3. Technical approach

As described above, the goal of this work is to develop a miniature instrument based on optical fiber sensors to perform DMA in situ during composite manufacturing. The approach is to attach a fiber optic strain gage to a miniature actuator, as illustrated in Figure 3-1. Candidate materials for the actuator include piezoelectric, magnetostrictive, and shape memory alloy materials. Piezoelectric actuators were chosen due to their ability to produce large forces and measurable strains, and due to their compatibility with typical composite manufacturing methods, including autoclaves, RTM, and compression molding.

By immersing the sensor/actuator assembly into a curing thermoset resin and applying a steady-state sinusoidally varying excitation to the actuator, a time-varying shear stress will be transferred from the actuator into the adjacent resin. The mechanical response of the resin and the sensor/actuator assembly will be constrained by the changing mechanical impedance of the resin. Therefore, the rheological properties of the resin, which can be related to the mechanical impedance by the sensor geometry, can be determined by monitoring the strain response of the sensor assembly through the optical fiber strain gage.

![Figure 3-1. Prototype rheometer sensor.](image)

3.1 Fiber Optic Strain Gage

An optical fiber sensor design was chosen for the strain gage element of the rheometer because the small size of the fiber presents a good potential for miniaturization. Optical fibers may be embedded in fiber reinforced composites with little or no change in the
mechanical properties of the composite if the sensor is aligned with the reinforcing fibers.\(^{27}\) In addition, the use of optical fiber strain gages improves immunity to electromagnetic interference, which may plague resistive strain gages in a typical manufacturing environment. Other attributes of optical fibers that commend them for this application include elimination of ground loops, excellent strain resolution, and an ability to operate at high temperatures. In principle, the embedded viscoelasticity sensor can be used as a strain gage after fabrication of the composite part, in order to monitor internal strains.

The optical fiber sensor used for measuring strain in the viscoelasticity sensor is a modification of the extrinsic Fabry-Perot interferometer (EFPI) design.\(^{28}\) The standard EFPI sensor is a phase-modulated sensor that uses optical path length difference in an interferometric cavity to measure strain, as illustrated in Figure 3-2. A single-mode fiber, used as an input/output fiber, and a multimode fiber, used purely as a reflector, form an air gap that acts as a low-finesse Fabry-Perot cavity. The hollow core fiber, a glass capillary tube, serves to align the two fibers collinearly. As the sensor is strained, the silica tube and hence the air gap changes in length, which causes a change in the phase difference between the reference and sensing reflections \(R_1\) and \(R_2\). The intensity of the light monitored at the output of a coupler by a photodetector responds sinusoidally to a linear strain field.

Two different systems for determining the strain using an EFPI sensor were used in the course of the research reported here. In the first system, a light source with a broad spectral width, such as a superluminescent light emitting diode (SLED) was used to replace the laser diode shown in Figure 3-2. Since the coherence length of the light emitted by the source is inversely proportional to the source spectral width, interference effects within the EFPI gap can be eliminated by choosing the spectral width so that the coherence length is less than the round-trip length of the reflection that transits the cavity. In that case, no interference results, and the intensity of light monitored at the photodetector depends on the optical losses in the cavity.

---


Since the light emitted by the input/output fiber into the cavity is divergent due to diffraction effects resulting from the small core diameter (9 μm) of the fiber, the amount of light returned to the photodetector will depend on the separation of the input/output fiber and the reflector fiber. As depicted in Figure 3-3, the divergent light is intercepted and reflected by the reflector fiber. Only that part of the reflected light that is captured by the core of the input/output fiber is returned to the photodetector, and the remainder is lost. If the reflector fiber is moved closer to the input/output fiber, the total spread of the light is reduced, and the percentage of light captured by the core is increased. The light loss suffered in the cavity has been characterized analytically, and may be expressed as\

\[
\Gamma_s (dB) = -10 \log \left( \frac{1}{1 + \left( \frac{2s}{k_s w^2} \right)} \right),
\]

(2)

---

where

\[ \Gamma_s \] is the optical loss in decibels,

\[ s \] is the longitudinal gap between the fiber ends,

\[ k_g = \frac{2\pi}{\lambda} \] is the propagation constant of the light in the gap, and

\[ w \] is the mode field radius of the fiber for a single electromagnetic mode

propagating at wavelength \( \lambda \), and is defined as the radial distance from the axis of

the fiber at which the intensity \( E^2 \) is \( 1/e^2 \) from that at the axis of the fiber.

This expression neglects the loss due to Fresnel reflections, which do not change as the gap
distance \( s \) varies, and accounts for the loss due to divergence of light from the input fiber.
The mode field radius depends on the wavelength of light, the refractive index profile, and
the core diameter, and is therefore fixed for a given fiber at a single wavelength.

Therefore, a one-to-one relationship exists between the optical power detected by the
photodetector and the separation between the input/output fiber and the reflector fiber. This
relationship may be used to determine the fiber gap, given the optical power incident on the
photodetector. For the investigation of the viscoelasticity research early in the research,
this intensity-based sensor design was used to measure the displacement \( \Delta L \). If the fibers
are attached to the actuator with two spots of adhesive separated by a distance \( L_g \) (the gage
length), then the relative strain in the actuator is \( \Delta L / L_g \), where \( \Delta L \) is the change in
separation \( s \). The main disadvantage of this method of interrogating EFPI strain gages is
that any additional losses between the sensor and photodetector, such as bend-induced
losses in the optical fiber or losses due to connector misalignment, will be interpreted as a
change in the cavity gap, resulting in errors. For this reason, an alternate method of sensor
interrogation, which is less sensitive to loss-induced errors, was employed in the latter half
of the research.
Figure 3-3. Divergence of light emitted from single mode input/output fiber, leading to longitudinal separation loss in air gap cavity. The glass capillary alignment tube is omitted to clarify the illustration of light divergence.

In the second approach to EFPI interrogation, called the absolute fiber sensor system, the light reflected from the cavity is measured using a spectrometer, as shown in Figure 3-4. If a broad spectrum of light, such as white light, is injected into a Fabry Perot cavity, the spectrum of the light reflected from the cavity has a modulation imposed on it that is uniquely determined by the cavity length.

If a light source with a broad spectral output, such as white light or a light emitting diode (LED) is used as a source for the EFPI, the exact air gap separation \( s \) can be determined. Because the optical path length of the cavity changes with the wavelength, the phase difference between the two reflections \( R_1 \) and \( R_2 \) is a function of the wavelength. The result is a signal that varies with wavelength. Mathematically, it can be shown that if the wavelength of the light injected into the EFPI is scanned from \( \lambda_1 \) to \( \lambda_2 \), then the change \( \Delta \phi \) in measured phase of the output signal determines the length of the cavity by

\[
\text{s} = \frac{\Delta \phi \cdot \lambda_1 \cdot \lambda_2}{4\pi \cdot \Delta \lambda},
\]

\( \text{(3)} \)

---

where $\Delta \lambda$ is the difference of the two wavelengths to be scanned. By scanning the phase difference accumulated between $\lambda_1$ and $\lambda_2$, one can apply Equation 3 to compute the corresponding gap length.

In the absolute fiber sensor system, a broadband light-emitting diode (LED) is employed as the optical source. The LED light is first injected into one leg of a 50/50 coupler, so that approximately half of the light travels to the sensor head where it is modulated by the Fabry-Perot cavity. The reflected light travels back from the sensor head, gets split in half again at the 50-50 coupler, and enters a spectrum analyzer. In the spectrum analyzer, the different wavelengths composing the LED spectrum are resolved using a diffraction grating and continuously measured using a self-scanning CCD array interfaced with a personal computer. The personal computer uses an algorithm to determine the exact Fabry-Perot cavity length, and hence strain, from the intensity modulated CCD signal. This algorithm finds two peaks in the output spectrum of the sensor, determines the wavelength of the two peaks, and then calculates the cavity length $s$ from
\[ s = \frac{\lambda_1 \lambda_2}{2(\lambda_1 - \lambda_2)} \] (4)

The wavelengths \( \lambda_1 \) and \( \lambda_2 \) represent any two maxima in the intensity-modulated LED spectra as shown Figure 3-5. The signal shown in Figure 3-5 represents the data derived in a typical scan from the CCD array. This data array is transmitted to a personal computer where, after each scan, a sorting algorithm determines values for \( \lambda_1 \) and \( \lambda_2 \) and computes the exact Fabry-Perot cavity length using Equation 4. Upon application of a load to the sensor, the strain may be calculated by dividing the length change in the sensor by the known gage length.

![Figure 3-5. Spectrum output by EFPI sensor using a broadband source.](image)

An additional modification to the standard EFPI strain gage was employed. The illustration of the standard sensor in Figure 3-2 shows that a high modulus epoxy is used to seal the glass capillary tube and attach the optical fibers to the glass tube. As the analysis below will show, the epoxy seal influences the stiffness of the strain gage, which in turn affects the accuracy of the rheometer. In order to reduce the strain gage stiffness, the epoxy on one end of the tube was replaced with a silicone elastomer, as shown in Figure 3-1 above.
3.2 Mechanical model of rheology sensor

For the sensor under investigation here, the applied force and resulting displacement are measured at the same surface. In this case, the analysis is best performed using the principles of mechanical immitances such as impedance, which relates mechanical forces applied to a system to the consequent velocity. The mechanical impedance of a mechanical element is defined as the ratio of an applied steady-state force, expressed as a phasor quantity, to the resulting velocity, again expressed as a phasor. Alternately, the force and velocity may be transformed into the Laplace domain, in which case the mechanical impedance will be expressed as a Laplace transformed quantity.

The mechanical impedance of a complex system may be calculated by first determining the impedances of the individual components of the system. The effective impedance of the system is then found by applying rules for the combination of impedance values which are drawn from operational analysis for electrical networks. From the effective impedance of the system, the mechanical response (displacement or velocity) of the system can be derived for an arbitrary response.

Two electromechanical analogies are commonly used to determine mechanical impedances of mechanical elements. In the first, force is analogous to voltage in an electrical circuit, and velocity is analogous to electrical current. This approach is termed the electrostatic analogy, since electrostatic force is proportional to voltage. In the second, force is analogous to electrical current, and velocity is analogous to voltage. This approach is named the electromagnetic analogy, because an electromagnet’s force is proportional to the current through the windings.

In the development that follows, the electrostatic analogy will be used in the analysis of the viscoelasticity sensor. Because mechanical forces add in parallel, while voltages add in series, the rules for combination of electrostatic mechanical impedances are converse of


those for electrical impedances. That is, electrostatic mechanical impedances add in parallel, while electrical impedances add in series. This is a consequence of the fact that force acts through a mechanical element, while voltage is measured across an electrical element.

Mechanical immitances which influence the dynamic response of a mechanical system include the inertance, the elastance, and the frictance of the system. Inertance, which expresses the mass or moment of inertia of an element, is analogous to inductance under the electrostatic analogy. The elastance, which may be depicted as the stiffness of an ideal spring, equals that part of the ratio of force to displacement which is in phase with displacement, and is analogous to the reciprocal of capacitance \((1/C)\) in an electrical circuit. Similarly, frictance may be thought of as the mechanical (viscous) resistance of an ideal dashpot, and is analogous to resistance in an electrical circuit. The frictance of an element is equal to that part of the ratio of force to velocity which is in phase with the velocity.

A force circuit diagram for the rheology sensor immersed in a viscoelastic medium is illustrated in Figure 3-6. The driving force \(f(\omega)\) is supplied by a piezoelectric plate of thickness \(t\), length \(L\), and area \(A=\text{LW}\), as shown in Figure 3-7. The velocity \(v(\omega)\) is measured across the length of the piezoelectric plate, where \(\omega\) is the radian frequency of the applied force. The optical fiber strain gage together with the actuator contributes an inertance \(I_s\), due to the mass of the glass tube and the actuator. The mechanical impedance of the silicone rubber adhesive joining the optical fiber reflector to the glass capillary tube is accounted for by the elastance \(E_s\) and the frictance \(F_s\). The driving force acts in parallel with the complex mechanical impedance \(Z_r^*\) of the resin. Since the piezoelectric actuator drives the system, its elastance and inertance do not influence the response of the system, and are not accounted for in the force circuit diagram. Inertial effects of the viscoelastic medium are neglected in this development.

![Figure 3-6. Force circuit diagram for rheology sensor immersed in a viscoelastic resin, showing elastance \(E_s\), frictance \(F_s\), and inertance \(I_s\) of the fiber strain gage, and mechanical impedance \(Z_r^*\) of the viscoelastic resin. The subscript \(s\) denotes sensor, the subscript \(r\) denotes resin, and the superscript * indicates a complex quantity.](image-url)
L\textunderscore Wepoxy\textunderscore silicone rubber

Figure 3-7. Sensor geometry and dimensions. The sensor gage length is $L$, and the surface area of the actuator is $A= LW$.

Since mechanical impedances add in parallel, the mechanical impedance of the system (sensor in resin) is seen to be

$$Z_m^* = R_m + jX_m = \frac{1}{j\omega}(E_s + E_r) + (F_s + F_r) + j\omega I_s,$$

where the real and reactive components of the complex impedance $Z_r^*$ are $R_m$ and $X_m$, respectively.

We seek expressions relating the observable outputs of the sensor to the viscoelastic properties of the resin through the instrument constants. In this case, the outputs are the strain amplitude of the sensor (or equivalently the displacement amplitude, since the strain gage length is known) and the phase lag between the applied force and measured strain.

The viscoelastic properties of the resin and sensor system may be expressed generally by respondance functions relating stress and strain. By definition, the relaxance $Q(s)$ of a system is the ratio of Laplace transform of stress to Laplace transform of strain,

$$Q(s) = \sigma(s)/\varepsilon(s),$$

while the retardance $U(s) = 1/Q(s)$ is the reciprocal of the relaxance. The harmonic respondance functions, which are used in the analysis of steady state vibrations, can be found by substituting the phasor variable $j\omega$ for the Laplace variable $s$. The harmonic
respondance of a system, which is independent of geometry, is related to the mechanical immitance of the system, by the expression

\[ Q'(\omega) = j\omega b Z_m^* \]  

(7)

where \( b \) is a geometric form factor transforming impedance (ratio of force to velocity) to relaxance (ratio of stress to strain). Consider the placement of the actuator between two fixed boundaries of infinite extent, as indicated in Figure 3-8. The form factor \( b \) relates impedance to relaxance by

\[ b f = \frac{\tau}{\dot{\gamma}}, \]  

(8)

where \( v \) is the velocity of the actuator, \( \tau \) is the stress, \( \dot{\gamma} \) is the shear strain rate, and \( f \) is the force applied by the actuator. Since

\[ \frac{\tau}{\dot{\gamma}} = \frac{(f/A)}{\left(\frac{v}{c_1} + \frac{v}{c_2}\right)} = f \frac{c_1 c_2}{v A(c_1 + c_2)}, \]  

(9)

then

\[ b = \frac{c_1 c_2}{A(c_1 + c_2)}, \]  

(10)

where \( A \) is the area of the broad surface of the actuator, and \( c_1 \) and \( c_2 \) are the two distances separating the actuator from the upper and lower boundaries, respectively. If \( c_1 \approx c_2 = c \), then

\[ b = \frac{c}{2A}, \]  

(11)
and the relationship between relaxance and impedance is

\[ Q^*(\omega) = j\omega \frac{C}{\Sigma} Z_m^*. \] (12)

For the geometry in Figure 3-8, in which the specimen is sheared between the actuator and the boundaries, the appropriate relaxance is the complex shear modulus \( G^*(\omega) \).

![Figure 3-8. Cross-sectional view of rheometer actuator showing separation of actuator from adjacent boundaries. The sinusoidal steady-state force is \( f(\omega) \).](image)

Applying this transformation to the force circuit diagram yields the stress circuit diagram of Figure 3-9, in which the complex modulus \( G^* \) of the resin takes the place of mechanical impedance \( Z_m^* \). Like mechanical impedances, relaxances add in parallel, because stress acts through the mechanical elements, while displacement acts across the elements.

![Figure 3-9. Stress circuit diagram for rheological sensor immersed in viscoelastic resin with complex modulus \( G^* \).](image)
Combining the elements from the stress circuit diagram gives the respondance of the sensor immersed in resin as

\[
\frac{\tau(\omega)}{\gamma(\omega)} = b(E_s + j\omega F_s - \omega^2 I_s) + G^*.
\] (13)

In order to relate the sensor output to the actuator's input to the actuator, it is necessary to derive an expression for the stress developed by the actuator in response to an applied voltage. The piezoelectric coefficient \(d_{ij}\) associates the strain developed in the \(i\) direction with an electric field impressed across the piezoelectric material in the \(j\) direction. For the lead-zirconate-titanate (PZT) planar wafers used in this study, the two electrodes are deposited over the large faces of the wafers, so that the electric field is developed across the thickness, which is by custom taken to be the “1” direction. The piezoelectric strain of interest is perpendicular to the applied field, and is in the long direction of the wafer, in the “3” direction. Therefore

\[
d_{31} = \frac{\text{strain developed}}{\text{applied field}} = \frac{\varepsilon}{V / t},
\] (14)

where the applied voltage is \(V\), across the actuator thickness \(t\). The voltage applied to the actuator can be expressed as a phasor, so that

\[
V(t) = \text{Re} \left\{ V_0 e^{j\omega t} \right\},
\] (15)

where, by convention, the \(\text{Re}\{\}\) notation will be omitted, although it should be understood that the physical, measurable quantities are derived by taking the real part of the phasor. The piezo strain induced by the sinusoidal voltage is then

\[
\varepsilon(t) = d_{31} \frac{V_0}{T} e^{j\omega t}.
\] (16)
The shear stress developed in the piezoelectric actuator and applied to the adjacent resin is

\[ \tau(t) = Y_{33}\varepsilon(t). \quad (17) \]

Here, the \( Y_{33} \) modulus coupling longitudinal stress in the long axis of the actuator to strain in the same direction was chosen for the anisotropic PZT material. So the stress generated by the actuator as a result of the applied voltage is

\[ \tau(t) = \frac{Y_{33}d_{31}}{t}V_0e^{j\omega t}. \quad (18) \]

The shear force developed by the shear stress is

\[ f(t) = \frac{A}{t}Y_{33}d_{31}V_0e^{j\omega t}. \quad (19) \]

where \( A \) is the surface area of the piezoelectric transducer through which the shear stress is applied to the resin. The steady state shear strain \( \gamma(t) \) that results in the resin lags the stress by a phase angle \( \phi \) so that

\[ \gamma(t) = \gamma_0 e^{j(\omega t - \phi)}, \quad (20) \]

and the resulting displacement at the tip of the actuator is thus

\[ \Delta x(t) = L\gamma_0 e^{j(\omega t - \phi)}, \quad (21) \]

where \( L \) is the gage length of the fiber optic strain gage shown in Figure 3-7. For this result, one end of the sensor is considered to be the fixed reference, and the displacement
(change in length) of the other end is measured by $\Delta x(t)$. An alternate form of Equation (8), which relates mechanical impedance and relaxance, is

$$b \frac{f}{\Delta x} = \frac{\tau}{\gamma}, \quad (22)$$

where the velocity and strain rate in Equation 5 have been converted to displacement and shear strain, respectively, by integrating with respect to time. We then find that

$$\frac{\tau}{\gamma} = b \frac{f}{\Delta x} = b\left(\frac{A}{tL}Y_{33}d_{31}V_0e^{j\omega t}\right) = \left(\frac{bA}{tL}\left(\frac{V_0}{\epsilon_0}\right)Y_{33}d_{31}\right)e^{j\phi}. \quad (23)$$

Setting Equation (23) equal to Equation (13) gives

$$\frac{bA}{tL}\left(\frac{V_0}{\epsilon_0}\right)Y_{33}d_{31}e^{j\phi} = bE_s + j\omega F_s - \omega^2 I_s + G^* \quad (24)$$

From this expression, we find that the complex rheology of the sensor can be related to the sensor outputs by the instrument constants. Using Euler’s identity gives

$$G^* = G' + jG'' = \left(\frac{bA}{tL}\left(\frac{V_0}{\epsilon_0}\right)Y_{33}d_{31}\right)(\cos \phi + j \sin \phi) - b\{E_s + j\omega F_s - \omega^2 I_s\}. \quad (25)$$

Separating the real and imaginary parts and substituting for the value of $b$ derived in Equation (11) yields the storage modulus

\[34\] ibidem
\[ G' = \left( \frac{c}{2L} \right) \left( \frac{V_0}{\varepsilon_0} \right) Y_{33} d_{31} \cos \phi - \frac{c}{2A} E_s - \omega^2 \frac{c}{2A} I_s \]  

(26)

and the loss modulus

\[ G'' = \left( \frac{c}{2L} \right) \left( \frac{V_0}{\varepsilon_0} \right) Y_{33} d_{31} \sin \phi - \omega \frac{c}{2A} F_s. \]  

(27)

These results indicate that to derive the respondance of the resin, it is necessary to scale the sensor output by a factor that accounts for the sensor geometry and proximity to boundaries. Specifically, increasing the separation \( c \) of the sensor from adjacent boundaries reduces the resulting shear strain which results from a fixed displacement of the sensor.

As the actuator vibrates, nearby boundaries that are perpendicular to the direction of motion of the actuator may result in the bulk compression of the resin between the boundary and the actuator and fiber cross-section. These end effects were neglected in the analysis to derive an approximate prediction of the sensor performance, but in practice calibration of the sensor may be necessary to account for errors due to end effects.\(^{35}\) However, due to the small cross sectional area of the actuator and fiber (about \( 10^{-6} \) m\(^2 \) for the prototype sensors used in this research) relative to the area of the shearing surface of the actuator (about \( 10^{-4} \) m\(^2 \) for the prototypes), compressional effects are not expected to contribute significantly to the sensor output.

In addition, the measurement suffers from loss of accuracy due to the subtraction of terms that do not contain sensor outputs, but rather mechanical parameters relating to the silicone elastomer that seals the fiber optic strain gage. Since those terms include the factor \( I/A \), miniaturization of the sensor, with consequent reduction of actuator area, will increase the sensitivity of the sensor to these terms. For this reason, it will be important to minimize the elastance of the elastomer. In addition, the measurement of storage modulus will lose accuracy due to the mass of the sensor assembly, multiplied by the geometrical factor and the square of the radian frequency. Fortunately, miniaturization should reduce the sensor output.

mass, and the operation of the sensor will be limited to frequencies of less than 100 Hz, reducing the influence of this term.

Consider that the loss tangent is defined as

\[
\tan \delta = \frac{G''}{G'}.
\]  

(28)

Rewriting Equations 26 and 27 as

\[
\left( \frac{c}{2\pi L} \right) \left( \frac{Y_{33} d_{31}}{\varepsilon_0} \right) \sin \phi = G'' + \omega \frac{c}{2A} F_s
\]

(29)

and

\[
\left( \frac{c}{2\pi L} \right) \left( \frac{Y_{33} d_{31}}{\varepsilon_0} \right) \cos \phi = G' + \frac{c}{2A} E_s - \omega^2 \frac{c}{2A} I_s,
\]

(20)

the ratio of the two equations may be calculated to produce

\[
\tan \phi = \frac{G'' + \omega \frac{c}{2A} F_s}{G' + \frac{c}{2A} E_s - \omega^2 \frac{c}{2A} I_s} \approx \tan \delta.
\]

(31)

Therefore, if the terms involving the instrument constants \( F_s, E_s, \) and \( I_s \) can be shown to be negligible with respect to the resin moduli \( G' \) and \( G'' \), then the loss tangent can be found by taking the tangent of the phase difference \( \phi \) between the actuator excitation and the resulting strain measured by the strain gage. Also, if \( c \ll A \), which would be expected for the case in which the sensor is embedded in a composite prepreg so that the reinforcing fibers are in close proximity to the sensor, then \( \tan \phi \approx \tan \delta \).
From Equation 25, the absolute value of the complex shear modulus, commonly referred to as the absolute shear modulus, is

$$|G^*| = \left( \frac{c}{2L} \right) \left( \frac{V_0}{\varepsilon_0} \right) Y_{33} d_{31} \left( E_s + \frac{c}{2A} \left( F_s + \omega F_s - \omega^2 I_s \right) \right).$$

(32)

Again, for a sensor embedded in a composite prepreg, $c \ll A$, so that the silicone rubber immitance terms may be neglected, and the amplitude of the complex shear modulus will be

$$|G^*| = \left( \frac{c}{2L} \right) \left( \frac{V_0}{\varepsilon_0} \right) Y_{33} d_{31}.$$  

(33)

In that case, then, the absolute shear modulus will be independent of the phase difference $\phi$, inversely proportional to the strain amplitude $\varepsilon_0$, and proportional to the boundary separation distance $c$. In a composite prepreg, the boundary separation will not change during processing by more than a few percent, while the change in modulus is typically several orders of magnitude. Therefore, the relative change in strain amplitude can be monitored during cure of a composite laminate for changes indicative of significant rheological changes. Decreases in strain amplitude of 50% or more can be attributed to the increase in modulus during gelation of the matrix.

### 3.3 Determination of Instrument Constants

To determine the magnitude of the terms involving the mechanical impedance of the elastomer, consider the motion of an annular volume of silicone rubber bonding the glass capillary alignment tube to the reflector fiber in a fiber strain gage, as shown in Figure 3-10. In the typical assembly of the sensor, the length $L_s$ of the silicone rubber is approximately 1 mm.
Figure 3-10. Cross section of interface between silicone rubber and glass components of fiber optic strain gage. The silicone forms an annulus around the reflector fiber.

The motion of the silicone rubber is related to the applied force \( f(t) \) in the time domain by

\[
f(t) = I_s \ddot{x} + F_s \dot{x} + E_s x,
\]  

(34)

where \( x(t) \) is the displacement of the free end of the silicone rubber, along the axis of the reflector fiber.

In the frequency domain, the equivalent relationship is

\[
f(\omega) = -\omega^2 I_s x(\omega) + j\omega F_s x(\omega) + E_s x(\omega),
\]  

(35)

so that the stress applied to the elastomer is related to the resulting strain by

\[
\frac{f(\omega)}{x(\omega)} \frac{A_s}{L_s} = \frac{\tau(\omega)}{\gamma(\omega)} = G_s^* = G_s' + jG_s'' = \frac{L_s}{A_s} \left[ (E_s - \omega^2 I_s) + j\omega F_s \right],
\]  

(36)
where $A_s$ and $L_s$ refer to the area of the silicone elastomer in contact with the reflector fiber, and to the length of the elastomer along the fiber, respectively. $G_s^*$ is the complex shear modulus of the silicone elastomer. The subscript $s$ is used on the modulus to indicate that it refers to the properties of the silicone elastomer, and not to the modulus of the viscoelastic fluid being tested. From Equation 36, the frictance is related to the loss modulus $G_s^\prime\prime$ by

$$F_s = \frac{A_s}{\omega L_s} G_s^\prime\prime.$$  \hspace{1cm} (37)

Therefore the term in Equation 32 containing the frictance becomes

$$\omega c^2 A \frac{c}{2A} F_s = \frac{c}{2A} \left(\frac{A_s}{L_s}\right) G_s^\prime\prime.$$  \hspace{1cm} (38)

Using a Rheometrics 800 Dynamic Spectrometer, the loss modulus for the silicone rubber used for assembly of the prototype sensors and cured at 200°C was found to be approximately $10^6$ Pa. For a length of silicone rubber of one millimeter, the area of the silicone rubber contacting the reflector fiber is

$$A_s = L_s \left(\pi \times 125 \ \mu m\right) = 3.9 \times 10^{-7} \text{m}^2.$$  \hspace{1cm} (38)

For a sensor employing an actuator with a shearing surface area of 5 mm x 15 mm,

$$\omega c^2 A \frac{c}{2A} F_s \approx 2.6 \times 10^6 c,$$  \hspace{1cm} (39)

where the units are Pa if $c$ is expressed as meters. Therefore, for the frictance in Equation 32 to be negligible with respect to the loss modulus of the test specimen, we require that

$$G_s^\prime\prime > c \left(2.6 \times 10^6 \frac{Pa}{m}\right),$$  \hspace{1cm} (40)

or
In a prepreg material, the reinforcement fibers will be separated from the sensor by distances on the order of 10 μm (see Appendix D). In that case, this criterion for minimizing the influence of the silicone rubber friction on the sensor output would be satisfied for

\[G'' >> 26 \text{ Pa},\]  

which should be satisfied for most thermoset resins, with the possible exception of low viscosity systems used for resin transfer molding.

Similarly, Equation 34 also states that

\[E_s = \frac{A_s}{L_s} G_s' + \omega^2 I_s.\]  

The inertance \(I_s\) of the moving elements of the sensor was determined by measuring the masses of the moving elements of the sensor, namely the actuator, the glass capillary tube and fibers, and the silicone rubber, and was found to be 0.10 ± 0.01 gram. The storage modulus \(G_s'\) of the silicone rubber was measured using DMA to be approximately 6x10^5 Pa. For the low frequencies typically used for dynamic rheological measurements (0.1 Hz to 10 Hz), the second term in the right side of Equation 43 can be shown to be several orders of magnitude less than the first term, and therefore may be neglected. In that case, the term in Equation 32 containing the elastance becomes

\[\frac{c}{2A} E_s - \omega^2 \frac{c}{2A} I_s \simeq \frac{c}{2A} E_s = c \frac{A_s}{L_s} G_s',\]  

\[c \ll \frac{G''}{2.6 \cdot 10^6 \text{ Pa/m}},\]  

(41)
with a value of approximately

\[
\frac{c}{2A}\left(\frac{A_s}{L_s}\right)G_s' = c\left(1.6 \times 10^6 \frac{Pa}{m}\right).
\]  

(45)

In order for the elastance and inertance in Equation 33 to be negligible with respect to the storage modulus of the test specimen, we require that

\[
G' >> \frac{c}{2A}E_s - \omega^2 \frac{c}{2A}I_s,
\]  

(46)

or

\[
G' >> c\left(1.6 \times 10^6 \frac{Pa}{m}\right),
\]  

(47)

from which the requirement for the actuator/boundary separation is determined to be

\[
c << \frac{G'}{1.6 \times 10^6 \frac{Pa}{m}}.
\]  

(48)

For a boundary separation of one micrometer in a prepreg lay-up, the silicone elastance term is insignificant if \(G' >> 16 \text{ Pa}\), which will be satisfied for most thermoset resins.

Of the constants in Equations 26 and 27, most may be determined by the dimensions of the sensor and the materials used in assembly of the sensor. Of the latter, the properties of the PZT-5A material used for the actuator determine the \(d_{31}\) piezoelectric coefficient and the tensile modulus \(Y_{33}\), which may be obtained from characterizations by the material’s manufacturer. The dimensional parameters related to the sensor geometry include actuator thickness \(t\), the actuator area \(A\), the strain sensor gage length \(L\), and the separation \(c\) of the actuator from the adjacent boundary. The inertance \(I_s\) is given by the mass of the vibrating components in the sensor assembly, which can be measured using a sensitive balance. The mechanical immitances \(E_s\) and \(F_s\), which were estimated above through the use of laboratory measurements of the complex modulus of the elastomer, can be determined by
the method of Massa and Schrag, in which the resonant frequency of the sensor in air is related to the elastance of the sensor.\textsuperscript{36} Newton’s Second Law applied to the rheometer in a viscoelastic resin relates its motion to the applied force, giving

\[
f(t) = m\ddot{x} + \left( Z_r^* + F_j \right) \dot{x} + E_s x.
\]  \hspace{1cm} (49)

Here $Z_r^*$ is the complex mechanical impedance of the resin, $F_j$ and $E_s$ are the friction and elastance, respectively, of the silicone elastomer, and $m$ is the mass of the moving elements of the sensor. From Equation 21, the resulting displacement is

\[
x(t) = L \gamma_0 e^{j(\omega t - \phi)} = x_0 e^{j(\omega t - \phi)}
\]  \hspace{1cm} (50)

and the applied force from Equation 19 is

\[
f(t) = A_t Y_{33} d_{31} V_0 e^{j(\omega t - \phi)} = f_0 e^{j(\omega t - \phi)}.
\]  \hspace{1cm} (51)

Substituting Equations 50 and 51 into Equation 49 gives

\[
\frac{f_0 e^{j\phi}}{x_0} = -\omega m + E_s + j\omega \left( Z_r^* + F_j \right).
\]  \hspace{1cm} (52)

Separating the complex impedance $Z_r^*$ into its real part $R_r$ and imaginary part $X_r$ leads to two equations,

\[
\frac{f_0}{x_0} \sin \phi = \omega (R_r + F_j),
\]  \hspace{1cm} (53)

and

\[ \frac{f_0}{X_0} \cos \phi = -\omega^2 m + E_s + \omega X_r. \]  

(54)

Taking the ratio of Equation 53 to Equation 54 yields

\[ \tan \phi = \frac{\omega(R_r + F_s)}{-\omega^2 m + E_s - \omega X_r}. \]  

(55)

For the sensor vibrating freely in air, \( R_r = X_r = 0 \), and

\[ \tan \phi = \frac{\omega F_s}{-\omega^2 m + E_s}. \]  

(56)

At resonance, the reactive component of the mechanical impedance equals zero, so that \( \phi = 90^\circ \) and

\[ E_s - \omega^2 m = 0, \]  

(57)
yielding

\[ E_s = m\omega_r^2. \]  

(58)

Since the mass of the moving elements can be determined by measurement, the elastance of the silicone rubber may be calculated after determining the resonance frequency \( \omega_r \) of the sensor in air.

Due to the low mass of the assembly, the resonant frequency of the assembly is likely to exceed 50 kHz (the lowest resonant frequency of a 0.18 mm x 5 mm x 15 mm PZT plate is 93 kHz, and the addition of the optical fiber sensor lowers the frequency by one percent). To facilitate determination of \( E_s \), the assembly may be loaded by attachment of additional mass to reduce the resonant frequency of additional mass to reduce the resonant frequency. Since the resonant frequency scales inversely with the square root of the mass of the
sensor, an additional 8.5 g must be added to the 0.1 g sensor to reduce the resonant frequency to 10 kHz.

### 3.4 Applicability of Analysis by Mechanical Impedance

The application of the principles of mechanical impedance to the determination of the rheological properties of a material may be overly simplistic if the distance separating the actuator from adjacent boundaries is much greater than the wavelength of the stress wave resulting from the actuator vibration. In that case, if the attenuation of the propagating stress wave is low enough, standing waves between the actuator and adjacent boundaries result, and the contribution of the resin properties to the mechanical impedance of the system will depend on the relative phase between the actuator vibration and the reflected wave. Analysis of that condition is best handled using methods germane to acoustic wave propagation. According to Philippoff, if

\[
2\pi c > > \lambda, \tag{59}
\]

where \( c \) is distance separating the actuator from an adjacent fixed boundary and \( \lambda \) is the wavelength of the stress wave in the viscoelastic medium, then phase difference between the actuator vibration and the reflected waves become important.\(^{37}\) Since the wavelength \( \lambda \) is given by

\[
\lambda = \frac{v_a}{(\omega/2\pi)}, \tag{60}
\]

where the acoustic velocity of the longitudinal stress wave in the resin may be approximated by

\[\]

as the square root of the ratio of the Young’s modulus of the resin to its density. For the
uncured Devcon Two-Ton epoxy used in the experimental studies, a density of 1.1 g/cm³
was measured. The Young’s modulus at 10 Hz for the same resin was found to be 10 Pa
using dynamic mechanical analysis. From these values, the acoustic wavelength for the
uncured resin at 10 Hz is estimated to be 9.5 mm. As pointed out above, the separation of
the sensor from adjacent reinforcing fibers in a prepreg is expected to be much less than
one millimeter, so that criterion of Equation 57 will not be satisfied, and the use of
mechanical impedance methods to model the sensor/resin interaction is justified.

3.5 Influence of Composite Reinforcement Fibers on Sensor Output

The analysis presented in this section has so far concentrated on descriptions of outputs of
the sensor immersed in neat resins, while one intended application for the sensor is the
determination of viscoelastic properties of thermoset resins with reinforcing fibers. The
first consequence of the addition of these reinforcing fibers to the resin is that exact
measurement of values of $G'$ and $G''$ is expected to be not possible, since the unknown
spacing of fibers from the sensor surface renders calculation of the moduli through
Equations 23 and 24 unrealizable. As explained in Section 3.2 above, the sensor can be
used to derive an approximation of the loss tangent during cure, as well as relative changes
in the absolute shear modulus during cure, despite the presence of the reinforcing fibers.

In addition, the presence of reinforcing fibers can be expected to decrease slightly the time
at which gelation is observed, relative to the gel time measured for a neat resin. Hedvat has
presented an analysis which demonstrates that determination of transition temperatures
from dynamic thermomechanical spectra obtained by Torsional Braid Analysis (TBA) is
sensitive to the modulus of the torsional braid used in the test.38 TBA permits measurement
of viscoelastic properties of low modulus polymers by depositing the material to be tested
on an unreactive braid, which provides mechanical support for dynamic torsional tests.
Outputs of the tests are measures of relative rigidity and log decrement, which are
proportional to shear modulus $G$ and loss tangent, respectively. Hedvat’s analysis shows

\[ v_a = \sqrt{\frac{E}{\rho}}, \]  

\[ (61) \]

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38 Schowl Hedvat, "Influence of the Braid on Transition Temperatures as Obtained by
that the loss tangent of a polymer deposited on the torsional braid fibers can be expected to decrease in magnitude relative to the value of the loss tangent of the polymer alone. In addition, the temperature at which the peak in loss tangent occurs due to a transition such as glass transition shifts to lower temperatures with the addition of the torsional braid. Since the analysis may be extended to include any reinforcing fibers in general, a similar shift of the loss tangent peak to lower temperatures may be expected in fiber-reinforced composites. By the time-temperature superposition principle for polymers, the shift to lower temperatures is equivalent to a shift of the loss tangent peak to lower frequencies in the frequency domain. In the time domain, the shift of the loss tangent peak to lower frequencies will result in the shift of the glass transition to earlier times in the cure cycle.