Chapter 3. Mathematical Model of Electromagnetic Brakes

3.1. Introduction

Precise mathematical models of the brakes are important for the purpose of simulation and control. In this chapter we review different models available for electromagnetic brakes and propose a new model which has better performance in least-squares sense.

3.2. General Description

The electromagnetic brake is a relatively primitive mechanism, yet it employs complex electromagnetic and thermal phenomena. As a result, the calculation of brake torque is a complex task. Both empirical and analytical approaches have been applied.

To explain the magnetic function of an electromagnetic retarder, the Maxwell principles may be applied to the following physical arrangement: a ferro-magnetic disc with a permeability, μ, and an electric conductivity, ρ, rotates at the face of a ring of magnetic poles of alternate polarity. Each pole produces a magnetic excitation flux, N_0, which is proportional to the excitation current within the coil as long as the core is not saturated. The lines of magnetic flux, N, form loops within the disc through the very small air gap which is arranged between the discs and the poles. When the disc rotates, as a first approximation, this flux varies in a sinusoidal function of time at a given point within the disc according to the following expression:

$$\varphi = \varphi_0 \sin \frac{pN}{60} t$$

where:

p = number of pairs of poles

N = revolutions per minute of the disc

t = time variable in seconds

Alternating eddy currents are created within the disc with a strength proportional to the flux, N, and these currents wind themselves around the lines of flux (see Figure 3.1). The electric conductivity, D, of the disc material causes these eddy currents to produce heat within the disc. If a magnetic system is rotated about an axis normal to a conducting sheet, the field of induced eddy currents will set up a retarding torque on the system which is proportional to its angular speed (Smythe 1989). The braking torque is generally also a function of the flux and the excitation current.

3.3. Models Available in the Literature

There are three models proposed in the literature on eddy current brakes (see Smythe 1942, Schiber 1974, Wouterse 1991). The approaches to solve the problem are different.

3.3.1. W.R. Smythe’s Model

Smythe’s approach (Smythe 1942) is to treat the problem as a disc of finite radius and obtained a closed-form solution of torque calculation by means of a reflection procedure (the magnetic field due to eddy currents which appears from either side of the sheet, is modeled by a pair of images receding with uniform velocity) specifically suited to the geometry of the problem. The first step is to calculate the magnetic induction, B, produced by the eddy currents induced in a rotating disk by a long right circular cylinder pole piece. The eddy currents are generated not only by the changes in the magnetic induction of the external field, but also by the changes of the magnetic induction of eddy currents elsewhere in the sheet. After deriving the stream function of a point, which is the current flowing through any cross section of the rotating disk from the point to its edge, the torque can be calculated by integrating the product of the radial component of the current by the magnetic induction and by the lever arm and integrating over the area of the pole piece. Since there is a demagnetizing effect such that permeable pole pieces of an electromagnet short-circuit the flux of the eddy current, if we represent φ_0 = the flux penetrating the rotating disk, T = brake torque, ω = angular velocity, φ_0 = flux penetrating the rotating disk at rest, D= constant coefficient, depending on pole arrangement, R = reluctance of the electromagnet, β = constant coefficient, $\gamma = 10^{-9}/\rho$ and ρ is the volume resistivity of the disk, the total flux when disk is in motion would be

$$\varphi = \varphi_0 - \frac{\beta^2\gamma^2\omega^2\varphi}{R} = \frac{R\varphi_0}{R+\beta^2\gamma^2\omega^2} \qquad (3.1)$$

and $\frac{\beta^2\gamma^2\omega^2\varphi}{R}$ represents the demagnetizing flux attained by dividing the demagnetizing magnetomotive force by the reluctance of the electromagnet. The final integration result of the brake torque is:

$$T = \omega\gamma\varphi^2 D = \frac{\omega\gamma R^2\varphi_0{}^2 D}{(R+\beta^2\gamma^2\omega^2)^2} \qquad (3.2)$$

This model is good at low speed but torque decreases too fast in high speed compared with the experimental curve (see Figure 3.2). The asymptotic behavior shows a fall-off of the torque more rapid than ω^{-1} in the high speed region, which is in contradiction with experimental results. Smythe pointed out that this behavior could be due to other conditions, such as the degree of saturation of the iron in the magnet which will upset the assumed relations between magnetomotive force and flux (φ) and may modify equations (3.1) and (3.2).

3.3.2. D. Schieber's Model

Schieber adapted a general method of solution to a rotating system which is different from Smythe's approach (Schiber 1974). The result is:

$$T = \frac{1}{2}\sigma\delta\omega\pi r^2 m^2 B_Z^2 \left[1 - \frac{(r/a)^2}{\{1-(m/a)^2\}^2}\right] \qquad (3.3)$$

where

σ = electrical conductivity of the rotating disk

δ = sheet thickness rotating disk

ω = angular velocity

π = constant coefficient

r = radius of electromagnet

m = distance of disc axis from pole-face center

a = disk radius

Bz = z component of magnetic flux density, z axis is the direction of the center of the electromagnetic pole

This formula is for low speed only. Schieber found out that his result is very close to Smythe's result at low speed and that it is valid for a linearly moving strip as well as a rotating disc. Schieber did not investigate the high speed region.

3.3.3. J.H. Wouterse's Model

Based on the works of Schieber and Smythe, Wouterse tried to find the global solution for the torque in the high-speed region as well as the low-speed region (Wouterse 1991). He observed that when the disc of an eddy current brake is moved, an electrical field $\mathbf{E}=\mathbf{n}\times\mathbf{B}$ is induced perpendicular to both the tangential speed of the rotating disk ν and the magnetic induction $\mathbf{B}$. If the speed is low with respect to the critical speed where the maximum torque occurred, the magnetic induction caused by the current pattern is negligible compared with the original induction $\mathbf{B}_0$ at zero velocity, and the magnetic induction perpendicular to the plane of the disc may be assumed to be equal to $\mathbf{B}_0$. Based on this observation, Wouterse proposed the expression for low speed:

$$\text{Fe} = \frac{1}{4}\frac{\pi}{\rho}D^2 d B_0^2 c \nu$$

$$c = \frac{1}{2}\left[1 - \frac{1}{4}\frac{1}{\left(1+\frac{r_1}{A}\right)^2\left(\frac{A-r_1}{D}\right)^2}\right] \tag{3.4}$$

where Fe is the braking force and v is the tangential speed. The other variables are parameters that can be evaluated based on different types of eddy current brakes. The formula completely agrees with Smythe's result in the low speed region.

Wouterse observed the similarity between an eddy current disc brake and a DC current-fed induction machine. The original flux remains uninfluenced by the rotor current-generated magnetic field for low speeds, and a speed-proportional torque is generated. The behavior is dominated by the current source character of the rotor circuit for very high speed, pushing away the original main flux into the leakage path, perpendicular to the teeth. Wouterse's study on the air gap magnetic field at different speeds produced three remarkable phenomena:

- At very low speeds, the field differs only slightly from the field at zero speed.

- At the speed at which the maximum dragging force is exerted, the mean induction under the pole is already significantly less than $\mathbf{B}_0$.

- At higher speeds, the magnetic induction tends to further decrease. Based on this observation, Wouterse proposed the following solution at the high speed region:

$$Fe(\nu) = \hat{Fe}\frac{2}{\frac{\nu_k}{\nu} + \frac{\nu}{\nu_k}}$$

with

$$\hat{Fe} = \frac{1}{\mu_0}\sqrt{(\frac{c}{\xi})\frac{\pi}{4}D^2 B_0^2}\sqrt{(\frac{x}{D})}$$

and

$$\nu_k = \frac{2}{\mu_0}\sqrt{(\frac{1}{c\xi})\frac{\rho}{d}}\sqrt{\frac{x}{D}} \qquad (3.5)$$

where

ρ = specific resistance of disc material

d = disc thickness

D = diameter of soft iron pole, for non-circular pole shape D denotes the diameter of the circle with the same area as pole face

ξ = ratio of zone width, in asymptotic current distribution around poles, to air gap

c = proportionality factor, ratio of total disk contour (outward curve) resistance to resistance of disk contour (outward curve) part under pole

ν = tangential speed of the rotating disk, measured at center of pole

ν_k = critical speed, i.e., speed at which exerted force is maximum

B_0 = air gap induction at zero speed

x = air gap between pole faces including disc thickness or coordinate perpendicular to air gap

r_1 = distance from center of disc to center of pole

Wouterse also made use of another known phenomenon of the high speed region in his proposal: the drag force becomes proportional to ν^{-1}. He modified the model at the high speed to make this characteristic explicit. The model turns out to be much closer to the experimental result in the high speed region.

3.4. Modified Model Proposed

While Wouterse's model gives a global solution which is good at high speed as well as at low speed, it has to use two different expressions for low-speed and high-speed regions. From a simulation or control perspective, there are difficulties involved in determining the critical speed or transitional region at which to split the low and high speed regions. As Wouterse pointed out in his paper, the proportionality factor ξ in equation (3.5) is not exactly known. It is estimated to have a value of about unity. The 10-20% estimated error of ξ would cause about a 10% error on equation (3.5). A uniform model is needed to represent the function at both regions in one expression and reduce the estimation error further.

Our approach is to modify Smythe's model according to Wouterse's observation. As Smythe himself pointed out, his model gives too rapid a roll-off at high speeds because the degree of saturation of the iron in the magnet upset many of his assumptions. To overcome this problem, we treat reluctance (R in formula (3.1)) as a function of speed instead of as a constant for representing the aggregate result of all those side effects that upset Smythe's assumptions to deduct his formula. This aggregate effect can be called "reluctance effect." The expression of reluctance should also reflect Wouterse's observations on the high speed region:

(a) The drag force becomes proportional to ν^{-1};

(b) The original magnetic induction under the pole tends to be canceled by the current induced around it in the disc.

We found that to satisfy all these observations, the reluctance of the electromagnet has to be proportional to ω asymptotically. To take this requirement into consideration and keep number of parameters limited. We found that to represent reluctance as $R = \frac{C_1 + C_2\omega + C_3\omega^3}{1 + C_4\omega^2}$ is a good approximation which satisfies the above requirement. The estimation of the C_i values for a specific type of electromagnetic brake has to been done with the close cooperation of the brake manufacturers.

Substituting this reluctance function in Smythe's formula, we are proposing the following uniform model which conforms to the experimental values (see Smythe 1942, Schiber 1974, Wouterse 1991, Omega Technologies 1996) for the electromagnetic brake operation at low as well as high speed:

$$T = \frac{k_1\omega}{\left(1 + \frac{k_2\omega^2 + k_3\omega^4}{1 + k_4\omega + k_5\omega^3}\right)^2} \tag{3.6}$$

This model is much closer to the experimental result and agrees with all the proposed models except for the high velocity range of Smythe's formula, which is inaccurate anyway. This model represents the correct behavior at high speeds. Parameters k_1-k_5 can be evaluated for specific types of electromagnetic brakes.

3.5. Comparison of Models

Different models have been evaluated based on data chart given by Omega Technologies (see Figure 3.3). A specific type, CC 250, is randomly selected to do the evaluation.

Since the modified model is a nonlinear function of angular speed and five unknown parameters need to be determined, the least squares method is used to get the values of k_1-k_5 for our modified model. Given a set of experimental data (Omega Technologies, 1996), we want to summarize the data by fitting them into the modified model that depends on the parameters k_1-k_5. We use the least squares as a maximum likelihood estimator. Suppose that we are fitting N data points (ω_i, T_i) to the model that has five adjustable parameters k_i, k =1,...,5 (see equation (3.5)). The model predicts a functional relationship between the measured independent and dependent variables, $T(\omega) = T(\omega; k1 \ldots k5)$, where the dependence on the parameters is indicated explicitly on the right-hand side. The objective function of the least squares fit is:

$$\text{minimize over } k1 \ldots k5 : \sum_{i=1}^{N} [T_i - T(\omega; k1 \ldots k5)]^2$$

Since we do not have the facility to evaluate the parameters for Smythe's model and Wouterse's model based on their formula, these parameters are assumed to be unknown and estimated by using least squares method. By applying the least squares method on the new modified model, Smythe's model, and J.H. Wouterse's model (at high speed) by using the same set of data (CC250, Omega Technologies), we obtain the results shown in Figure 3.1. It can be seen that the new model has better performance in approximating the original curve in the least-squares sense.

How to solve the parameters for the new modified model analytically is still an unsolved problem because we do not have the necessary equipment to do the measurement and test. Detailed test and analysis on "reluctance effect" have to be performed to make analytic solution feasible. Modified model should have better performance because it represent the reluctance as a changing value instead of a constant. Modified model is also a "global" solution which is suitable for simulation and control application.

3.6. Summary

A new model is proposed in this section that has better performance in a least-squares sense compared with all the models available in the literature. While the parameters for the new models can be estimated based on a least-squares fit, how to estimate the parameters analytically is open for further research. For simulation and control purposes of this thesis, the new model can be used.

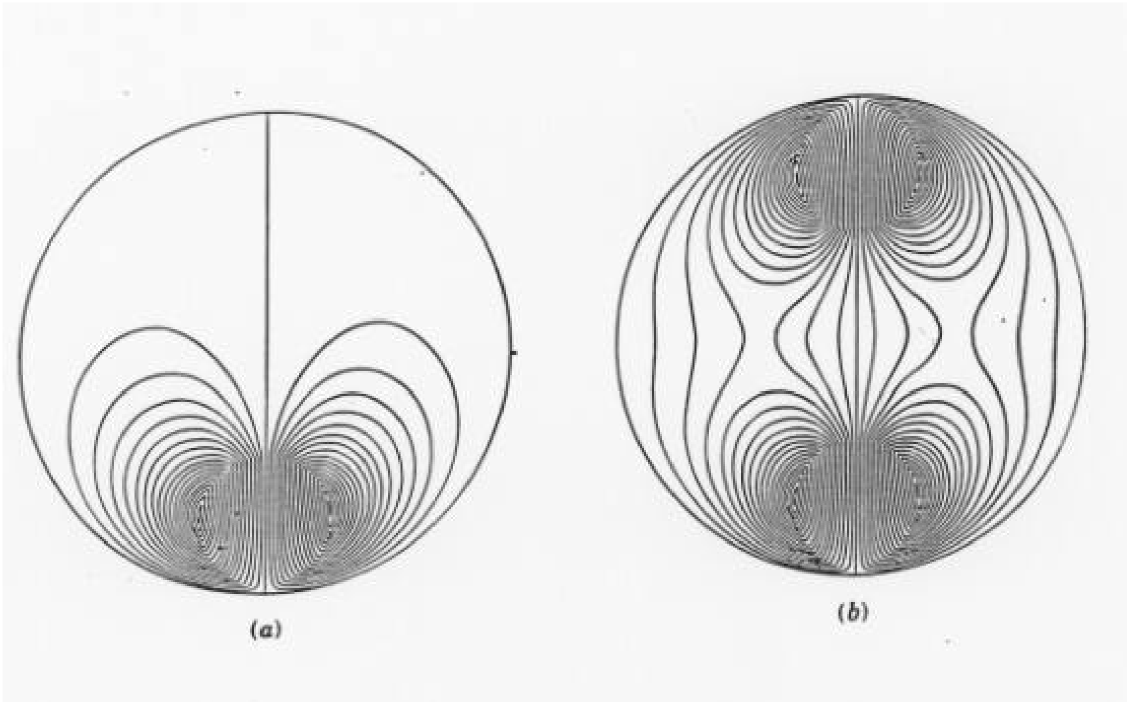

Figure 3.1. Eddy Currents Distribution Diagram for Electromagnetic Brake (Smythe, 1942)

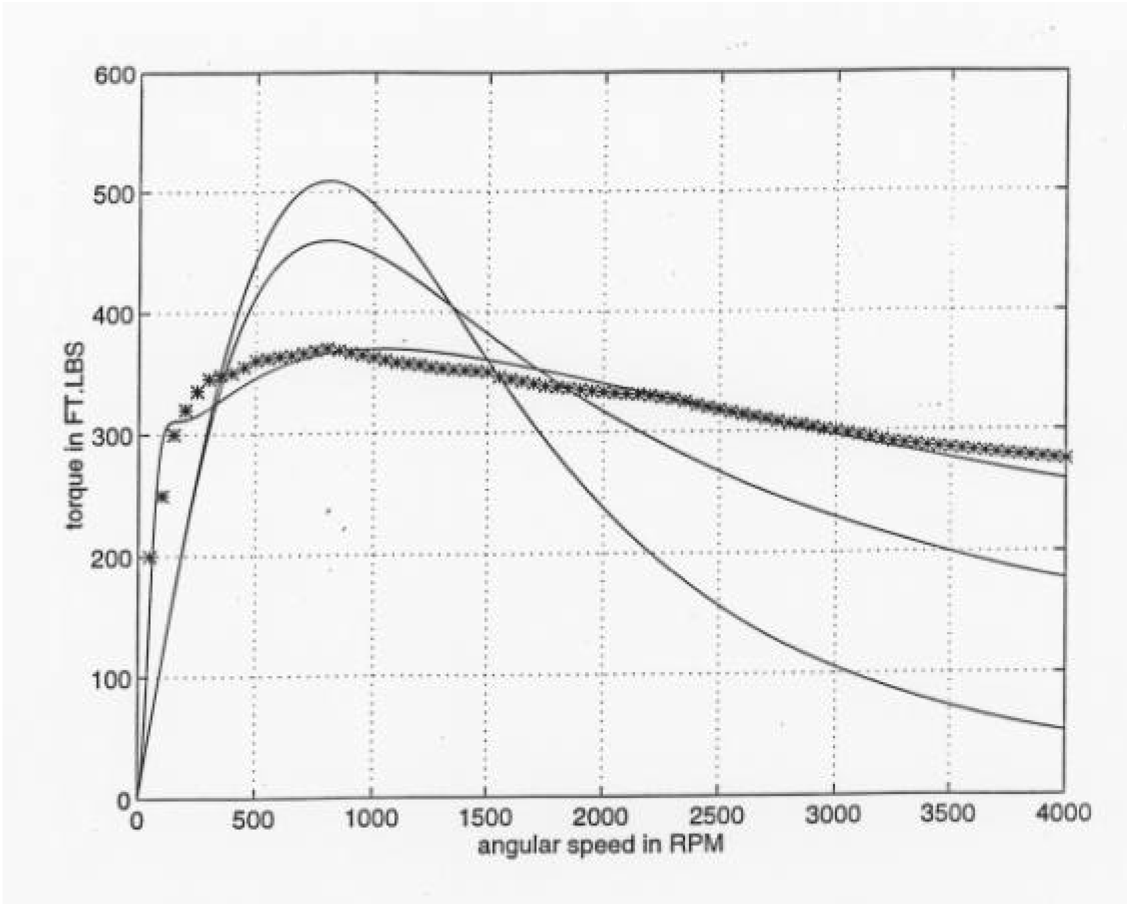

Figure 3.2. Performance Comparison among Different Static Models.
("*" curve -- Experimental data (from Omega Technologies);
dark curve -- proposed model;
thin curves -- Smythe's Model and Wouterse's Model;
(Smythe's model has a higher peak)

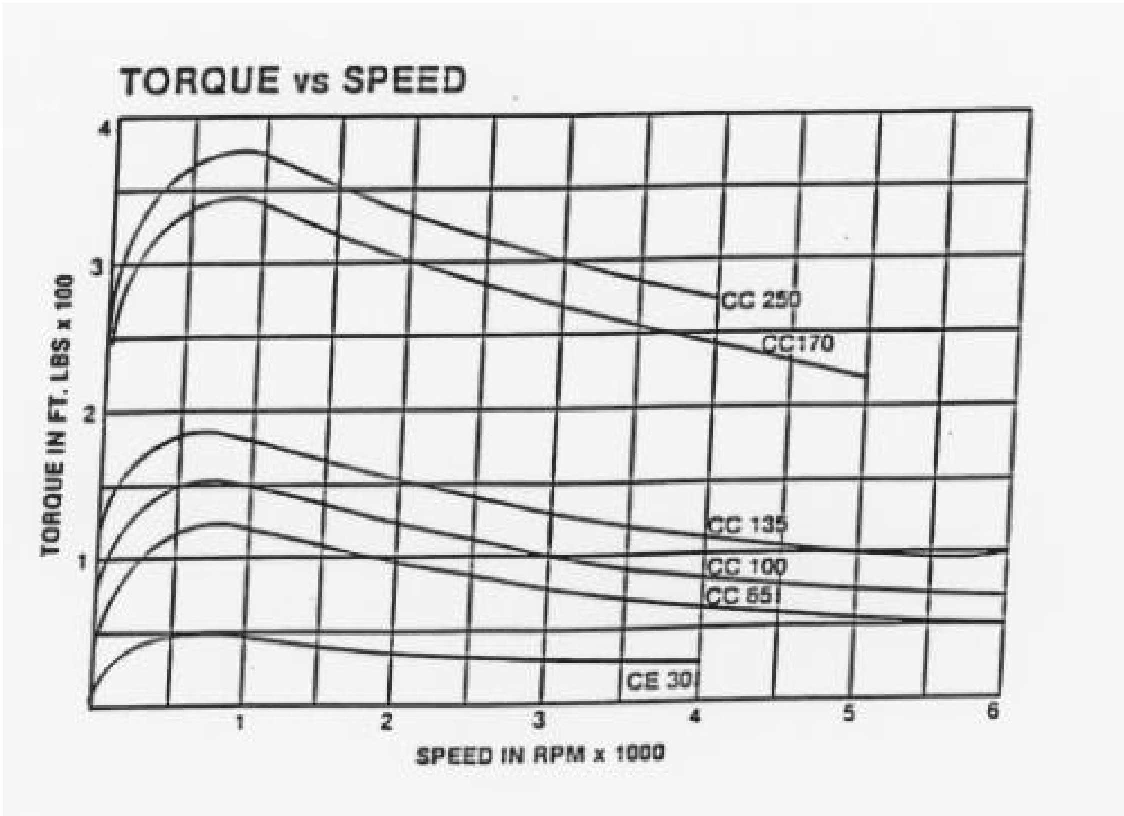

Figure 3.3. Torque Versus Speed Diagram for CC250 type Electromagnetic Brake. (Omega Technologies, 1996)