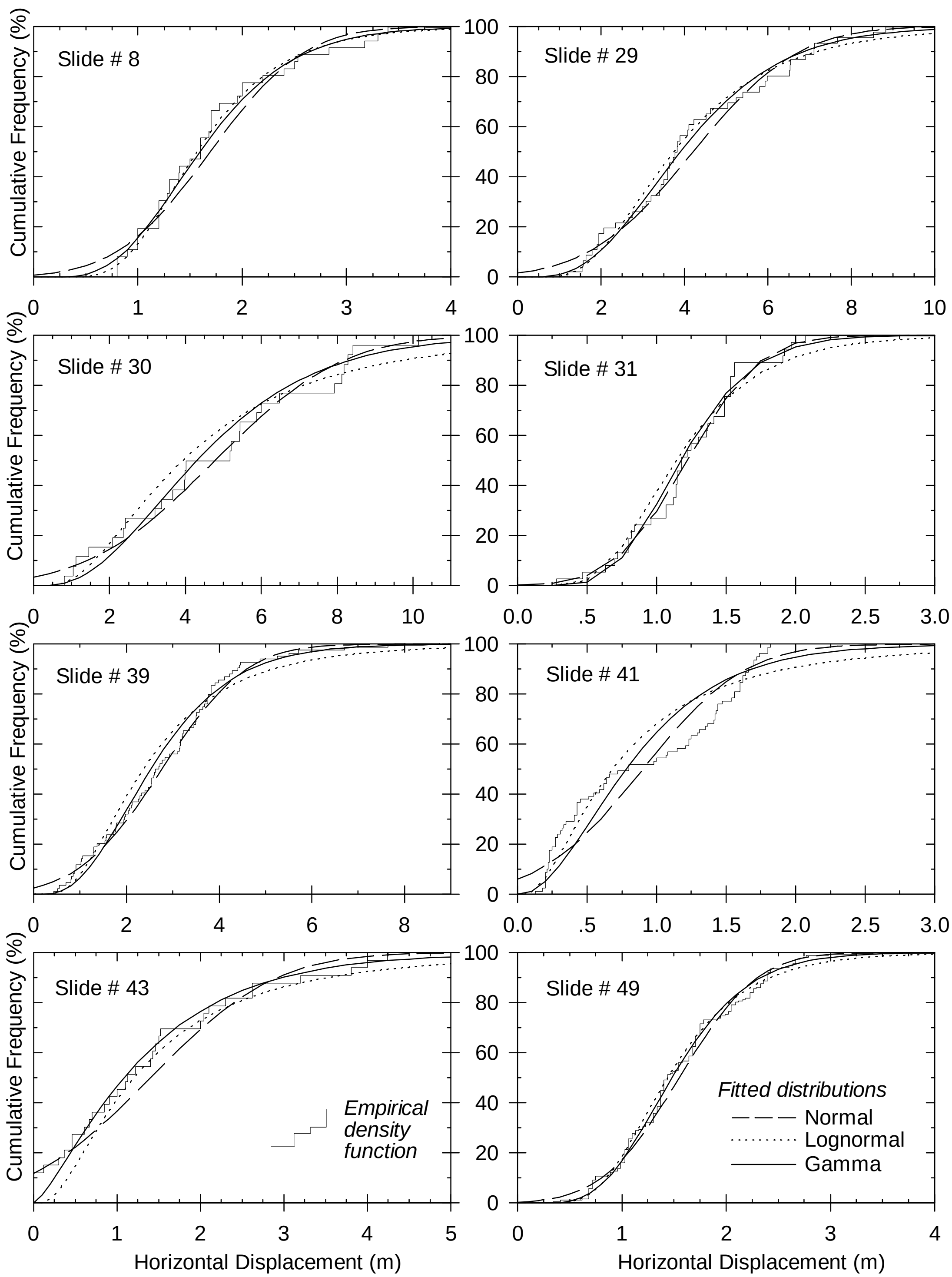

Figure 8.4. Empirical density functions of measured horizontal displacements with fitted statistical distributions.

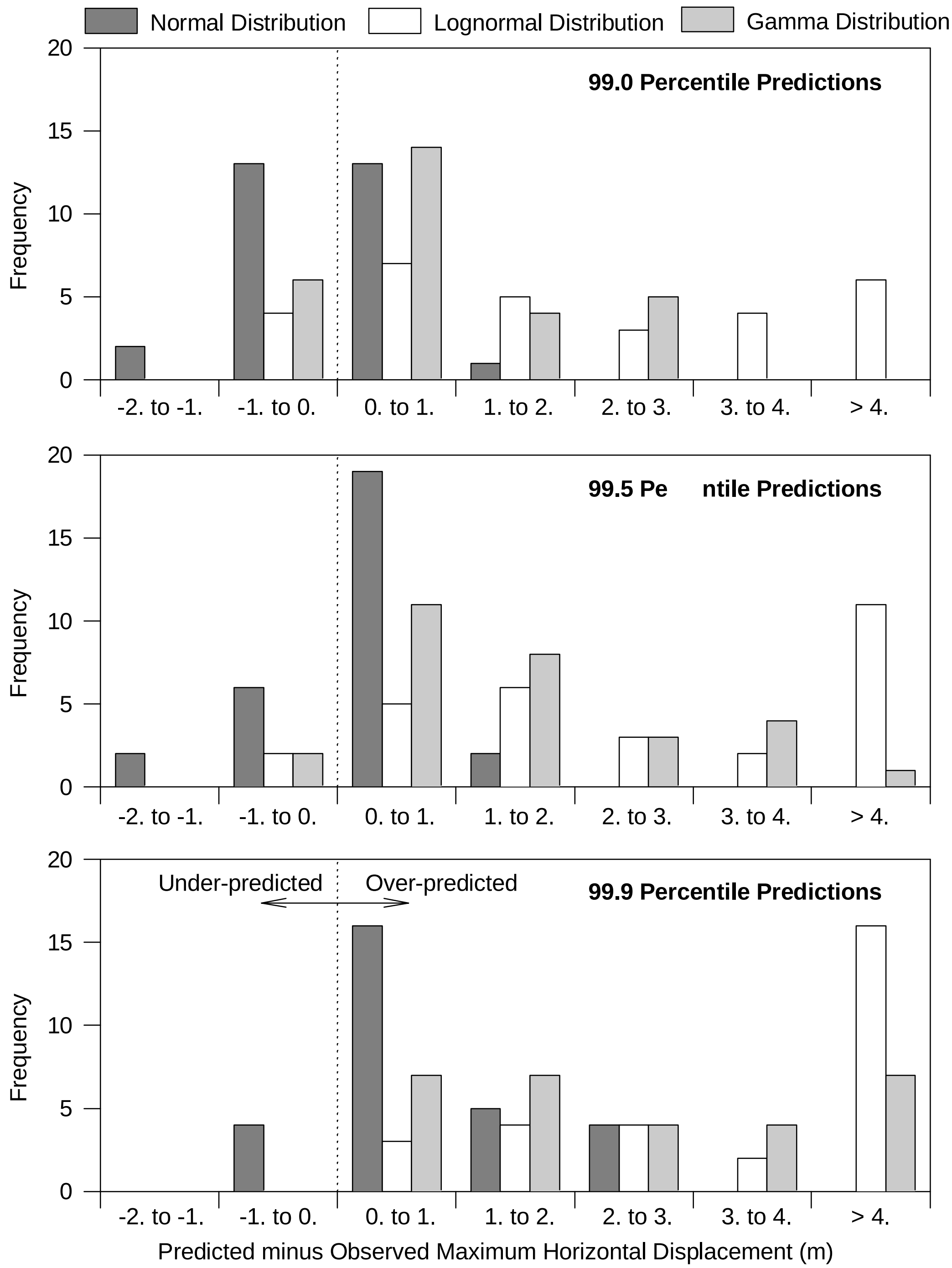

Figure 8.5. Histograms of errors in the maximum horizontal displacement predicted at various percentiles of three statistical distributions.

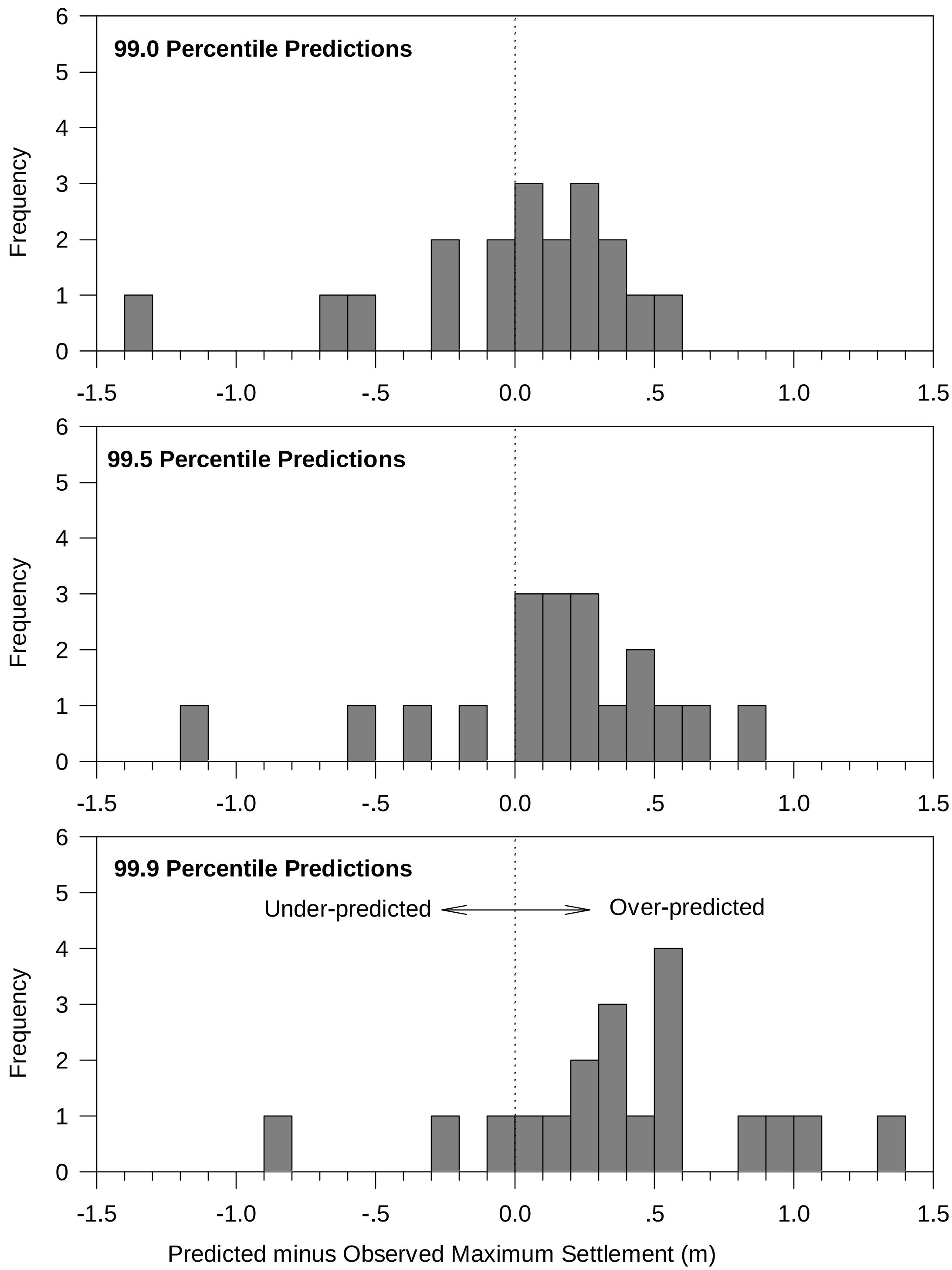

Figure 8.6. Histograms of errors in the maximum settlement predicted at various percentiles of the normal distribution.

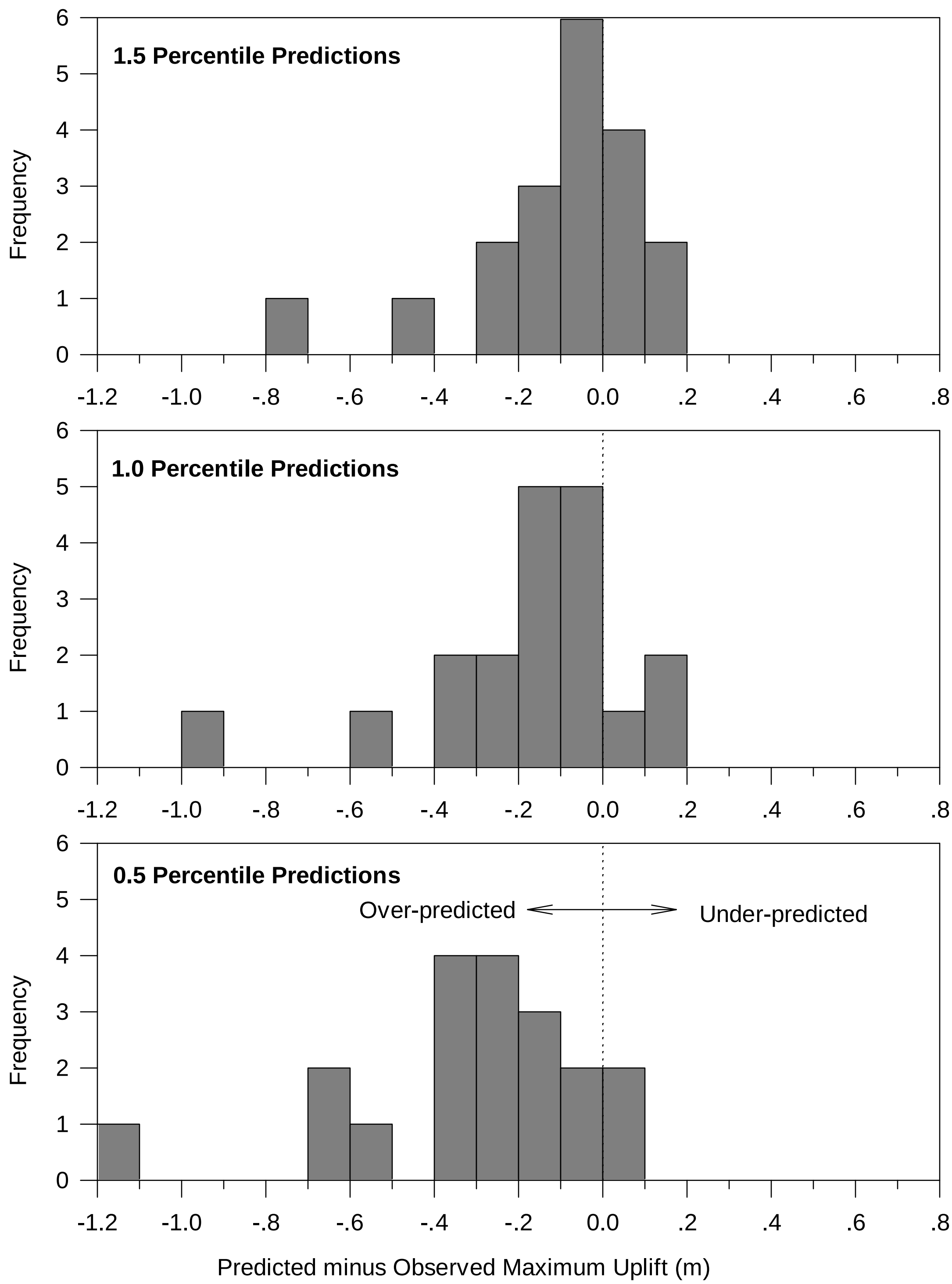

Figure 8.7. Histograms of errors in the maximum uplift predicted at various percentiles of the normal distribution.