CHAPTER 5. WEDGE-SHAPE DIELECTRIC WAVEGUIDE BOUNDED BY CONDUCTING PLANES

5.1 INTRODUCTION

Dielectric waveguides with conducting boundaries have important applications at microwave, millimeter wave, and optical frequencies. They are used as low-loss transmission media, as elements of integrated circuit devices, and in a variety of other devices such as polarizer, mode analyzer, and mode filter. Propagation characteristics of planar optical waveguides with metal boundaries have been studied by many researchers [12]-[13], and [15]. The investigation of waveguides involving both planar and curved boundaries is more complicated and has been carried out using experimental and numerical techniques.

In this chapter, attention is focused on a wedge-shape waveguide consisting of two conducting plane boundaries with the interior of the wedge partially filled with a dielectric material. The remaining portion of the wedge interior is free space or is occupied by another dielectric material of lower dielectric constant. The dielectric-free space boundary is assumed to be circularly cylindrical. A special case of this waveguide corresponding to a wedge angle of 180^o, that is a semi-circular rod backed by a conducting plane, has been examined before [16].

It is the aim of this chapter to present a comprehensive analysis of wedge-shape dielectric waveguides bounded by conducting planes and with arbitrary wedge angles. Propagation properties of guided modes are studied. Field solutions, dispersion relations, cutoff conditions, and conductor and dielectric losses are examined. The analysis presented here reveals that these wedge-shape waveguides support only TE and hybrid HE- and EH-type modes; thus, TM modes do not exist. Dispersion characteristics and normalized dielectric and conductor loss coefficients for several lower-order modes and several wedge angles are presented.

5.2 FIELD SOLUTIONS AND CHARACTERISTIC EQUATIONS

Let us consider a wedge-shape dielectric waveguide bounded by two conducting planes at $\varphi = 0$ and $\varphi = \varphi_o$. The dielectric region $0 < \varphi < \varphi_o$ consists of a core of radius a and relative permittivity ε_{r1} and a cladding of relative permittivity $\varepsilon_{r2} < \varepsilon_{r1}$. For microwave and millimeter wave applications, the cladding is usually air with $\varepsilon_{r2} = 1$, but for application at optical frequencies it consists of some dielectric material. Both core and cladding are assumed to be homogeneous, isotropic and nonmagnetic with permeability μ_o. Furthermore, the cladding and the conducting planes are assumed to extend to infinity in the radial direction. Figure 5.1 illustrates the geometry of the waveguide. A cylindrical coordinate system (r,φ,z) is chosen and propagation of electromagnetic fields along the positive z-direction is considered. The time and z-dependences of fields are assumed to be as $e^{j(\omega t-\beta z)}$, where β is the axial propagation constant and ω is the angular frequency. This term, which is common to all field components, is dropped from the solutions.

To determine the field solutions in the region $0 \leq \varphi \leq \varphi_o$, it suffices to solve the wave equation for the axial components, then calculate transverse components and impose the appropriate boundary conditions. At first, solutions corresponding to perfect conducting planes and lossless dielectrics are obtained. Conductor and dielectric losses will be calculated using perturbation techniques. Field solutions in this wedge-shape dielectric waveguide have the same mathematical form as those of a complete circular dielectric waveguide, but will be subject to additional boundary conditions at $\varphi = 0$ and $\varphi = \varphi_o$. These additional boundary conditions require that axial and radial components of the electric field vanish at $\varphi = 0$ and $\varphi = \varphi_o$. For guided modes, the propagation constant β

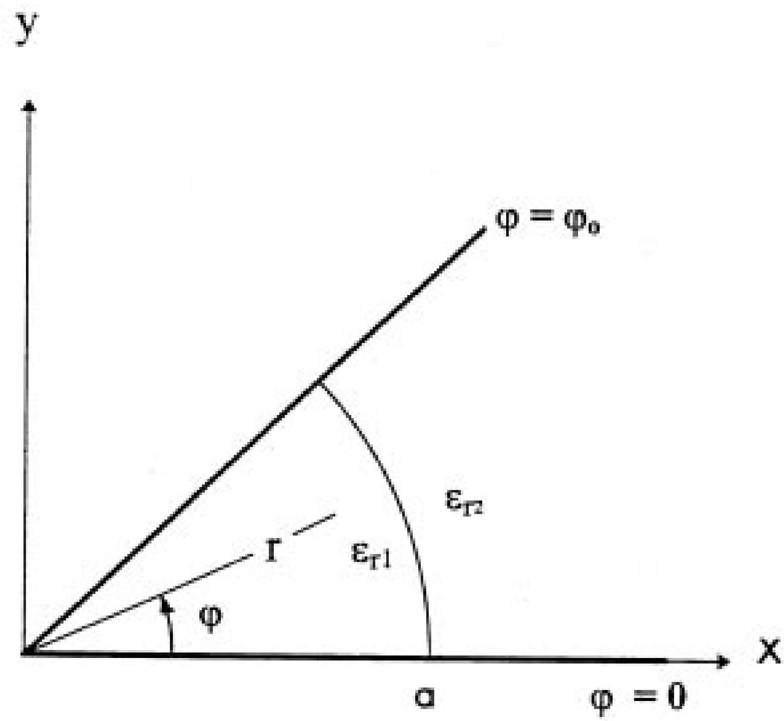

Figure 5.1 Geometry and coordinates for a wedge-shape dielectric waveguide bounded by conducting planes at $\varphi = 0$ and $\varphi = \varphi_o$.

lies in the range $k_o(\varepsilon_{r2})^{1/2} < \beta < k_o(\varepsilon_{r1})^{1/2}$ where $k_o = 2\pi/\lambda$; λ being the free-space wavelength. The solution of wave equation for axial field components E_z and H_z, which are bounded everywhere and represent guided modes, are summarized as [115],

$$E_z = A_1 J_\nu(ur)\cos(\nu\varphi + \overline{\varphi}_o), \qquad r < a \tag{5.1a}$$

$$= A_2 K_\nu(wr)\cos(\nu\varphi + \overline{\varphi}_o), \qquad r > a \tag{5.1b}$$

$$H_z = B_1 J_\nu(ur)\sin(\nu\varphi + \overline{\varphi}_o), \qquad r < a \tag{5.2a}$$

$$= B_2 K_\nu(wr)\sin(\nu\varphi + \overline{\varphi}_o), \qquad r > a \tag{5.2b}$$

where J_ν is the Bessel function of the first kind, K_ν is the modified Bessel function of the second kind, A_1, A_2, B_1 and B_2 are amplitude coefficients, ν is the azimuthal number, $\overline{\varphi}_o$ is a constant phase term, and

$$u = k_o\,(\varepsilon_{r1} - \overline{\beta}^2)^{1/2} \tag{5.3a}$$

$$w = k_o\,(\overline{\beta}^2 - \varepsilon_{r2})^{1/2} \tag{5.3b}$$

with $\overline{\beta} = \beta/k_o$ is the normalized propagation constant. The transverse field components are readily obtained by substituting (5.1) and (5.2) into

$$\mathbf{E_t} = (j/q_i)[\omega\mu_o \mathbf{a_z} \times \nabla_\mathbf{z} H_z - \beta\nabla_\mathbf{z} E_z] \tag{5.4a}$$

$$\mathbf{H_t} = (-j/q_i)[(\omega\,\varepsilon_o\,\varepsilon_{ri})\,\mathbf{a_z} \times \nabla_\mathbf{z} E_z + \beta\nabla_\mathbf{z} H_z] \tag{5.4b}$$

where $q_i = k_o^2\varepsilon_{ri} - \beta^2$ with i =1 and 2 corresponding to the core and cladding regions, respectively. The results for transverse field expressions are summarized in (B.9)-(B.12), in Appendix B.

The waveguide under consideration has three boundaries; $\varphi = 0$, $\varphi = \varphi_o$, and $r = a$. The first two are conducting boundaries and the tangential components of the electric field (E_z and E_r) must vanish on them. The third is a boundary between the core and cladding at which the tangential components of both electric and magnetic fields must be continuous. Imposition of boundary conditions at $r = a$; namely, continuity of E_z, E_φ, H_z and H_φ at $r = a$, results in a characteristic equation which is identical in format to that of a complete circular dielectric waveguide. The results is expressed as [115],

$$(\eta_1+\eta_2)(\varepsilon_{r1}\eta_1+\varepsilon_{r2}\eta_2) = (\nu\ \bar{\beta})^2(V/UW)^4 \tag{5.5}$$

where

$$\eta_1 = J'_\nu(U)/[U\, J_\nu(U)] \tag{5.6a}$$

$$\eta_2 = K'_\nu(W)/[W\, K_\nu(W)] \tag{5.6b}$$

and $U = u\, a$, $W = w\, a$, and $V^2 = U^2 + W^2 = (ak_o)^2(\varepsilon_{r1}-\varepsilon_{r2})$. The boundary condition at $\varphi = 0$ results in $\cos(\bar{\varphi}_o) = 0$ with the solution expressed as

$$\bar{\varphi}_o = l\pi + \pi/2, \qquad l \text{ an integer} \tag{5.7}$$

Consequently, the φ-dependence of fields are as $\sin(\nu\varphi)$ for E_z, E_r, H_φ, and as $\cos(\nu\varphi)$ for H_z, E_φ, and H_r. The boundary condition at $\varphi = \varphi_o$ leads to $\sin(\nu\varphi_o) = 0$. Hence, $\nu\varphi_o = m\pi$; m being an integer, and thus ν must obey the following relation,

$$\nu = m\pi/\varphi_o \tag{5.8}$$

5.3 GUIDED MODE SOLUTIONS

5.3.1 TE and TM modes

When $\nu = 0$, from (5.1), (5.4), and (5.7),it is concluded that $E_z = E_r = H_\varphi = 0$, that is, TM modes cannot exist. For TE modes, on the other hand, the field components are H_z, E_φ, and H_r, none of which is subject to boundary conditions at $\varphi = 0$ and $\varphi = \varphi_o$. The TE mode solutions for any arbitrary value of φ_o are identical to those of a complete circular dielectric waveguide with the only difference that here fields are confined to the region $0 \leq \varphi \leq \varphi_o$, while for the complete circular dielectric waveguide they exist in the entire region $0 \leq \varphi \leq 2\pi$. The characteristic equation for TE modes is obtained from (5.5) with $\nu = 0$. The result is

$$J_1(U)/[U\, J_0(U)] + K_1(W)/[W\, K_0(W)] = 0 \tag{5.9}$$

The cutoff condition for TE modes is expressed as $J_0(V_c) = 0$, which is obtained from (5.9) in the limit of $W \rightarrow 0$. Here, $V_c = (ak_{oc})(\varepsilon_{r1}-\varepsilon_{r2})^{1/2}$ is the normalized cutoff frequency.

5.3.2 Hybrid HE and EH Modes

When $\nu \neq 0$, all six components of fields exist and the modes are hybrid. They are classified as HE and EH corresponding to two distinct solutions of (5.5). The hybrid mode designation is based on the same criterion introduced in [115]. These modes, when exist, will be identical to odd modes in a complete circular dielectric waveguide for integer values of ν only. When ν is non-integer, the modes assume distributions different from circular waveguides. Some special cases are considered for more detailed examination. Cutoff conditions for HE and EH modes are obtained from (5.5) in the limit of $W \rightarrow 0$. The results are summarized as

$$\varepsilon_{r2} V_c J_\nu(V_c) - (\varepsilon_{r2}+\varepsilon_{r1})(\nu-1) J_{\nu-1}(V_c) = 0, \quad \text{for } HE_{\nu p} \text{ modes} \tag{5.10a}$$

$$J_\nu(V_c) = 0, \quad \text{for } EH_{\nu p} \text{ modes} \tag{5.10b}$$

5.4 SPECIAL CASES

5.4.1 $\varphi_o = \pi$

When $\varphi_o = \pi$, the waveguide is semicircular, and from (5.8), $\nu = m = 1, 2, 3 \ldots$ is obtained. The hybrid modes of circular dielectric waveguide are even and odd HE and EH modes; where even and odd correspond to φ-dependences as $\cos(\nu\varphi)$ and $\sin(\nu\varphi)$ for E_z, respectively. This even and odd designation is arbitrary because it depends on the choice of field component. For a semicircular waveguide, the φ-dependence of E_z can only assume the form $\sin(\nu\varphi)$, implying that the modes are of odd HE and EH types. The fundamental mode of this waveguide is HE_{11} with zero cutoff frequency.

5.4.2 $\varphi_o = \pi/2$

When $\varphi_o = \pi/2$, $\nu = 2m = 2, 4, 6 \ldots$ In this case only odd hybrid modes of $HE_{2m,p}$ and $EH_{2m,p}$ types such as HE_{21}, HE_{41}, EH_{21}, EH_{41}, etc..., can exist. The fundamental mode of this waveguide is thus no longer the HE_{11} mode. It is, in fact, TE_{01} mode with a normalized cutoff frequency of $V_c = 2.405$.

5.4.3 $\varphi_o = \pi/n$, n an integer > 2

In this case, from (5.9) we have $\nu = mn$. Generally, the larger the value of n the smaller the number of modes that can be supported by the guide for the same frequency of operation. The fundamental mode of all guides with $\varphi_o = \pi/n$, n = 2, 3, 4..., is the TE_{01} .

5.4.4 Arbitrary φ_o

For the general case of arbitrary φ_o such that ν is not an integer, with the exception of TE_{0p} modes, the fields are described by Bessel and modified Bessel functions of non-integer orders. As an example, we consider the case of $\varphi_o = 2\pi/3$. From (5.8), $\nu = 3m/2$ where m = 0, 1, 2 ... For m = 0, we have $\nu = 0$ and the modes are TE_{0p}. For $m = 1, \nu = 3/2$ and the field solutions involve the following functions

$$J_\nu(ur) = J_{3/2}(ur) = (2/\pi ur)^{1/2}[\sin(ur)/(ur) - \cos(ur)] \tag{5.11a}$$

$$K_\nu(wr) = K_{3/2}(wr) = (\pi/2wr)^{1/2}(1 + 1/wr)e^{-wr} \tag{5.11b}$$

To determine the characteristic equation, η_1 and η_2 as defined in (5.6a) and (5.6b) are first calculated. Using (5.11a) and (5.11b) in (5.6a) and (5.6b), yields

$$\eta_1 = [1/(1-U\mathrm{Cot}(U))]-3/(2U^2) \tag{5.12a}$$

and

$$\eta_2 = - 1/(1+W) - 3/(2W^2) \tag{5.12b}$$

Now, combining (5.5), (5.12a) and (5.12b), the characteristic equation for hybrid modes with $\nu = 3/2$ is obtained. Letting $W \to 0$ in the resulting characteristic equation, the corresponding cutoff conditions are obtained as

$$V_c\,\mathrm{Cot}(V_c) = (\varepsilon_{r1}-\varepsilon_{r2})/(2\varepsilon_{r2}), \quad \text{for HE modes} \tag{5.13a}$$

$$V_c\,\mathrm{Cot}(V_c) = 1,\ V_c \neq 0, \quad \text{for EH modes} \tag{5.13b}$$

In summary, from the examination of above cases it is concluded that the number, types, and properties of modes supported by a wedge-shape waveguide are dependent upon the wedge angle φ_o with the exception of TE_{0p} modes which always exist above their respective cutoff frequencies for an arbitrary φ_o and all their propagation properties (but the conductor loss) are independent of φ_o.

5.5 WAVEGUIDE LOSSES

In practical situations the conducting walls have finite conductivity and dielectric materials have complex permittivity with small imaginary parts, with the implication that propagating modes suffer power losses. These losses may be calculated using standard perturbation techniques.

5.5.1 Conductor Loss

The attenuation coefficient α_c, due to conductor loss, is obtained from $\alpha_c = P_{lc} /2P$, where P_{lc} is the power dissipated per unit length of the conducting boundaries and P is the power flow. P_{lc} is calculated as [116]

$$P_{lc} = (1/2)R_s \int_{Sc} |J_s|^2 \, ds = R_s \int_0^{\infty} (|H_z|^2 + |H_r|^2) \, dr \tag{5.14}$$

where $R_s = (\omega\mu_o/2\sigma)^{1/2}$ is the surface resistance of each conducting wall per unit length, J_s is the surface current density, and S_c represents the surface of conducting walls per unit length of waveguide. The power flow, P, is obtained from

$$P = (1/2)\mathrm{Re} \int_{Sw} (\mathbf{E} \times \mathbf{H^*}).\mathbf{ds} = (1/2) \int_0^{\varphi} \int_0^{\infty} (E_r H_\varphi^* - E_\varphi H_r^*) r dr d\varphi \tag{5.15}$$

where S_w is cross sectional area of the wedge waveguide. The results for P and P_{lc} are given by (B.21) and (B.22), respectively, in Appendix B.

5.5.2 Dielectric Loss

The attenuation coefficient α_d, due to dielectric losses, is obtained from $\alpha_d = P_{ld} /2P$, where P_{ld}, is the power dissipated per unit length of the dielectric materials and P is the power flow given in (B.16). P_{ld} is obtained from

$$P_{ld} = (1/2)\int_{Sw} \sigma_d |\mathbf{E}|^2 \, ds = (1/2)\int_0^{\varphi} \int_0^{a} \sigma_d(|E_r|^2 + |E_\varphi|^2 + |E_z|^2) r\,dr\,d\varphi \qquad (5.16)$$

where σ_d is the conductivity of the dielectric and equals σ_{d1} in the core and σ_{d2} in the cladding. The expression for P_{ld} is given by (B.23), in Appendix B.

5.6 NUMERICAL RESULTS AND DISCUSSION

Propagation properties, including axial propagation constant, cutoff frequency, conductor loss, dielectric loss, and frequency range for single-mode operation, are evaluated for the fundamental mode of several wedge-shape dielectric waveguides with conducting boundaries. A typical value of 2.25 is chosen for the dielectric constant of the core material (ε_{r1}), and the cladding is assumed to be air with $\varepsilon_{r2} = 1$. To make the results less parameter specific, the following normalized quantities are defined:

Normalized frequency: $ak_o = a\omega(\mu_o\varepsilon_o)^{1/2}$ (5.17)

Normalized propagation constant: $b = (\beta^2/k_o^2 - \varepsilon_{r2})/(\varepsilon_{r1} - \varepsilon_{r2})$ (5.18)

Normalized conductor loss: $\overline{\alpha}_c = (a\varphi_o/R_s)\alpha_c$ (5.19)

Normalized dielectric loss: $\overline{\alpha}_d = (1/\sigma_d)\alpha_d$ (5.20)

The normalized propagation constant is calculated by solving the characteristic equation (5.5) using numerical techniques. Figure 5.2 illustrates variations of normalized propagation constant versus normalized frequency for the fundamental modes of waveguides with wedge angle $\varphi_o = \pi$, $2\pi/3$, and π/n; $n \geq 2$. For the first two values of φ_o the fundamental mode is the first HE type mode, whereas for $\varphi_o = \pi/n$; $n \geq 2$, TE_{01} is the

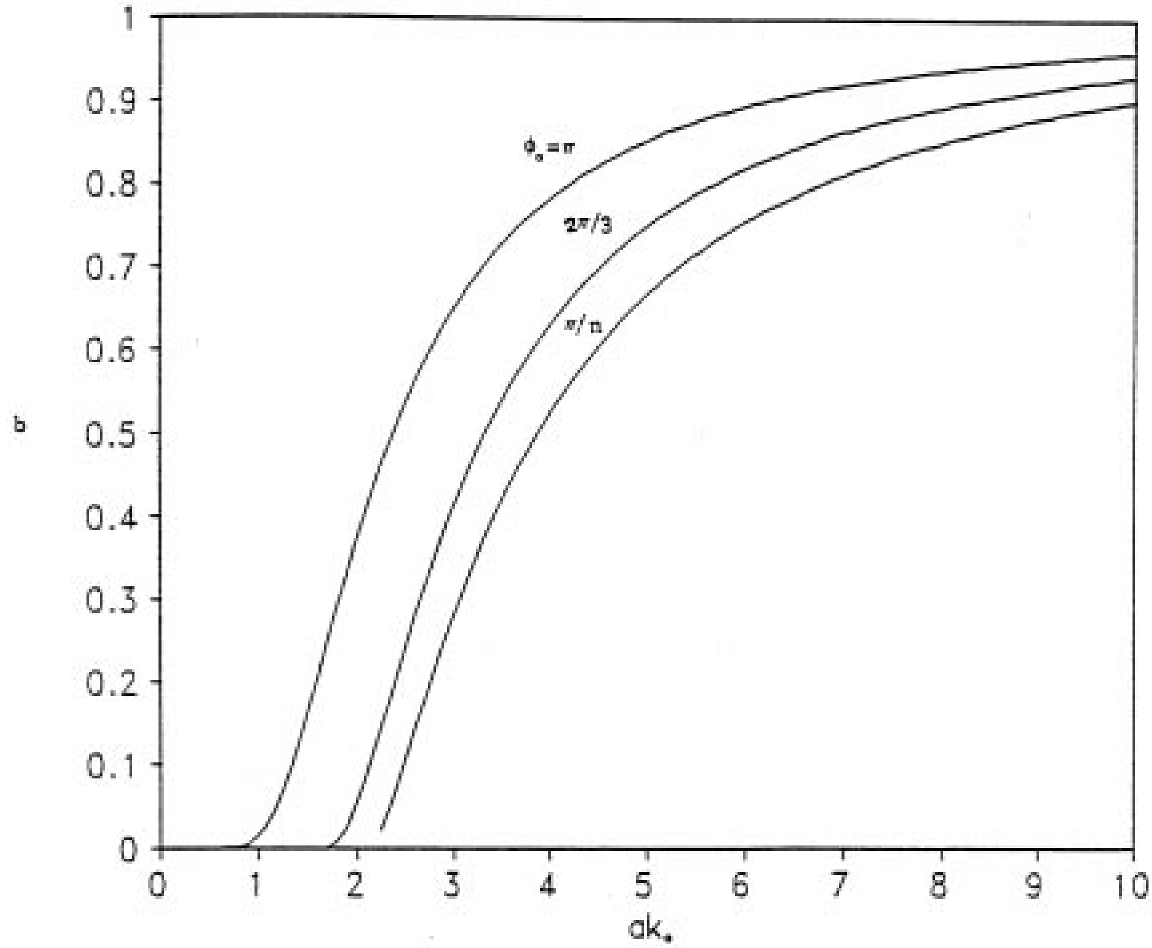

Figure 5.2 Normalized propagation constant, b, versus normalized frequency, ak_o, for the fundamental modes of wedge-shape dielectric waveguide with $\varphi_o = \pi$, $2\pi/3$, and π/n; $n \geq 2$, and with $\varepsilon_{r1} = 2.25$, and $\varepsilon_{r2} = 1$. The fundamental modes for these values of φ_o are HE_{11}, $HE_{3/2,1}$, and TE_{01}, respectively.

Table 5.1 Single-Mode Frequency Range For Different Wedge Angles.

Wedge Angle (φ_o)	**Fundamental Mode**	**Single-Mode Frequency Range $ak_o = a\omega(\mu_o\varepsilon_o)^{1/}$**
π	HE_{11}	$0.626 \leq ak_o \leq 2.151$
$2\pi/3$	$HE_{3/2,1}$	$0.914 \leq ak_o \leq 2.151$
$\pi/2$	TE_{01}	$2.151 \leq ak_o \leq 2.501$
$\pi/3$	TE_{01}	$2.151 \leq ak_o \leq 3.832$
$\pi/4$	TE_{01}	$2.151 \leq ak_o \leq 4.937$

fundamental mode. Only the HE mode of waveguide with $\varphi_o = \pi$ has a theoretical zero cutoff frequency, but as is evident from Figure 5.2 this mode exhibits a practical cutoff value of $ak_o \approx 0.6261$.

To determine the frequency range for single-mode operation, the cutoff frequency of the second mode is needed. For waveguides with $\varphi_o = \pi$ and $2\pi/3$, the second mode is TE_{01} with a cutoff normalized frequency of $ak_{oc} = 2.151$, while for waveguides with $\varphi_o = \pi/n$; n = 2, 3, and 4, the second modes are HE_{21}, HE_{31}, and TE_{02} with cutoff frequencies of ak_{oc} = 2.501, 3.832, and 4.937, respectively. Table 5.1 summarizes the single-mode frequency range for the above values of wedge angle. The maximum possible frequency range for single-mode operation that can be obtained for a wedge-shape waveguide is limited to the cutoffs of TE_{01} and TE_{02} modes, which is $2.151 < ak_o < 4.937$. This is achieved when no other modes, lying between TE_{01} and TE_{02} modes, can be excited. Thus, compared to a hollow circular waveguide for which the single-mode frequency range is $1.841 < ak_o < 2.151$ [116], wedge-shape waveguides can be operated at higher frequencies or may have larger dimensions. For frequencies larger than 100 GHz, the diameter of a hollow circular waveguide becomes excessively small. The use of wedge-shape waveguide may relax this limitation.

Figures 5.3, 5.4, and 5.5 show variations of normalized attenuation coefficients $\overline{\alpha}_c$ and $\overline{\alpha}_d$ versus normalized frequency for the fundamental modes of waveguides with wedge angle $\varphi_o = \pi$, $2\pi/3$ and π/n, $n \geq 2$. It is noted that the attenuation curves in these figures exhibit similar behaviors. Near cutoff frequencies both conductor and dielectric loss coefficients rise sharply, reaching a maximum, and leveling off above the cutoffs for $ak_o > 5$. Examination of the actual conductor attenuation coefficient α_c, reveals that the conductor loss depends on the wedge angle φ_o in two ways; (i) directly proportional to φ_o^{-1} through the power flow given by (B.16) and, (ii) implicitly through the dependence of fields on φ_o. The dielectric loss coefficient α_d, on the other hand, might be influenced by the wedge angle only through the dependence of fields on φ_o. For the special case of $\varphi_o = \pi/n$, n an integer, the dielectric losses of all modes are independent of the wedge angle, while their conductor losses are simply inversely proportional to φ_o. This is because for this special case the azimuthal number ν is an integer and modal fields, when they exist, are independent of the wedge angle. For TE_{0p} modes, this property is maintained for waveguides with arbitrary wedge angles. Accordingly, the $\overline{\alpha}_d$ curve in Figure 5.4 can also be used to evaluate the dielectric loss of TE_{01} modes when $\varphi_o = \pi$, $2\pi/3$, or any arbitrary value.

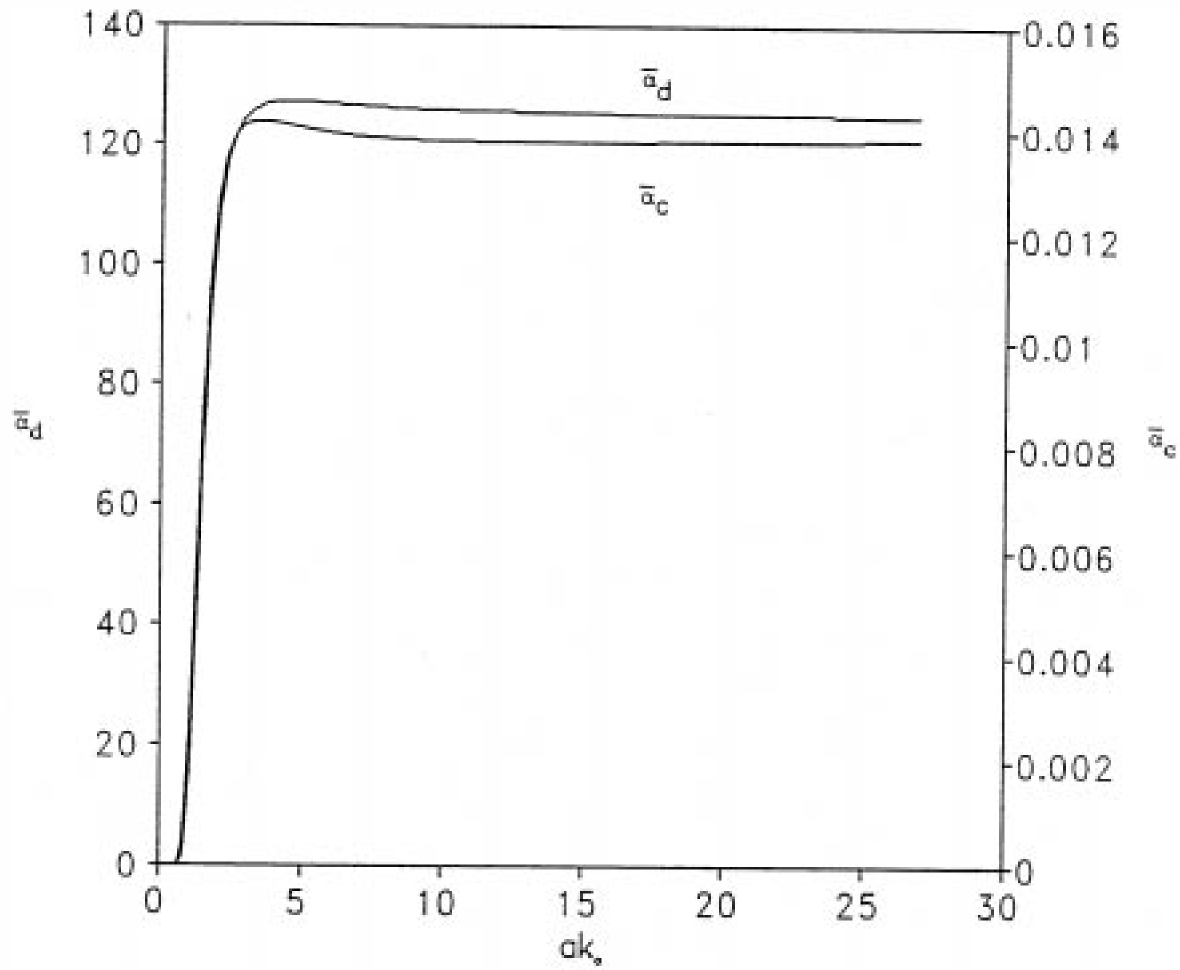

Figure 5.3 Variations of normalized attenuation coefficients of conductor, $\overline{\alpha}_c$, and dielectric, $\overline{\alpha}_d$, versus normalized frequency, ak_o, for the HE_{11} mode in a wedge-shape dielectric waveguide with $\varphi_o = \pi$, and $\varepsilon_{r1} = 2.25$, and $\varepsilon_{r2} = 1$.

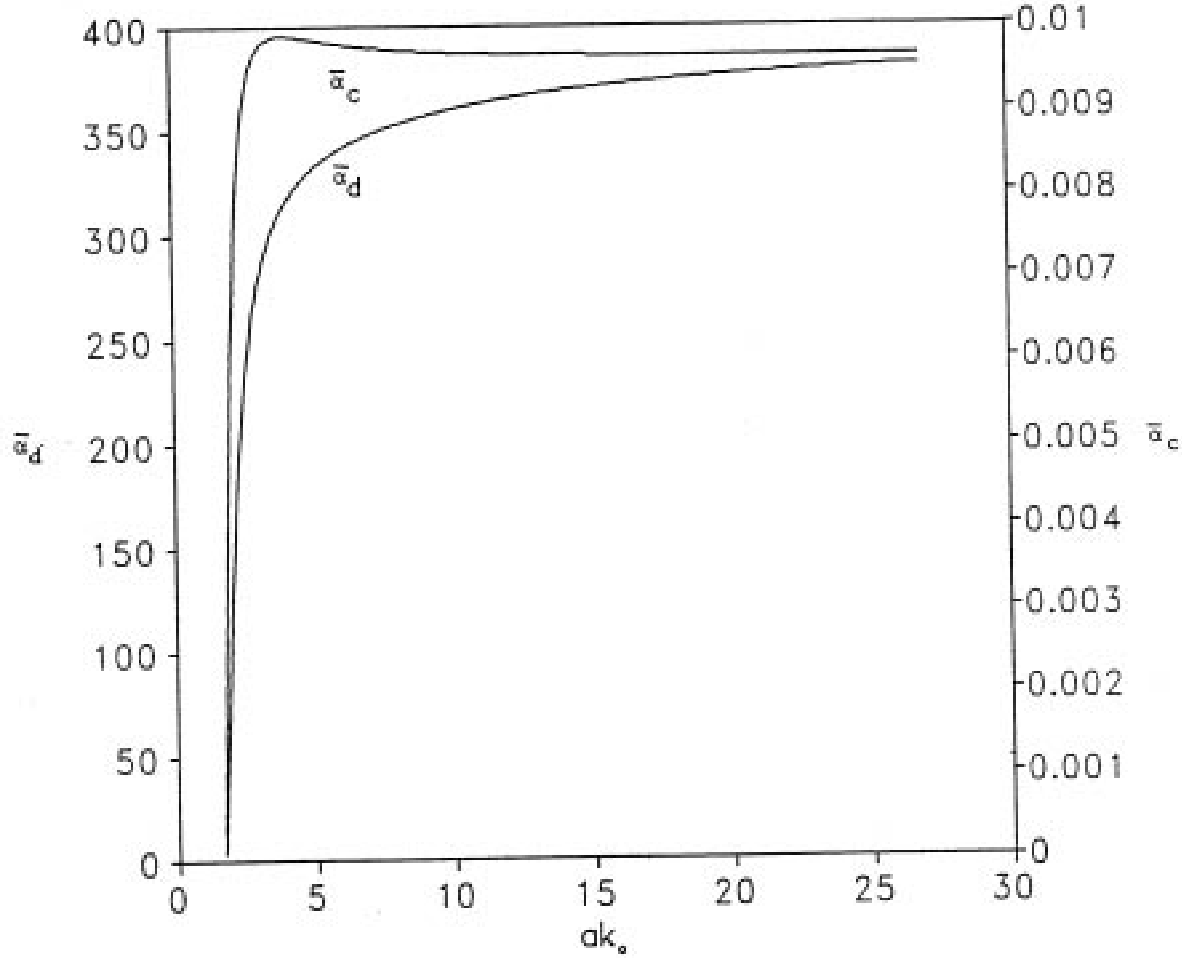

Figure 5.4 Variations of normalized attenuation coefficients of conductor, $\overline{\alpha}_c$, and dielectric, $\overline{\alpha}_d$, versus normalized frequency, ak_o, for the $HE_{3/2,1}$ mode in a wedge-shape dielectric waveguide with $\varphi_o = 2\pi/3$, and $\varepsilon_{r1} = 2.25$, and $\varepsilon_{r2} = 1$.

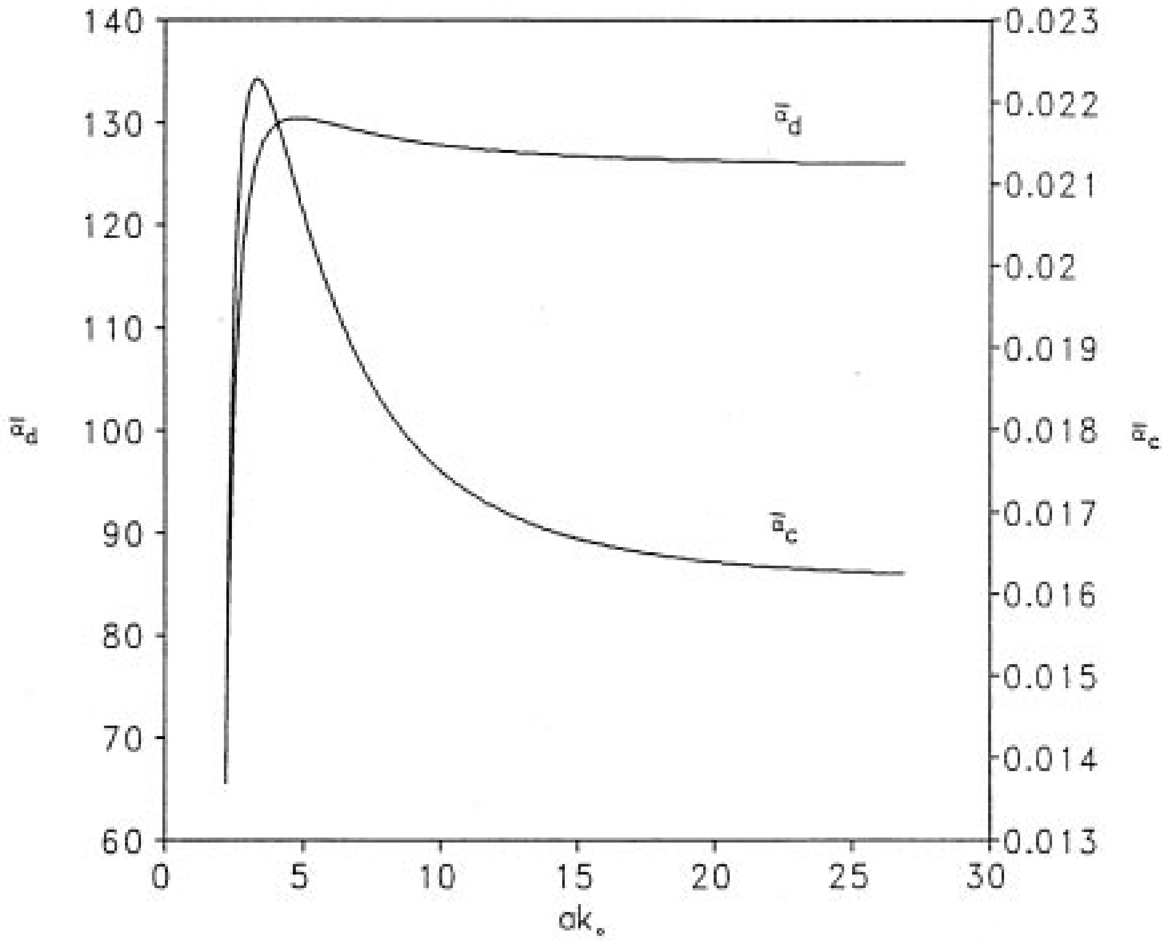

Figure 5.5 Variations of normalized attenuation coefficients of conductor, $\bar{\alpha}_c$, and dielectric, $\bar{\alpha}_d$, versus normalized frequency, ak_o, for the TE_{01} mode in a wedge-shape dielectric waveguide with $\varphi_o = \pi/n$; n an integer, and $\varepsilon_{r1} = 2.25$, and $\varepsilon_{r2} = 1$.