CHAPTER 2
REVIEW OF LITERATURE

During the first half of the 20th century, the relations used in designing irrigation canals and describing the geometry of channels were purely empirical in nature. It was only in the 1950's that investigators started to approach the subject from an analytical perspective.

Glover and Florey (1951) derived the geometry of a channel whose boundary is composed of particles that are all on the threshold of motion. They considered the balance of forces acting on a given particle on the channel boundary, and specified that the fluid shear stress acting on the particle is equal to the critical stress at that point, i.e., the stress at which the particle is on the verge of motion. Theoretically, the resulting threshold channel cross-section is the optimum shape for a given discharge, since it has the minimum cross-sectional area that a channel can have without erosion of the boundary material occuring. Their derivation led to a continuously curving channel boundary which is described by a cosine function. This is the classic 'cosine' profile found in the literature of open channel flow (e.g. Henderson, 1966; Raudkivi, 1990).

Parker (1978) noted that the derivation of the cosine-profile of a threshold channel did not take into account the effect of lateral momentum-diffusion. Because of cross-channel velocity gradients, momentum is diffused from the center of the channel toward its banks. This causes a redistribution of stresses along the boundary and results in a shape different from the cosine profile. He developed an expression for momentum-diffusion based on work done by Lundgren and Jonsson (1964), which gives the value of shear stress τ exerted by the fluid at a given point on the boundary of the channel. Parker suggested that the shear stress can be expressed as

$$t = rgS\frac{dA}{dP} - \frac{d}{dP}\left[\int_0^{D_n} r\overline{u'v'}dz + \int_0^{D_n} rUVdz\right] \qquad (1)$$

where

ρ = mass density of water
g = acceleration due to gravity
A = cross-sectional area
D_n = normal depth
P = wetted perimeter
U = mean downstream velocity
u’ = fluctuating downstream velocity
V = mean cross-stream velocity
v’ = fluctuating cross-stream velocity
z = normal distance from channel bed

The first term on the right-hand side of Equation 1 is the shear stress value given by the area method. This has to be corrected for the effects of lateral momentum-diffusion (second term) and secondary currents (third term). Lundgren and Jonsson suggested that the effect of secondary currents is the weaker of the two and may be neglected in a first-order analysis. Tominaga, Nezu, Ezaki, and Nakagawa (1989), among others, have observed that in the case of straight channels, the maximum velocity of secondary currents does not exceed 1.5% of the maximum downstream velocity. Thus, the simplification is justified. Furthermore, in order to achieve closure, Lundgren and Jonsson used two assumptions, with the limitation that the channel have small lateral bed curvature. The first assumption is that surfaces of vanishing shear stress and momentum flux are orthogonal to the isovels (Leighly, 1932). The second is that the logarithmic rough-wall law for flat-beds and pipes applies to the entire flow along normals to the bed (Keulegan, 1938). Diplas and Vigilar (1992) numerically solved this momentum-diffusion equation in conjunction with the force-balance equation developed by Ikeda (1982). The new stress distribution monotonically decreases as one proceeds from the center of the channel to the water margin. The resulting profile is still one that is continuously curving, but is now described by a fifth degree polynomial. Figure 2.1 shows this profile, and the corresponding actual and critical stress distributions. The two distributions coincide at each and every point; satisfying the condition for a threshold channel. Figure 2.2 shows the cosine profile and its corresponding actual and critical stress distributions. It can be seen that actual stress exceeds critical stress in part of the boundary, and is below critical stress over the rest of the boundary. This indicates that a cosine profile is unstable, i.e., it will experience erosion. It is therefore not representative of a threshold channel.

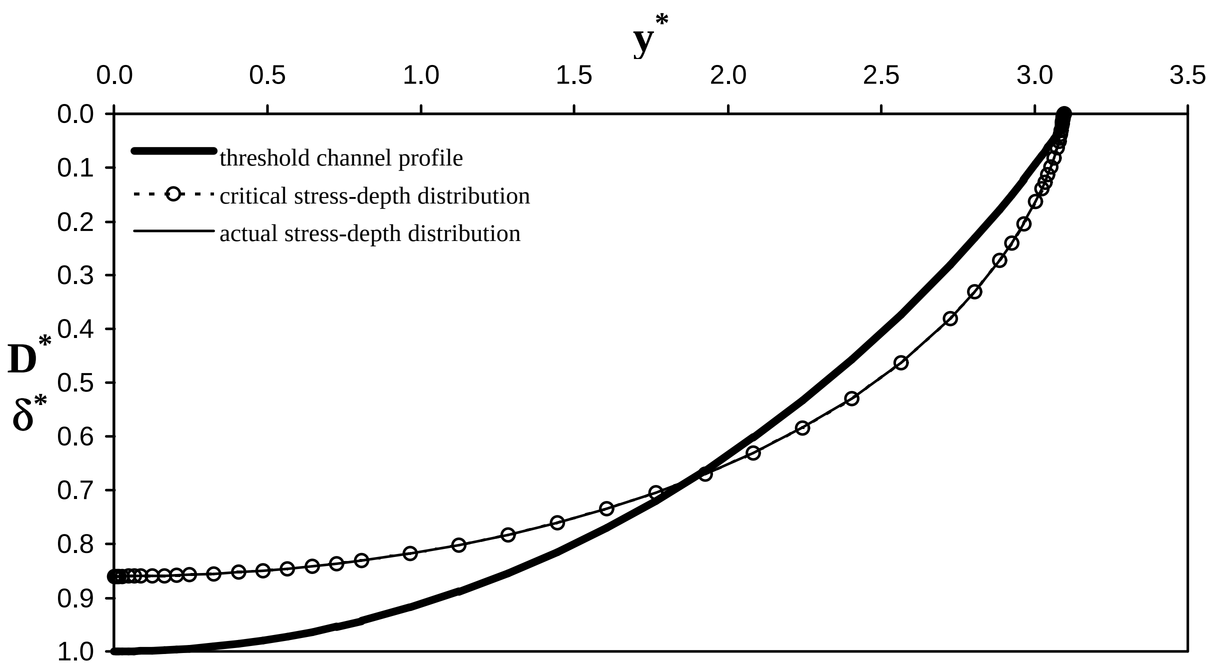

Figure 2.1. Threshold channel profile and stress distribution

It was also shown that for a particular sediment mixture, the cross-sectional area of the threshold channel is 2.2 times larger than that of a cosine cross-section, and conveys a discharge 2.33 times greater (Vigilar and Diplas, 1992). Figure 2.3 is a visual comparison of the two cross-sections. It can be seen that accounting for momentum-diffusion has a significant effect on the geometry of a threshold channel.

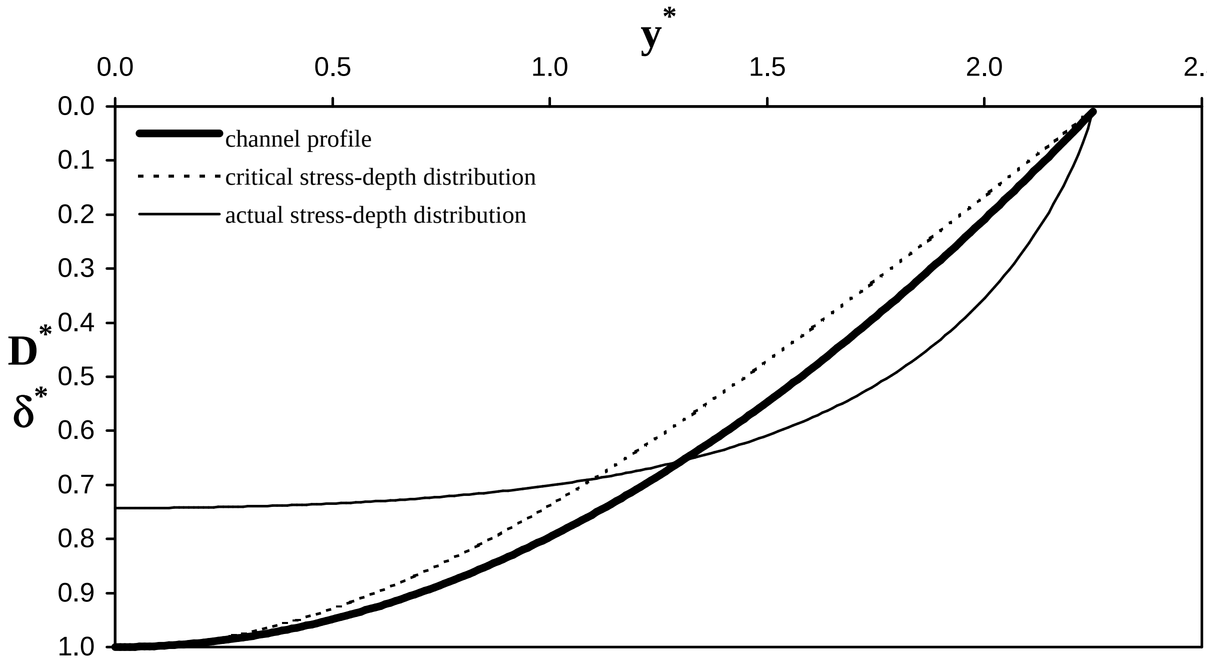

Figure 2.2. Cosine channel profile and stress distribution

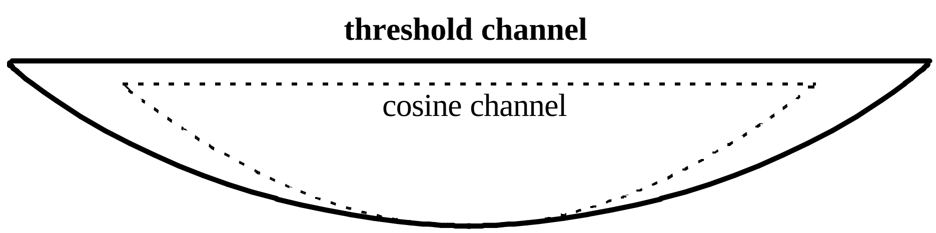

Figure 2.3. Comparison between threshold channel profile based on momentum-diffusion approach and corresponding cosine channel profile

The problem with the threshold channel is that it does not adequately represent most rivers. Natural rivers and streams are observed to transport sediment not only along the bed, but in suspension as well (e.g. Kellerhals, 1967; Parker, 1978). Among others, Wolman and Brush (1961) and Stebbings (1963) have demonstrated in laboratory flumes that stable channels are capable of transporting sediment. Some rivers are composed of sediment of such coarseness that their suspension is precluded. However, they are still observed to transport considerable amounts of sediment along their beds while maintaining stable banks. Threshold channels, on the other hand, cannot support sediment motion since all particles along its boundary are static. Its continuously curving shape is incompatible with sediment transport.

To see this, assume that a threshold profile can be used to represent the cross-section of either river. If this were so, there would have to be a small area at the center of the channel over which the stress is slightly above critical, to allow bedload motion in that small area. Since particles on the channel boundary are subjected not only to the force of the flowing water but also to the force of gravity (which has a lateral component directed down the bank slope), the particles set in motion will not only tend to move downstream but toward the center of the channel as well. This triggers erosion of the channel boundary and results in a widening of the channel. As long as there is bedload motion, the channel will continue to widen (Parker, 1978). This is contrary to the observation that rivers maintain stable banks while supporting sediment transport.

On the other hand, if the channel were stable, the sediment load would have to vanish. This was first explained by Hirano (1973) and is supported by Stebbings' (1963) experiments which showed that an initially trapezoidal cross-section eventually assumed a continuously curving threshold channel profile when it was starved of sediment load.

Thus, a threshold channel is incompatible with sediment transport. Assuming that a threshold channel adequately represents natural rivers would mean that these rivers cannot be stable and support sediment transport at the same time, when in fact, such behavior is observed so often in nature. This contradiction is known as the 'stable-channel paradox.'

A more realistic representation of gravel and sand-silt rivers would be a cross-section with a flat-bed region, attached to two curving banks (Figure 2.4). For such a geometry, the stress distribution that results from consideration of momentum-diffusion is still monotonically decreasing from the channel center to the water margin. The difference is that now, it is possible for the fluid shear stresses to be above critical over the flat-bed region, equal to critical stress at the junction point of the flat-bed and bank regions, and at or below critical stress over the entire bank region. This allows sediment motion but confines it to the flat-bed region, while maintaining stability in the bank region; the 'stable bank, mobile bed' phenomenon observed in so many rivers. It is the consideration of momentum-diffusion coupled with this 'curved bank, flat bed' geometry that leads to the resolution of the 'stable-channel paradox.'

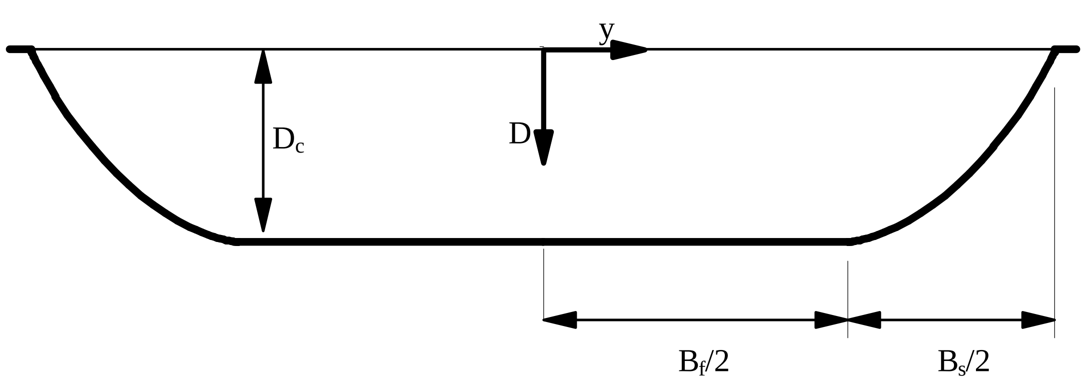

Figure 2.4. Definition diagram

An optimal stable channel is a channel whose banks experience critical stress everywhere (Note that the actual shear stress decreases as one proceeds from the junction point to the water margin, however, the value of critical stress decreases as well. The objective will be to determine the bank shape such that the critical and actual shear stresses decrease at the same rate). The determination of its geometry will not be a trivial matter of attaching just any flat-bed width to banks which conform to the shape of a threshold channel. For channels that are not sufficiently wide, the presence of the flat-bed region will cause less momentum to be diffused toward the banks. This means that the banks, which experience critical stresses for the threshold channel case, would now experience stresses which are below critical. They are thus expected to be narrower. It can be expected that within a certain flat bed-width range, the shape of the banks will vary depending on the amount of momentum that is laterally diffused from the flat-bed region.

Several investigators have developed models for predicting the geometry of optimal stable channels. Pizzuto (1990) and Kovacs and Parker (1994) have developed models that generate equilibrium channels by assuming initial channel profiles, and then simulating their evolution with time. Parker (1978) came up with a model whose formulation incorporates the effects of momentum-diffusion, and used singular perturbation techniques to obtain a stable channel geometry. He assumed that a cosine profile would sufficiently represent the banks of his stable channel (the bank solution). He then derived the stress distribution over the flat-bed region (the bed solution) using singular perturbation techniques; matching bed and bank solutions by requiring that at the junction point, stress and its gradient be continuous.

Although Parker assumes that momentum-diffusion occurs over the entire channel region and accounts for it in his governing equations, his choice of the cosine profile to approximate the stable channel's bank shape seems to be inconsistent with this assumption. Recall that for the threshold channel case, the cosine profile was derived when the effects of momentum-diffusion diffusion were neglected. Inclusion of momentum-diffusion in the formulation revealed the cosine shape to be unstable when considering such a channel. It is reasonable to suspect that a cosine bank profile would be unstable for this 'stable bank, mobile bed' case as well.

The present work, consistently accounts for momentum-diffusion, and derives optimal stable channel profiles directly, by solving the governing equations at equilibrium. The bank profile and stress distribution are determined by solving the coupled equations of fluid momentum-diffusion and particle force-balance (the bank solution). The width of the flat-bed region is determined by solving the fluid momentum-diffusion equation over the flat-bed region (the bed solution), while making sure that matching conditions at the junction of the flat-bed and bank regions are satisfied. Once the channel geometry has been determined, the complete channel boundary shear stress distribution will have been determined as well.

It must be noted that the emphasis of this work is on channels that convey sediment of such size that their suspension is precluded. The case of a channel that involves sediment transport over its bed as well as in suspension will be discussed in the penultimate chapter.