

**PREVENTIVE MAINTENANCE FOR A  
MULTI-TASK SYSTEM**

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# PREVENTIVE MAINTENANCE PLANNING FOR A MULTI-TASK SYSTEM

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## (ABSTRACT)

This research models the behavior of a multi-task system with respect to time. The type of multi-task system considered here is one in which not all system components are required to perform each task. Each component may, however, be used for more than one task. Also, it is possible that some of the components may be required for every task that the system performs.

The components that are required for a subset of the tasks are considered to be intermittently demanded components and those components required for every task are continuously demanded components. This modeling approach assumes that the system is subject to a Modified Age Replacement Policy (MARP.) With a MARP the intermittently demanded components are maintained during their idle periods and the continuously demanded components are replaced according to their age replacement times.

A renewal theory approach is used to develop an availability expression for the multi-task system. Past research has focused on systems consisting of continuously demanded components and typically does not distinguish between

elapsed clock time and elapsed operating time in the renewal density function expressions. This research recognizes that the operational age of an intermittently demanded component is different than the chronological age of the component. The renewal density function and availability measures are modeled using joint density functions defined on both clock time and operating time.

The expressions are evaluated numerically using a multidimensional numerical integration routine. The results show logical behavior of the joint density functions used to define the availability measure. The availability measure also displays behavior consistent with the definition of a multi-task system. This model is an important development in the need for stochastic models of highly complex systems. The model is also a first step in defining performance measures for systems composed of both intermittently demanded components and continuously demanded components.

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## Dedication

I dedicate this work to my sons: William Samuel and Eric Michael. They gave the greatest sacrifice of living without their mother for a year. I hope this work will inspire in them the same perseverance and quest for knowledge that my parents have inspired in me.

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