Chapter 2

Signal Detection in Laser-Based Systems

The main objective of this dissertation is to optimize system performance at the photodetection end. Since the optical signal is typically at its weakest power level when it arrives at the receiver, it is important to account for the various noise mechanisms which affect the signal-to-noise ratio (SNR) and degrade the fidelity of the communicated waveform. This chapter presents a brief discussion on the detection and amplification schemes commonly used in optical fiber systems. System performance, as characterized by the average receiver sensitivity (in photons/bit) is analyzed for both On-Off Keying (OOK) and Frequency-Shift Keying (FSK) transmission, for both PIN and optical preamplifier receivers. This analysis provides us with a reference with which to compare the performance of noise-like, spectrum-sliced systems, provided in later chapters.

2.1 Optoelectronic Receivers

The basic role of the photodetector is to convert the incident optical signal to its electrical counterpart for recovering the data communicated through the optical system. The resulting electrical current, known as the primary photocurrent I_s , is directly proportional to the incident optical power level P_s [1]. That is

$$I_s = P_s \tag{2.1}$$

where the constant of proportionality is the responsivity of the photodetector, in units of amperes/watt. The other fundamental quantity that is used in conjunction with photodetectors is the quantum efficiency () which represents the photon to electron conversion efficiency of the

photodetector and is defined as the ratio of the electron-generation rate for a given photon incidence rate [1].

$$=\frac{I_s/q}{P_s/h}=\frac{h}{q} \tag{2.2}$$

where h is the photon energy and q is the electronic charge. The quantum efficiency and the responsivity are thus related through Eqn. (2.2); typical quantum efficiencies are on the order of 60-70%.

Photodetectors fabricated from semiconductor materials such as InGaAs are exclusively used in practical lightwave systems because of their high quantum efficiency and sensitivity, fast response, low-noise, small size comparable with fiber dimensions, and their compatibility at the 1.55 µm centered lowest attenuation window of the fiber. Commercially available optical receivers include PIN (positive-intrinsic-negative polarity of constituent semiconductor materials) photodiodes, and avalanche photodiodes (APD) which realize photon to electron conversion through the avalanching mechanism. APDs possess higher effective responsivities than PIN diodes because the avalanching mechanism acts like a chain reaction to provide an internal gain through an electron-level phenomenon known as impact ionization.

Fig. 2.1 illustrates the different types of receivers commonly used in optical fiber systems [32]. Detection in the system can be incoherent, referred to as direct detection, or coherent which requires the presence of a laser diode local oscillator at the receiver, similar to radio communication. Direct detection is used in most practical systems since it is difficult to generate a stable carrier oscillator (laser) at the receiver which is matched with that at the transmitter. Fiber systems employ direct detection with amplitude shift keying (ASK) commonly called on-off-keying (OOK) or Intensity Modulation (IM) since the laser is turned 'on' to transmit a '1' and turned 'off' to indicate a '0.' The detection process is actually the measurement of energy by an integrate-and-dump electronic filter with integration time *T* and a bandwidth which is given by the reciprocal data rate. Coherent detection systems, on the other hand, are based on boosting the weak transmitted signal by combining it with the strong signal from the local oscillator. Due to the non-linear nature

of the photodetector, a strong beat signal is formed at the difference frequency. Depending on the value of this intermediate frequency (IF), coherent systems are referred to as homodyne (IF = 0) or heterodyne (finite IF). Although coherent receivers require a lower SNR for a given bit-error-rate (BER) performance as compared to direct detection receivers, the latter has been consistently adopted for practical deployment because of its simplicity and lower cost. Moreover the recent advent of all-optical amplification has made the SNR advantage of coherent demodulation insignificant.

2.1.1 Noise limitations in optical receivers

The process of photodetection, like most physical processes, is not ideal and inherently adds some noise to the system. Hence fluctuations in the primary photocurrent are observed even if the incident optical power level remains constant. Although Eqn. (2.1) relates I_s to P_s through a constant, it is true only if I_s is interpreted to be the average photocurrent (during that particular bit interval). The fundamental noise mechanisms responsible for current fluctuations in optical receivers are shot noise (Schottky noise) and thermal noise [1,5].

Shot noise arises due to the statistical nature of photon to electron conversion and also because the photons that arrive at the photodetector do so at random times described by a Poisson distribution. The spectral density $S_{sh}(\)$ of shot noise is assumed to be constant or white, with a noise variance $\frac{2}{sh}$ given by

$$\frac{2}{sh} = S_{sh}()d = 2qI_sB_e,$$
 (2.3)

where B_e is the effective electronic noise bandwidth of the receiver and q is the electronic charge. Sometimes the shot noise term is augmented with another fluctuating term, known as dark current shot-noise, which results from stray light incident on the detector and/or spurious thermally-generated electron-hole pairs. Shot noise puts the upper (quantum) limit on the best possible receiver sensitivity.

Practical receivers include electronic components for the amplification and demodulation of the weak optical signal that is photodetected. Random thermal motion of the conductors in these electronic components manifests itself as a fluctuating noise component even in the absence of any incident optical power. This noise component, referred to as thermal noise or Nyquist noise, severely degrades the detection sensitivity of the optoelectronic receiver, and in most practical cases dominates over shot noise. For fiber-based systems, the thermal noise is nearly white (up to 1 THz) and is usual modeled as a stationary Gaussian random process with a variance of [33]

$$c_{th}^{2} = S_{th}()d = (4V_{T}qB_{e}/Z) = 8 V_{T}C_{T}qB_{e}^{2}$$
 (2.4)

where $S_{th}(\)$ is the corresponding power spectral density, V_T is thermal potential (product of Boltzmann's constant and the temperature (300 K) divide by the electronic charge), Z the effective noise impedance, and C_T the effective noise capacitance of the receiver related to Z by [33]

$$C_T = \begin{pmatrix} 1/2 & ZB_e \end{pmatrix}. \tag{2.5}$$

For example, with $C_T = 0.1$ pF and $B_e = 2.5$ GHz, $_{th} = 255$ nA.

2.1.2 Normalized Receiver Sensitivity

The performance metric for an optoelectronic receiver is the minimum amount of optical power needed to distinguish between the reception of a mark (1) or space (0) bit [5]. The random noise in the system causes the detection process to commit an occasional error which is specified in terms of the bit-error-rate (BER). Typical BERs that are achievable with optical fiber systems are better than 10^{-9} ; thus the receiver sensitivity is usually specified in terms of the average power required to maintain a BER of 10^{-9} (1 error per billion bits received). Although the SNR value is an important parameter while assigning the power budget of the system, it is dependent on the data rate and system-dependent noise levels. A normalized performance metric that is commonly used to specify receiver sensitivity is in terms of the number of photons required to make a confident decision, per bit interval. The sensitivity in photons/bit is related to the received optical power by

$$N_p = \frac{\text{photons}}{\text{bit}} = \frac{\text{energy/sec}}{(\text{energy/photon})(\text{bits/sec})} = \frac{P_s}{(h)R_p}$$
 (2.6)

where R_b is the bit rate. In most cases, the *average* sensitivity (\overline{N}_p) , rather than peak sensitivity, is specified. Expressing Eqn. (2.6) in terms of its average value, and using Eqn. (2.2) to relate sensitivity in terms of the photocurrent we get

$$I_s = 2\overline{N}_p qR_h. (2.7)$$

The factor of 2 on the RHS is because the average receiver sensitivity for OOK demodulation is half that of the peak sensitivity since the laser is turned off while transmitting a 0 bit. This factor is, however, absent in frequency-shift-keyed (FSK) fiber systems since the average sensitivity equals the peak sensitivity.

In the quantum limit, \overline{N}_p equals 10 photons/bit to satisfy a BER of 10^{-9} [1]. In most practical receivers, however, thermal noise dominates and the required receiver sensitivities are on the order of a few thousand photons/bit, as shown in Section 2.3.

2.2 Signal Detection

The major purpose of the photoreceiver is to detect the signal that is being communicated. A general figure of merit of how well the receiver can do this task is the received electrical SNR. Although the received SNR is a meaningful quantity in analog systems, digital systems rely on the finite probability of error that characterizes the detection and estimation mechanism [5]. In digital systems, the signals are referred to as mark (1) and space (0), to account for the different modulation formats that are commonly used. Modulation formats that have been demonstrated with direct detection include amplitude shift keying (ASK) where the laser is turned on and off to transmit a 1 or a 0, respectively (hence the alternate term on-off keying), frequency shift keying (FSK) where two different frequencies are used to represent a 1 and a 0, and differential phase shift keying (DPSK) in which the data is differentially encoded on to two phase-differing carriers. In this dissertation we are primarily concerned with OOK and FSK modulation. This section presents

analytical expressions for calculating the receiver sensitivity required for maintaining a particular bit-error-rate (BER) performance. Expressing the receiver sensitivity in terms of average photons/bit helps in the performance comparison of the different modulation formats, in terms of the average power they require at the receiver.

The direct detection receiver was introduced in Section 2.1; a schematic is shown in Fig. 2.2. To briefly summarize its functionality - the receiver consists of the photodetector (PIN or APD), which converts the optical data into electrical pulses. The electrical pulses, after an optional preamplification stage, are passed through an electronic integrate-and-dump filter of bandwidth B_e . This circuit acts as a low-pass filter and integrates the bit energy over the bit period. In an OOK system, the integrated energy is then compared with some reference value, and depending on whether the energy is greater or less than the reference value, a decision is made as to whether the transmitted signal was a 1 or a 0, respectively. FSK and DPSK signals, on the other hand, are first converted to OOK by using appropriate filters or interferometers, and then detected following the usual OOK procedure.

2.2.1 Probabilistic Analysis for Digital Lightwave System Performance

As mentioned before, evaluation of the error probability is an appropriate performance metric for digital direct detection systems. Using the standard axioms developed for a maximum *a posteriori* (MAP) receiver the error probability P_e may be defined as

$$P_e = \Pr(0)\Pr(1|0) + \Pr(1)\Pr(0|1)$$
(2.8)

where Pr(1) and Pr(0) refer to the *a priori* probabilities of receiving a 1 or a 0, respectively. The conditional probabilities Pr(1|0) and Pr(0|1) refer to the respective probability of falsely detecting the first argument, when the second argument was transmitted. If we assume that the probability distributions of the photoelectron count (n) of symbols 1 and 0 are $P_n(1)$ and $P_n(0)$, respectively, then the conditional probabilities may be specified in terms of the pre-determined decision or threshold level S_{th} as

$$\Pr(1|0) = P_x^0 \tag{2.9}$$

and

$$Pr(0|1) = P_x^{1}. (2.10)$$

If the photoelectron counts are large, the discrete probability distributions may be replaced with continuous probability density functions (PDF), and the summations with integrals. Hence the error probability may be expressed in terms of the threshold level as

$$P_e = \Pr(0) P_x^0(x) dx + \Pr(1) P_x^1(x) dx.$$
 (2.11)

The optimum threshold level minimizes the probability of error and may be obtained by differentiating both sides of Eqn. (2.11) wrt to S_{th} , and we get

$$\frac{P_e}{S_{th}} = 0 = -\Pr(0)P_x^0(S_{th}) + \Pr(1)P_x^1(S_{th})$$
 (2.12)

or

$$\frac{P_x^1(S_{th})}{P_x^0(S_{th})} = \frac{\Pr(0)}{\Pr(1)}.$$
 (2.13)

Hence the optimum threshold depends on both the pdf's of the mark and space bits, as well as their *a priori* transmission probabilities. For equal *a priori* probabilities, the threshold is evaluated by equating the two conditional pdf's.

Since the major source of degradation in photoreceivers is the thermal noise, it is a good approximation to assume that the continuous PDFs are, in fact, Gaussian. This error analysis was first developed by Personick at AT&T Bell Labs in 1973 [34], and is summarized here. The corresponding PDFs are shown in Fig. 2.3. Assuming Gaussian statistics for the signal and noise terms, the conditional probabilities may be expressed as [5]

$$P(1|0) = \frac{1}{\sqrt{2} - \frac{2}{0}} \exp \frac{(\mu_0 - x)^2}{2 - \frac{2}{0}} dx$$
 (2.14)

and

$$P(0|1) = \frac{1}{\sqrt{2} - \frac{2}{1}} \exp \frac{(\mu_1 - x)^2}{2 - \frac{2}{1}} dx$$
 (2.15)

where $\mu_{1(0)}$ is the mean value of the photocurrent when a 1(0) is received and $\frac{2}{1(0)}$ is the noise variance (average squared deviation from the mean) when a 1(0) is received. A change in variables leads to

$$\Pr(1|0) = \frac{1}{\sqrt{2}} \exp -\frac{x^2}{2} dx \tag{2.16}$$

and

$$Pr(0|1) = \frac{1}{\sqrt{2}} \exp -\frac{x^2}{2} dx$$
 (2.17)

where $Q_0 = (S_{th} - \mu_0)/_0$ and $Q_1 = (\mu_1 - S_{th})/_1$. Now minimizing the expression for error probability, as given by Eqn. (2.11), would involve setting the differential equal to zero in order to calculate the optimal S_{th} . However this is often numerically intractable and an easier technique is to assume that the error probability is minimized when the shaded areas in Fig. 2.3 (conditional probabilities) are equal. Although this yields a sub-optimal estimate, the number obtained is conservative and serves the purpose for most practical cases of interest.

Equating $Q_1 = Q_0 = Q$ leads to

$$S_{th} = \frac{{}_{1}\mu_{0} + {}_{0}\mu_{1}}{{}_{0} + {}_{1}} \tag{2.18}$$

using which the parameter of interest Q, often known as the Personick Q factor, is given as

$$Q = \frac{\mu_1 - \mu_0}{\frac{1}{1} + \frac{1}{0}} . {(2.19)}$$

The BER is then calculated by [5]

$$P_e = \frac{1}{\sqrt{2}} \exp(-\frac{x^2}{2}) dx = \frac{1}{2} \operatorname{erfc} \frac{Q}{\sqrt{2}}$$
 (2.20)

A plot of BER as a function of the Q factor is shown in Fig. 2.4. For example a 10^{-9} error probability means a Q value of approximately 6. The square of the Q factor may be interpreted as the equivalent SNR of the receiver. The second point to note here is that the exact probability distributions for the 0 and 1 symbols are NOT Gaussian if the shot noise is significant. However the Gaussian approximation is generally valid for PIN receivers since the thermal noise (Gaussian random variable) dominates. Moreover, even if the Gaussian approximation were exact, due to reasons mentioned before, Eq. (2.20) yields a sub-optimal estimate. Hence it should be kept in mind that the receiver sensitivity calculated in the following sub-sections is conservative, and the use of exact probability distributions will provide a better estimate.

2.2.2 Shot Noise/Quantum Limit to Receiver Sensitivity

It is often instructive to develop fundamental bounds on the best system performance achievable. This not only helps the designer compare system performance with the ideal limits, but also aids in setting reasonable limits on efforts to improve performance. Shannon's laws of information theory are one such example [32]. Following the same line of thought, this section briefly summarizes the analysis for the ideal lightwave receiver sensitivity, often referred to as the quantum or the shotnoise limit.

i) Shot-noise-limited receiver sensitivity for OOK transmission

Photons arrive at the receiver at an average rate per second of [32]

$$r = N_p R_b. (2.21)$$

Assuming randomness in the *arrival times* of such photons, their arrival rate may be characterized using the Poisson distribution as

$$Pr(N) = \frac{(rT)^N \exp(-rT)}{N!}$$
(2.22)

which gives the probability of arrival of exactly *N* photons arriving at the photodetector, in time *T* seconds. This idealized OOK system using a photon counting receiver can then be constructed by transmitting light during a 1 state, and no light during a 0 state. The probability densities are hence discrete consisting of impulses with the decision threshold set between 0 and the first impulse. As explained by Green [32], this photon counting receiver, reliably estimates a single photon arrival to correspond to the 1 state. However when no photons are detected, it may not only be due to a 0 transmission, but also due to the erroneous situation of a 1 transmission with the received light being so weak so as being incapable of contributing any photons. Hence no error is made during a photon detection, but a finite error probability is associated with no-photon detection. Thus

$$P_{e} = \frac{1}{2}(0) + \frac{1}{2} \Pr(0 \text{ photons arrive, but a 1 bit was sent})$$

$$= \frac{1}{2} \exp(-rT) = \frac{1}{2} \exp(-r/R_{b}) = \frac{1}{2} \exp(-N_{p})$$
(2.23)

For example, an average (\overline{N}_p) of 10 photons are required to maintain $P_e=10^{-9}$ for OOK transmission using an ideal receiver with equiprobable 1 and 0 transmission. (Note that for OOK $\overline{N}_p=N_p/2$).

It is often numerically convenient to assume that the pdf of the received signal and noise at the decision circuit are Gaussian distributed. Although the result obtained is conservative, it often provides a quick insight into receiver sensitivity, and yields very good estimates (in most cases) when the noise in the system increases.

For OOK, in the shot noise limit, the various moments may be written as

$$\mu_{I(1)} = I_s \qquad \qquad \mu_{I(0)} = 0 \tag{2.24}$$

$$\frac{2}{I(1)} = 2qI_sB_e \qquad \qquad \frac{2}{I(0)} = 0 \tag{2.25}$$

Here the subscripts denote the received (1/0) bit. For OOK

$$\mu_{1/0} = \mu_{I(1/0)} \tag{2.26}$$

$$_{1/0} = _{I(1/0)} \tag{2.27}$$

using which the receiver sensitivity may be expressed, assuming unity quantum efficiency, in terms of the Q factor as

$$\overline{N}_p = Q^2 \frac{B_e}{R_b} \,. \tag{2.28}$$

For $B_e = (R_b/2)$, and at $P_e = 10^{-9}$ (Q=6), this gives 18 photons per bit. Comparing this number with the result (10 ph/b) obtained using the Poisson distribution illustrates the conservative nature of the Gaussian distribution.

Another interesting aspect of the above analysis is that randomness may not only be associated with the photon arrival times, but also may be linked to the statistics of photon to electron conversion by the photodetector [35]. Assuming that the photodetector has a quantum efficiency of <1, the receiver sensitivity evaluated above has to be corrected by the corresponding factor. For example for OOK, the sensitivity becomes $\overline{N}_p = 10/$ photoelectrons/bit. This correction is necessary when the decision is made electrically, rather than optically.

ii) Shot-noise-limited receiver sensitivity for FSK transmission

For FSK transmission, light is transmitted in both 1 and 0 states, and hence the photocurrent may be written as

$$I_s = N_p \quad qR_b = \overline{N}_p \quad qR_b \,. \tag{2.29}$$

The error probability, (at different frequencies/wavelengths), is given by

$$P_{e} = \exp(-rT) = \exp(-N_{p}) = \exp(-\overline{N}_{p})$$
 (2.30)

which says that an average of 21 photons are required to maintain $P_e = 10^{-9}$.

Now if we use the Gaussian approximation for FSK, in the shot-noise limit, the various moments may be written as (referring to Eqns. (2.24) and (2.25)

$$\mu_1 = \mu_{I(1)} - \mu_{I(0)}$$
 $\mu_0 = \mu_{I(0)} - \mu_{I(1)}$ (2.31)

and

$$_{1} = _{0} = \sqrt{_{I(1)}^{2} + _{I(0)}^{2}}$$
 (2.32)

using which the receiver sensitivity may be expressed in terms of the Q factor (=1) as

$$\overline{N}_p = 2Q^2 \frac{B_e}{R_b} \,. \tag{2.33}$$

Hence for $B_e = (R_b/2)$, and at $P_e = 10^{-9}$ (Q=6), this gives 36 photons/bit.

2.2.3 Practical/Thermal Noise Limited Photoreceivers

Practical receivers operate far from the ideal limits described in the last section and are the subject of the following discussion. Receiver sensitivities for both OOK and FSK transmission systems are evaluated, assuming the receiver thermal noise dominates over the shot noise components. This is a valid approximation when one takes the ratio of Eqns. (2.4) and (2.3) and uses $I_s = 2\overline{N}_P \ qR_b$, to get

$$\frac{\frac{2}{th}}{\frac{2}{sh}} = \frac{1}{\overline{N}_p} 2 V_T \frac{C_T}{q} \frac{B_e}{R_h}$$
 (2.34)

which using practical values of $C_T = 0.1 \text{ pF}$, = 0.7, and $(B_e / R_b) = 0.5 \text{ yields}$

$$\frac{\frac{2}{th}}{\frac{2}{sh}} \frac{72930}{\overline{N}_p}. \tag{2.35}$$

Hence, in practical receivers, and at low to moderate incident optical power levels at the photodetector, thermal noise dominates over shot noise.

i) Receiver Sensitivity for OOK Transmission

For the On-Off Keying (OOK) modulation scheme under consideration, the mean values may be expressed as $\mu_1 = I_s$, $\mu_0 = 0$ (*i.e.* no optical power transmitted during a zero bit, infinite extinction ratio), where I_s is the received signal photocurrent at the decision circuit. The sources of noise in

the system, as discussed before, are the shot noise and the thermal noise. The corresponding noise variances are

$${}_{1}^{2} = 2qI_{s}B_{e} + 8 V_{T}C_{T}qB_{e}^{2}, (2.36)$$

and

$$_{0}^{2} = 8 V_{T} C_{T} q B_{e}^{2}, (2.37)$$

where q is the electronic charge, B_e is the electrical bandwidth of the PIN receiver, C_T is the receiver noise capacitance, T is the operating temperature, and k_B is the Boltzmann's constant.

Using the statistical moments described above and neglecting the shot-noise component, the Q factor may be written as

$$Q = \frac{I_s}{2\sqrt{8 \ V_T C_T q B_e^2}} \ . \tag{2.38}$$

Hence using Eqn. (2.38) and $B_e = (R_b/2)$, we obtain

$$\overline{N}_p = \frac{Q}{2} \sqrt{\frac{8 \ V_T C_T}{q}} \ . \tag{2.39}$$

Eqn. (2.39) is the major result of this section and it shows the dependence of the receiver sensitivity on the receiver parameters and the bit rate, at a given error probability.

For optical fiber systems, the receiver sensitivity is usually specified at a bit error rate (BER) of 10^{-9} , where Q is approximately 6. Assuming $C_T = 0.1$ pF, = 70% or 0.7, the receiver sensitivity is 2732 photons/bit. It is interesting to compare this thermal noise-limited receiver sensitivity with the shot noise limit of 10 photons/bit. Thermal noise dominates in practical PIN receivers and the best sensitivity that may be obtained is on of the order of a few thousand photons/bit.

ii) Receiver Sensitivity for FSK Transmission

Fig. 2.5 shows a schematic of the receiver that is typically used for a frequency-shift-keyed (FSK) system. It comprises two bandpass filters at different center frequencies for the mark and space.

The outputs of the BPF are passed through square-law detectors (photodetectors) and then through the electrical low-pass integrate-and-dump filters. The output from one of the channels is sign-reversed before it is added to the other channel's output so that the decision is 1 if the sign of the sum of the two channels is positive, and 0 if it is negative.

The receiver sensitivity analysis is similar to that of the on-off-keying case except that here $\mu_1 = I_s$ and $\mu_0 = -I_s$. Moreover the noise variances for the 1 and 0 bits are equal since the system is basically symmetrical (*i.e.* both the channels add noise to the system) and balanced (decision threshold = 0) and may be written as

$${}_{1}^{2} = {}_{0}^{2} = 2qI_{s}B_{e} + 8 V_{T}C_{T}qB_{e}^{2}$$
 (2.40)

The *Q* factor then becomes

$$Q = \frac{I_s}{\sqrt{2(8 \ V_T C_T q B_e^2)}} \ . \tag{2.41}$$

and using Eqn. (2.41) and $B_e = (R_b/2)$, the receiver sensitivity is

$$\overline{N}_p = \frac{Q}{\sqrt{2}} \sqrt{\frac{8 \ V_T C_T}{q}} \ .$$
 (2.42)

Thus assuming C_T =0.1 pF, =70% or 0.7, the receiver sensitivity is 3864 photons/bit...

iii) OOK vs. FSK

It is interesting to compare the receiver sensitivities possible using OOK and FSK transmission in the receivers described above. First of all, FSK will perform poorer, as compared to OOK (using identical receivers), since noise is added during both 1 and 0 transmission. In the shot noise limit, no photons may be produced external to the transmitter and hence only the 1 bit is associated with noise for OOK. This directly implies double the noise for FSK, and hence a 3 dB power penalty. This was observed assuming both Poisson and Gaussian statistics earlier.

However, when the thermal noise case is taken into consideration, current *may indeed be produced* external to the transmitter (by receiver circuitry) which adds noise during the 0 bit in

OOK transmission. This in turn lowers the relative penalty between OOK and FSK, as seen in the earlier section where the relative penalty was calculated to be 1.5 dB.

The other interesting aspect of the above analysis is that there is *no* relative penalty when one compares electrical energy per bit in OOK and FSK systems employing incoherent detection. The reason why we find the penalty in the optical case is because unlike an electrical system, we are interested here in the *optical energy per bit*. The square-law detector results in the electrical energy at the output being proportional to the *square* of the optical energy at the input. Now for the same BER, in OOK the electrical energy in the on-state is 3 dB greater than the electrical bit energy in FSK. This implies that the optical energy is 1.5 dB greater which results in OOK requiring 1.5 dB lower average optical power than FSK.

2.3 Erbium-Doped Fiber Amplifiers

Optical fiber systems are power limited due to the finite attenuation per unit length of the optical fiber cable, as well as the excess loss due to the intermediate optical components. Commercially available optical fiber cable for contemporary 1.55 μ m fiber systems have an average attenuation of 0.2 dB/km. If we assume that the transmitter laser injects 1 mW (0 dBm) of power into the fiber, and the average detection sensitivity of the receiver is 3000 photons/bit, then the power budget for a 1 Gb/s signal is only 34.1 dB. Assigning 5 dB to coupler/splice losses etc., the maximum distance that the link may span is only 29/0.2 = 145 km. (Note that in practical cases, however, the fiber dispersion comes into play and reduces this number). It is, however, clear that some form of signal amplification or regeneration is essential for the system to operate error free over longer distances. The first and second generation lightwave systems operating at 0.8 μ m and 1.3 μ m, respectively, employed periodic electronic regenerators to rejuvenate the signal every 30-50 kilometers. With the deployment of the third generation lightwave systems at 1.55 μ m, which had lower intrinsic attenuation (0.20 dB/km as compared to 0.36 dB/km as at 1300 nm), it was possible to increase this inter-repeater distance by another few tens of kilometers. Future lightwave systems, however, will operate virtually *electronics-free* over transoceanic distances (~ 9000 km). This is being made

possible by the advent of optical amplifiers, which is probably the key enabling technology for next generation fiber optic systems. This section briefly describes the basic principle of operation and applications of optical amplifiers [1,5,6,12].

Optical amplifiers employ the mechanism of stimulated emission to amplify the incident optical power, similar to lasers. The main ingredients of such amplifiers include a gain medium, which exhibits photon emission in the wavelength range of interest, and a pumping mechanism (optical or electrical) which is used to create population inversion and increase the number of excited state electrons. Optical amplifiers fabricated with semiconductor materials have been used in some high speed fiber communication systems and networks. A recent class of fiber amplifiers employs rare-earth ions, such as that of erbium, neodymium, and ytterbium as the gain medium to realize fiber amplifiers operating at different wavelengths covering the visible to the infrared region. Of particular interest among such devices have been erbium-doped fiber amplifiers (EDFAs) because of their properties to operate at 1.55 µm, the wavelength region in which fiber loss is minimum. The major advantages of employing EDFAs, as opposed to electronic regenerators, are as follows:

- Since the amplifier is able to produce extremely high optical gain levels, it is possible to have large (~70-100 km) spacing between consecutive optical amplifier repeaters in the long-haul transmission system. Moreover the amplifier also helps to combat splitting losses at taps and couplers in local access networks.
- Due to the large bandwidth and the slow gain dynamics of amplification, the amplifier is
 virtually transparent to the data rate or format. Hence the installed system may be upgraded
 without changing the intermediate repeater equipment.
- The EDFA provides a broadband, relatively flat, gain spectrum which makes it possible to use multiple wavelengths. Each wavelength acts as an autonomous lightpath and can carry different forms of voice, video, and data traffic, thus providing a rich diversity to the system. EDFAs are an indispensable element of next generation WDM systems.
- Increased availability and reliability offers the possibility of network cost optimization.

2.3.1 Gain Dynamics and Noise Figure

The energy level diagram of erbium ions in silica-based optical fibers and the absorption and gain spectra of an EDFA is shown in Fig. 2.6. Efficient pumping is achieved by using semiconductor lasers operating near 0.98 μ m and 1.48 μ m. As shown in the figure the erbium ions doped in the silica fiber form a three-level laser system. The amplification occurs on the ${}^4I_{13/2}$ ${}^4I_{15/2}$ transition of the erbium (Er $^{3+}$) ions, with a bandwidth of about 40 nm centered around 1.54 μ m. The pumped erbium ions decay to the ${}^4I_{13/2}$ level, which is known as a metastable level since it has a long lifetime of about 10 ms. Co-doping the fiber with ytterbium (Yb), which has a broad absorption band between 900 and 1100 nm allows the use of the 1064 nm line of the Nd:YAG laser or the 1047 nm line of the Nd:YLF laser diodes as pump sources, which have higher power levels as compared to conventional semiconductor 980 or 1480 nm laser diodes.

Recently researchers at Lucent Technologies (Murray Hill, NJ) and SDL (San Jose, CA) demonstrated a three-stage configuration of a diode pumped erbium/ytterbium (Er/Yb) fiber amplifier with a 4W output [36]. Cladding-pumped Nd:YAG fiber lasers at 1064 nm were used as the pump source, with appropriate pump reflectors, for the three stages (co-pumped and counterpumped) of the fiber amplifier.

The major source of noise in EDFAs is due to spontaneous emissions from the metastable and higher energy Er^{3+} states to the lower (ground) state. This amplified spontaneous emission noise (ASE) adds to the signal, thus degrading the signal-to-noise ratio in conventional fiber communication systems. The single-sided, single-polarization spectral density of ASE noise (S_{sp}) is nearly constant (white) and can be written as [1]

$$S_{sp}(\) = (G-1)n_{sp}h$$
 (2.43)

where is the optical frequency, G is the gain of the amplifier, h is Planck's constant, and n_{sp} is referred to as the spontaneous emission factor and is a measure of the incompleteness in population inversion which gives rise to the spontaneous emission $(n_{sp} - 1)$.

An important figure of merit for the fiber amplifier is its noise figure (F_n), which is given by the ratio of the output SNR to the input SNR, and which is related to the spontaneous emission factor by the relation $F_n = 2n_{sp}$. Even if we assume ideal conditions with perfect population inversion (n_{sp} =1), the noise figure penalty is at least 3 dB. Hence the SNR is apparently degraded with the use of the amplifier. The reason for this seemingly degradation of performance (ideal electrical amplifiers have a noise figure of 0 dB) is due to the fact that the noise figure is actually an optical quantity, whereas the SNR is measured electrically [37]. Moreover performance comparison with electrical quantities is based on the assumption that the receiver used to measure SNR is operating in the ideal quantum limit, which is not possible practically. Commercially available fiber amplifiers have noise figures of 4-6 dB [38].

2.3.2 Amplifier Applications

Erbium-doped fiber amplifiers are being deployed extensively in three different configurations -- as power boosters, in-line regenerators, and as preamplifiers at the photodetector [32]. A schematic of the three applications is shown in Fig. 2.7. Power amplifiers are used just after the laser to boost the optical power level that is injected into the optical fiber. The amplifier is operated in saturation to yield high output powers, but which is achieved at the expense of an inferior noise figure. Since the doped fiber is typically several tens of meters long, the pump power decreases along the fiber length. Moreover, when in saturation, an increase in the input signal level drains the erbium ions to increase the output power, which compresses the gain of the amplifier. Hence the pump signal is made to propagate in a direction opposite to that of the signal being boosted, thus providing the maximum gain when the amplified signal just exits the amplifier and enters the long-haul fiber system. Similarly the use of a dual pumping configuration allows larger gains to be achieved. A typical value of output power for the booster amplifier is > + 12 dBm (~16 mW). Isolators are used with the amplifier booster module to prevent any oscillations that may be caused by the reflection of the amplified signal [40].

Amplifiers are also used as in-line repeaters in long-haul fiber systems. For example, the next generation of transoceanic fiber systems (1996) between the US and Europe, known as TAT (Transatlantic Telephone) 12/13, networks will employ EDFAs as in-line amplifiers with gains of 10-16 dB and spaced ~ 50 km apart [41]. Since the input signal is relatively weak when it reaches the in-line amplifier, it is necessary to have a low-noise figure. Moreover, such amplifiers often employ bandpass filters to reduce the spontaneous-spontaneous beat noise which corrupts the system SNR. The optical output power is around + 14 dBm which provides a power budget of 20-24 dB for the fiber segment until the next repeater. Moreover operating the amplifier in the gain compression range provides automatic gain control to the system.

When used as a preamplifier at the receiver end of the link, the EDFA improves the overall receiver sensitivity and permits a SNR that is virtually independent of the receiver thermal noise. Unlike the power amplifier, the optical preamplifier is characterized by a low noise figure and the addition of a band-pass filter to reduce the level of spontaneous-spontaneous beat noise. The filter also aids in rejecting the residual pump power (from the preamplifier pump source) that may cause extra noise at the detector. Hence the forward pumped configuration, as shown in Fig. 2.7, is preferred.

2.4 Photodetection of Optically-Amplified Signals

As described in the previous sub-section, the fiber amplifier not only amplifies the signal but also degrades the system (optical) SNR performance by adding ASE noise to the system. The unmodulated bandpass field term at the input to the photodetector may be written as [37]

$$E(t) = [A + x(t)]\cos\left[ct + (t)\right] - y(t)\sin\left[ct + (t)\right]$$
(2.44)

where x and y are the in-phase (relative to the deterministic signal A) and quadrature phase components of the additive ASE noise, and (t) is the laser phase noise. Assuming the ASE noise to be uniformly distributed over a bandwidth B_o , x and y are baseband processes, each with a bandwidth B_o /2. Since in practical systems the signal polarization is uncontrolled, the output of

the square law detector will contain other terms due to the orthogonal polarizations of the preamplifier ASE. Hence the output of the detector, in units of current, is

$$I = \frac{1}{2} \left[(A + x)^2 + y^2 + \tilde{x}^2 + \tilde{y}^2 \right] + I_{ckt}$$

$$= \frac{1}{2} \left[(A^2) + (2Ax) + (x^2 + y^2 + \tilde{x}^2 + \tilde{y}^2) \right] + I_{ckt}$$

$$= I_s + I_{s-ASE} + I_{ASE-ASE} + I_{ckt}$$
(2.45)

where \tilde{x} , \tilde{y} represent the orthogonal polarizations, and I_{ckt} is the electrical shot noise and thermal noise generated in the receiver circuit. The other terms comprising the last part of Eq. (2.45) are the signal (I_s) and various beat noise terms that arise because of the square-law detection (mixing between signal and noise) process and are referred to as:

 I_{s-ASE} - signal-spontaneous beat noise, and

 $I_{ASE-ASE}$ - spontaneous-spontaneous beat noise.

These beat noise terms, due to the high gain of the fiber preamplifier, dominate over the receiver electrical or circuit noise term, and thus put the upper limit on the best possible receiver sensitivity. Although the spontaneous-spontaneous noise can be minimized by placing a narrow optical filter just after the optical preamplifier, the signal-spontaneous beat noise occurs within the modulation bandwidth of the signal, and hence cannot be minimized.

Now the total electrical noise is given by the sum of the variances (2) of the individual beat noise currents which fall within the electrical bandwidth of the receiver. Assuming the ASE noise to be approximately white over the bandwidth B_o of the optical filter following the preamplifier, and using Eq. (2.43), the average noise current due to the ASE may be written as

$$var(x) = \int_{x}^{2} = n_{sp} \ q(G-1)B_{o} = I_{ASE}/2.$$
 (2.46)

To write the above expression we note that the two phases (x, y) and the two polarizations (\tilde{x}, \tilde{y}) of the ASE noise are independent, zero-mean, and identically distributed Gaussian random variables. The variance in the signal-spontaneous beat noise is then

$$var(I_{s-ASE}) = var(Ax) = {2 \atop s-ASE} = A^{2}[var(x)] (2B_{e}/B_{o})$$
 (2.47)

where the filtering factor $(2B_e/B_o)$ is a consequence of the receiver electrical bandwidth B_e being at least equal to the modulation bandwidth $B_o/2$. From Eq. (2.45), we note that $I_s=A^2/2$, and hence the expression for the variance becomes

$$\frac{2}{s - ASE} = 2I_s I_{ASE} (B_e / B_o). \tag{2.48}$$

Now to derive the expression for the variance of the spontaneous-spontaneous beat noise current we have to evaluate expressions of the form var (x^2) . We do that by [37] noting that for a Gaussian process $var(x^2) = 2[var(x)]^2$, and hence

$$var(I_{ASE-ASE}) = {}^{2}_{ASE-ASE} = var[(x^{2} + y^{2} + \tilde{x}^{2} + \tilde{y}^{2})/2].$$
 (2.49)

Since the various x and y components are identical and independent, the above equation reduces to

$${}_{ASE-ASE}^{2} = 2\text{var}(x^{2}) = 4[\text{var}(x)]^{2}.$$
 (2.50)

Using Eqs. (2.46) and (2.50) we finally get

$${}_{ASE-ASE}^{2} = I_{ASE}^{2} (B_{e} / B_{o}). {(2.51)}$$

2.4.1 Improvement in Receiver Sensitivity

It is relatively straightforward to prove [4,42,43] that the incorporation of an optical preamplifier prior to square-law photodetection results in the enhancement of the signal-to-noise ratio (SNR) at the receiver, thereby decreasing in the average power required at the receiver to satisfy a given error probability. As shown using semiclassical photodetection techniques by Arnaud [42], the maximum achievable SNR when a preamplifier is used in the system is dictated by the effect of the beating of the signal with the spontaneous emission from the preamplifier. Here we briefly

summarize a simple analytical reasoning for the enhancement in SNR by optical preamplification, after [5].

As shown in Section 2.3.1, the noise characteristics of an optical amplifier are represented by its noise figure, F_n , which is always greater than unity and reaches the quantum-limit of 3 dB in the high gain regime. Since the noise figure is a measure of the SNR degradation in the system, a value higher than unity implies that the incorporation of the amplifier will always degrade the SNR, which is contradictory to the objective of SNR enhancement. This apparent contradiction is discussed here. If S_{\min} represents the minimum signal power S that can be detected with confidence, than the minimum SNR required at the photodetector is given by S_{\min}/N , where N is the electrical noise power. (Typical value of S_{\min}/N in fiber optic systems are around 20 dB). If an optical preamplifier with (optical) gain G is incorporated before the detector, the received SNR=SNR' becomes

$$SNR' = \frac{G^2 S}{N + G^2 N_{ASE}}$$
 (2.52)

where G^2N_{ASE} represents the amplified spontaneous emission noise added by the amplifier to the background noise. The new minimum detectable signal S'_{\min} at the receiver is now evaluated from the equation

$$\frac{G^2 S'_{\min}}{N + G^2 N_{ASE}} = \frac{S_{\min}}{N} \,. \tag{2.53}$$

Hence an improvement in detection sensitivity of the receiver is now possible if the new minimum S'_{\min} is calculated to be lower than the previous minimum S'_{\min} . That is when

$$\frac{S_{\min}}{S'_{\min}} = \frac{G^2}{1 + (G^2 N_{ASE} / N)}$$
 (2.54)

which becomes feasible when the gain of the amplifier is very high. If the G^2N_{ASE} term is dominated by the signal-ASE beat noise components and high gain assumed, then $G^2>1+(G^2N_{ASE}/N)$ when the input power is *less* than 400 μ W (-4 dBm) [5]; this condition is

satisfied in all practical cases of interest. Moreover if the input signals with and without the preamplifier are assumed to be identical ($S'_{\min} = S_{\min}$), then it is straightforward to note from Eqns. (2.50) through (2.52) that the *electrical* SNR is enhanced with the preamplifier (note that the *optical* SNR is inherently degraded by at least the noise figure (> 3 dB) by the preamplifier).

It is interesting to note that the previous analysis assumes that the background receiver electrical noise is dominated by thermal noise. As shown in [5], *no* improvement in electrical SNR is possible when an optical preamplifier is used with an ideal, shot-noise limited detector. Moreover realistic avalanche photodiodes do not offer the same potential for SNR enhancement, as do optical preamplifier-based PIN photodiodes. The application of optical preamplification to SNR enhancement in binary ASK, or OOK, and FSK based optical fiber systems is discussed in the next subsections.

2.4.2 Receiver Sensitivity of Optically Preamplified OOK System

As shown in the previous section, the incorporation of optical preamplification at the receiver leads to a lower detection sensitivity required at the photodetector. Although the optical preamplifier adds noise to the system, which actually degrades the *optical* SNR by at least 3 dB [37], the high gain that it provides improves the electrical SNR at the decision circuit. A lower detection sensitivity at the receiver translates to a higher power budget which means that a longer un-repeatered length of the fiber can be used in the link.

Assuming Gaussian statistics for the received photocurrent, the first moments may be expressed as $\mu_1 = I_s + I_{ASE}$ and $\mu_0 = I_{ASE}$, where I_s is the received signal photocurrent and I_{ASE} is the photocurrent due to the amplified spontaneous emission noise (ASE) from the optical preamplifier. It should be noted that the ASE photocurrent is independent of the signal and thus exists during both the mark (1) and space (0) bit duration. Since the gain of the optical preamplifier is usually very large, the electrical noise in the receiver (thermal and shot noise) may be safely neglected, as compared to the noise generated by the preamplifier. As outlined earlier, the noise contribution from the optical amplifier consists of two main terms which are the photocurrent

equivalents of the signal-spontaneous-beat noise (${2 \atop s-ASE}$) and the spontaneous-spontaneous-beat noise (${2 \atop ASE-ASE}$). Hence using Eqns. (2.48) and (2.51), the noise variances for the 1 and 0 bits may be written as

$${}_{1}^{2} = 2I_{s}I_{ASE}\frac{B_{e}}{B_{o}} + I_{ASE}^{2}\frac{B_{e}}{B_{o}}$$
 (2.55)

and

$$_{0}^{2} = I_{ASE}^{2} \frac{B_{e}}{B_{o}}$$
 (2.56)

where B_e and B_o are the electrical and optical bandwidths, respectively.

Expressing the photocurrent in terms of the average receiver sensitivity in photons/bit and the preamplifier gain G as

$$I_s = 2\overline{N}_p \ qR_pG \tag{2.57}$$

and the spontaneous photocurrent as

$$I_{ASE} = 2n_{sp} \ q(G-1)B_o \tag{2.58}$$

the receiver sensitivity may be determined. Using Eqn. (2.58), the receiver sensitivity in terms of the average photons/bit is

$$\overline{N}_p = 2n_{sp}Q^2 + 1 + \frac{1}{Q}\sqrt{\frac{B_o T}{B_o T}}$$
 (2.59)

Here is the ratio of the electrical bandwidth of the receiver to the bit rate. The above expression has been derived assuming that no polarization filter is placed after the optical preamplifier, which is usually the case in practical fiber optic systems. However, if a polarization filter is included in the system, there is no effect on the first term on the RHS of Eqn. (2.59) since it represents the signal spontaneous photocurrent with the spontaneous noise being in the *same* polarization as the signal. However the term under the square root sign, which represents the spontaneous-spontaneous noise, is reduced by a factor of half since the polarization filter effectively

cancels the spontaneous noise in the polarization orthogonal to its own. Throughout this work, we have assumed the *absence* of the polarization filter in the optical preamplifier receiver.

Hence for a 10^{-9} error probability (Q=6), $n_{sp}=1$ (ideal amplifier with a noise figure of 3 dB), and an electrical bandwidth which is half of the data rate (=0.5), the receiver sensitivity is $\overline{N}_P=36+6\sqrt{2B_oT}$, where T is the reciprocal bit rate. For $B_oT=1$, the receiver sensitivity is 44.5 photons/bit. The factor of 2 inside the square root sign is absent when a polarization filter is used, resulting in a sensitivity of 42 photons/bit. A more practical estimate of receiver sensitivity is obtained by using more realistic values of amplifier noise figure (6 dB, which corresponds to $n_{sp}=2$) and electrical components (=0.7). Using these practical values yields a receiver sensitivity of $\overline{N}_P=100.8+20\sqrt{B_oT}$, (2.60)

which for $B_oT=1$ equals 120.8 photons/bit (compare this with the 2732 photons/bit obtained without the use of the preamplifier). It should be noted, however, that a Gaussian approximation for the photocurrent statistics is a conservative estimate. An exact analysis, which employs Rayleigh and Rician probability distributions, yields a value which is slightly lower than the numbers mentioned above [4].

2.4.3 Receiver Sensitivity of Optically Preamplified FSK System

The receiver structure here is the same as that assumed for OOK, except with the incorporation of an optical preamplifier before the signal plus noise is converted to baseband by the two BPF. Thus $\mu_1 = -\mu_0 = I_s + I_{ASE}$. The noise variances are equal, as mentioned in the last section, and consist of the signal-spontaneous beat noise current and *twice* the spontaneous-spontaneous beat noise current. The factor of two arises because both the channels contribute to the signal-independent ASE photocurrent. Hence following an analysis similar to that of the OOK case, the receiver sensitivity may be expressed as

$$\overline{N}_p = 2n_{sp}Q^2 + 1 + \sqrt{1 + \frac{2B_oT}{Q^2}}$$
 (2.61)

Hence for a 10^{-9} error probability (Q=6), $n_{sp}=1$ (ideal amplifier with a noise figure of 3 dB), and an electrical bandwidth which is half of the data rate (=0.5), the receiver sensitivity is $\overline{N}_p = 36(1+0.33\sqrt{9+B_oT})$ which for $B_oT=1$ gives 74 photons/bit. A more practical estimate of receiver sensitivity is obtained by using realistic values of amplifier noise figure (6 dB, which corresponds to $n_{sp}=2$) and electrical components (=0.7). Using these practical values yields a receiver sensitivity of

$$\overline{N}_p = 100.8 + 28.4\sqrt{12.6 + B_o T}$$
, (2.62)

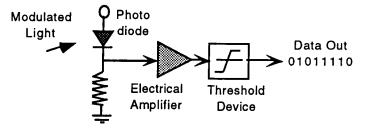
which for $B_oT = 1$ equals 205.52 photons/bit (compare this with the 3864 photons/bit obtained *without* the use of the preamplifier).

The relative penalty between OOK and FSK, as shown in this section, raises the interesting question: how good is the Gaussian approximation for calculating the receiver sensitivity for OOK and FSK? The question to ask before that is, how close are the actual distributions of the signal and noise to the Gaussian statistics. In an optical preamplifier-based system, there are three photocurrent components: a) the signal photocurrent, b) the signal-noise beat photocurrent, and c) the noise-noise beat photocurrent. Assuming that the ASE noise is additive white Gaussian prior to photodetection, and the signal is not random, (a) gives us just a dc term, (b) results in a Gaussian distribution, and (c) results in a chi-squared distribution. Hence for OOK, in a 1 state the equivalent noise distribution is really the convolution of the Gaussian and the chi-squared, whereas the 0 state distribution is chi-squared only (neglecting thermal noise). But for FSK, since signal transmission occurs during both the 1 and 0 states, the corresponding distributions are the convolution results of the chi-squared plus Gaussian. The main point of this discussion is that the Gaussian approximation only yields an order-of-magnitude estimates of receiver sensitivity and appears to be more conservative for FSK, than for OOK. For more exact results, more exact distributions need to be used, as done by Marcuse [44,45].

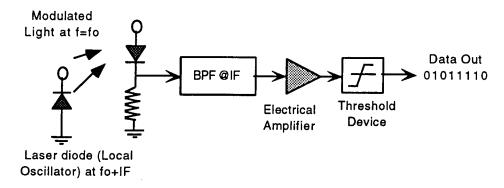
Fig. 2.8 shows a comparison of the receiver sensitivity for optically-preamplified, deterministic-source-based, OOK and FSK systems. As shown in the figure, the required sensitivity increases when B_oT is increased. These results are important since they set the upper limit on the best receiver sensitivity that may be obtained with a noise-like source.

2.5 Summary

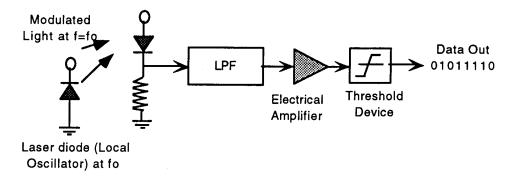
In this chapter we have discussed the receiver performance of lightwave systems that employ coherent, deterministic laser sources to transmit information. Both PIN and optical preamplifier receivers were discussed. In the next chapter we will replace the laser source with a spectrum-sliced source and analyze the difference in system performance.



(a) Direct detection receiver



(b) Coherent-heterodyne receiver



(c) Coherent-homodyne receiver

Fig. 2.1: *Schematic of lightwave receiver configurations.*

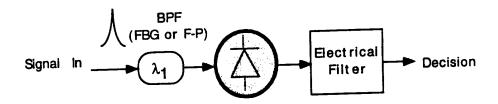


Fig. 2.2: Schematic of the direct detection (DD) or incoherent receiver.

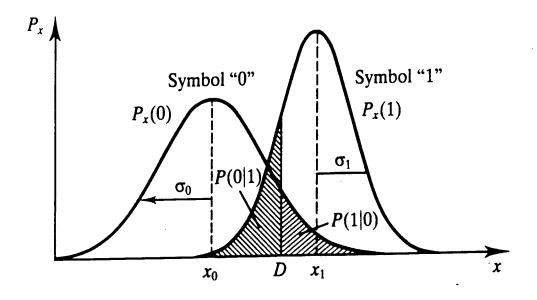


Fig. 2.3: Gaussian probability distributions corresponding to the photodetection of digital symbols.

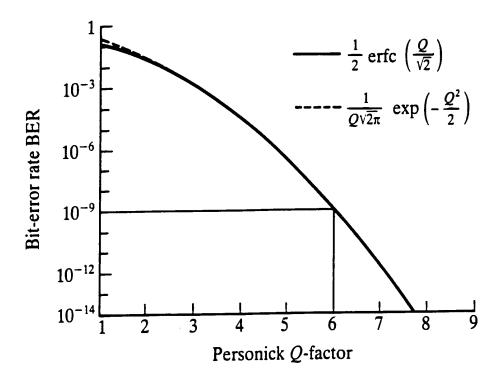
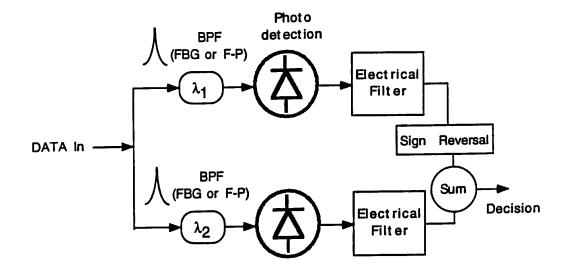
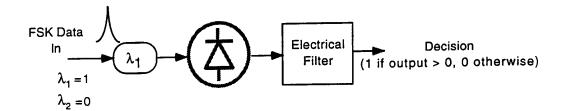


Fig. 2.4: Bit error rate (BER) as a function of the Q factor.



(a) Dual Filter FSK receiver



(b) Single-Filter, FSK to ASK Discriminator

Fig. 2.5: Typical receivers for FSK detection.

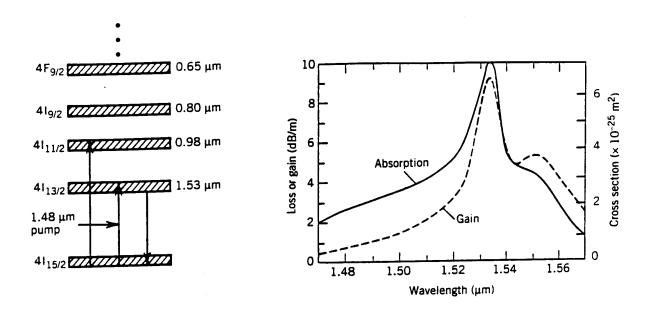
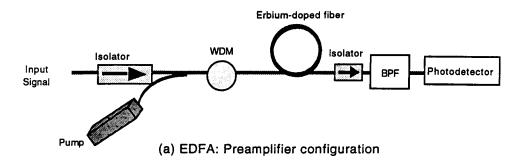
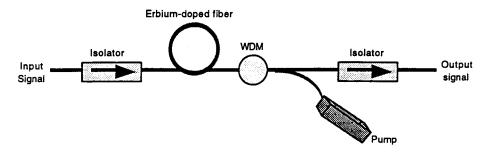
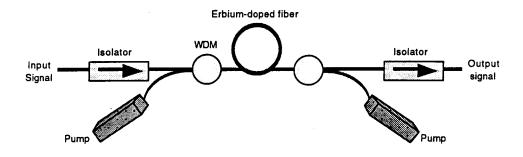


Fig. 2.6: Energy levels and typical absorption/emission spectra of erbium-doped silica fiber [1].





(b) EDFA: Backward-pumped power booster configuration



(c) EDFA: Dual-pumped power booster configuration

Fig. 2.7: Application configurations of the erbium-doped fiber amplifier (EDFA).

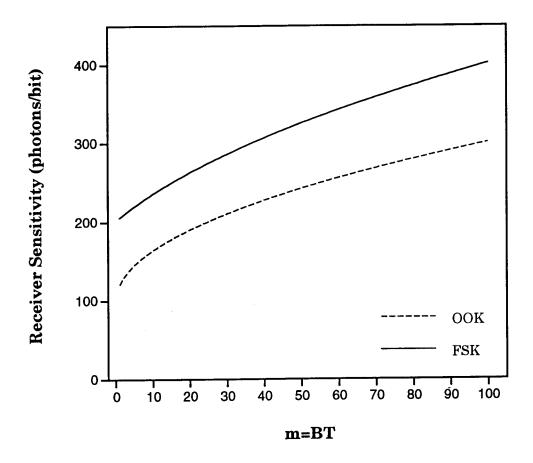


Fig. 2.8: Comparison of receiver sensitivity of optically preamplified, laser-based, OOK and FSK systems.