Chapter 11

EPOLLS Model for Vertical Displacement

11.1. Development of Vertical-EPOLLS Model for Average Vertical Displacement.

Developing an empirical method for predicting *horizontal* displacements on a lateral spread is the primary objective of this study. Significant vertical displacements of lesser magnitude, including both settlement and uplift, also occur on a liquefaction-induced lateral spread. To round out the EPOLLS model, the *Vertical-EPOLLS* (or *V-EPOLLS*) component was developed for predicting vertical displacements of the ground surface. Like the EPOLLS model for horizontal displacements, developed in Chapters 9 and 10, the Vertical-EPOLLS model is a linear regression model fit to data in the EPOLLS database. However, the V-EPOLLS model is considerably less accurate and reliable than the model for horizontal movements and is only appropriate for roughly estimating the possible magnitude of vertical displacements.

The Vertical-EPOLLS model suffers from deficiencies in the data used to fit the model. Vertical displacements on the case study lateral spreads were determined from the movement of reference points such as manhole covers, fence posts, etc. These measured vertical displacements may be greater than that of the surrounding ground surface due to sinking of the reference structure or less if the structure is supported on a deeper stratum. In general, the measured vertical displacements give a less reliable indication of free-field surface deformations than the measured horizontal displacements. Moreover, for the 78 lateral spreads in the EPOLLS database, the average vertical displacement is known for only 36 case studies. These 36 case studies represent a relatively narrow range of site conditions, and most of this data is from lateral spreads in Niigata, Japan. In general, the available data set on vertical displacements is smaller and less reliable than the data on horizontal deformations.

Selection of model regressors.

Preliminary analyses showed that a good model for the average vertical displacement (*Avg_Vert*) could not be defined without parameters describing the subsurface soil conditions. That is, regression models for *Avg_Vert* using only the seismological, topographical, and geometrical EPOLLS parameters yield very low values of R^2. Clearly, insufficient data is available for developing a multi-component model for the average vertical displacement. Hence, the Vertical-EPOLLS model has only one equation for *Avg_Vert*, which uses parameters determined from soil boring logs.

Intuitively, one would expect that a large-deformation lateral spread would produce large horizontal and vertical displacements. Indeed, a simple scatter plot of the average vertical and horizontal displacements suggests a likely correlation. That is, larger vertical displacements occur on lateral spreads experiencing larger horizontal deformations. The predicted average horizontal displacement was therefore considered as a candidate regressor variable in defining a model for *Avg_Vert*.

Regressor variables for the Vertical-EPOLLS model must be selected from the available parameters in the database. This problem is greatly hampered by the relatively small number of EPOLLS case studies where *Avg_Vert* is known. Of these, most of the candidate parameters are defined for only about two dozen case studies. Inclusion of any one parameter may result in the loss of a few case studies where that parameter value is unknown. Working with a small data set, the loss of one or two observations can significantly impact the fitted model. For example, including the variable *Avg-N160_Cap* gives a model with a very high R^2, but apparently only because four case studies with an unknown value of *Avg-N160_Cap* are dropped from the model fit.

Because of these limitations in the available data, the apparent significance of each regressor variable is difficult to judge and the automated model selection procedures, described in Section D.5, are of little use in defining the best Vertical-EPOLLS model. Numerous candidate models, using various combinations of the available parameters, were examined in a manual search for models giving a high value of R^2 for the maximum possible number of case studies. Engineering judgment was used throughout this analysis to identify potential combinations of variables for the V-EPOLLS model or to eliminate some regressors from further consideration. Many of the potential model parameters were eliminated from consideration during this process because their values are unknown in too many case studies.

In the end, the chosen Vertical-EPOLLS model included three regressor variables:

$$Avg\_Vert = f\,(Avg\_Horz_R,\ Avg\text{-}Thick\_Liq,\ Avg\text{-}Z\_MnFS) \qquad (11.1)$$

The average horizontal displacement predicted with the Regional-EPOLLS model component (Avg_Horz_R) incorporates the effects of the seismological parameters into a single variable in the V-EPOLLS model. The other two parameters represent the average liquefied thickness and average depth to the weakest soil sublayer across the area of the lateral spread. All three variables in Equation 11.1 are significant to greater than the 70% level in partial *F*-tests, but the intercept term in this model was significant to only the 27% level and was therefore dropped.

The model represented by Equation 11.1 gives R^2=0.245 for 25 case studies. Obviously, this is very low and indicates a very poor model for *Avg_Vert*. Slightly better models were found, but the small improvement in R^2 did not justify the additional effort that would be required in future applications to define the necessary model parameters. As noted above, inclusion of some regressors gave much higher R^2 values, but only because three to four additional case studies with missing parameter values were dropped from fitting the model. Although the fit is obviously poor, Equation 11.1 appears to give the best possible model for predicting the average vertical displacement for the EPOLLS data set.

Evaluation for influential observations and multicollinearity.

Statistical tests (see Section D.7) consistently indicated that only one case study, Slide No. 40, was an influential observation in fitting the model in Equation 11.1. However, this case study is relatively well documented and was, therefore, retained in fitting the V-EPOLLS model. Statistical tests for possible multicollinearity, described in Section D.6, were also undertaken but did not indicate any problems.

Vertical-EPOLLS component for Avg_Vert.

The Vertical-EPOLLS model was fit to the data from 25 case studies as indicated in Table 11.1. The fitted regression model is:

$$Avg\_Vert = 0.0656 Avg\_Horz_R + 0.0284 Avg\text{-}Thick\_Liq + 0.0329 Avg\text{-}Z\_MnFS \qquad (11.2)$$

Each of the model parameters are defined fully in Table 11.3. In a global *F*-test, this regression is significant to a 90.3% level and each regressor variable is significant to greater than the 70% level in partial *F*-tests. For the 25 case studies used to fit Equation 11.2, R^2=0.245 and $\bar{R}^2$=0.176. The model residuals, plotted in Figure 11.1a, appear uniformly dispersed about zero, indicating a satisfactory performance. Partial regression plots (see Section D.8) are shown in Figure 11.2 and do not indicate any need for transformations of the regressor variables.

With R^2=0.245, the V-EPOLLS model for *Avg_Vert* explains only 25% of the observed variation in the average displacements. Obviously, this is a poor model capable of giving only an approximate estimate of the average vertical displacements on a lateral spread. Performance of the Vertical-EPOLLS model is discussed further in Section 11.3.

11.2. Development of Vertical-EPOLLS Model for Variation of Vertical Displacement.

Selection of model regressors.

Despite the poor performance of the Vertical-EPOLLS model for *Avg_Vert*, a complementary model for estimating the standard deviation of the vertical displacements (*StD_Vert*) was developed. In searching for the best regressors to include in this model component, consideration was limited to a few variables that would be readily available when using the EPOLLS model. Since the V-EPOLLS component for *Avg_Vert* is not a very good model, there is little to be gained from a more accurate model for *StD_Vert*. The additional effort required to define other parameters would not be justified in predicting *StD_Vert*.

A simple, reasonably good model for *StD_Vert* was found to include the following variables:

$$StD\_Vert = f\,(Avg\_Horz_R,\; Rng\text{-}Z\_MnFS) \qquad (11.3)$$

As also used in Equation 11.2, Avg_Horz_R is the average horizontal displacement predicted with the Regional-EPOLLS model. The variable *Rng-Z_MnFS* represents the range in depths to the weakest liquefiable soil across the site. That is, considering soil conditions at all points across the lateral spread, *Rng-Z_MnFS* is the maximum minus the minimum depth to the lowest factor of safety against liquefaction. Both variables in Equation 11.3 are significant to greater than a 99% level in partial *F*-tests, but the intercept term in this model was not significant and was therefore dropped.

Evaluation for influential observations and multicollinearity.

Statistical tests with the model in Equation 11.3 consistently indicated that Slide No. 40 was a highly influential observation. However, as was the case for the *Avg_Vert* model, this case study was retained in fitting the model for *StD_Vert* because it is relatively well-documented. In addition, no indications of multicollinearity were found with this model.

Vertical-EPOLLS component for StD_Vert.

The fitted Vertical-EPOLLS model for *StD_Vert* is:

$$StD\_Vert = 0.158\,Avg\_Horz_R + 0.0388\,Rng\text{-}Z\_MnFS \qquad (11.4)$$

Each of the model parameters are defined fully in Table 11.3. In a global *F*-test, this regression is significant to a 99.5% level and each regressor variable is significant to greater than the 99.9% level in partial *F*-tests. For the 17 case studies used to fit Equation 11.4, R^2=0.512 and $\bar{R}^2$=0.480. Plotted in Figure 11.1b, the model residuals are uniformly dispersed about zero, indicating a satisfactory performance. The partial regression plots shown in Figure 11.3 do not indicate the need for transformations of the regressor variables.

The Vertical-EPOLLS component for *StD_Vert* was fit to the data from 17 case studies as indicated in Table 11.2. Note that eleven of these case studies, or two-thirds of the data used, are lateral spreads in Niigata, Japan. Hence, the relatively high value of R^2=0.512 probably results directly from the narrow range of conditions represented by the data used to fit this model. In reality, the V-EPOLLS model is probably much less reliable in predicting *StD_Vert* than indicated by R^2 and $\overline{R}^2$, and more comparable to the accuracy of the model for predicting *Avg_Vert*.

11.3. Evaluation of EPOLLS Model for Vertical Displacement.

The Vertical-EPOLLS model is comprised of Equation 11.2 and 11.4. Introducing a different notation allows these equations to be written as:

$$Avg\_Vert = (65.6 Avg\_Horz_R + 28.4 H_{liq} + 32.9 Z_{FSmin}) \; / \; 1000. \qquad (11.5)$$

$$StD\_Vert = (158 Avg\_Horz_R + 38.8 \Delta Z_{FSmin}) \; / \; 1000. \qquad (11.6)$$

where all of these variables are defined in Table 11.3 and Avg_Horz_R is computed with Equation 9.20.

For the 25 case studies with no missing parameters, Equation 11.5 yields R^2=0.245 and $\overline{R}^2$=0.176. The predicted *Avg_Vert* for these case studies are plotted against the average of the measured vertical displacements in Figure 11.4a. Recall that a "perfect" prediction would produce points on a 45° line on this plot and that R^2 is a numerical measure of the scatter about this line. Even though R^2 is very low for this model, the predicted average vertical displacement is within ± 0.50 m in 84% (21 out of 25) case studies. On the other hand, using the same random split of the database indicated in Table 9.4, a double cross-validation analysis produced R^2_p values of 0.170 and -0.014. These values clearly indicate the poor quality and unreliable nature of the V-EPOLLS model for *Avg_Vert*.

Equation 11.6 yields R^2=0.512 and $\overline{R}^2$=0.480 for the 17 available case study lateral spreads with no missing parameter values. The corresponding scatter plot of the predicted versus observed *StD_Vert* is shown in Figure 11.4b. However, the apparent good performance of this model component is probably misleading because 65% of the this data (11 out of 17 case studies) was acquired from sites in Niigata, Japan. Hence, the V-EPOLLS model for *StD_Vert* is thought to mostly reflect the conditions at the Niigata sites and not the general behavior of lateral spreads. This conclusion is further supported by a double cross-validation analysis that produced R^2_p values of 0.357 and -0.518, which clearly indicates the poor quality of the V-EPOLLS model for *StD_Vert*.

Maximum likely settlement and uplift can be estimated conservatively at the 99.5 and 1.0 percentile, respectively, of the normal distribution. Calculation of a maximum displacement with the normal distribution is done in a very similar manner as demonstrated in Section 10.4 for horizontal displacements with the gamma distribution. Recall that uplift or heaving is measured as a negative vertical displacement. Because the Vertical-EPOLLS model components are not very good at predicting the average and standard deviation of the vertical displacements, very poor predictions of the maximum displacement are expected. Scatter plots of the predicted versus observed maximum vertical displacements are shown in Figure 11.5a (settlement) and 11.5b (uplift). In 75% of the available case studies, the error in the predicted maximum settlement is less than 1.0 m. Similarly, the error in predicted maximum uplift is less than 0.50 m in 71% of the available case studies.

To reiterate, the Vertical-EPOLLS model gives generally poor predictions of the average and standard deviation of vertical displacements on a lateral spread. The poor model performance results from the limited amount of available data that is taken mostly from lateral spreads in Niigata, Japan. Better empirical methods for predicting liquefaction-induced settlements are described briefly in Section 4.5. However, when used in conjunction with the Geotechnical-EPOLLS model for predicting horizontal deformations, the Vertical-EPOLLS model is appropriate for making rough approximations of the magnitude of vertical displacements. These quick and simple estimates can be made with little additional effort, although the reliability of the Vertical-EPOLLS model predictions is poor.

11.4. Criteria for Model Predictions.

For completeness, criteria for making predictions with the Vertical-EPOLLS model are given here. Even though the model should be used only to get approximate estimates of vertical displacements, predictions should still not require extrapolation beyond the range of the parameters used to fit the model. The valid limits of the four parameters in Equations 11.5 and 11.6 are given in Table 11.4; vertical displacement predictions should not be made outside these bounds. In addition, the range of values used to fit the model are indicated in the histograms of the variables shown in Figure 11.6.

The diagnostic test for *hidden extrapolation*, described in Section D.9, requires values of h_{max} given in Table 11.5 for the V-EPOLLS model components. Parameters required to compute the prediction interval on *Avg_Vert*, using Equation D.27, are given in Table 11.6. Both of these calculations require the matrix of regressor values for the prediction, [x], and the matrix $([X]^T[X])^{-1}$ used to fit the model. For the two components of the V-EPOLLS model, these are:

- Vertical-EPOLLS for *Avg_Vert*:

$$[x] = [Avg\_Horz_R \quad H_{liq} \quad Z_{FSmin}]; \quad ([X])^T[X])^{-1} = \begin{bmatrix} .0311 & -.00153 & -.00635 \\ & .00106 & -.000829 \\ symmetric & & .00367 \end{bmatrix} \tag{11.7}$$

- Vertical-EPOLLS for *StD_Vert*:

$$[x] = [Avg\_Horz_R \quad \Delta Z_{FSmin}]; \quad ([X])^T[X])^{-1} = \begin{bmatrix} .0220 & -.00533 \\ -.00533 & .00254 \end{bmatrix} \tag{11.8}$$

Table 11.1. Data used to fit and evaluate the EPOLLS model for average vertical displacement.

Slide_ID	Earthquake	Avg_Vert	EQ_Mw	Fault_Dist	Accel_max	Duration	(Avg_Horz)-R	Avg-Thick_Liq	Avg-Z_MnFS
1	1906: San Francisco, California	0.60	7.7	13.0	0.44	45.	0.71	2.7	3.3
2	1906: San Francisco, California	0.90	7.7	12.0	0.46	45.	0.66	4.4	4.0
13	1964: Prince William Sound, Alaska	1.00	9.2	35.0	0.47	87.	0.81	27.6	8.6
18	1964: Prince William Sound, Alaska	0.27	9.2	60.0	0.31	75.	1.13	7.5	4.5
26	1964: Niigata, Japan	0.65	7.6	16.0	0.16	19.	2.78	9.2	7.0
27	1964: Niigata, Japan	1.23	7.6	15.0	0.16	19.	2.83	11.7	4.5
28	1964: Niigata, Japan	0.14	7.6	15.0	0.16	19.	2.83	2.2	2.8
29	1964: Niigata, Japan	1.13	7.6	13.0	0.17	19.	2.84	11.3	6.9
30	1964: Niigata, Japan	1.14	7.6	14.0	0.17	19.	2.79	8.7	8.7
31	1964: Niigata, Japan	0.04	7.6	14.0	0.17	19.	2.79	8.7	6.4
32	1964: Niigata, Japan	1.04	7.6	14.0	0.17	19.	2.79	9.1	5.5
35	1964: Niigata, Japan	0.61	7.6	11.0	0.17	19.	2.93	10.7	9.4
37	1964: Niigata, Japan	0.08	7.6	12.0	0.17	19.	2.88	6.2	6.7
38	1964: Niigata, Japan	0.52	7.6	12.0	0.17	19.	2.88	4.6	5.4
39	1964: Niigata, Japan	0.29	7.6	11.0	0.18	19.	2.85	3.8	2.9
40	1971: San Fernando, California	0.80	6.7	0.5	0.50	15.	0.41	2.1	12.4
41	1971: San Fernando, California	0.25	6.7	1.0	0.50	14.	0.41	1.8	6.3
44	1979: Imperial Valley, California	0.00	6.5	4.4	0.40	12.	0.52	4.1	3.6
54	1989: Loma Prieta, California	0.30	7.0	65.0	0.17	8.	0.60	2.2	4.0
55	1989: Loma Prieta, California	0.20	7.0	12.0	0.25	14.	1.47	4.0	9.1
56	1989: Loma Prieta, California	0.08	7.0	1.0	0.39	11.	1.14	5.2	7.6
102	1990: Luzon, Philippines	0.50	7.6	65.0	0.20	16.	0.92	17.2	2.1
103	1990: Luzon, Philippines	0.50	7.6	65.0	0.20	16.	0.92	15.7	2.8
113	1993: Hokkaido Nansei-oki, Japan	0.62	7.7	15.0	0.25	60.	1.17	5.8	2.6
114	1993: Hokkaido Nansei-oki, Japan	0.25	7.7	15.0	0.25	60.	1.17	6.3	3.8

Table 11.2. Data used to fit and evaluate the EPOLLS model for standard deviation of vertical displacements.

Slide_ID	Earthquake	StD_Vert	EQ_Mw	Fault_Dist	Accel_max	Duration	(Avg_Horz)-R	Rng-Z_MnFS
1	1906: San Francisco, California	0.50	7.7	13.0	0.44	45.	0.71	0.9
2	1906: San Francisco, California	0.50	7.7	12.0	0.46	45.	0.66	4.4
26	1964: Niigata, Japan	0.64	7.6	16.0	0.16	19.	2.78	7.0
27	1964: Niigata, Japan	0.59	7.6	15.0	0.16	19.	2.83	4.0
28	1964: Niigata, Japan	0.61	7.6	15.0	0.16	19.	2.83	1.5
29	1964: Niigata, Japan	0.87	7.6	13.0	0.17	19.	2.84	14.0
30	1964: Niigata, Japan	1.03	7.6	14.0	0.17	19.	2.79	6.8
31	1964: Niigata, Japan	0.47	7.6	14.0	0.17	19.	2.79	1.5
32	1964: Niigata, Japan	0.61	7.6	14.0	0.17	19.	2.79	7.0
35	1964: Niigata, Japan	0.62	7.6	11.0	0.17	19.	2.93	4.8
37	1964: Niigata, Japan	0.59	7.6	12.0	0.17	19.	2.88	6.7
38	1964: Niigata, Japan	0.78	7.6	12.0	0.17	19.	2.88	5.4
39	1964: Niigata, Japan	0.50	7.6	11.0	0.18	19.	2.85	2.3
40	1971: San Fernando, California	0.79	6.7	0.5	0.50	15.	0.41	16.0
41	1971: San Fernando, California	0.24	6.7	1.0	0.50	14.	0.41	5.6
56	1989: Loma Prieta, California	0.07	7.0	1.0	0.39	11.	1.14	4.7
114	1993: Hokkaido Nansei-oki, Japan	0.15	7.7	15.0	0.25	60.	1.17	2.5

Table 11.3. Definition of variables used in the EPOLLS model for vertical displacements.

Variable	Units	Field Name in Database	Definition
Avg_Horz_R	m	--	average horizontal displacement predicted with Regional-EPOLLS model (Equations 9.17 and 9.20): $Avg\_Horz_R = (D_R - 2.21)^2 + 0.149$ $D_R = \{613M_w - 13.9R_f - 2420A_{max} - 11.4T_d \} / 1000.$
M_w	--	*EQ_Mw*	moment magnitude of earthquake
R_f	km	*Fault_Dist*	shortest horizontal distance from site to the surface projection of the fault rupture or zone of seismic energy release
A_{max}	g	*Accel_max*	peak horizontal acceleration at the ground surface of the site that would occur in the absence of excess pore pressures or liquefaction generated by the earthquake
T_d	sec	*Duration*	duration of strong earthquake motions at the site, defined as time between the first and last occurrence of a surface acceleration ≥ 0.05 g
H_{liq}	m	*Avg-Thick_Liq*	average thickness of liquefied soil: • use the gross liquefied thickness for a major soil unit that liquefies with possible thin, unliquefied seams • use the summation of the liquefied thicknesses when thin sublayers in the soil profile liquefy
Z_{FSmin}	m	*Avg-Z_MnFS*	average depth to the minimum factor of safety in potentially liquefiable soil
ΔZ_{FSmin}	m	*Rng-Z_MnFS*	range in depth (maximum minus minimum across site) to the minimum factor of safety in potentially liquefiable soil

Table 11.4. Limiting range of EPOLLS model parameters for predicting vertical displacements.

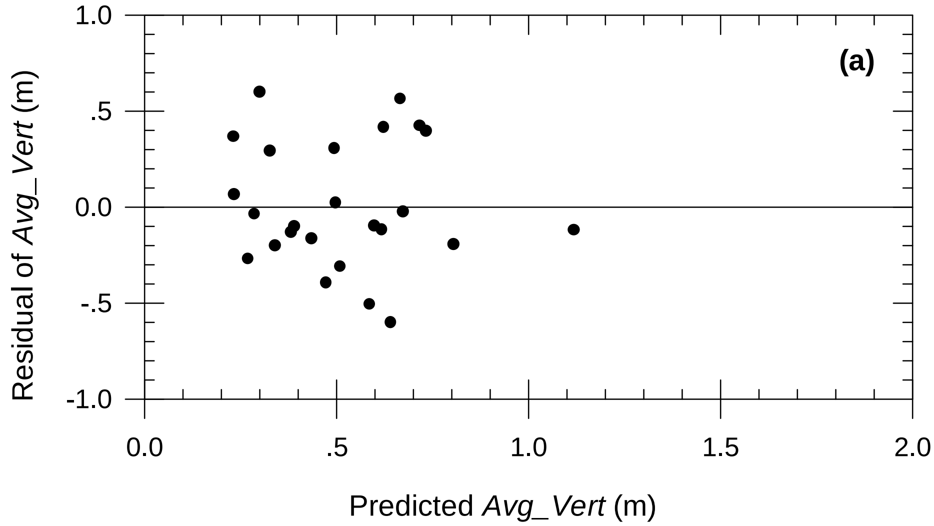
Variable	Units	Minimum Value	Maximum Value
Avg_Horz_R	m	0.4	2.9
H_{liq}	m	1.8	17.2
Z_{FSmin}	m	2.1	12.4
ΔZ_{FSmin}	m	0.9	16.0

Table 11.5. Values of h_{max} for testing for hidden extrapolation in the Vertical-EPOLLS model.

Model Component:	*Avg_Vert*	*StD_Vert*
h_{max} =	0.55	0.59

Table 11.6. Parameters for computing prediction intervals on the average vertical displacement.

Model Component:	Vertical-EPOLLS
MSE =	0.119
(n - p) =	22
(1-α)	$t_{\alpha/2,n-p}$ *for (1-α)% prediction interval*
50%	0.686
80%	1.321
90%	1.717
95%	2.074
99%	2.819



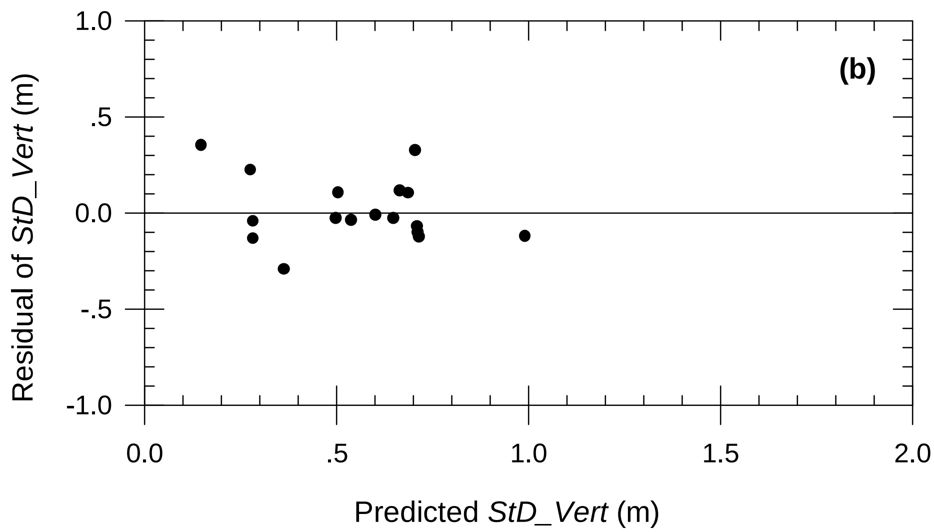


Figure 11.1. Residuals of the fitted Vertical-EPOLLS model for (a) average and (b) standard deviation of vertical displacements.

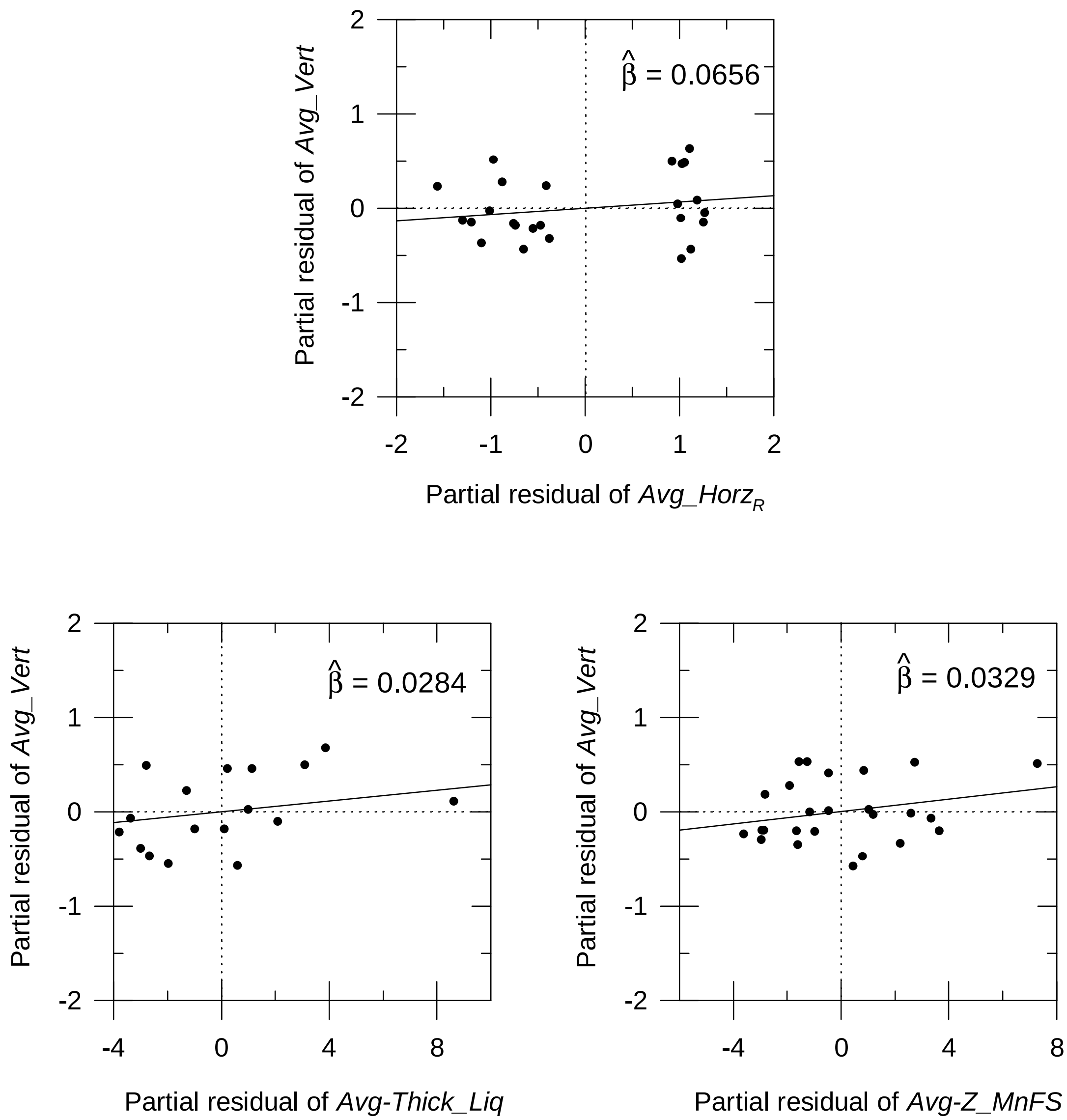


Figure 11.2. Partial regression plots for regressor variables in Vertical-EPOLLS model for average vertical displacement.

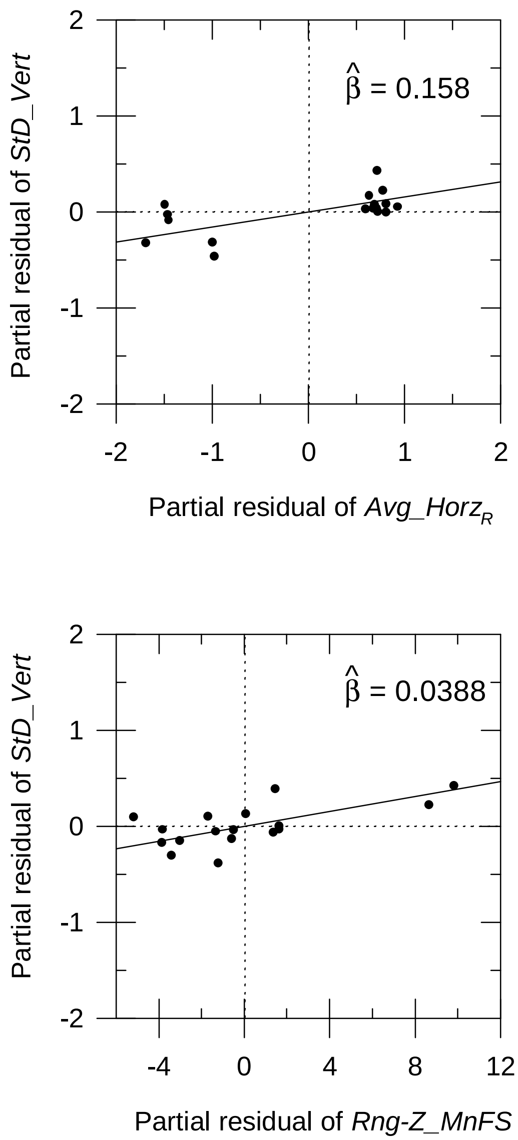


Figure 11.3. Partial regression plots for regressor variables in Vertical-EPOLLS model for standard deviation of vertical displacements.

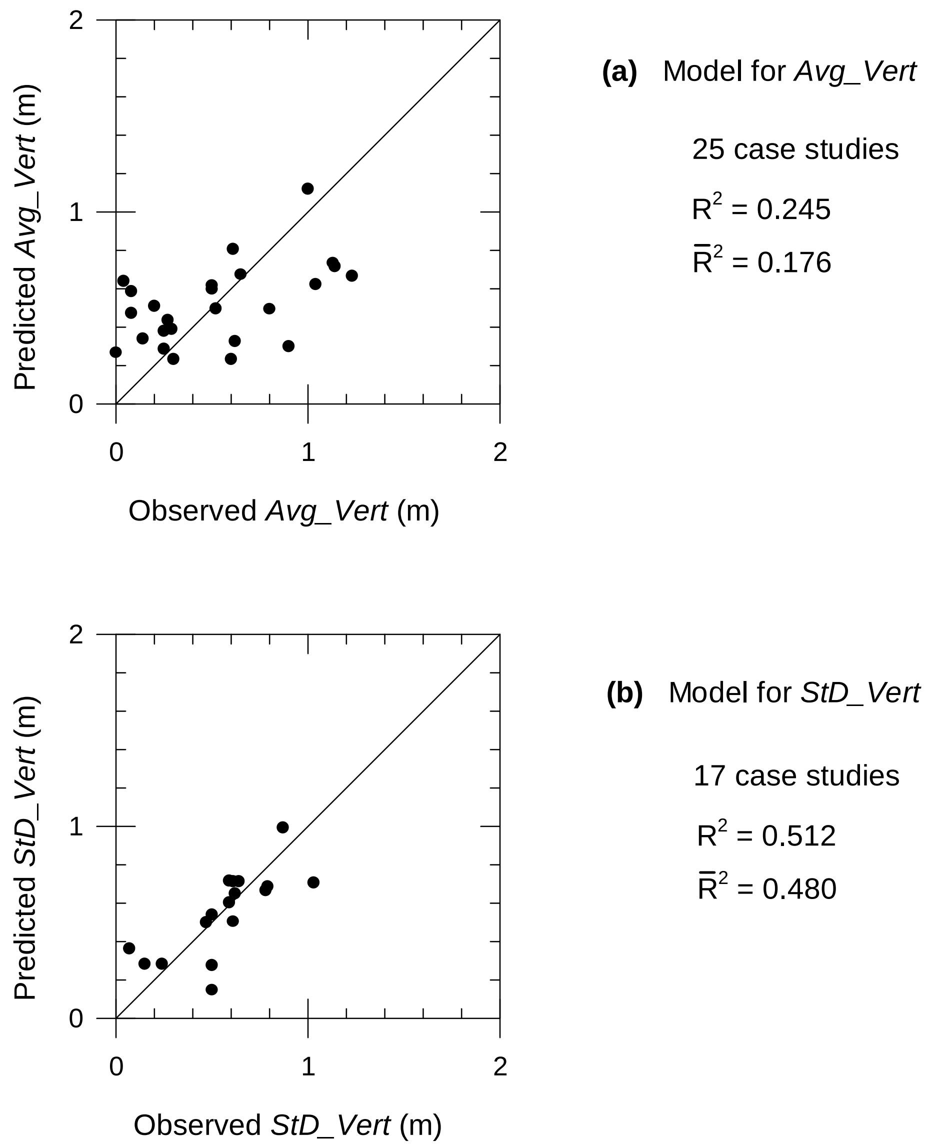


Figure 11.4. Performance of the Vertical-EPOLLS model in predicting (a) average and (b) standard deviation of vertical displacements.

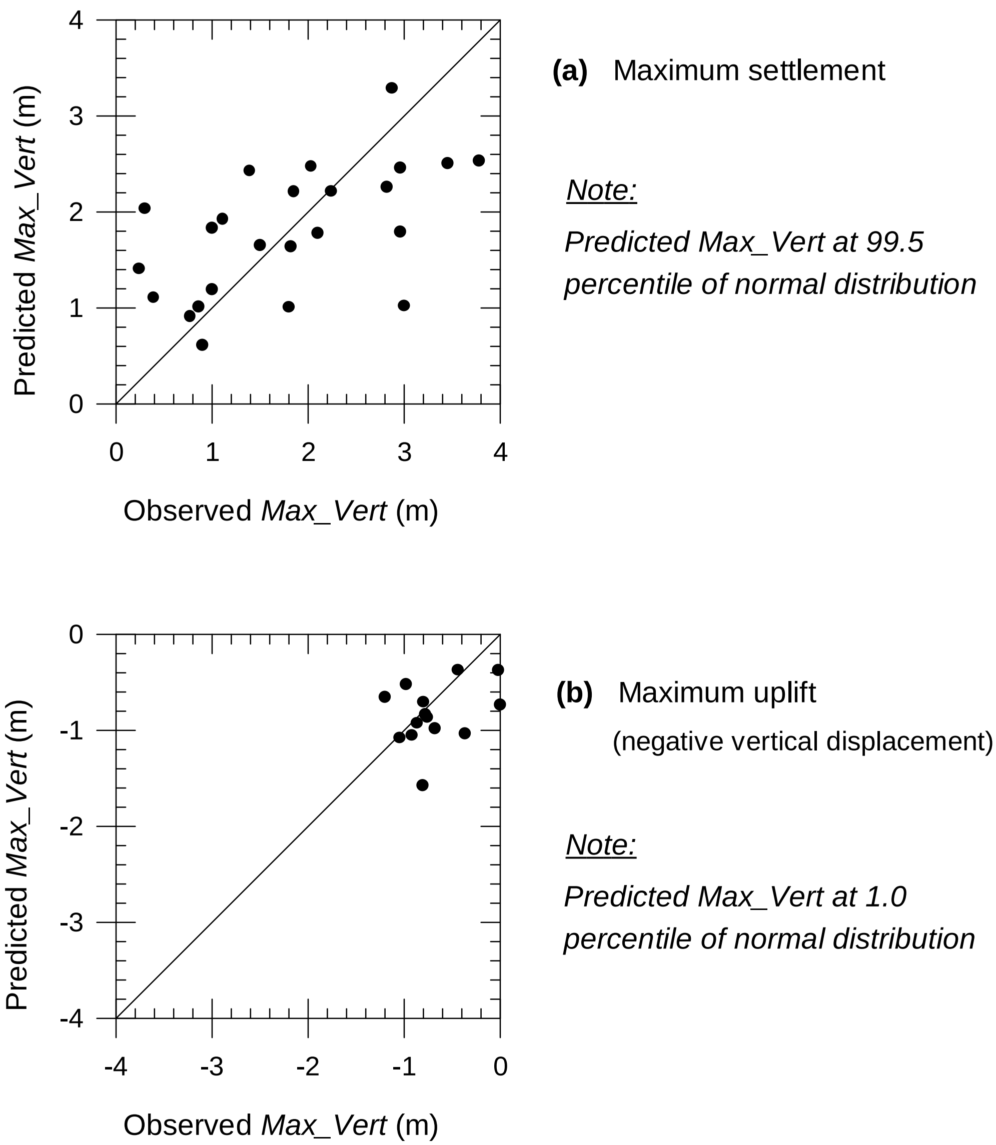


Figure 11.5. Performance of the Vertical-EPOLLS model in predicting (a) maximum settlement and (b) maximum uplift.

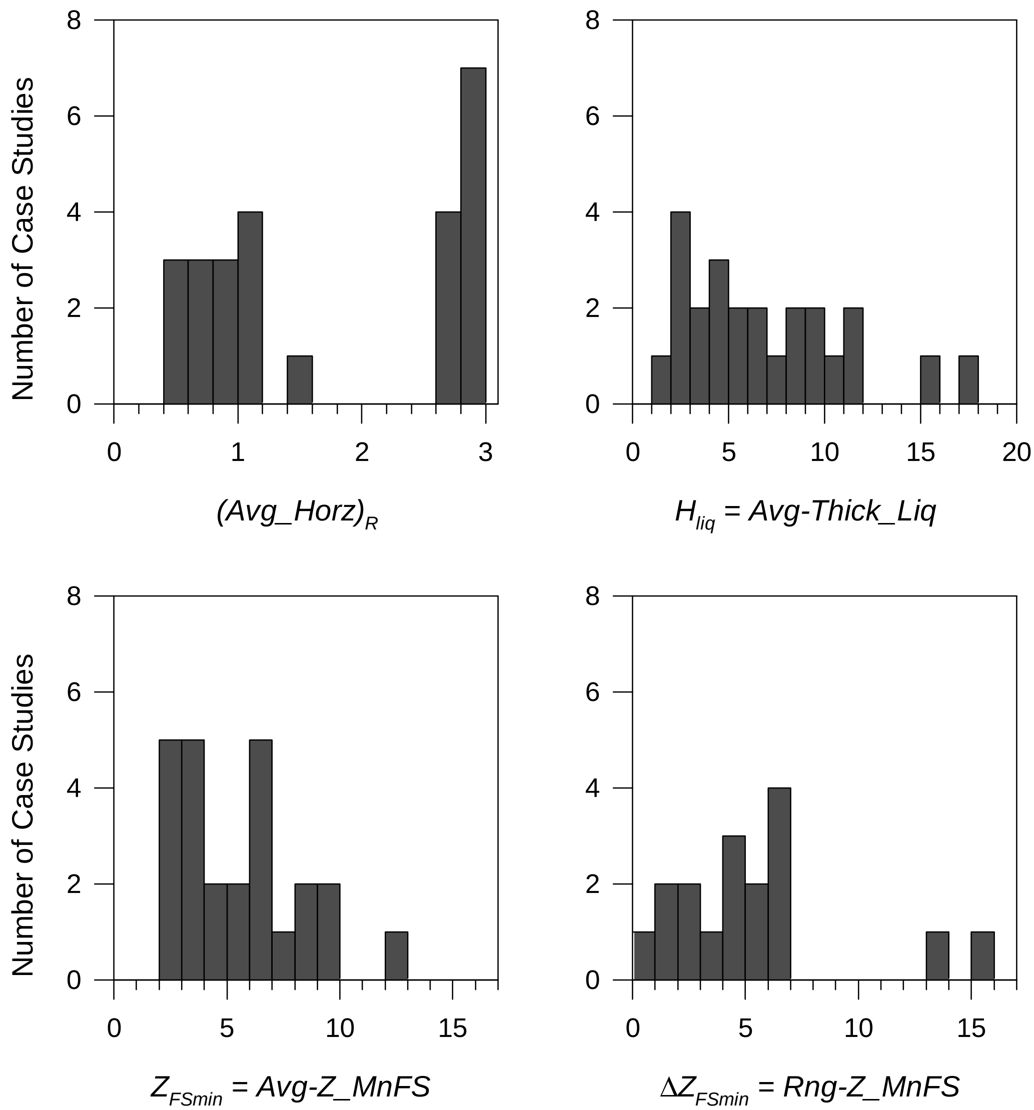


Figure 11.6. Histograms of data used to fit the Vertical-EPOLLS model.