Chapter 4

Spectrum-Sliced WDM Using FSK Transmission

The focus of this chapter is to consider the case of spectral-slicing when Frequency-Shift Keying (FSK) is used as the modulation scheme. Although the majority of lightwave systems use conventional On-Off Keying (OOK) or intensity modulation, FSK is preferred in certain situations and this chapter will discuss where transmission using FSK may prove advantageous in the context of a noise-like spectrum-sliced system. The mathematical formulation of this problem is followed by an analysis of the receiver sensitivity and comparison with the results for OOK discussed in the previous chapter.

4.1 Introduction

FSK is a modulation scheme where the 1 and 0 bits are defined in terms of two distinct wavelengths. In conventional FSK systems, these wavelengths are generated by slight variations in the bias current of a laser diode which produces a change in the output wavelength[1]. Since amplitude variations in the bias current are kept to a minimum, the source does not exhibit any chirp, as is typical in directly-modulated OOK transmission. Hence the output waveform has a very small spectral spread which helps in minimizing dispersion penalties. Since the optical phase varies continuously as the drive current to the laser is changed, this scheme is often termed continuous-phase FSK or CPFSK. This provides a narrower spectrum than switching between two different lasers at discrete wavelengths. The continuous phase also results in absence of phase or amplitude discontinuities at bit boundaries, which allows narrower channel spacings.

[1] Two figures of merit for lasers used in an FSK system are a wide and uniform modulation bandwidth, and a high frequency-modulation (FM) response as measured in units of frequency deviation per mA of injected current (MHz/mA). These two characteristics are usually inversely related. [1]

It should be noted, however, that this advantage of lower chirp in FSK systems becomes insignificant when external modulators are used with OOK transmission. In systems employing external modulators, the laser is kept on at all times (resulting in negligible source chirp), and the output is intensity modulated by an external modulator such as those fabricated using $LiNbO_3$. This, obviously, adds to the cost of the OOK system. Hence in systems where smaller modulation currents and direct modulation is desirable, FSK is often considered to be the modulation format of choice. Of course, external modulation may also be used in conjunction with FSK where instead of complete extinction (as in OOK), the modulator provides the appropriate phase modulation to transmit the desired wavelength (bit).

The demodulation of FSK signals in incoherent or direct detection systems is usually carried out using two bandpass filters, each centered at the mark and space wavelength, respectively. In single channel systems, the expense of two filters may be avoided by conversion of FSK to OOK prior to demodulation, using a single bandpass optical filter. The filter blocks one of the tones (wavelengths) and hence may be used as the demodulator in a binary transmission system. Typical FSK demodulation schemes used in incoherent detection optical systems were discussed before in Chapter 2, and are illustrated in Fig. 2.5.

Since the information in an FSK signal is carried in terms of the spectral content, and not absolute amplitude (as in OOK), the detection procedure for FSK is classified as being balanced. This means that the threshold, in the case of binary transmission, is set at zero. As shown in Fig. 2.5a, the outputs of the bandpass filters are differenced, and the sign of the difference determines whether a 1 or a 0 bit was transmitted. Hence the receiver circuit is simplified since, unlike OOK, the requirements of optimizing the detection threshold are eliminated. Kaminow [59] proposed and demonstrated the application of FSK with direct detection for optical multiple access WDM networks using passive star couplers for simultaneous access of all channels.

4.1.1 Why Spectrum-Sliced FSK ?

In this chapter we will discuss the analysis of spectrally-sliced WDM systems that employ FSK modulation. Generally, FSK transmission needs twice the number of distinct wavelength bands per binary WDM channel, than does OOK. Hence the consideration of FSK for spectrum-slicing may seem a bit self-defeating since SSWDM systems are not very bandwidth efficient. However, as we will see from the analysis presented herein, FSK has the potential to be advantageous in certain WDM networking situations, as compared to OOK. **Our analysis shows that FSK transmission in a noise-like system is accompanied by lower peak-power requirements. This analysis is, to the best of our knowledge, the first such proposal and motivation for using FSK transmission in a spectrum-sliced WDM environment.**

This chapter is organized as follows. Section 4.2 discusses the receiver structure and mathematical formulation for FSK in a spectrum-sliced scenario, with PIN detection. These results are then compared with receiver sensitivities obtained using an optical preamplifier receiver in Section 4.3. The preamplifier receiver is analyzed using both exact probability distributions, as well as the commonly used Gaussian approximation. Section 4.4 presents a comparison of results obtained for OOK transmission in the previous chapter, with results obtained for FSK transmission in this chapter. Finally a discussion and some closing comments are listed in Section 4.5.

4.2 FSK Receiver Structure for PIN Detection

The receiver structure for FSK detection assumed for the following analysis is the dual-filter combination shown in Fig. 2.5a. We now employ the Gaussian approximation and the exact analysis to evaluate the required photodetection sensitivity for a spectrum-sliced WDM system that uses FSK transmission.

4.2.1 Gaussian Approximation

From the receiver structure, and assuming that a 1 is transmitted

$$\mu_{I(1)} = 2\sigma^2 \qquad \mu_{I(0)} = 0 \tag{4.1}$$

$$\sigma^2_{I(1)} = \frac{2}{m}\sigma^4 + \sigma^2_g \qquad \sigma^2_{I(0)} = \sigma^2_g \tag{4.2}$$

where σ_g^2 is the noise current due to receiver thermal noise, $\mu_{I(1/0)}$ is the mean value of the photocurrent when a (1/0) bit is transmitted, and $\sigma_{I(1/0)}^2$ is the corresponding noise variance. Since the test statistic is formed by differencing the outputs of the two filters, it follows that

$$\mu_1 = \mu_{I(1)} - \mu_{I(0)} = 2\sigma^2 \qquad \mu_0 = \mu_{I(0)} - \mu_{I(1)} = -2\sigma^2 \tag{4.3}$$

and

$$\sigma_1^2 = \sigma_0^2 = \sigma_{I(1)}^2 + \sigma_{I(0)}^2 = \frac{2}{m}\sigma^4 + 2\sigma_g^2 . \tag{4.4}$$

Hence we can express the Q factor as

$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} = \frac{4\sigma^2}{2\sqrt{\frac{2}{m}\sigma^4 + 2\sigma_g^2}} \tag{4.5}$$

Solving for σ^2 we obtain

$$\sigma^2 = \frac{\sigma_g Q}{\sqrt{2}\sqrt{1-\left(Q^2/2m\right)}} . \tag{4.6}$$

We also know that for FSK, the peak and average signal power (photocurrent) is the same. That is

$$2\sigma^2 = \overline{N}_p \eta q R_b \tag{4.7}$$

Hence using Eqns. (4.6) and (4.7), we can solve for the average receiver sensitivity in photons per bit as

$$\overline{N}_p = 2\left(\frac{B_e}{R_b}\right)\left(\frac{Q}{\sqrt{2}\eta}\sqrt{\frac{8\pi V_T C_T}{q}}\right)\Big/\sqrt{1-\left(Q^2/2m\right)} . \tag{4.8}$$

Assuming that $B_e = R_b/2$ we then obtain

$$\overline{N}_p = \left(\frac{Q}{\sqrt{2}\eta}\sqrt{\frac{8\pi V_T C_T}{q}}\right)\Big/\sqrt{1-\left(Q^2/2m\right)} . \tag{4.9}$$

Evaluating Eq. (4.9) for our reference case $C_T = 0.1$ pF, $\eta = 0.7$, and $P_e = 10^{-9}$ $(Q = 6)$, we obtain

$$\overline{N}_p = 3864\Big/\sqrt{1-(18/m)} . \tag{4.10}$$

The factor of 18 in the denominator is identical to the OOK case and implies an error-floor (under this Gaussian approximation), at $m = 18$. Fig. 4.1 graphs the receiver sensitivity as a function of m, following Eq. (4.10). As a reference, the figure also shows the average receiver sensitivity for a laser source and a thermal noise limited PIN receiver detection. As the graph illustrates, a) the receiver sensitivity for a spectrum-sliced signal is strongly dependent on the operating m value (product of optical bandwidth and date rate), BUT the sensitivity when a laser source is used is constant, being dictated by receiver thermal noise which is independent of m, and b) the sensitivity obtained using the spectrum-sliced source approach the laser source results at very high values of m.

4.2.2 Exact Analysis

An exact analysis approach for evaluating FSK PIN detection is considerably more difficult to evaluate than the Gaussian approximation. Referring to Fig. 2.5a and assuming the signal is present in channel 1, the output of that channel is the spectrum-sliced signal power (chi-squared after PIN detection) added with the channel thermal noise (Gaussian). Hence the pdf of the resultant distribution is the convolution of the chi-squared with the Gaussian, and is similar to the expression obtained in Section 3.2 for a 1 bit of OOK transmission. The pdf of the unoccupied FSK channel is Gaussian since only thermal noise is present in that channel.

Hence the random process Z at the decision circuit may be represented in terms of the chi-squared process Y (due to the spectrum-sliced signal) and the same-channel (X_1) and other-channel (X_2) thermal noise processes as

$$Z = Y + X_1 - X_2 \tag{4.12}$$

since the decision is based on differencing the outputs of the two channels. Now the objective is to find the pdf associated with Z. One possible technique to do this is through the use of the characteristic functions. Using Eq. (4.12), the characteristic function of Z is given by

$$\Psi_Z(z) = \mathrm{E}[\exp(jwz)] = \mathrm{E}\left[\exp\left(jw\left(y + x_1 - x_2\right)\right)\right] \tag{4.13}$$

where E[..] refers to the expected value of the argument. The corresponding pdf may then be evaluated, using the corresponding pdf's for the equivalent chi-squared (Eq. (3.10)) and the Gaussian distribution, by the inverse Fourier transform of the characteristic function. The resultant pdf is of the same form as that evaluated earlier (see Section 3.4.2) and involves hypergeometric functions. The error probability is then, assuming signal in channel 1, given by

$$P_e = \Pr\left(Y + X_1 - X_2 < 0\right).$$

Evaluation of the error probability thus involves the non-trivial task of integrating hypergeometric functions. This was not carried further since it is the spectrum-sliced case, not the deterministic case, that is the primary focus of this dissertation.

4.3 Optical Preamplifier Receiver for FSK Detection

In this section we will focus on the major contribution of this chapter which is the detailed analysis and evaluation of the receiver sensitivity of spectrum-sliced FSK systems employing optical preamplifier receivers. Results using both the exact (chi-squared) and the Gaussian probability distributions for the signal and noise photocurrents are presented.

4.3.1 Gaussian Approximation

Using the methodology outlined in Section 2.1, it now follows that the mean and variances of the photocurrent for FSK transmission are given by

$$\mu_1 = 2\sigma_s^2 \qquad\qquad \mu_0 = -\mu_1 \tag{4.15}$$

and

$$\sqrt{\text{var1}} = \sqrt{\text{var0}} = \sqrt{\frac{2}{m}\left(\sigma_s^2 + \sigma_n^2\right)^2 + \frac{2}{m}\left(\sigma_n^2\right)^2} \tag{4.16}$$

which allows the Q factor to be written as

$$Q = \frac{2\left(2\sigma_s^2\right)}{2\sqrt{\frac{2}{m}\left(\sigma_s^2 + \sigma_n^2\right)^2 + \frac{2}{m}\left(\sigma_n^2\right)^2}}\,. \tag{4.17}$$

For FSK

$$\sigma_s^2 = \frac{\overline{N}_p}{2}\eta q R_b G \tag{4.18}$$

and

$$\sigma_n^2 = n_{sp}\eta q (G-1) B_o . \tag{4.19}$$

using which the receiver sensitivity may be expressed as

$$\overline{N}_p = \frac{2n_{sp} m Q^2}{2m - Q^2}\left[1 + \frac{1}{Q}\sqrt{4m - Q^2}\right]. \tag{4.20}$$

For example for n_{sp} = 2, and $P_e = 10^{-9}$ (Q = 6), the average receiver sensitivity as a function of m is

$$\overline{N}_p = \frac{72m}{m-18}\left[1 + \frac{1}{3}\sqrt{2m-18}\right] \tag{4.21}$$

and is plotted in Fig. 4.2. Like OOK, the Gaussian approximation indicates an error-floor at m = 18. As a reference the corresponding average receiver sensitivity ($\overline{N}_p$) when a laser source is employed are also shown. The points to note here are: a) $\overline{N}_p$ for both the laser and the spectrum-sliced source depend upon m, but the noise-like source yields poorer performance at low values of m, b) like the PIN case, $\overline{N}_p$ for both the transmitter configurations starts becoming similar at very high values of m- since at high m values (large optical bandwidth), receiver noise (optical preamplifier ASE), common to both systems, starts becoming the dominant noise mechanism, and c) although $\overline{N}_p$ for the laser source increases when m is increased, the spectrum-sliced source shows the existence of an optimum m at which the receiver sensitivity is minimized. As discussed in the previous chapter, this optimum is a consequence of signal energy fluctuations dominating at low m and the optical preamplifier ASE dominating at high m.

Note that the appearance of the error floor is dependent more on the statistical approximation (*e.g.* Gaussian) used to represent signal and noise terms, and subsequent BER calculation, rather than the actual transmission scheme. Hence it is definitely possible (and also verified by the exact analysis in the next subsection), that FSK may not show any error floors. The advantage then of using the Gaussian approximation is only to simplify BER analysis and to obtain order of magnitude numbers for receiver sensitivity.

4.3.2 Exact Analysis

The receiver structure for this analysis is illustrated in Fig. 4.3. It is essentially similar to the dual-filter configuration shown in Fig. 2.5a except that it incorporates optical preamplifiers before the signal is input to the two channels. Let us denote the output of the two channels as S_1 and S_0. The corresponding chi-squared pdf associated with each output was derived earlier (Section 3.2), and is restated here as a reference.

$$P(S) = \frac{\left(m/\sigma^2\right)}{(2m-1)!} S^{2m-1} \exp\left(mS/\sigma^2\right). \tag{4.22}$$

Assuming that a 1 was transmitted, the probability of error

$$\begin{aligned} P_e &= \Pr\left\{S_1 - S_0 \ < \ 0\right\} \\ &= \Pr\left\{S_1 \ < \ S_0\right\} \end{aligned} \tag{4.23}$$

since the decision is based on the sign (+ or -) of the difference of the two outputs. Now if say we take any random $S_0 = x$ and $S_1 = y$, then an error occurs when, during a 1 transmission, $y < x$.

This may be expressed in terms of the joint pdf of S_1 and S_0 as

$$P_e = \iint_{y<x} \Pr\left(S_1 = y,\ S_0 = x\right) dxdy. \tag{4.24}$$

Since S_1 and S_0 are independent, the joint pdf turns into a product and we get

$$\begin{aligned} P_e &= \iint_{y<x} \Pr\left(S_1 = y\right) \Pr\left(S_0 = x\right) dxdy \\ &= \int_0^\infty \left[\int_0^x \Pr\left(S_1 = y\right) dy\right] \Pr\left(S_0 = x\right) dx \end{aligned} \tag{4.25}$$

where the pdf given in Eq. (4.21) may be evoked to write

$$P_e = \frac{m^{4m}}{[(2m-1)!]^2} \int_0^\infty \left[\int_0^x \frac{1}{\sigma_1^{4m}} y^{2m-1} \exp\left(-my/\sigma_1^2\right) dy \right] \frac{1}{\sigma_0^{4m}} x^{2m-1} \exp\left(-mx/\sigma_0^2\right) dx . \qquad (4.26)$$

Using $u = \left(my/\sigma_1^2\right)$ and $v = \left(mx/\sigma_0^2\right)$ and defining

$$\beta = \sigma_0^2/\sigma_1^2 \qquad (4.27)$$

we can simplify Eq. (4.25) to get

$$P_e = \frac{1}{[(2m-1)!]^2} \int_0^\infty v^{2m-1} \exp(-v) dv \int_0^{v\beta} u^{2m-1} \exp(-u) du . \qquad (4.28)$$

A simplified form of Eq. (4.29) was obtained by Jacobs [60] (as shown in Appendix B) who showed that Eq. (4.28) can be evaluated in the form of a series as

$$P_e = 1 - \frac{1}{(1+\beta)^{2m}} \sum_{i=0}^{2m-1} {}^{2m-1+i}C_i \left(\frac{\beta}{1+\beta} \right)^i . \qquad (4.29)$$

In the above expression we have used the combinatorial function

$${}^nC_r = \frac{n!}{r!(n-r)!} \qquad (4.30)$$

An approximate simplified form of Eq. (4.29) is also described in Appendix B and the final form is mentioned here as

$$P_e \approx \frac{1}{\sqrt{8\pi m}} \frac{(4\beta)^{2m}}{(1+\beta)^{4m-1}} . \qquad (4.31)$$

Now for FSK (assuming that the signal is in channel 1)

$$\sigma_1^2 = \sigma_s^2 + \sigma_n^2 \qquad \sigma_0^2 = \sigma_n^2 \qquad (4.32)$$

where the subscripts s and n refer to the signal and noise photocurrents, respectively, and were described earlier in Eqns. (4.18) and (4.19), respectively. Hence using Eq. (4.27), β was expressed as (with the gain of the preamplifier $G >> 1$ and $n_{sp} = 2$)

$$\beta = \frac{\sigma_n^2}{\sigma_s^2 + \sigma_n^2} = \frac{1}{1 + \left(\overline{N}_p/4m\right)} \qquad (4.33)$$

The above expression was substituted in Eq. (4.31) to calculate the receiver sensitivity in photons per bit and is plotted in Fig. 4.4. For example at $P_e = 10^{-9}$, average required receiver sensitivity is 240 photons/bit. Fig. 4.5 illustrates how the optimum m and the corresponding receiver sensitivity vary as a function of the error probability. Figure 4.6 graphs a comparison between the receiver sensitivities obtained using the Gaussian and the exact (chi-squared) probability distributions. It is noted that the Gaussian yields very conservative numbers for low values at m but that it approaches the exact sensitivity for higher values of m. The reason for this is, of course, the central limit theorem which predicts that the average of a large number of random samples tend to be Gaussian distributed.

4.4 FSK vs. OOK

As described in the earlier sections of this chapter, spectrum-sliced FSK transmission presents a number of similarities compared with OOK transmission. In this section we summarize the similarities and differences between both spectrum-sliced and conventional (laser-based) OOK and FSK transmission, especially in the context of wavelength division multiplexing. The purpose of this comparison is to highlight situations where one transmission scheme may be better in terms of transmission performance (power budget, peak power requirements) over the other. We first summarize results from laser-based systems employing PIN or preamplifier receivers (Chapter 2), and then focus on noise-like spectrum-sliced systems (Chapters 3 and 4). The section is concluded with a numerical example comparing the transmission performance of OOK and FSK under a similar set of operating conditions.

4.4.1 Deterministic Laser-Based Systems

Although not discussed extensively in this work, coherent optical fiber systems offer the highest receiver sensitivity, as compared to direct detection or incoherent receivers. This is due to two factors [1]: a) shot noise from the high power local oscillator (laser) results makes the receiver thermal noise insignificant, and b) the IF electrical filter allows very high frequency selectivity, and in turn, a very high rejection of the background noise. However, in spite of these advantages, contemporary systems prefer the use of direct detection methods which, although possessing poorer theoretical sensitivity, result in a simple detector circuit and do not require coherent sources. Moreover the advent of the erbium-doped optical amplifier as receiver preamplifiers has made the shot-noise advantage of coherent receivers less significant. In this work we have concentrated entirely on direct detection receivers but mention coherent receivers here for the sake of completeness.

Table 1 summarizes the *power efficiency* comparison or in other words the differences between the average receiver sensitivity ($\overline{N}_p$ in photons/bit) required for OOK and FSK. The numbers mentioned provide an appreciation for relative order of magnitude. The following parameters have been assumed for preparing this table:

a) Noise capacitance (C_T): 0.1 pF

b) Electrical bandwidth (B_e) to Bit Rate (R_b) ratio (γ): 0.5

c) For optical preamplifier: Gain (G) >> 1, and spontaneous emission noise figure (n_{sp}) = 2

d) No polarization filter assumed after preamplifier

e) Bit error rate =10^{-9} (Q = 6)

f) Penalty is defined as the ratio of average photons per bit needed for FSK/OOK

Table 4.1: *Average Receiver Sensitivity for OOK vs. FSK, Laser-Based Systems.*

Type of System	OOK (ph/b)	FSK (ph/b)	Penalty (dB)
Shot Noise Limit, Photon Counting Receiver, Poisson Distribution	10	20	3 dB
Thermal Noise Limited PIN Receiver, Gaussian Distribution	2732	3864	1.5 dB
Optical Preamplifier Based Receiver	$100.8 + 20\sqrt{m}$	$100.8 + 28.4\sqrt{12.6 + m}$	For m=1: 2.31 dB m=100: 1.2 dB m very large: 1.5 dB

It is clear that, under the assumptions listed above, system performance (in terms of receiver sensitivity), for thermal noise-limited PIN receivers only depends on the receiver noise capacitance and is independent of the receiver optical bandwidth. This result if true for both OOK and FSK. For optical preamplifier receiver, on the other hand, receiver performance degrades as the optical bandwidth is increased. This is shown in Fig. 4.7 which compares the receiver sensitivity as a function of m for OOK and FSK for a laser source and an optical preamplifier receiver. As $m \to \infty$, both Eq. (2.60) and (2.62) indicate that $\bar{N}_p$ increases proportional to $\sqrt{m}$, with the proportionality constant being a factor $\sqrt{2}$ (1.5 dB) higher for FSK than for OOK. (It is interesting that the 1.5 dB differential is observed for the case of PIN detection also and, as discussed before in Section 2.2.3, is a consequence of the optical energy per bit requirement being higher for FSK than for OOK.)

4.4.2: Spectrum-sliced Noise-Like Systems

One of the objectives of this chapter is to understand the relative performance of FSK and OOK systems, and to provide specific instances where FSK performs better than OOK, in the context of a spectrum-sliced WDM system.

i) PIN detection

As discussed In Section 4.2, for PIN detection, and using the Gaussian approximation, the receiver sensitivities for OOK and FSK both result in a power penalty, wrt to the deterministic case. The receiver sensitivities are:

$$\bar{N}_{p,\mathrm{OOK}} = \frac{2732}{1-18/m} \qquad \bar{N}_{p,\mathrm{FSK}} = \frac{3864}{\sqrt{1-18/m}} \tag{4.34}$$

which yields a relative power penalty (in dB) of

$$\alpha_{\mathrm{rel}}(m) = 1.5 + 5\log(1-18/m). \tag{4.35}$$

This is plotted in Fig. 4.8 which says that FSK is actually better than OOK, in terms of receiver sensitivity, for small values of m. However as m increases, the log term in the equation approaches zero, and the relative power penalty returns to the deterministic case of 1.5 dB. Also, $\overline{N}_p$ varies as $1/\sqrt{1-Q^2/2m}$ for FSK whereas for OOK the variation is as $1/\left(1-Q^2/2m\right)$ which means that although FSK performs poorer as m is decreased, the variation is slower than it is for OOK. Again it needs to be pointed out that this prediction of better performance with FSK (at lower values of m) may only be a consequence of using the Gaussian approximation to represent signal and noise terms. What this analysis does say with confidence is that as the m value is increased, the Gaussian approximation may be relied upon to calculate fairly accurate numbers for receiver sensitivity. The other point to note from Eq. (4.32), is that: **a) PIN detection in spectrum-sliced systems, with either OOK or FSK, does NOT result in any optimum bandwidth, and b) receiver sensitivities required using a PIN receiver are on the order of a few thousand photons/bit.**

ii) Optical preamplifier receiver detection

The corresponding receiver sensitivities when one employs an optical preamplifier receiver for OOK or FSK are shown in Fig. 4.9, at various error probabilities. Both average and peak receiver sensitivity are shown. The following conclusions may be drawn from the figure.

1. Both OOK and FSK provide a greater than 10 dB power advantage (in photons/bit) compared to the corresponding PIN cases - $\overline{N}_p$ with the preamplifier receiver is a few hundred photons/bit, whereas with the PIN receiver it is a few thousand photons/bit.

2. **Although FSK requires a higher average optical power at the optimum, the peak power required is lower than OOK.** This advantage is significant when one realizes that practical broadband noise sources, such as semiconductor and doped fiber-amplifiers do not

produce a large amount of ASE power. Moreover the total output power and the gain-flatness of ASE sources are often inversely related. The best reported ASE source for spectrum-slicing provides a 110 mW output over a 25 nm bandwidth. Distributed feedback (DFB) and fiber lasers, on the other hand, yield output powers on the order of several Watts. Hence for spectrum-sliced systems to be competitive with conventional laser-based WDM systems, power efficiency is a major requirement. As we have shown, at least analytically, FSK provides an advantage over OOK for spectrum-slicing since peak, rather than average power, is a more important parameter in this situation.

3. Figure 4.10 shows a comparison between the optimum m for FSK and OOK. The observation here is that **the optimum *m* for FSK (**$m_{opt,FSK}$**) occurs at a lower value than that for OOK (**$m_{opt,OOK}$**). It should be kept in mind, however, that FSK needs twice the number of bands per WDM channel.** As the numerical results show

$$\left(m_{FSK-WDM} = 2m_{opt,FSK}\right) > \left(m_{OOK-WDM} = m_{opt,OOK}\right) \qquad (4.36)$$

where $m_{(FSK/OOK)-WDM}$ is the optimum value of m and hence the optical bandwidth required to implement a WDM channel. Eq. (4.36) says that even though FSK needs a lower value of m to transmit a 1 or a 0, the total channel bandwidth (1 bandwidth + 0 bandwidth) required will be larger than that for OOK. A larger bandwidth spread means that FSK transmission will “see” a higher dispersion coefficient and hence larger dispersion penalties (under this simplistic analysis).

4.4.3 Numerical Example

It is interesting to compare the power budget and transmission capacity achievable with spectrum-sliced OOK and FSK under a given set of operating conditions. Let us consider the following system parameters:

a) Broadband noise source which can output 100 mW uniformly over a 25 nm bandwidth,

b) The optical preamplifier receiver ($n_{sp} = 2$) with an optical filter that can be tuned to maximize either transmission capacity (lower required bandwidth per WDM channel) or power budget (increase optical power transmitted per channel),

c) The bit rate per WDM channel is 2.5 Gb/s, and

d) The error probability requirement is set at $P_e = 10^{-9}$.

i) Power Budget

As we saw in Chapter 3, power budget increases as we increase the bandwidth of the optical filter. We now arbitrarily tune the optical filter to be 1 nm wide and define power budget as the difference of the *average* (over the 1 and 0 bits) transmitted power and the *average* required receiver sensitivity. As we will see in this section, the results are independent of this initial assumption of receiver (WDM channel) optical bandwidth.

Now for OOK we know that the peak power is twice the average power, hence the average power available from a 100 mW broadband source is really 50 mW, with the average power spectral density being 50/25 = 2 mW/nm. Thus the transmitted power per WDM channel = 2 mW/nm X 1 nm = 2 mW, and in dBm units is

$$\text{Transmitted Power} = 10\left[\log_{10} 2\right] = 3.01 \text{ dBm} \tag{4.37}$$

The required receiver sensitivity is then noted from the $\overline{N}_p$ vs. m curve (Fig. 3.3). When R_b = 2.5 Gb/s and B_o = 125 GHz (1 nm), the m is 50. For m = 50, the required $\overline{N}_p$ is 236.22 photons per bit. Hence at the receiver

$$\text{Receiver sensitivity (dBm)} = \overline{N}_p h\nu R_b \text{ (in dBm units)}$$

$$= 10\log_{10}\left(\overline{N}_p\right) - 94.95 = -71.22 \text{ dBm} \tag{4.38}$$

Hence the available power budget is 3.01 - (-71.22) = 74.22 dB.

For FSK, peak power is the same as the average power, which means that now the transmitter uses all of its power all the time, unlike OOK where the 0 transmission meant zero transmitted power. Thus the transmitted power in the 1 nm wide channel is now 4 mW. At the receiver, for FSK, we note from Fig. 4.4 that the required $\overline{N}_p$ is 271.57 photons at m = 50. These numbers yield a power budget of 6.02 - (- 70.61) = 76.63 dB. **Comparing with the number calculated for OOK, we find that FSK provides an improvement of 2.41 dB in power budget when an identical noise source is spectrum sliced at the transmitter end. As Fig. 4.9 illustrates, FSK and OOK require about the same $\overline{N}_p$, irrespective of the operating *m*, which says that the improvement in power budget with FSK discussed in this section, is nearly independent of the optical bandwidth of the WDM channel.**

ii) Transmission Capacity

Transmission capacity in lightwave WDM systems is often increased by increasing the spectral efficiency of the transmitted signals, measured in bits per sec per unit bandwidth (~ 1/m). According to the requirements we have set in this section, the data rate per channel is 2.5 Gb/s, hence to increase transmission capacity, one would have to reduce the optical bandwidth of a single WDM channel, so as to pack a greater number of channels within the 25 nm gain bandwidth available from the EDF-ASE signal source. At the same time we also know that decreasing the optical bandwidth will increase the power penalty at the receiver, indeed the motivation for using an optical preamplifier receiver is to minimize this power penalty (wrt to PIN detection). As was shown in this work, operation at an optimum bandwidth will allow us reduce this power penalty. Hence in this section we will compare the transmission capacities achievable with an FSK and OOK system, both of which are implemented using identical transmitter and receiver parameters, as listed in the beginning of this section, but in addition, each channel is now set to be at the optimum bandwidth. To consider a realistic system we assume that the ratio of channel spacing to optical bandwidth is about 3.

For an OOK system, the optimum bandwidth is given by the product of the optimum m (at the assumed error probability), and the bit rate. This yields 33 X 2.5 GHz, or 82.5 GHz. At 1550 nm, this corresponds to an optical bandwidth of 82.5/125 = 0.66 nm or considering the guard band also, a per channel bandwidth of 0.66 X 3 = 1.98 nm. The total system capacity is calculated as follows:

$$\begin{aligned}\text{Transmission Capacity} &= (\text{Total bandwidth / bandwidth per channel}) \text{ X Bit rate/channel} \\ &= (25 / 1.98) \text{ X } 2.5 \text{ Gb/s} \\ &= 30 \text{ Gb/s} \end{aligned} \tag{4.39}$$

Performing a similar calculation for FSK and noting that the bandwidth per channel will be twice that calculated with the optimum *m* (21 at $P_e = 10^{-9}$), we get a transmission capacity of about 25 Gb/s. Comparing this number with the 30 Gb/s achievable with OOK demonstrates that FSK is not as bandwidth efficient as OOK.

4.5 Summary

To summarize, we have discussed the analysis of spectrum-sliced FSK systems in this chapter using both the conventional Gaussian approximation and the exact (chi-squared) analysis. The obtained results were compared with the corresponding results for OOK from the previous chapter, and it was found that FSK shows certain advantages compared with OOK. The eventual practical use of FSK in a spectrum-sliced system will, of course, depend on system parameters which include the local and total dispersion settings. But it is of indeed of interest to show that a competitive transmission option, other than OOK, does exist when spectrum-sliced systems are designed and implemented.

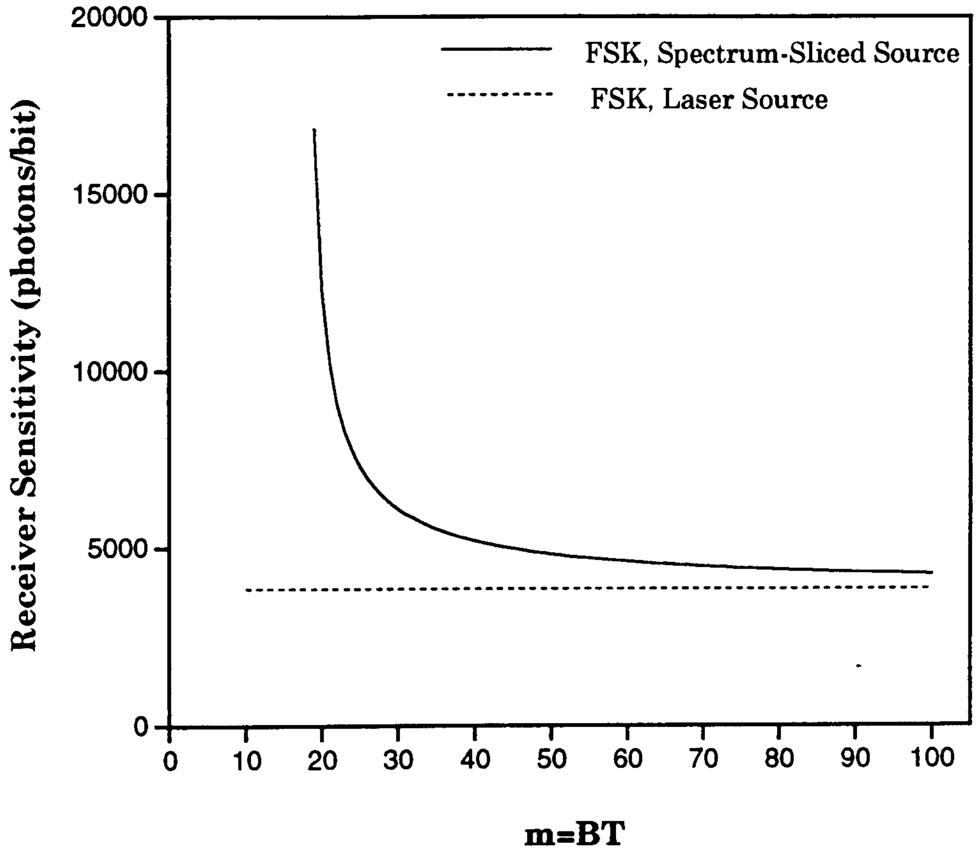

Fig. 4.1: *Receiver sensitivity for a PIN receiver (*$C_T = 0.1$pF, $\eta = 0.7$*) using the Gaussian approximation. Results are plotted for a noise-like (spectrum-sliced) source with FSK transmission. Also shown, as a reference, is the sensitivity when a coherent laser is used as the transmitter.*

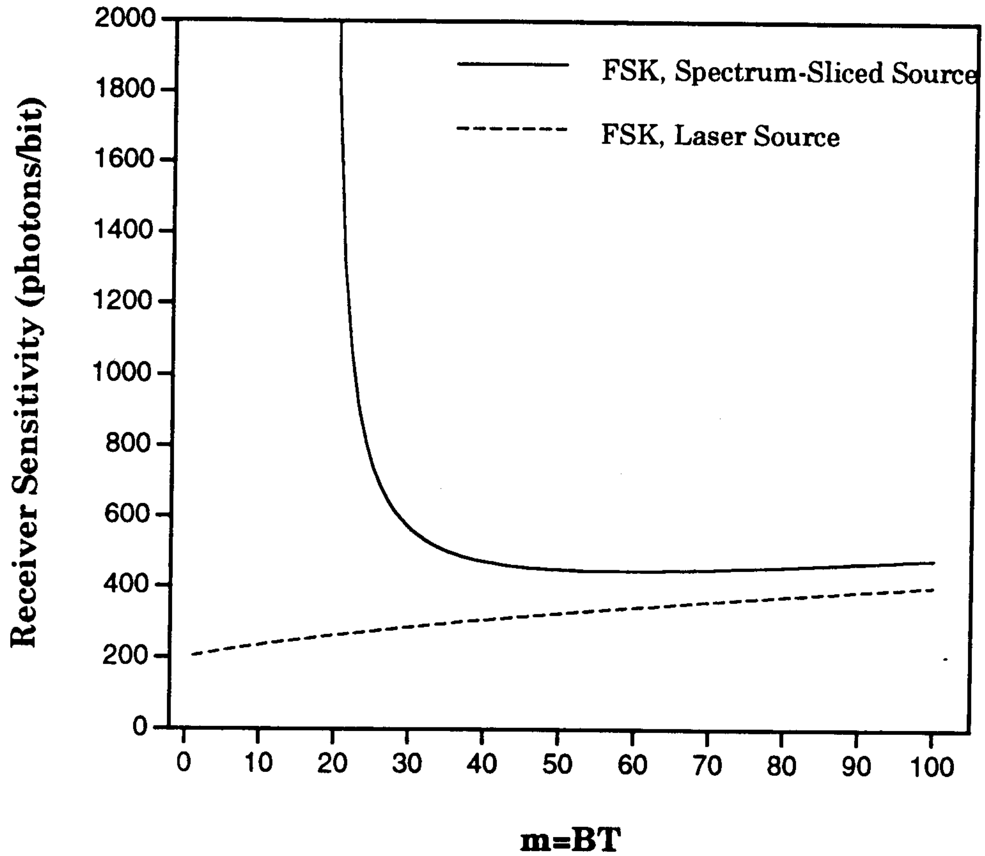

Fig. 4.2: *Receiver sensitivity for an optical preamplifier receiver (*$n_{sp} = 2$ *) using the Gaussian approximation. Results are plotted for a noise-like (spectrum-sliced) source with FSK transmission. Also shown, as a reference, is the sensitivity when a coherent laser is used as the transmitter.*

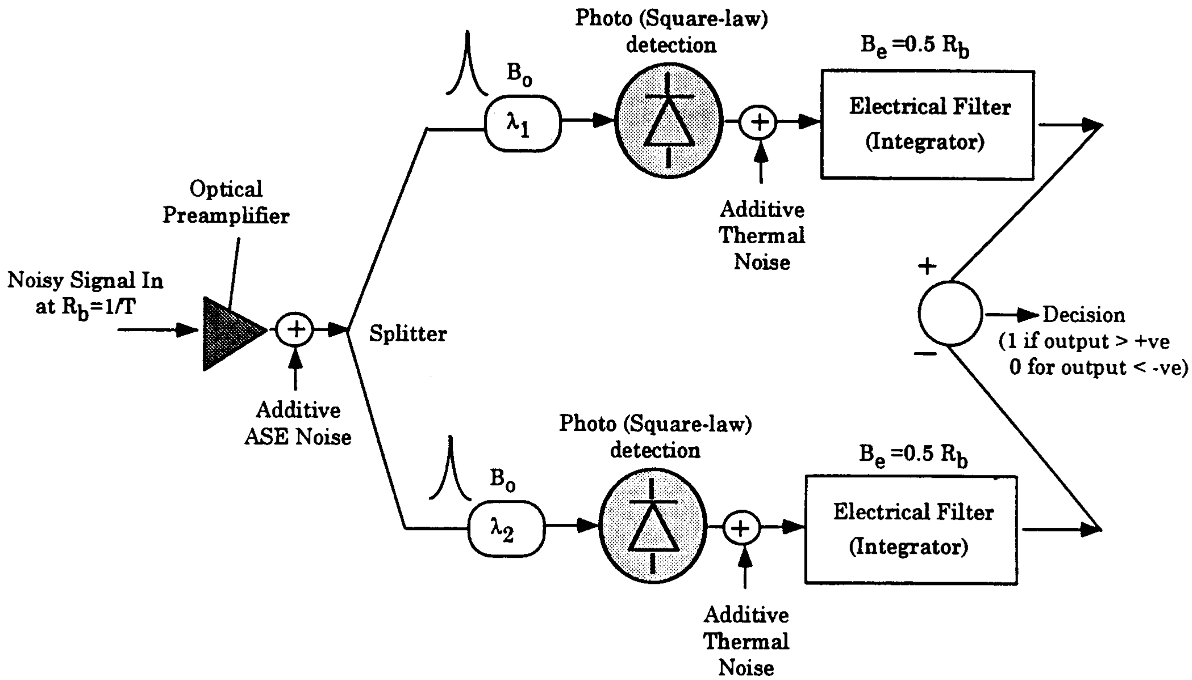

Fig. 4.3: *Receiver structure for FSK analysis.*

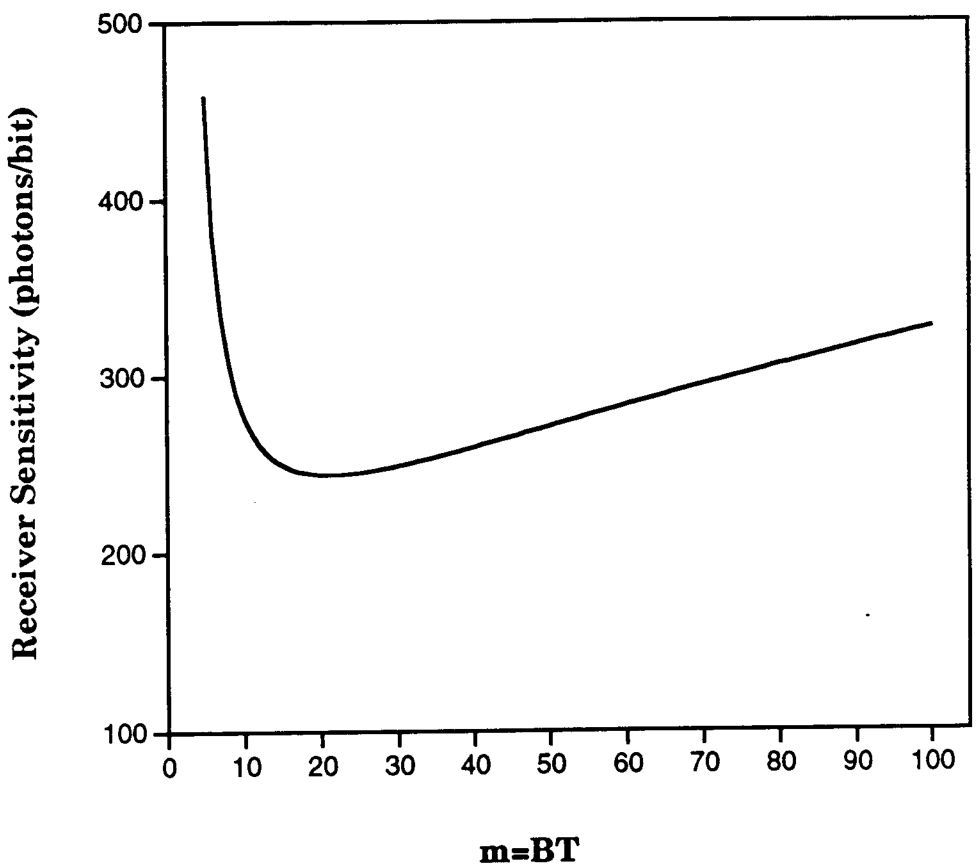

Fig. 4.4: *Receiver sensitivity for an optical preamplifier receiver (*$n_{sp} = 2$*) using the exact (chi-square) analysis. Results are plotted for a noise-like (spectrum-sliced) source with FSK transmission.*

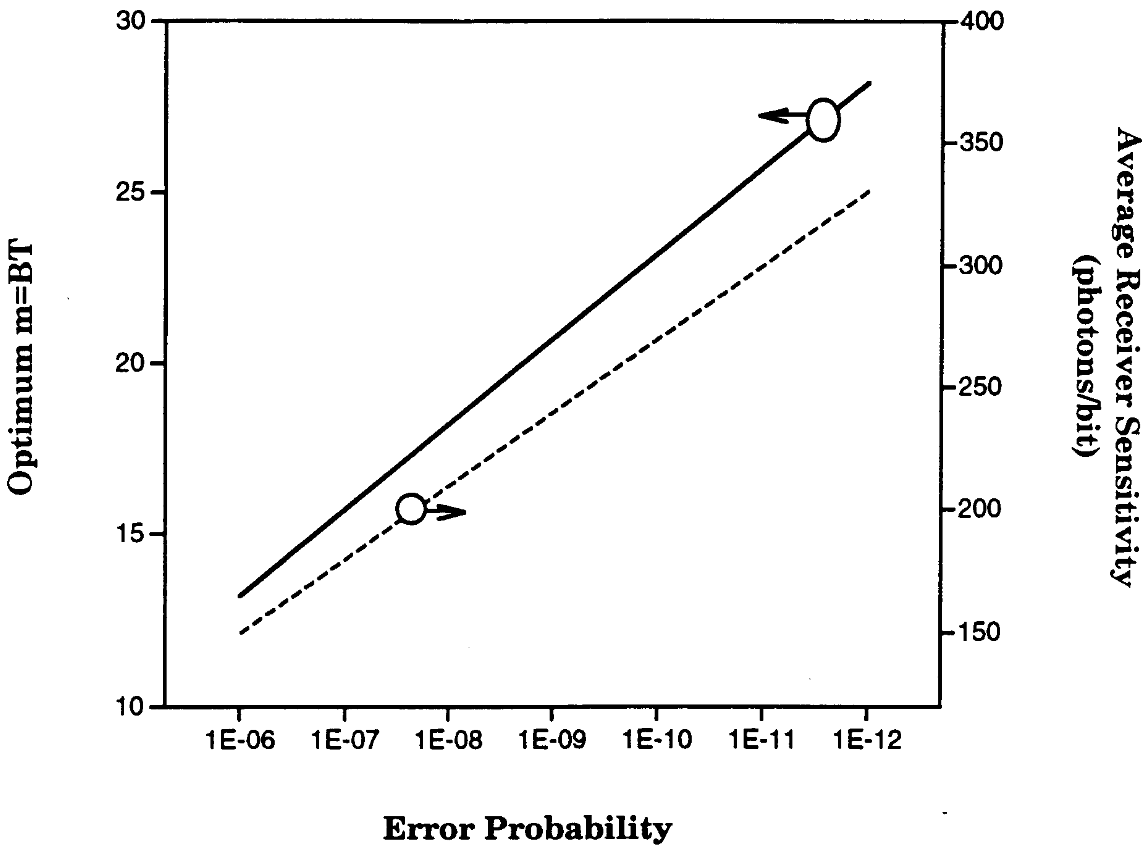

Fig. 4.5: *Optimum* $m = B_o T$ *and the corresponding minimum average receiver sensitivity* $\overline{N}_P$ *(in photons/bit), evaluated at different error probabilities, for FSK transmission and optical preamplifier receiver detection.*

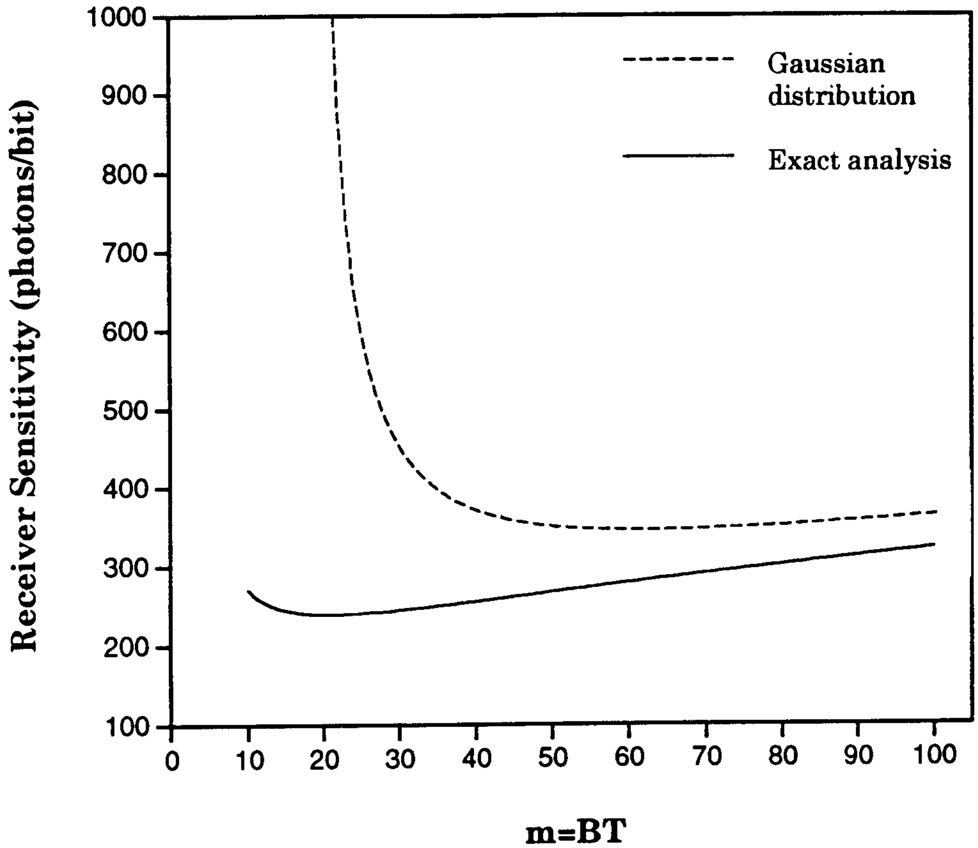

Fig. 4.6: *Comparison of receiver sensitivity results using the Gaussian and exact probability distributions, FSK transmission.*

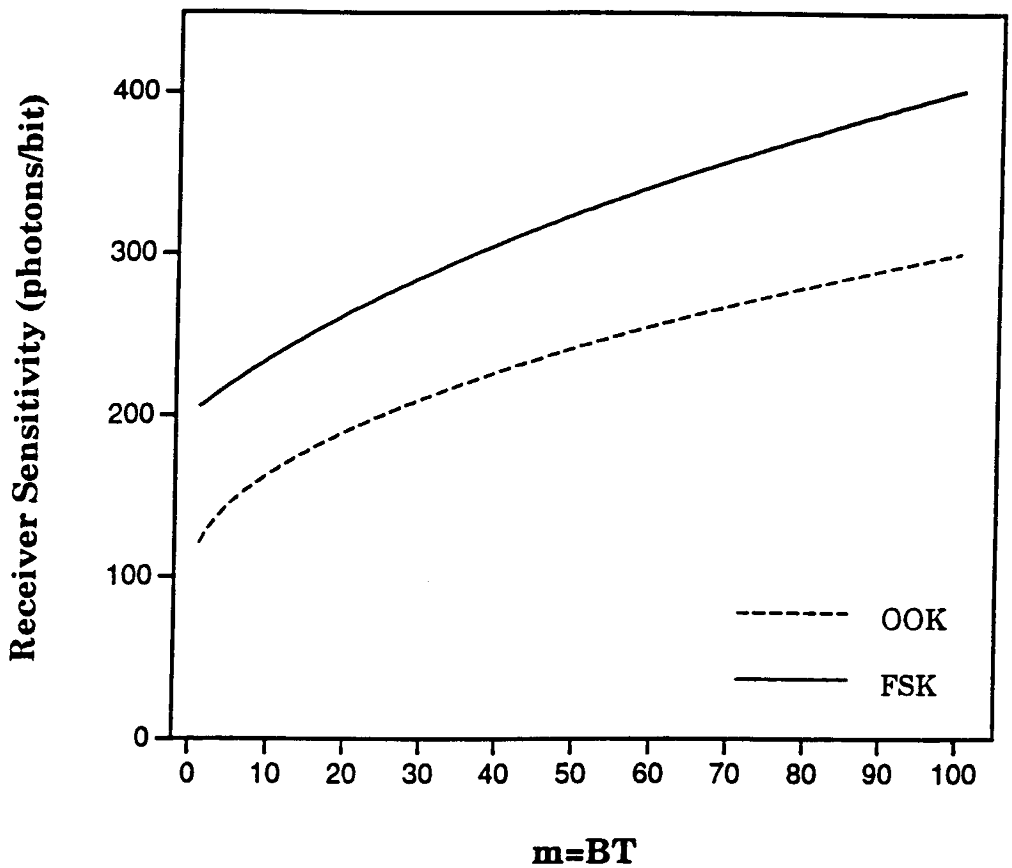

Fig. 4.7: *Comparison of the average receiver sensitivity of FSK and OOK systems using laser transmitters and optical preamplifier receivers, under the Gaussian approximation.*

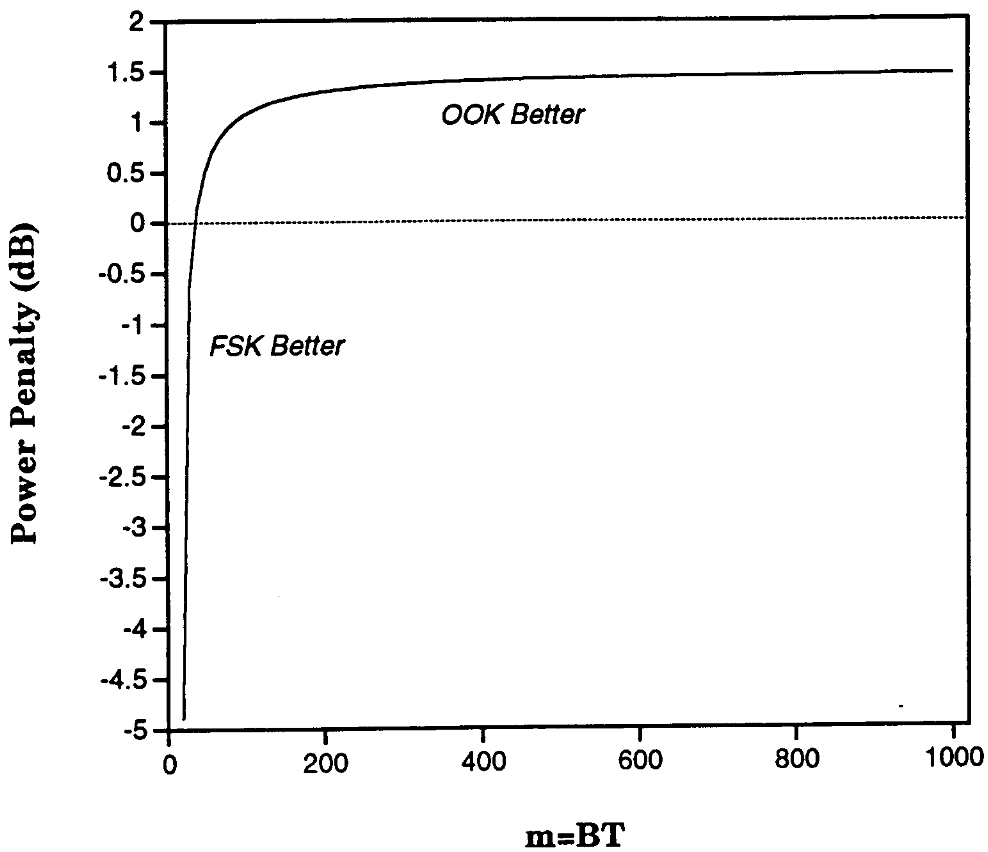

Fig. 4.8: *Power penalty between OOK and FSK, as a function of m, under the Gaussian approximation.*

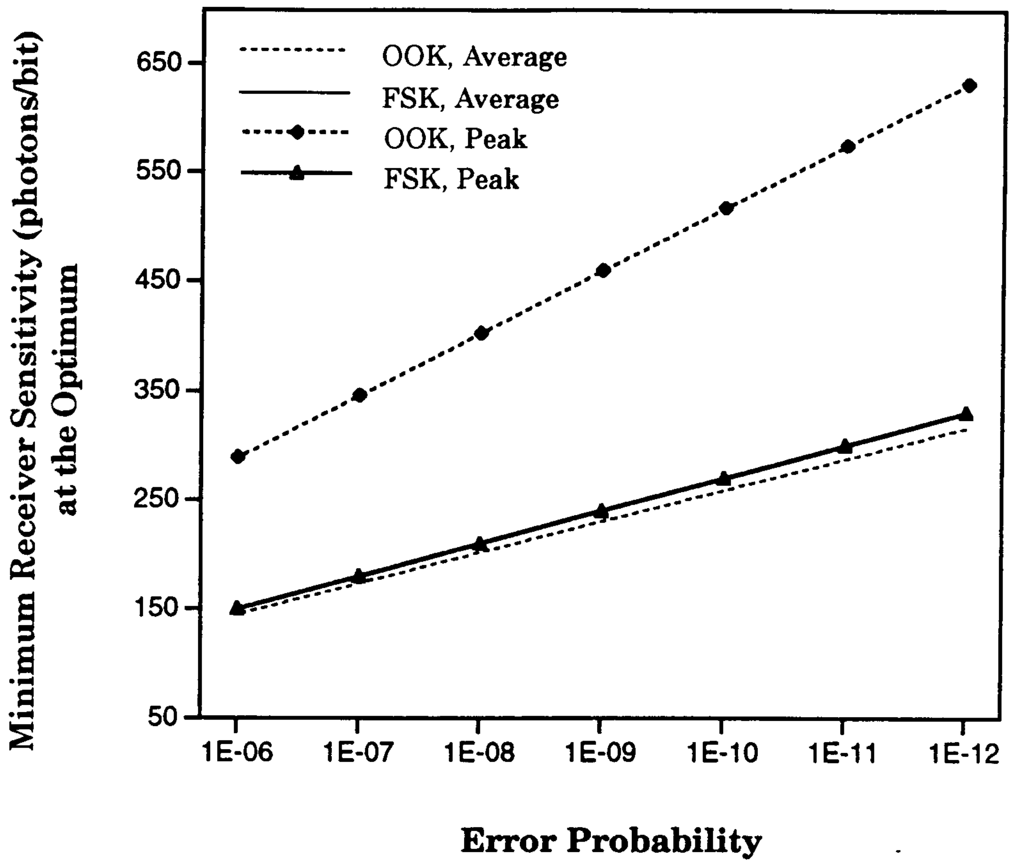

Fig. 4.9: *Average and peak receiver sensitivity (in photons/bit), evaluated at different error probabilities using exact (chi-square) statistics, for OOK and FSK transmission and optical preamplifier receiver detection.*

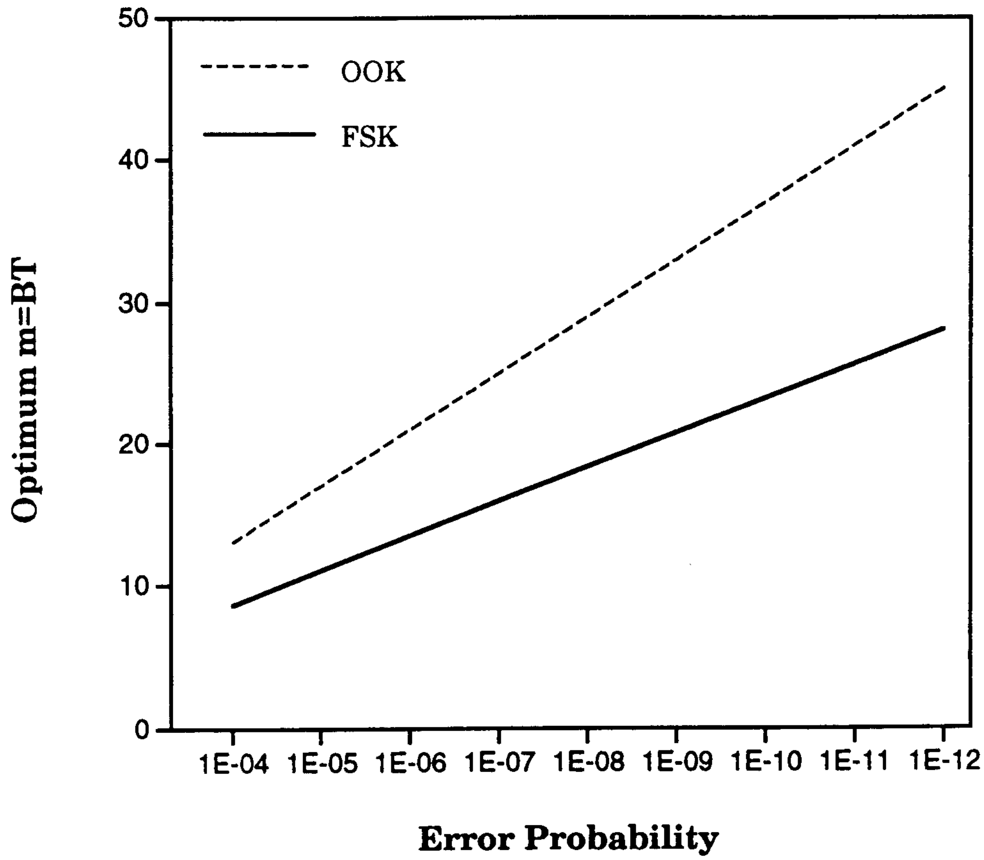

Fig. 4.10: *Optimum* $m = B_o T$ *evaluated at different error probabilities using exact (chi-square) statistics, for OOK and FSK transmission and optical preamplifier receiver detection.*