APPENDIX A

The parameters used in the characteristic equation determined in Chapter 4 are summarized as

$$\eta_1 = \frac{U_1 \, Z'_{01}(U_1)}{Z_{01}(U_1)} \quad , U_1 = u_1 a_1 \tag{A.1}$$

$$\eta_2 = \frac{U_2 \, Z'_{02}(U_2)}{Z_{02}(U_2)} \quad , U_2 = u_2 a_1 \tag{A.2}$$

$$\eta_3 = \frac{U_2 \, \bar{Z}'_{02}(U_2)}{\bar{Z}_{02}(U_2)} \tag{A.3}$$

$$\eta_4 = \frac{\bar{U}_2 \, \bar{Z}'_{02}(\bar{U}_2)}{Z_{02}(\bar{U}_2)} \quad , \bar{U}_2 = u_2 a_2 \tag{A.4}$$

$$\eta_5 = \frac{\bar{U}_2 \, \bar{Z}'_{02}(\bar{U}_2)}{\bar{Z}_{02}(\bar{U}_2)} \tag{A.5}$$

$$\eta_6 = \frac{U_3 \, Z'_{03}(U_3)}{Z_{03}(U_3)} \quad , U_3 = u_3 a_2 \tag{A.6}$$

$$\eta_7 = \frac{U_3 \, \bar{Z}'_{03}(U_3)}{\bar{Z}_{03}(U_3)} \tag{A.7}$$

$$\eta_8 = \frac{\bar{U}_3\, Z'_{03}(\bar{U}_3)}{Z_{03}(\bar{U}_3)} \quad , \quad \bar{U}_3 = u_3\alpha_3 \tag{A.8}$$

$$\eta_9 = \frac{\bar{U}_3\, \bar{Z}'_{03}(\bar{U}_3)}{\bar{Z}_{03}(\bar{U}_3)} \tag{A.9}$$

$$\eta_{10} = \frac{U_4\, \bar{Z}'_{04}(U_4)}{\bar{Z}_{04}(U_4)} \quad , \; U_4 = u_4\alpha_3 \tag{A.10}$$

$$\xi_1 = \frac{Z_{02}(U_2)\, \bar{Z}_{02}(\bar{U}_2)}{Z_{02}(\bar{U}_2)\, \bar{Z}_{02}(U_2)} \quad , \qquad \xi_2 = \frac{Z_{03}(U_3)\, \bar{Z}_{03}(\bar{U}_3)}{Z_{03}(\bar{U}_3)\, \bar{Z}_{03}(U_3)} \quad , \tag{A.11}$$

The field coefficients are A_i, B_{i+1}, C_i, and D_{i+1}; where i = 1, 2, and 3. A_i and B_{i+1} are determined in terms of A_1 and shown in Chapter 4. The rest of the amplitude coefficients, C_i and D_{i+1}, are also determined in terms of A_1 and are summarized as

$$C_1 = \frac{(T_2T_{13} - T_3T_{12})\, D_2 - T_2S_1}{T_1T_{12} - T_2T_{11}} \tag{A.12a}$$

$$C_2 = \frac{(T_3T_{11} - T_1T_{13})\, D_2 + T_1S_1}{T_1T_{12} - T_2T_{11}} \tag{A.12b}$$

$$C_3 = \frac{(T_9T_{20} - T_{10}T_{19})\, D_4 - T_9S_3}{T_8T_{19} - T_9T_{18}} \tag{A.12c}$$

$$D_2 = \frac{(\bar{S}_1 \bar{T}_4 - \bar{S}_2 \bar{T}_2)}{\bar{T}_1 \bar{T}_4 - \bar{T}_2 \bar{T}_3} \quad \text{(A.13a)}$$

$$D_3 = \frac{(T_{10}T_{18} - T_8T_{20}) D_4 + T_8S_3}{T_8T_{19} - T_9T_{18}} \quad \text{(A.13b)}$$

$$D_4 = \frac{(\bar{S}_2 \bar{T}_1 - \bar{S}_1 \bar{T}_3)}{\bar{T}_1 \bar{T}_4 - \bar{T}_2 \bar{T}_3} \quad \text{(A.13c)}$$

where

$T_1 = Z_{21}(U_1)$, $T_{11} = U_1 Z'_{21}(U_1)$

$T_2 = -Z_{22}(U_2)$, $T_{12} = -U_2 Z'_{22}(U_2)$

$T_3 = -\bar{Z}_{22}(U_2)$, $T_{13} = -U_2 \bar{Z}'_{22}(U_2)$

$T_4 = Z_{22}(\bar{U}_2)$, $T_{14} = \bar{U}_2 Z'_{22}(\bar{U}_2)$

$T_5 = \bar{Z}_{22}(\bar{U}_2)$, $T_{15} = \bar{U}_2 \bar{Z}'_{22}(\bar{U}_2)$

$T_6 = -Z_{23}(U_3)$, $T_{16} = -U_3 Z'_{23}(U_3)$

$T_7 = -\bar{Z}_{23}(U_3)$, $T_{17} = -U_3 \bar{Z}'_{23}(U_3)$

$T_8 = Z_{23}(\bar{U}_3)$, $T_{18} = \bar{U}_3 Z'_{23}(\bar{U}_3)$

$T_9 = \bar{Z}_{23}(\bar{U}_3)$, $T_{19} = \bar{U}_3 \bar{Z}'_{23}(\bar{U}_3)$

$T_{10} = -\bar{Z}_{24}(U_4)$, $T_{20} = -U_4 \bar{Z}'_{24}(U_4)$,

$\bar{T}_1 = T_5 + T_4 \hat{T}_1$

$\bar{T}_2 = T_6 \hat{T}_3 + T_7 \hat{T}_5$

$\bar{T}_3 = T_{15} + T_{14} \hat{T}_1$

$\bar{T}_4 = T_{16} \hat{T}_3 + T_{17} \hat{T}_5$,

$\hat{T}_1 = (T_3T_{11} - T_1T_{13}) /(T_1T_{12} - T_2T_{11})$

$\hat{T}_2 = T_1/(T_1T_{12} - T_2T_{11})$

$\hat{T}_3 = (T_9T_{20} - T_{10}T_{19}) /(T_8T_{19} - T_9T_{18})$

$\hat{T}_4 = -T_9 /(T_8T_{19} - T_9T_{18})$

$\hat{T}_5 = (T_{10}T_{18} - T_8T_{20}) /(T_8T_{19} - T_9T_{18})$

$\hat{T}_6 = T_8/(T_8T_{19} - T_9T_{18}),$

$S_1 = -A_1U_1^2Z''_{01}(U_1) + A_2U_2^2Z''_{02}(U_2) + B_2U_2^2\bar{Z}''_{02}(U_2)$

$S_2 = -A_2 \bar{U}_2^2Z''_{02}(\bar{U}_2) - B_2 \bar{U}_2^2\bar{Z}''_{02}(\bar{U}_2) + A_3U_3^2Z''_{03}(U_3) + B_3U_3^2\bar{Z}''_{03}(U_3)$

$S_3 = -A_3 \bar{U}_3^2Z''_{03}(\bar{U}_3) - B_3 \bar{U}_3^2\bar{Z}''_{02}(\bar{U}_3) + B_4U_4^2\bar{Z}''_{04}(U_4),$

and

$\bar{S}_1 = - [T_4\hat{T}_2 S_1+ (T_6 \hat{T}_4 + T_7 \hat{T}_6)S_3]$

$\bar{S}_2 = - [T_{14}\hat{T}_2 S_1+ (T_{16} \hat{T}_4 + T_{17} \hat{T}_6)S_3] + S_2.$

APPENDIX B

Some parameters used in the field expressions are summarized as

$f_1(r) = (-jk_o/u^2)[A_1(\bar{\beta}u)J'_\nu(ur) + B_1(\nu Z_o/r)J_\nu(ur)],$ (B.1)

$f_2(r) = (jk_o/w^2)[A_2(\bar{\beta}w)K'_\nu(wr) + B_2(\nu Z_o/r)K_\nu(wr)],$ (B.2)

$f_3(r) = (-jk_o/u^2)[A_1(\bar{\beta}u\nu/r)J_\nu(ur) + B_1(uZ_o)J'_\nu(ur)],$ (B.3)

$f_4(r) = (jk_o/w^2)[A_2(\bar{\beta}\nu/r)K_\nu(wr) + B_2(wZ_o)K'_\nu(wr)],$ (B.4)

$g_1(r) = (jk_o/u^2)[A_1(n_1^2\nu/Z_or)J_\nu(ur) + B_1(\bar{\beta}u)J'_\nu(ur)],$ (B.5)

$g_2(r) = (-jk_o/w^2)[A_2(n_2^2\nu/Z_or)K_\nu(wr) + B_2(\bar{\beta}w)K'_\nu(wr)],$ (B.6)

$g_3(r) = (-jk_o/u^2)[A_1(n_1^2u/Z_o)J'_\nu(ur) + B_1(\bar{\beta}\nu/r)J_\nu(ur)],$ (B.7)

$g_4(r) = (jk_o/w^2)[A_2(n_2^2w/Z_o)K'_\nu(wr) + B_2(\bar{\beta}\nu/r)K_\nu(wr)],$ (B.8)

where $Z_o = (\mu_o/\varepsilon_o)^{1/2}$.

The transverse field components are summarized as

$E_r = f_1(r)\cos(\nu\varphi + \bar{\varphi}_o),$ $r < a$ (B.9a)

$E_r = f_2(r)\cos(\nu\varphi + \bar{\varphi}_o),$ $r > a$ (B.9b)

$E_\varphi = -f_3(r)\sin(\nu\varphi + \bar{\varphi}_o),$ $r < a$ (B.10a)

$E_\varphi = -f_4(r)\sin(\nu\varphi + \bar{\varphi}_o),$ $r > a$ (B.10b)

$H_r = -g_1(r)\sin(\nu\varphi + \bar{\varphi}_o),$ $r < a$ (B.11a)

$H_r = -g_2(r)\sin(\nu\varphi + \bar{\varphi}_o),$ $r > a$ (B.11b)

$H_\varphi = g_3(r)\cos(\nu\varphi + \bar{\varphi}_o),$ $r < a$ (B.12a)

$H_\varphi = g_4(r)\cos(\nu\varphi + \bar{\varphi}_o),$ $r > a$ (B.12b)

The following coefficients are defined as

$$Q_1 = \bar{\beta}\, k_o^2(1/u^2)(Z_oB_1^2 + n_1^2A_1^2/Z_o) \qquad \text{(B.13)}$$

$$Q_2 = (1/u^4)[(2k_o^2 /u^2)A_1B_1 (\bar{\beta}^2+ n_1^2) -2Q_1] \qquad \text{(B.14)}$$

$$Q_3 = \bar{\beta}\, k_o^2(1/w^2)(Z_oB_2^2 + n_2^2A_2^2/Z_o) \qquad \text{(B.15)}$$

$$Q_4 = (1/w^2)[(2k_o^2 /w^2)A_2B_2 (\bar{\beta}^2+ n_2^2) -2Q_3]. \qquad \text{(B.16)}$$

The field coefficients are A_1, B_1, A_2 and B_2. Here, we choose B_1 as the independent coefficient and use the boundary conditions at $r = a$ to express B_2, A_1 and A_2 in terms of B_1. From (5.2),

$$B_1 J_\nu(U) = B_2 K_\nu(W)$$

then $$B_2 = B_1J_\nu(U)/K_\nu(W) \qquad \text{(B.17)}$$

From (5.1),

$$A_1 J_\nu(U) = A_2 K_\nu(W)$$

then $$A_2 = A_1J_\nu(U)/K_\nu(W) \qquad \text{(B.18)}$$

using boundary conditions for E_φ and H_φ, and substituting (B.17) and (B.18) in these expressions, we can obtain the following,

$$A_1 = -(Z_o/\nu)(1/ \bar{\beta}) (UW/V)^2(\eta_1+\eta_2)B_1 \qquad \text{(B.19)}$$

and $$A_2 = -(Z_o/\nu)(1/ \bar{\beta}) [J_\nu(U)/K_\nu(W)](\eta_1+\eta_2)B_1. \qquad \text{(B.20)}$$

Calculation of Power Flow, P:

To calculate the power flow, P, we write

$$(\mathbf{E}\times\mathbf{H}^*).\mathbf{a_z} = E_r H_\varphi^* - E_\varphi H_r^*$$

The expression for power flow is obtained by substituting the field components in the expression above, which yields

$$P = (\varphi_o/8)\{ Q_1 \alpha^2[J^2_{\nu-1}(U) - J_\nu(U) J_{\nu-2}(U)] + \nu Q_2 J^2_\nu(U) + Q_3 \alpha^2[-K^2_{\nu-1}(W) + K_\nu(W) K_{\nu-2}(W)] - \nu Q_4 K^2_\nu(W) \}. \quad \text{(B.21)}$$

Calculation of Power Loss, P_l :

To calculate the power loss, P_l , we start with

$$\mathbf{J_s} = \mathbf{a_n} \times \mathbf{H}$$

$$\mathbf{J_s} = \mathbf{Js_1} = \mathbf{a}_\varphi \times \mathbf{H} = -H_r\mathbf{a_z} + H_z\mathbf{a_r}, \qquad \varphi = 0$$

$$\mathbf{J_s} = \mathbf{Js_2} = -\mathbf{a}_\varphi \times \mathbf{H} = H_r\mathbf{a_z} - H_z\mathbf{a_r}, \qquad \varphi = \varphi_o$$

$$|\mathbf{J_s}|^2 = |H_r|^2 + |H_z|^2.$$

After simplification, the expression for conductor power loss is obtained as

$$P_{lc} = Rs\left\{ \int_0^a (B_1 J^2_\nu(Ur) + |g_1(r)|^2)dr + \int_a^\infty (B_2 K^2_\nu(Wr) + |g_2(r)|^2 \right\}dr. \quad \text{(B.22)}$$

The expression for dielectric power loss, P_{ld} , is obtained as

$$P_{ld} = (1/2)\,\sigma_d \int_0^\varphi \cos^2(\nu\varphi)d\varphi \int_0^a [A^2_1 J^2_\nu(ur)rdr + |f_3(r)|^2 + |f_1(r)|^2]\,rdr. \quad \text{(B.23)}$$