Scenarios from Miller's Algebra Classes:
At 10:20 a.m. on November 14, students enter Mrs. Miller's classroom. She is stationed near the doorway and tells them to begin work on two problems shown on the board. Students work as she takes roll.

Solve for x: $4x - 3y = 2$
Solve for b: $A = \frac{1}{2}bh$

The daily objective indicates that students will graph solutions to inequalities and review chapters 3 and 6. Mrs. Miller is circulating and responding to student questions. "Yes." "Yes." She nods and offers suggestions, telling one student "It's still wrong." He asks if he should change his equation from $A = \frac{1}{2}bh$ to $2A=bh$. Miller moves to another student and then returns to the boy to ask "Is $\frac{2A}{h}$ the same as 2Ah?" He smiles and says "No. I see." At 10:29 she calls students' attention to the board as she begins to discuss the two problems. Four students respond as Mrs. Miller works through the first transformation. As she begins problem #2, she calls on a student to tell "what this other equation represents."

> "I try to start off at the bell getting started on something in terms of either a review of what they've done before or something that is going to lead into what we're doing today to get them jump-started. During that time, I'm doing the administrative stuff and I also try to go around and check their homework and/or check their answers to that activity while folks are finishing up."

"The area of a triangle" is his reply.
"Brenda, tell us how to begin."
"I don't know."
"What strategy did we use? Why don't you 'box out the variable'?" says Miller as she draws a different-colored square around the b. Hearing no response, she asks another girl to review her notes for ideas. "What operations are indicated here? (pointing to the formula)"
"Multiplication."
"And what is *b* multiplied by in this member of the equation?
"By $\frac{1}{2}$ and h."
"So what might we do?"
"Divide by $\frac{1}{2}$ and h?"
"Good idea! Who can handle this math? Randy?"
Randy advises that "dividing by $\frac{1}{2}$ is the same as multiplying by the reciprocal (2.) The simplified result is b=$\frac{2A}{h}$."

Mrs. Miller asks for other approaches. One student explains her two-step process of "multiplying through by 2 and then in the next step, dividing by h." Miller tells the class that "both methods are good ways to think about the solution. Now, your homework was to solve inequalities. Here are two. Please solve them, graph the solution, and raise your hand when you are finished. I'll check your work. Does anyone remember anything about solving inequalities that might be different from solving equations?"

    "Changing the inequality sign when you multiply" says one girl.
    "When you multiply or divide by what" asks Miller.
    "A negative number."

It is 10:42 and the teacher is moving about the classroom. Four hands are raised to show that they are finished. Students chat quietly as they wait for Mrs. Miller to check their work.

Mrs. Miller moves to the overhead projector and questions students about the first problem [$2x + 5 \leq -13$] She transforms the inequality by subtracting 5 from each member and then asks one boy if she should reverse the inequality sign.
    A boy says "No" and Miller asks Tracy for the next step.
    Tracy says "I don't know," but Miller prods "Yes you do."
    "Multiply or divide," responds Tracy.
    "Which?"
    "Divide."
    "By what? What are we multiplying by in this inequality?"
    "By 2. So divide by 2."
    "Right." Mrs. Miller models the operation and writes $x \leq -9$. How could we check this, Cathy?
    Mia responds quickly "Put -9 for x" and Cathy repeats her.
    Mrs. Miller smiles and adds "Let's do it." As she checks the inequality statement by substitution she asks the class "Is this the only answer I have?" Several students respond "No."
    "What else could be a correct answer? What else?"
    "-10" and "-12" are suggested.
    Miller agrees and reminds students of ways to check their work. Mia and her partner slap a "high five" to signify their correct contribution to the discussion.

Mrs. Miller moves to the next problem, calling on Jacob to assist her with the first step (distributive property.) As he multiplies correctly, she reminds the class that some had made errors in the distributive property on the last quiz. When Jacob completes the distributive process, he tells Miller that the next step is to simplify. She calls on Theresa to transform

the inequality (by adding 8 to both members) and then asks Kenneth to finish the problem (by subtracting x from both.) When the inequality has been transformed to 11 $\leq$ x she talks with students about her preference for writing x $\geq$ 11 in order to reduce the likelihood of confusion when they graph their solution. "Did anyone solve this inequality using a different point of view?" When no one responds, she asks Mike who quietly says "No, I didn't. Oh, yes I did." She encourages him to describe his process (distributing, then adding -2x and -3 to both members to produce -x $\leq$ -11.) The final transformation (dividing by -1) illustrates a need to reverse the inequality sign.

The time is 10:53 and all students are on task except one girl who arrives late and promptly puts her head down on the desk. Mrs. Miller asks students to take out homework assignments and she begins to call on them to state orally their solutions. All students refer to papers as they respond. When the correct answers have been read she asks "Are there any we need to work together?" Students ask for help with five problems and Mrs. Miller asks five students each to take a problem and work it on the board. The equations are literal equations and students have several questions. Mrs. Miller talks about the relationship of the distributive property to the equation S = L - rL which they are to solve for L. As she shows them the connection between 2r + 3r and either r(2+3) or (2+3)r they begin to see that L-rL may be rewritten as L(1-r) or (1-r)L. At that point, the final division by (1-r) seems clear to the group. Mrs. Miller reminds them that they may need to use creative thinking to solve literal equations. They discuss solving A -P = Prt for r. One boy suggests two separate steps--first dividing by P then dividing by t to produce the solution. Cathy suggests dividing by P and t (ie., Pt) at once. Mrs. Miller validates Cathy's response as accurate and 'quicker.' Two students ask Miller whether the final solution is best left r = $\frac{(A-P)}{Pt}$ or simplified to read r=$\frac{A}{Pt}-\frac{1}{t}$? Mrs. Miller indicates that both are acceptable and suggests that they will "see this problem or one similar to it again."

At 11:13 three students have their heads down. Mrs. Miller redirects students to the rest of the homework assignment. Again, she calls on students to respond orally. In quick succession she calls on Brett, Randy, Mike, Matt, Franklin, David, Jacob, Brenda, Cathy, Kenneth, and Mia. Most answers are correct although Miller occasionally makes suggestions or probes for further explanation of the procedures used. Franklin and

David answer a second question and Mrs. Miller says "I think we need to see #22 and #30 worked on the board. Any others?" Kenneth calls for #29. Miller assigns #22 to Brett (who missed his problem earlier) and #30 to Brenda (who does not have her paper.) Miller takes a seat at Brett's desk and all students are attentive to classmates working at the board. When Brett and Brenda are finished, Miller asks the rest of the class if the answers are correct. A few students say "No"

> "We usually check and go over homework. I have a question-and-answer time for homework problems. Usually the homework leads into your next lesson."

and she goes to the board, asking for suggestions from the class. One girl points out an error in copying the problem and a boy says "What's there is correct, but the problem's not finished. He still needs to divide by -3." Mrs. Miller asks him to finish the problem with the corrections and he quickly does so.

"Anybody see any mistakes on #30?" she asks the class.

"I see a multiplication error in the distributive property" says a student.

"Brenda, please correct your problem." Five classmates offer suggestions for making the corrections needed to complete #30.

Mrs. Miller uses #30 to review graphing procedures. "Let's take $x \geq -3$ as our illustration. Which values solve this inequality?"

"Everything from -3 on" says one girl. Heads nod as her classmates seem to agree. Miller quickly sketches a number line and labels points -2, 0, and 4 telling them "keep the increments on your number line the same." She places an open circle at -3 and shades the number line toward the right.

Brett asks "What if x is greater than or equal to a fraction? Like $x \geq \frac{3}{5}$?"

"Where is $\frac{3}{5}$ located on the number line?" asks Miller. Brett is confused and suggests that the fraction "is between 4 and 5." Miller is puzzled and begins to draw models illustrating fractional parts of one whole, hoping to lead Brett to state that $\frac{3}{5}$ is smaller than one whole. When he decides that $\frac{3}{5}$ is between 0 and 1, Miller agrees and quickly sketches a number line showing increments of $\frac{1}{5}$ from -1 to +1. At that point, Brett describes how to complete the graph.

"Let's see your work on these two problems," says Mrs. Miller as she writes on the board

        Graph   (a) $\frac{1}{3} > x$

                (b) $x > 4$

It is 11:35 and Mrs. Miller is circulating, checking student work, and offering suggestions or assistance as needed. She smiles as she says to most students "a is not right; b is." She finds that Jacob has both graphs correct and she turns to retrace her steps, rechecking students' work to see what changes they have made. Hands are raised, indicating that students are ready with changes. Miller says "yes, yes," as she circulates.

I provide more individual feedback to each algebra student in the block schedule arrangement than in a 55-minute schedule. (Q11)

"Jacob, tell us how to graph $\frac{1}{3} > x$." When he does not respond she prods. "Remember what we can do to check? What could help us graph this?"

Cathy speaks up. "Change it so x is first. You know. $x < \frac{1}{3}$."

"Good" says Mrs. Miller, asking Mike to describe the finished graphs.

"You now have been introduced to everything on Monday's test. You will see multi-step equations of all kinds including ones with variables on both sides and ones needing the distributive property. There will be inequalities to solve and also [literal] equations to solve for one variable. To get us ready, look at the piece of paper I gave you when you entered class. It has a number and a color, for example 'green 2' or 'red 5.'" She points to various locations in the classroom where students should gather depending on their code. Students move to the new location and join a small group.

"Write all your names on one paper," Miller directs, "and then work the quiz problems on that paper. This is a 20-point activity. You may talk in your group, but not to me. I will deduct points if you consult anyone outside your group. On Monday, a problem like one of these will be given to everyone in your group. I will give 20 points to each group member who accurately solves the problem. Questions?"

Students begin working with their trio at 11:48. One group of girls asks questions of each other and talks about the four problems until class ends. A group of boys are busy discussing the problems. One of them advises the others to "change fractions to decimals because they are easier to work." A trio (two girls and one boy) includes a student who was tardy. The two girls spend time helping him catch up. Another group with two boys and one girl talk quietly and remain on task until the bell. Two groups have a pair (one boy, one girl.) In one pair,

the girl has been absent and missed a quiz. Her partner spends time explaining each problem in detail. In the other group, the boy is talking quickly and Mrs. Miller suggests that his partner "is not with you here. You both must agree on the solution to the problems. Put your work on one paper." A few minutes later, he is thumbing through a magazine and they are the only group not working.

Mrs. Miller circulates but offers no direct assistance. "It matters when you leave here that all of you are comfortable with this. Work together. Talk together."

> "I certainly do more group activities than I did in a 55-minute class. I don't know whether I would have anyway [if we hadn't gone to block scheduling] because the trend has been in that direction. I probably would have. It came along about the same time. Typically what happens with younger students, the more numbers you get [in the group], the less work is done. Two or three will do the work, so if you limit to less [than four] you are more likely to get them involved."

Students are working and each pair or trio selects a 'leader' to manage the interactions. Calculators are available if needed. Only one pair appears disengaged (11:55.) At noon Mike asks Mrs. Miller if there is homework. She points to an assignment on the board and announces that "the assignment will help you prepare for the test. Leave your group paper on your desk and I will collect them at the bell."

> "There is typically a homework assignment. One of the problems is, if you give them enough homework to practice all they need to practice, they are overwhelmed. Many times they see the assignment and just go 'I'm not going to do it.'"

A few minutes later the bell rings and students pick up their books and bags and leave the room. Mrs. Miller circulates to collect papers left on desks and puts the desks back into rows as her next class enters.

Several weeks later the 10:20 bell sounds in Mrs. Miller's algebra class. She opens class discussion with two questions. "What is the slope intercept equation? What is the standard form equation?" A student enters late and she tells him to "sign in and see me after school tomorrow." When another student enters two minutes later, she repeats the same message to him. One of the girls tells Mrs. Miller that she doesn't remember the two equations, but she has
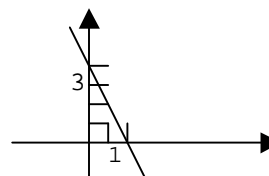
> "They sign in when they are tardy to show they have to make it up. The [assignment] calendar is there for students to look up what they missed."

them in her notebook.   Mrs. Miller commends that response, saying "That's right. You may need your notebook."

"Ask yourselves 'what are the characteristics of an equation in standard form?" While I check attendance, please do the following problems and I will come around and check your work.

Write an equation of the line in standard form:
1.   refer to the graph

2. (6,-1) ; m = 1/3

3. (1,-6) and (-3,2)

By 10:28 Mrs. Miller is circulating among the fifteen students. As she moves about, she questions. "Will this slope (pointing to the student's paper) get you where you want to go as you move along the line?"
"No."
"Why not?"
"I think it [slope] should be negative."
To another student: "Look at this problem. What does m represent?"
"Rise over run."
"And what does that mean?"
Miller pauses beside one girl to ask her why she isn't working. "But Mrs. Miller, I don't have my notebook or my book. I can't do my work." Mrs. Miller looks at her with a disapproving frown and goes to the board.
"Monica, how can we write an equation for #1."
"Don't ask me."
"What equation have we used in the past?"
"Slope, y-intercept equation."
"Do you know that [equation?]"
"y=mx + b"
"When we look at the graph, where do we see the line crossing the y-axis?"
"At 4."
"So where in the formula will that go?"
"For b."
"Connie, how do we find slope?"
"Use, like, a stairway." A boy adds "Use rise over run."
Mrs. Miller models the motion between two points on the graphed line as 'down 2 and over 1' and says "So what is the slope, Connie?"

"I tend to want to spend time with questions, and make sure the students understand and have full comprehension. It [the block schedule] gives you that time, whereas in a 55-minute class, I don't have time for any more questions. I have to go on with class."

        "-2/1"
        "And this goes where in the equation?"
        "m"
        "Now, Everett, let's change this equation [y = $\frac{-2}{1}$x + 4] to
standard form."
        "I don't know." Mrs. Miller asks for someone to help and a
girl advises Everett to "group the x and y terms on one side of
the equation."
        Everett does so and produces the standard form for the
linear equation [2x + y = 4].

Mrs. Miller addresses the class. "Is this complete? Are there
any fractions? (choral response indicating 'no') Is the
coefficient of x a positive number? (choral 'yes') "Go on to #2
and #3. I will continue to check." As she circulates, she asks
Randy to tell Shirley what "m" represents in #2. He says
"slope" and Miller moves along. The same girl from the earlier
interaction says "Mrs. Miller, I'd like to have my own paper and
my own book." When the teacher ignores her she asks "May I go
to the bathroom?" Miller says "No."

At 10:44 Mrs. Miller directs class attention to the board with a
question "How do I get an equation when I have a point and the
slope?"
        One student suggests "Plot the point and sketch the line
using the slope."
        "Let's do it," she says, sketching. "Does this give me the
information I need?"
        "No. You can't read the y-intercept. You've drawn it in
the 'wrong direction.'"
        "Could I go in another direction instead of up 1 and over
3? Suppose I went down one [-1]. What direction should I 'run'
to keep a positive slope?"
        "Left" says a boy.
        "Right" she says as she writes -1/-3. "So let's plot this.
What is the y-intercept?"
        "-3"
        "And the equation?"
        "y=$\frac{1}{3}$x -3"
        "Good. Randy, let's get this into standard form."
        "Subtract $\frac{1}{3}$x from both sides of the equation to get
$\frac{-1}{3}$x + y = -3"
        "Lori, are we finished."
        "No."
        "Do what?"

"Multiply each term by -3 so the coefficient will be a positive whole number."

Mrs. Miller glances at Connie, a stylish girl in the front of the class who is talking to her seatmate and examining her manicured nails. Her friend nudges her and she glances at the board.

Miller continues "Monica, what does the slope turn out to be in #3? (pause) Connie, help her."

"I have no idea" says Connie.

"Connie, look at what Monica has figured out [for slope.] She has $\frac{-8}{4}$ or -2. What does this mean?"

A boy answers for Connie. "Slope."

Connie repeats "slope."

"And where do we put slope in our equation" asks Mrs. Miller.

"b?" asks Connie. When Miller fails to respond, Connie says "No, it's m."

"Okay, Connie, -2 is m. So we get y = -2x. What else do we need?"

"b is -3."

"Is it? Look at the graph."

"Oh. It's -4"

Mrs. Miller writes y = -2x - 4 on the board. "So let's take it further to standard form. Everett--."

[About half the time,] my lesson demonstrates that learning algebra involves learning a specific set of rules. (Q39)

"2x + y = -4."

"Debbie, is there anything else?"

"No."

"Nice job! You should be ready for the quiz."

"Quiz?" says Connie in a puzzled tone.

"Where were you during the last class?" asks Mrs. Miller with a smile. "Let's all look at the homework and quickly check answers." It is 10:55 and two students have their heads down and are not working. Mrs. Miller randomly questions nine students about the homework. By 11:05 three additional students are not engaged. "Debbie, look at #22." Mrs. Miller sketches as Debbie directs her to plot (-4,-2) and apply the slope of to locate the y-intercept (at the origin.)

Debbie tells Miller "that's the point you need."

"So what equation can I write?" asks her teacher.

"y= $\frac{1}{2}$x + 0"

"Let's keep going to standard form."

"Subtract $\frac{1}{2}$x from both sides. That gives $\frac{-1}{2}$x + y =0."

"Now what? Are we finished?"

"No. We change $\frac{-1}{2}$x to a positive whole number coefficient."

"How could we?"

"Multiply by -2."
"Let's do it" urges Mrs. Miller.   "What happens?"
Debbie completes the problem as x - 2y = 0.
Mrs. Miller tells the class that they seem somewhat
confused when the y-intercept is zero.
"Could you do #23?" asks Debbie.
Miller writes (+3,-1); m=0 on the board. "Becky?"
"Plot the point (3, -1)"
As she does so, Miller asks "What does a line with zero
slope look like?"
One girl says "a straight line."
"What are you saying? Class, what does she mean?"
Some students call out "vertical"; others say "horizontal."
"Which is it?"
Debbie says "horizontal."
Mrs. Miller draws a horizontal line through (3,-1) saying
"and the equation of a horizontal line is..."
"y=-1" someone responds.
"That's correct."

Mrs. Miller turns to collect student papers and reminds a girl
to be sure to get her assignments from the calendar posted at
the back of the classroom.   She asks students to move back to
single desks, no pairs, and to take a sheet of graph paper to
use on the quiz she is handing them.   The time is 11:11 and
students have eleven problems to solve.   The
first two problems require students to *write
in words how to write an equation for a line
given one point and the slope or given two
points*.   The next seven problems provide
basic information (points and/or slopes) and
require students to *write an equation in
slope-intercept form*.   The last two problems
provide enough information for the students to *write a linear
equation in standard form*.   Most students work diligently.   Tom
asks to go to his locker and Mrs. Miller writes a pass.   Three
students put their heads down.   Mrs. Miller is checking papers
and recording information in her gradebook.   She does not notice
when Debbie raises her hand and then lowers it.   At 11:22 Debbie
raises her hand again and Miller goes to her.   Becky raises her
hand and gets a quiet comment from her teacher.   By 11:25
Everett is finished and then Lori and Debbie finish their quiz.
The three of them turn in their papers and three others quickly
follow.   A boy whose head was down turns in a blank paper.   Mrs.
Miller circulates, collecting finished papers.   Two other girls
finish.   Tom returns from his locker.   At 11:35 Connie puts her
paper on Miller's desk.   Most students are finished and the

> Often, I grade
> students using a
> paper and pencil
> assessment that
> involves solving
> open-ended problems.
> (Q57)

teacher is quietly distributing a calculator to each student. At 11:38 the last two students turn in their papers and Mrs. Miller tells them to use a sheet of graph paper to make a full-page set of coordinate axes. As they do so, she asks them to reset the graphing calculators, reviewing the process for those who may have forgotten. "Let's graph y=x. Describe the graph, Monica."

"It's a diagonal line."

"Anything else, Becky?" (Tom and Shirley are not participating and two others are asleep.)

"Through (0,0)"

"How can we 'see' the points on the line?"

"Trace the line."

"Okay, let's trace. Tell me what the points 'look like' Lori."

"X and Y are the same."

"Right! This is all 'old-hat' for you. Let's do something else. Today we will graph absolute value equations." As she says this, she writes   y1 = $|\,x\,|$

"Refresh our memory. What is absolute value Lucy?"

"The distance from zero."

"So, for example, the absolute value of 2 is what, Debbie?"

"Two."

"And the absolute value of -2?"

"Two. Two units from zero."

"Do you see *abs* on your calculator keypad?" Students nod and say "yes."

"Where is it?"

Debbie says, "in blue, above the x^{-1}. It's a 'second function.'"

Mrs. Miller directs students to press the blue key and then the x^{-1} key. She writes Y1=abs x on the board and tells them to graph it.

"It's vee-shaped" says Debbie.

Mrs. Miller adds the note 'v-shaped' on the board. She asks students if they notice a 'low point.'   "What do we call it?"   Hearing someone say "the bottom" she smiles and says "No. The <u>vertex</u> is the point at which this graph bends." She adds a second note to the board: 'vertex is (0,0).'

"Kevin, what can we say about the slope?"

"Left of the vertex it is negative; right of the vertex it is positive."

"Right!   Good!" and she writes this statement as the third note about the graph. "Now, if I want to graph this [on the coordinate grid,] give me some ordered pairs." She calls on Lori, Becky, Everett,

[More than half the time], my lesson plan presents an algebra concept using multiple representations (symbolic, number, and graphic.) (Q41)

Debbie, Randy, Shirley ("Oh, you're not doing this," Miller shakes her head), Kevin and Lucy who give various ordered pairs. "Let's connect the points you suggested. Here's a graphic representation. Tell me anything else you observe about the graph."

"The ordered pairs on the right have the same x and y; on the left, they are opposites."

Miller writes that statement on the board as students take notes. It is 11:55 when she asks someone to tell the slope on the left.

"-1" says Debbie.

"And on the right?"

"1." (Debbie)

"Good! Class, you are looking at a parent graph. Now, let's scroll [on our calculator] to Y2 and enter abs x. Then add 10 to it and graph y = *abs* x + 10. Tell me something about the graph."

Lucy: "It's shaped the same."

Connie: "It 'starts' at 10."

Miller: "Give me some points."

"(1,11)"; "(-6,16)"; "(0,10)"

Miller plots the points on the board as students call out. "Go back to y. Find Y3 and enter *abs* x -7." It is noon and Shirley decides to try this one. "Before you graph, try to predict."

All algebra students have access to graphing calculators if needed during algebra lessons. (Q17)

A student says "It will go down."

"Graph and let's see."

"It did!" says a student, referring to the graph's lowered position on the coordinate plane.

"What is making this happen?" asks Mrs. Miller.

Debbie says "whatever is added or subtracted."

Mrs. Miller circles the +10 and -7 in the two equations:

Y2 = *abs* x + 10
Y3 = *abs* x - 7

"Now try Y4=*abs* x +4. Make a prediction, Lori."

"Up 4."

"Is it up 4 when you graph?" asks Miller as the class responds "yes."

"So whatever we add or subtract moves the parent graph up or down. Let's try one more" she says, writing y = *abs* (x+3) as she speaks.

Randy: "It's gone to the left."

The bell interrupts this moment of exploration, discovery, and engagement. Mrs. Miller tells students to "leave the calculators on the desk and I'll collect them. There is no homework. We'll start here tomorrow."

Just one week later, the same group of students enter Mrs. Miller's class. As the bell rings, a girl who was socializing in the hall enters and asks to go back to her locker for her notebook. Mrs. Miller refuses the request and the girls mumbles, saying she needs her notebook, needs paper, needs a drink of water. Miller turns away saying "I am not going to deal with it. No." To the class, Mrs. Miller says "Please make a coordinate grid on your paper and graph the following..." as she writes on the board,

$$Y = \left| x \right| + 2$$

$$Y = \left| x - 3 \right|$$

$$Y = - \left| x + 2 \right| - 3$$

The grid on the board shows the graph of y = *abs* x (absolute value of x.) As Mrs. Miller takes roll, students work on the assignment. The one girl continues to mumble. At 10:32, Mrs. Miller says "Okay, let's do these graphs together. Connie, on the first problem, we do what?"

"Go up 2 units on the y-axis."

"That's a good start. Let's explain this. Where is +2 in relation to the absolute value of x?"

"Outside the absolute value sign. So we take our graph (absolute value of x) and move it up 2 units on the y-axis."

Mrs. Miller quickly sketches what Connie describes. "Right. The vertex slides up 2 units. Good. How about the second graph? Describe what happens, Becky."

"Go to the right 3 units with your vertex."

Mrs. Miller makes the slide as described. As she locates the vertex (0,3) she asks "Where do I put other points?"

"Use slope 1 to fill in. Over 1, up 1, and so on."

"That's right. Rick, do the third one."

"Your vertex is left 2 and down 3."

It is 10:35 as Mrs. Miller locates the vertex on the coordinate grid. "Then what? The peak or turning point of your graph is (-2,-3). After that your graph should be the 'usual' absolute value. What is the slope?"

Rick: "1 and -1"

"There are some things they learn better because they are able to spend more continuous time with it. Such as graphing. That is something you would normally break up over several days and you can spend one class period on it and [the students] understand it."

"Correct" says Mrs. Miller as she finishes the sketch. "Let's check homework."   She calls students by name as they refer to graphs in the textbook. They match absolute value graphs to equations, telling how they make the match, and identify the coordinates of the vertex.   After a quick oral check she informs students that the exam will include this material.   She reminds them that all incomplete work is due Friday. "If there are no questions, let's do today's work."

Students have no questions so Mrs. Miller writes *'Measures of Central Tendency'* on the board.   "Does anybody know what this is?" (no response) "We don't use this phrase much. However, you may be familiar with the four measures of central tendency." She writes *mean, median, mode, range*.   "To calculate these, you need data. What's data?"
     Connie says "Data is a collection of information."
     "That's right."   Miller writes *'a collection of information (numbers)'*   "What is the mean?"
     Stephanie says "average."
     Miller: "Rick, how do we find an average?"
     "Add all your numbers and divide."   Miller writes *'add all the data entries and divide by the number of entries.'*
     "Let's take the class grades on our last test."
She writes [83,78,22,61,85,93,0,92,90,78,87]
Debbie suggests that they may need calculators. It's 10:56 and Mrs. Miller tells students to use the graphing calculator for this lesson. "I'm not checking your addition and division. You may bring your own calculator. Go ahead and find the average. Raise your hand and I'll check."   Mrs. Miller begins to circulate. "The average is what, Sonya?"
     "67.3333"
     "How did you get that?" asks Miller.   Sonya explains correctly.
     "Is this a good average for our class?"   There are several "no's."
     "So the zero really hurt our average?"
     Kevin recalculates.   "Without the zero, our average would be 73.4"
     "Do you think there are any other numbers that are really affecting our average?"
     "22 and 39" says a boy.
     "You're right.   Sometimes extreme values can make a big difference in the mean or average. Let's look at the median." She writes *'the value in the middle when the data is listed numerically in order from least to greatest.'*   "Before you

[Frequently,] I require students to collect and use real data during algebra lessons. (O48)

answer any question about the data, it is important to order them. Do that first (on an exam or whenever you are answering questions about central tendency.)"

The class is working and Debbie is the first to say "There's an even number [of data] so there won't be a middle."

Miller asks the class to order the data and then says to Lori, "What's the number in the middle?"

"There isn't one. There are two...78 and 83."

"Right. When you have an even number of entries, you'll have two middle numbers. What can you do?"

Sonya: "Subtract?"

"If you do, you will get 7. Is 7 the middle value between them?"

"No."

"What could we do?"

Debbie: "Find the middle between them."

"The average?"

"Yes."

"Okay, let's average the two middle numbers. What do we get?"

Everett: "80.5"

Mrs. Miller adds a note to her definition of median on the board. *If there are two numbers in the middle, find the average of these numbers.* "Let's go on. What is the mode?"

Stephanie suggests that it would be any number you see twice.

Mrs. Miller says, "It could be. But suppose several numbers occur twice and another number occurs three times. What would the mode be?"

Stephanie: "The one we see the most."

Miller nods and writes mode is the number that occurs most frequently. "So what is our mode, Becky?"

"78."

"And the range, Rick?"

He reads from the textbook, "the difference from A to B."

"Right. But what does this mean? Debbie?"

"Difference is subtract."

"So what is the range? (from 0 to 93)"

"93 minus 0 is 93."

Mrs. Miller comments "I don't like my class range to be 93. With such a wide range, it tells me that someone learned a whole lot and some didn't learn much at all. I'd like the range to be much closer so all are learning about the same amount. Let's look at some ages. Give me your ages." She writes the ages on the board as the surveys the students. [17,18,15,16,16,14, 16,15,14,16,15,16,14,>18 (Miller), and >18 (observer)]. "We are going to make a frequency chart. List each entry once and use

tally marks to show how many times each entry occurs." She demonstrates as she speaks.

Only two students are not engaged and working with this real data set. Mrs. Miller prompts those students to get involved. "The purpose of our frequency chart is to organize the data so you can answer questions about it. What age appears most?" Students say "16." "And what age least?"

     Choral: "17"; "18"

     "A line plot is another way to show our data."

     Sonya says "We have lots of notes today." She laughs as she adds "You are making us hostile." Mrs. Miller smiles and continues.

     "In a line plot, you use a number line to show how many times an entry occurs (using x's.) Machines can do this for us. But you need to know what the machine is doing so we'll do this with paper and pencil." (Again, she models by producing the frequency chart.)

     At 11:26 Mrs. Miller begins to discuss stem and leaf plots. She directs the class to reconsider the first set of data on test grades. "Does anyone know this? (stem and leaf plots)"

     Sonya says "I think there is a vertical line. Does it look like a tree?"

     "I don't think so. But you may be starting to remember. You should order your stems (tens) numerically." She places 9, 8, 7, 6, 3, 2, 0 in the tens section of the chart. "The leaves complete the grades from the quiz. A '3' in the ones column beside the 9-stem becomes the leaf for 93, and so on" as she finishes the chart. Clearly, most grades are above..?"

     "60"

     "This graph can easily tell me that some are not doing well. But, out of twelve grades, half are a C or better."

At 11:35 Mrs. Miller questions students using an oral summary. "What is the mean? Define the median. Is there anything important about order for the median? What is the mode?" After hearing accurate responses she says "For the next 20 minutes, I have an assignment for you." She makes the assignment from the textbook and adds "As you work, I'll be returning papers. Let me know if I can help you."

Students begin working, using the calculators and occasionally stopping the teacher to ask about a problem, their grade, the exam, or some other question. Mrs. Miller answers questions and points out errors. The next fifteen minutes are used for one-to-one time with Kevin, Becky, Debbie, Stephanie, Monica, and Justin. Two students work together and four work independently.

At noon Miller hands each student a paper, saying "This is information we covered early this year. It will help you prepare for your exam. Remember to bring your books to the exam so that I can collect them and reissue them for second semester." She collects the calculators and reminds them to finish the statistics problems for homework. "There will be a notebook check on Friday, a check of problems assigned, and a review of two more topics." At 12:05 the bell rings and students hurry from the room as they move to the next block class or lunch.

Teacher Background
Mrs. Miller received an undergraduate degree in elementary education and added an endorsement in secondary mathematics some years ago. Her teaching experience includes middle level mathematics and science prior to teaching high school mathematics for the past ten years. She has taught algebra every year in either a fifty-minute daily schedule or the hundred-minute alternate day block schedule. Mrs. Miller indicates that she does not feel successful as an algebra teacher in the block schedule. She "still feels unprepared" to teach in the block schedule. "If there is a successful way to do it, I'm not aware of it. The block schedule does not lend itself to my teaching style. I've been trained to do the 45-50 minute block where you present, do guided practice, and [students] practice. The curriculum hasn't been changed in terms of what we have to cover. In a block schedule there's so much material to cover during the period. The sponge effect [student ability to absorb] is less than the amount of material you have to give out. I certainly have adapted to it [the block schedule] but you have your set way of doing it [teaching.] I still teach the same way I have always taught." However, when questioned, she strongly agreed that the block schedule challenges her to try new instructional strategies.

Distinctive Features of the Case

"Jump-starting the lesson"
Mrs. Miller prefers to begin her lessons with an opening focus that requires students to work a few problems. "I try to get started on something at the bell that will review what they've done before or lead into what we're doing today." These problems are usually not graded but are review or maintenance activities. They may be related to the homework, a prelude to the new lesson, or preparation for a quiz.

Student Engagement

Mrs. Miller uses many auditory strategies with much oral communication required of students. The oral communication taking place during questioning is supported by visual illustrations at the board. Miller's practice is to engage multiple students with responses to questions and in problem solving. She uses definitions, terminology, rules and formulae that represent the language of algebra. Her questions require students to understand and use that language themselves. Her requirement that students take notes also helps to keep the learners engaged throughout the lesson.

> I require students to take notes in class. (Q33)

Mrs. Miller routinely assigns homework but seldom permits them to use class time to begin homework. There is a clear message that algebra class involves bell-to-bell participation in working exercises or problems, responding to questions, sharing information with others, taking notes, and completing assignments and assessments. The majority of the social interactions involve individual exchanges: student-to-student or student-to-teacher. Mrs. Miller admits she is "not particularly a groupie-type of teacher. If there was something I could do to get them to understand that they need to practice this to really understand it and for it to become a permanent part of their memory. By practice, I mean not necessarily homework. Pay attention in class and do something with me. Take notes. Try and see where you can go from there. The ones who have tried have been successful. For those who have not even tried, there's no success."

Student accountability

Mrs. Miller maintains a relaxed, informal tone with her students. She has a no-nonsense attitude when students attempt to make excuses, get off task, or come to class unprepared. She has instituted classroom procedures requiring students who arrive tardy to sign in and meet with her after school. In addition, all assignments are posted on a monthly calendar in the classroom to assist students with catching up when they have been absent. She also puts assignments on the school's automated "Homework Hotline."

Mrs. Miller has seen no improvement in attendance or student achievement (grades) since the implementation of the block schedule. "In the block schedule, absenteeism is a killer, especially where you're covering two lessons in a class. If you are absent one day, you miss two concepts. If you're sick for a

three day period. . . I've had students fall from A's and B's to D's just because they missed two days in a row and didn't take the time to come for extra help."   She believes that the alternate day meeting schedule as well as the length of the class period present a difficult challenge for algebra students. "They don't have time for it to sink in and to figure out if they know how to do it before you move on to something else. When you go from Thursday to Monday, if they haven't mastered the concept, they've almost forgotten what you did on Thursday. You spend a lot of time reviewing."

Mrs. Miller sees student attention begin to drop off "right about one hour."   One afternoon when she was working with several students after school, they were talking about algebra being 'hard and boring.' One ninth-grade girl noted that "if I start to wander the least bit, I lose out on what's going on. For 45 minutes I can pretty well manage, but after 45 minutes, I find myself just going out. . .even if you get up and move or do something different. I find it very hard to concentrate." Mrs. Miller suspects this may be typical for ninth graders who enter high school having experienced math classes of much shorter duration.   She understands the student's point of view. "Actually, a lot of algebra is boring. It's just learning to do the process and if you don't find that challenging, it can be boring." She adds "You really need to break up the lesson and do alternative kinds of activities. Because I don't do that, I probably lose the students' attention."   She occasionally provides opportunities for students to use 'real' data in problem-solving, but most problems tend to represent examples of algebra procedures rather than 'real-life applications.' In her opinion, "the biggest detriment to me is keeping the kids on task. For the motivated student, the block schedule is probably good. But the student who is struggling with school, one who is not really thrilled about school and who is forced to sit for a long period of time in one subject...that might be detrimental."

Technology
Students have access to calculators (including graphing calculators) as needed or appropriate. Mrs. Miller believes that the block schedule has provided an opportunity for students to master graphing concepts more quickly than students did when class lengths were shorter.

Individual perspective
Mrs. Miller believes that the block schedule "provides time to do in-class discovery that you would not have time to do in a 50-minute period. In geometry, block scheduling really works nicely. It gives you plenty of time to do all the things you can do. But for algebra, it's a different story." She believes some mathematics classes may be better suited to a block schedule. She has noticed changes in the readiness level of students for algebra and is worried "because now everyone is required to take algebra." If students lack important skills, attendance and achievement may be affected. "My overall feeling is that there has not been a marked difference for me to say 'Wow! I'm not having as many absences and my [students'] grades have really gone up.' The biggest limitation is the concentration level or attention span of the students. Just being able to focus for that period of time is tough."

There is a personal challenge for the teacher in the block schedule. Mrs. Miller indicates that she "just goes all day! Every other day from 8 o'clock to 4 o'clock if students stay after school, I have gone straight through and I haven't stopped. I eat a quick lunch. I talk, I am up, I am going, I am on call the entire time. I do that every other day. And if it happens that on the day you have planning, you are needed for something. . .well, there's a real chance you go three straight days without a break the entire time you are at school. No time for planning or for grading. It is really exhausting." Not having a daily time for planning is the worst part of the block schedule for Mrs. Miller.

Advice to Algebra Teachers Beginning a Block Assignment
"There are two sides to every schedule. There's no perfect one unless some could be in the block schedule and others have two classes. I get the general feeling that the majority of the school, outside of the mathematics and foreign language department, are satisfied with the block." Her advice to algebra teachers embarking on a block assignment would be to "get training in teaching strategies, teaching activities, how to break the course up so it can be taught. You need actual algebra workshops on how to teach the material, how long to spend, where to put [concepts] into the schedule. You need to know how to teach a greater amount of material in one day and not be overwhelmed. Which materials should be used? There might be someone out there who has worked with it [block schedule] enough to know that 'it is best if you do graphing in one day' or 'what to do when teaching equations in the block

schedule.'"   When asked, Mrs. Miller said she would feel
qualified by her experiences to talk to others about algebra
lessons in a block schedule.   She hesitated before adding "I
could give them my experiences.   But I don't think we've been
successful enough.   I could tell them the things that did not
work, but what you need are things that work."

Student achievement
At year's end, seventeen students enrolled in Miller's classes
earned A, B, or C grades in algebra. Another five received a D
and seven failed (24%.)