CHAPTER FOUR

4. Data, results, and conclusions

4.1 Data

As part of a Southern Global Change Program project (Baldwin *et al.* 1993) linking a stand-level growth and yield model to an individual tree process model, the author and field crews randomly selected 34 loblolly pine (*Pinus taeda* L.) trees from representative unthinned plantations of varying ages in the Piedmont and Atlantic Coastal Plain of Virginia and North Carolina. The trees were felled and their crowns measured. Specific to this discussion, the crowns were divided into 6 sections (equidistant proportional to respective crown length), and one branch per section was randomly chosen for extensive measurement. Included in these measurements were the height above ground of the sample branch, the perpendicular distance from the bole to branch tip, and the perpendicular distance from the bole to the branch’s first foliage. Each tree's age, diameter at breast height (DBH), total height (HT) and height to live crown (HLC) were recorded. Crown length was subsequently calculated by subtracting HLC from HT.

A description of the mensurational attributes of the 34 sample trees can be found in Table 4.1. The region of origin for the trees in these data can be determined by the tree numbers shown in Table 4.1. Tree numbers in Table 4.1 beginning with “1” are from the Piedmont; tree numbers beginning with a “2” or “3” denote trees found in the Atlantic Coastal Plain.

Table 4.1. Descriptive statistics of the 34 trees used to describe crown profiles.

Tree	Age (yr)	DBH (cm)	HT (m)	HLC (m)
10101	10	14.1	10.3	4.5
10102	9	11.9	8.9	4.0
10103	10	17.3	10.4	3.4
10201	21	18.9	16.0	9.8
10301	17	17.1	14.3	8.4
10302	21	20.0	15.3	9.4
10303	22	17.7	12.6	6.7
10401	17	20.9	15.1	6.7
10501	18	11.6	12.2	7.3
10601	19	15.3	15.0	8.7
10701	17	18.3	14.8	9.2
10801	19	20.8	16.1	8.3
10901	29	23.8	18.7	11.0
10902	30	23.0	17.7	11.6
20101	15	17.7	14.3	5.8
20201	12	17.8	11.7	4.1
20301	12	18.5	13.0	4.3
20401	21	21.6	16.5	9.4
20501	16	23.4	16.2	7.4
20601	25	31.6	25.7	14.5
20701	14	17.3	15.2	8.9
20801	15	17.1	14.7	5.6
20901	14	19.7	16.4	8.1
30101	10	19.6	12.6	4.6
30201	11	15.3	12.0	6.3
30301	10	17.5	10.6	3.4
30401	11	15.6	11.1	4.2
30501	13	23.1	14.6	7.4
30601	19	16.4	13.6	7.3
30701	15	15.7	12.4	6.9
30801	14	17.0	12.6	5.5
30901	17	25.5	14.9	5.7
31001	22	18.5	16.1	10.0
31101	18	13.1	13.4	8.4

For all nonparametric regression fits herein, the heights above ground of the branches were normalized such that 0 corresponds to the height of the lowest sample branch and 1 to the tree top (the perpendicular distance at tree tip was set to 0). One can roughly trace out an estimate of the profile of the tree crown using these seven points. Bandwidth can then be thought of as a proportion of the distance between the lowest branch and tree tip. The normalized value of 0 was assigned to lowest sample branch because a crown radius at crown base was not available unless the sample branch actually occurred at crown base. The same condition holds for all fits of the Mohren model, equation (2.1).

For all fits of simple and/or multiple linear regression models, normalized x-values were used. For these fits, however, the normalized value of 0 was assigned to crown base. Thus the normalized X value of the lowest sample branch was >0 (unless said branch occurred at crown base). When the Baldwin model, equations (2.2) and (2.3), were applied to these data, the normalized X value of 0 again corresponded to base live crown. In these cases, even though a radius at base live crown was not required to either fit or use the models, proper application of said models required that 0 correspond to base live crown.

4.2 Nonparametric regression and bandwidth determination

Each of the three nonparametric procedures (kernel, local linear, and local quadratic regression) was used to describe outer crown profile of the trees. To show the effect of altering bandwidth, prespecified bandwidths of 0.50, 0.25 and 0.10 were used to describe crown profile for two sample trees. Figures 4.1a and 4.1b compare results for kernel regression fits for the two trees, Figures 4.1c and 4.1d compare results of local linear regression for the same two trees, and Figures 4.1e and 4.1f compare results of local quadratic regression, again for the same two trees. One can easily see the effect of altering the bandwidth from 0.50 (yielding an underfit) to 0.10 (yielding an overfit). Table 4.2 presents the statistics for the fits pictured in Figure 4.1.

Table 4.2. A comparison of fit criteria for varying bandwidth fits for kernel, local linear, and local quadratic regression for the two trees depicted throughout Figure 4.1.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr($\mathbf{W}$)
4.1a	Kernel	0.50	0.0809	0.7559	0.1473	1.8
4.1a	Kernel	0.25	0.0352	0.4363	0.1092	3.0
4.1a	Kernel	0.10	0.0033	0.4122	0.4675	6.1
4.1b	Kernel	0.50	0.0607	0.6023	0.1157	1.8
4.1b	Kernel	0.25	0.0196	0.2886	0.0728	3.0
4.1b	Kernel	0.10	0.0051	0.2141	0.1502	5.6
4.1c	Local Linear	0.50	0.0561	0.8277	0.1878	2.6
4.1c	Local Linear	0.25	0.0303	0.5357	0.1672	3.8
4.1c	Local Linear	0.10	0.0045	0.2701	0.4804	6.4
4.1d	Local Linear	0.50	0.0184	0.1521	0.0347	2.6
4.1d	Local Linear	0.25	0.0147	0.1945	0.0595	3.7
4.1d	Local Linear	0.10	0.0039	0.2374	0.4290	6.4
4.1e	Local Quadratic	0.50	0.0381	0.8154	0.2414	3.6
4.1e	Local Quadratic	0.25	0.0104	0.2383	0.1196	5.0
4.1e	Local Quadratic	0.10	0.0001	0.3112	34.8415	7.0
4.1f	Local Quadratic	0.50	0.0180	0.3960	0.1140	3.5
4.1f	Local Quadratic	0.25	0.0139	0.2619	0.1294	5.0
4.1f	Local Quadratic	0.10	0.0011	0.4081	13.5121	7.0

Note the roles the PRESS and the PRESS* criteria would play when determining the proper bandwidth. Selecting the fit with the smallest PRESS statistic can result with the user choosing an inappropriately small bandwidth. PRESS*, however, combines the properties of decreasing PRESS and increasing tr(**W**), and can lead to the choice of a more appropriate bandwidth. Use of bandwidths chosen via the minimum PRESS* criterion results in fits that achieve a proper balance between underfitting and overfitting in order to maintain a reasonably smooth shape without just "connecting the dots".

Figure 4.2 shows fits obtained by using the minimum PRESS* bandwidth for six trees. (Figures 4.3 through 4.11 depict the same type of figures for all 34 trees.) For comparative purposes, Figure 4.2a is the same tree represented in Figures 4.1a, 4.1c, and 4.2e, Figure 4.2b is the tree appearing in Figures 4.1b, 4.1d, and 4.1f, and Figures 4.2c-f are four additional trees not appearing in Figure 4.1. The inadequacy of kernel regression to fully capture boundary behavior is evident in these figures. Fit statistics are shown in Table 4.3. (Corresponding fit statistics for Figures 4.3 through 4.11 can be found in Table 4.4 through 4.12.)

Figure 4.2f suggests that these techniques may fail to capture some structure when the data sets are too "jagged". One might argue that there is no real structure to capture in such data, other than the near straight line achieved by the nonparametric techniques. This is accomplished by choosing a large bandwidth (possibly near 1 for 0-1 scaled data). A large bandwidth results because any reasonably smooth fit will be very high in bias (due to jagged data), and so to keep variance small, it is best to choose a large bandwidth (possibly unity). If one desired, an extremely small bandwidth could be specified in order

Table 4.3. A comparison of optimal (PRESS* criterion) nonparametric fits for the six trees depicted in Figure 4.2.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.2a	Kernel	0.29	0.0413	0.4641	0.1068	2.7
4.2a	Local Linear	0.18	0.0194	0.3331	0.1377	4.6
4.2a	Local Quadratic	0.28	0.0137	0.2495	0.1101	4.8
4.2b	Kernel	0.23	0.0173	0.2743	0.0724	3.1
4.2b	Local Linear	0.67	0.0192	0.1486	0.0320	2.4
4.2b	Local Quadratic	0.29	0.0150	0.2525	0.1057	4.6
4.2c	Kernel	0.18	0.0394	0.5713	0.1827	3.9
4.2c	Local Linear	0.27	0.0787	0.6608	0.1888	3.5
4.2c	Local Quadratic	0.44	0.0889	0.7814	0.2323	3.6
4.2d	Kernel	0.20	0.0507	0.7887	0.2335	3.6
4.2d	Local Linear	0.28	0.0666	0.5609	0.1596	3.5
4.2d	Local Quadratic	0.47	0.0752	0.6868	0.2002	3.6
4.2e	Kernel	0.25	0.0336	0.5148	0.1287	3.0
4.2e	Local Linear	1.00	0.0221	0.3122	0.0650	2.2
4.2e	Local Quadratic	0.13	0.0040	0.1451	0.1329	5.9
4.2f	Kernel	0.48	0.1437	1.3979	0.2672	1.8
4.2f	Local Linear	1.00	0.1049	0.8237	0.1718	2.2
4.2f	Local Quadratic	1.00	0.1251	1.9552	0.5044	3.2

Table 4.4. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.3.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.3a	Kernel	0.25	0.0336	0.5148	0.1287	3.0
4.3a	Local Linear	1.00	0.0221	0.3122	0.0650	2.2
4.3a	Local Quadratic	0.13	0.0040	0.1451	0.1329	5.9
4.3b	Kernel	0.30	0.0247	0.4841	0.1108	2.6
4.3b	Local Linear	0.69	0.0095	0.0790	0.0171	2.4
4.3b	Local Quadratic	0.78	0.0099	0.0890	0.0233	3.2
4.3c	Kernel	0.20	0.0233	0.5518	0.1624	3.6
4.3c	Local Linear	0.21	0.0057	0.0947	0.0329	4.1
4.3c	Local Quadratic	0.25	0.0029	0.0589	0.0290	5.0
4.3d	Kernel	0.29	0.0787	0.7916	0.1844	2.7
4.3d	Local Linear	1.00	0.0955	1.3143	0.2729	2.1
4.3d	Local Quadratic	0.25	0.0434	1.0353	0.4583	4.7

Table 4.5. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.4.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.4a	Kernel	0.48	0.1437	1.3979	0.2672	1.8
4.4a	Local Linear	1.00	0.1049	0.8237	0.1718	2.2
4.4a	Local Quadratic	1.00	0.1251	1.9552	0.5044	3.2
4.4b	Kernel	0.25	0.0104	0.1615	0.0406	3.0
4.4b	Local Linear	1.00	0.0059	0.0455	0.0094	2.1
4.4b	Local Quadratic	0.26	0.0043	0.0862	0.0398	4.8
4.4c	Kernel	0.27	0.0379	0.5421	0.1328	2.9
4.4c	Local Linear	0.50	0.0218	0.2566	0.0580	2.6
4.4c	Local Quadratic	1.00	0.0164	0.1975	0.0507	3.1
4.4d	Kernel	0.38	0.1265	1.4502	0.3035	2.2
4.4d	Local Linear	1.00	0.0794	0.8328	0.1719	2.2
4.4d	Local Quadratic	1.00	0.0983	1.5136	0.3907	3.1

Table 4.6. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.5.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.5a	Kernel	0.17	0.0225	0.3268	0.1034	3.8
4.5a	Local Linear	0.26	0.0570	0.5409	0.1647	3.7
4.5a	Local Quadratic	0.41	0.0494	0.5751	0.1867	3.9
4.5b	Kernel	0.25	0.0268	0.9964	0.2501	3.0
4.5b	Local Linear	0.24	0.0047	0.1158	0.0364	3.8
4.5b	Local Quadratic	0.41	0.0040	0.0370	0.0115	3.8
4.5c	Kernel	0.29	0.0413	0.4641	0.1068	2.7
4.5c	Local Linear	0.18	0.0194	0.3331	0.1377	4.6
4.5c	Local Quadratic	0.28	0.0137	0.2495	0.1101	4.8
4.5d	Kernel	0.29	0.0709	1.1210	0.2588	2.7
4.5d	Local Linear	0.52	0.0321	0.2638	0.0600	2.6
4.5d	Local Quadratic	1.00	0.0330	0.3493	0.0911	3.2

Table 4.7. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.6.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.6a	Kernel	0.28	0.1121	0.1331	0.3155	2.8
4.6a	Local Linear	1.00	0.0691	0.7858	0.1634	2.2
4.6a	Local Quadratic	1.00	0.0780	4.4480	1.1450	3.1
4.6b	Kernel	0.24	0.0630	0.8292	0.2165	3.2
4.6b	Local Linear	1.00	0.0716	0.8352	0.1737	2.2
4.6b	Local Quadratic	1.00	0.0829	2.2542	0.5754	3.1
4.6c	Kernel	0.33	0.1071	1.4511	0.3165	2.4
4.6c	Local Linear	0.23	0.0478	0.4010	0.0829	2.2
4.6c	Local Quadratic	N/A	N/A	N/A	N/A	N/A
4.6d	Kernel	0.20	0.0174	0.3423	0.1029	3.0
4.6d	Local Linear	1.00	0.0085	0.1031	0.0266	3.1
4.6d	Local Quadratic	0.23	0.0173	0.2743	0.0724	3.2

Table 4.8. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.7.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.7a	Kernel	0.23	0.0173	0.2743	0.0724	3.1
4.7a	Local Linear	0.67	0.0192	0.1486	0.0320	2.4
4.7a	Local Quadratic	0.29	0.0150	0.2525	0.1057	4.6
4.7b	Kernel	0.18	0.0394	0.5713	0.1827	3.9
4.7b	Local Linear	0.27	0.0787	0.6608	0.1888	3.5
4.7b	Local Quadratic	0.44	0.0889	0.7814	0.2323	3.6
4.7c	Kernel	0.12	0.0060	0.0884	0.0428	4.9
4.7c	Local Linear	1.00	0.1055	0.9508	0.1960	2.1
4.7c	Local Quadratic	1.00	0.0990	2.1101	0.5420	3.1
4.7d	Kernel	0.24	0.0699	0.8644	0.2196	3.1
4.7d	Local Linear	1.00	0.2067	2.8539	0.5889	2.1
4.7d	Local Quadratic	0.40	0.0531	2.9282	0.9552	3.9

Table 4.9. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.8.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.8a	Kernel	0.24	0.0190	0.2223	0.0572	3.1
4.8a	Local Linear	0.28	0.0180	0.1521	0.0441	3.5
4.8a	Local Quadratic	0.64	0.0197	0.2578	0.0710	3.4
4.8b	Kernel	0.23	0.0302	0.4017	0.1060	3.2
4.8b	Local Linear	0.26	0.0261	0.2726	0.0809	3.6
4.8b	Local Quadratic	0.39	0.0198	0.2236	0.0726	3.9
4.8c	Kernel	0.20	0.0144	0.2550	0.0780	3.7
4.8c	Local Linear	0.20	0.0116	0.1190	0.0428	4.2
4.8c	Local Quadratic	0.39	0.0135	0.2154	0.0697	3.9
4.8d	Kernel	0.28	0.0552	0.6242	0.1476	2.8
4.8d	Local Linear	1.00	0.0638	0.7777	0.1605	2.2
4.8d	Local Quadratic	0.87	0.0593	1.5964	0.4245	3.2

Table 4.10. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.9.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.9a	Kernel	0.29	0.0245	0.3181	0.0738	2.7
4.9a	Local Linear	0.52	0.0066	0.0586	0.0132	2.5
4.9a	Local Quadratic	1.00	0.0059	0.0646	0.0168	3.2
4.9b	Kernel	0.22	0.0248	0.3869	0.1082	3.4
4.9b	Local Linear	0.65	0.0326	0.2815	0.0609	2.4
4.9b	Local Quadratic	0.23	0.0103	0.2065	0.1233	5.3
4.9c	Kernel	0.29	0.0661	0.7417	0.1734	2.7
4.9c	Local Linear	0.23	0.0456	0.5355	0.1727	3.9
4.9c	Local Quadratic	0.38	0.0425	0.4348	0.1430	4.0
4.9d	Kernel	0.29	0.0543	1.0283	0.2393	2.7
4.9d	Local Linear	1.00	0.0280	0.2580	0.0537	2.2
4.9d	Local Quadratic	1.00	0.0324	1.0706	0.2744	3.1

Table 4.11. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.10.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.10a	Kernel	0.28	0.0518	0.6073	0.1415	2.7
4.10a	Local Linear	0.74	0.0690	1.0242	0.2156	2.2
4.10a	Local Quadratic	0.97	0.0355	0.9621	0.2525	2.3
4.10b	Kernel	0.36	0.1303	1.7855	0.3743	2.2
4.10b	Local Linear	1.00	0.0831	0.7357	0.1535	2.2
4.10b	Local Quadratic	N/A	N/A	N/A	N/A	N/A
4.10c	Kernel	0.23	0.0253	0.3132	0.0817	3.2
4.10c	Local Linear	1.00	0.0159	0.1516	0.0311	2.1
4.10c	Local Quadratic	1.00	0.0165	0.2384	0.0621	3.2
4.10d	Kernel	0.20	0.0507	0.7887	0.2335	3.6
4.10d	Local Linear	0.28	0.0666	0.5609	0.1596	3.5
4.10d	Local Quadratic	0.47	0.0752	0.6868	0.2002	3.6

Table 4.12. A comparison of fit criteria for optimal (via PRESS*) bandwidth fits for kernel, local linear, and local quadratic regression for the four trees depicted in Figure 4.11.

Figure	Fit	Bandwidth	$s^2_{(NP)}$	PRESS	PRESS*	tr(**W**)
4.11a	Kernel	0.16	0.0122	0.1950	0.0736	4.3
4.11a	Local Linear	1.00	0.0175	0.1647	0.0341	2.2
4.11a	Local Quadratic	1.00	0.0207	0.2514	0.0650	3.1
4.11b	Kernel	0.24	0.0350	0.4253	0.1066	3.1
4.11b	Local Linear	0.33	0.0274	0.2329	0.0619	3.2
4.11b	Local Quadratic	0.44	0.0236	0.3075	0.0961	3.8

to capture some of the extreme variability in the data. However doing so would have negative implications by increasing the variance of prediction $\sigma^2 \mathbf{WW}'$. This occurs because the weight matrix **W** approaches the identity matrix as the bandwidth approaches zero (giving all weight [$w_{ii} = 1$] to each point being fit), and $\sigma^2 \mathbf{WW}'$ is maximized when **W** equals the identity matrix.

For each of the crowns depicted in Figures 4.3 through 4.11, the optimal (PRESS* criterion) nonparametric fit was taken to be that fit which possessed the smallest PRESS* value. Those values can be evaluated from the data presented in Tables 4.4 through 4.12. The form of nonparametric regression deemed optimal varied from tree to tree.

4.3 Nonparametric regression versus linear regression

4.3.1 Results

One potential method to model the crown profiles would be to use ordinary least squares regression to approximate the shapes of the outer crown profiles. Shapes such as that depicted in Figure 4.6c might be represented by simple linear regression of crown radius on height above ground (recall all fits were performed with height above ground being normalized to a 0-1 scale even though the figures depict height above ground on the X-axes). Shapes such as that depicted in Figure 4.7d and 4.8b might be represented by multiple linear regression, with crown radius being a function of a linear and quadratic term of the normalized height above ground for the former, and linear, quadratic, and cubic terms for the latter.

Regression curves of all three forms (simple linear, and quadratic and cubic multiple linear regression) were fit to these data, tree by tree, and plotted with the optimal (PRESS* criterion) nonparametric fit. See Figures 4.12 through 4.20. Corresponding fit statistics can be found in Tables 4.13 through 4.21. The PRESS statistic can be used to compare the OLS fits with the nonparametric fits because the dataset does not change, nor are there any transformations made to the x's, as the method of fit changes.

Various levels of success were achieved with the OLS fits. When the crown profile was approximately linear, as in Figure 4.12b and 4.13b, there was little difference between the nonparametric fit and the corresponding OLS fits. For such shapes, OLS regression beyond the simple linear form is inappropriate. Interestingly, $s^2_{(NP)}$ for these crown profiles

Table 4.13. Fit statistics for the curves depicted in Figure 4.12.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.12a	Lin.	0.022	0.256	Loc. Lin.	0.0221	0.3122	0.0650
	Quad.	0.025	1.922				
	Cubic	0.014	4.583				
4.12b	Lin.	0.010	0.089	Loc. Lin.	0.0095	0.0790	0.0171
	Quad.	0.010	0.091				
	Cubic	0.012	0.107				
4.12c	Lin.	0.024	0.327	Loc. Quad.	0.0057	0.0947	0.0329
	Quad.	0.008	0.114				
	Cubic	0.008	0.269				
4.12d	Lin.	0.098	1.172	Kernel	0.0787	0.7916	0.1844
	Quad.	0.104	4.063				
	Cubic	0.049	3.104				

Table 4.14. Fit statistics for the curves depicted in Figure 4.13.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.13a	Lin.	0.101	0.804	Loc. Lin.	0.1049	0.8237	0.1718
	Quad.	0.123	1.663				
	Cubic	0.154	14.556				
4.13b	Lin.	0.005	0.043	Loc. Lin.	0.0059	0.0455	0.0094
	Quad.	0.007	0.083				
	Cubic	0.009	0.333				
4.13c	Lin.	0.032	0.352	Loc. Quad.	0.0164	0.1975	0.0507
	Quad.	0.016	0.181				
	Cubic	0.021	0.457				
4.13d	Lin.	0.077	0.780	Loc Lin.	0.0794	0.8328	0.1719
	Quad.	0.096	1.327				
	Cubic	0.124	4.437				

Table 4.15. Fit statistics for the curves depicted in Figure 4.14.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.14a	Lin.	0.094	0.917	Kernel	0.0225	0.3268	0.1034
	Quad.	0.133	1.800				
	Cubic	0.059	1.276				
4.14b	Lin.	0.025	0.395	Loc. Quad.	0.0040	0.0370	0.0115
	Quad.	0.007	0.147				
	Cubic	0.004	0.148				
4.14c	Lin.	0.066	0.734	Kernel	0.0413	0.4641	0.1068
	Quad.	0.067	1.306				
	Cubic	0.033	1.617				
4.14d	Lin.	0.035	0.323	Loc Lin.	0.0321	0.2638	0.0600
	Quad.	0.032	0.295				
	Cubic	0.041	4.137				

Table 4.16. Fit statistics for the curves depicted in Figure 4.15.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.15a	Lin.	0.068	0.700	Loc. Lin.	0.0691	0.7858	0.1634
	Quad.	0.080	3.420				
	Cubic	0.078	25.883				
4.15b	Lin.	0.069	0.740	Loc. Lin.	0.0716	0.8352	0.1737
	Quad.	0.086	2.105				
	Cubic	0.074	7.099				
4.15c	Lin.	0.046	0.389	Loc. Lin.	0.0478	0.4010	0.0829
	Quad.	0.057	0.486				
	Cubic	0.076	9.711				
4.15d	Lin.	0.032	0.335	Loc Lin.	0.0085	0.1031	0.0266
	Quad.	0.008	0.092				
	Cubic	0.010	0.353				

Table 4.17. Fit statistics for the curves depicted in Figure 4.16.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.16a	Lin.	0.022	0.178	Loc. Lin.	0.0192	0.1486	0.0320
	Quad.	0.018	0.303				
	Cubic	0.022	0.925				
4.16b	Lin.	0.107	0.882	Kernel	0.0394	0.5713	0.1827
	Quad.	0.133	1.675				
	Cubic	0.075	1.352				
4.16c	Lin.	0.110	0.980	Kernel	0.0060	0.0884	0.0428
	Quad.	0.101	1.967				
	Cubic	0.105	4.298				
4.16d	Lin.	0.219	2.694	Kernel	0.0699	0.8644	0.2196
	Quad.	0.200	6.213				
	Cubic	0.025	1.894				

Table 4.18. Fit statistics for the curves depicted in Figure 4.17.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.17a	Lin.	0.060	0.699	Loc. Lin.	0.0180	0.1521	0.0441
	Quad.	0.024	0.362				
	Cubic	0.018	1.068				
4.17b	Lin.	0.111	1.286	Loc. Quad.	0.0198	0.2236	0.0726
	Quad.	0.029	0.487				
	Cubic	0.017	0.190				
4.17c	Lin.	0.046	0.502	Loc. Lin.	0.0116	0.1190	0.0428
	Quad.	0.037	1.036				
	Cubic	0.012	0.515				
4.17d	Lin.	0.064	0.721	Kernel	0.0552	0.6242	0.1476
	Quad.	0.071	1.740				
	Cubic	0.128	1.978				

Table 4.19. Fit statistics for the curves depicted in Figure 4.18.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.18a	Lin.	0.008	0.077	Loc. Lin.	0.0066	0.0586	0.0132
	Quad.	0.006	0.056				
	Cubic	0.007	0.209				
4.18b	Lin.	0.039	0.339	Loc. Lin.	0.0326	0.2815	0.0609
	Quad.	0.030	0.482				
	Cubic	0.033	1.644				
4.18c	Lin.	0.120	1.249	Loc. Quad.	0.0425	0.4348	0.1430
	Quad.	0.094	1.390				
	Cubic	0.030	0.549				
4.18d	Lin.	0.027	0.236	Loc. Lin.	0.0280	0.2580	0.0537
	Quad.	0.033	0.943				
	Cubic	0.031	7.463				

Table 4.20. Fit statistics for the curves depicted in Figure 4.19.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.19a	Lin.	0.087	1.113	Kernel	0.0518	0.6073	0.1415
	Quad.	0.042	1.032				
	Cubic	0.023	0.832				
4.19b	Lin.	0.085	0.755	Loc. Lin.	0.0831	0.7357	0.1535
	Quad.	0.088	0.821				
	Cubic	0.111	24.529				
4.19c	Lin.	0.016	0.154	Loc. Lin.	0.0159	0.1516	0.0311
	Quad.	0.016	0.214				
	Cubic	0.020	0.511				
4.19d	Lin.	0.102	1.060	Loc. Lin.	0.0666	0.5609	0.1596
	Quad.	0.109	2.663				
	Cubic	0.067	5.150				

Table 4.21. Fit statistics for the curves depicted in Figure 4.20.

Figure	Parametric Fit	s^2	PRESS	Nonparametric Fit	$s^2_{(NP)}$	PRESS	PRESS*
4.20a	Lin.	0.017	0.159	Loc. Lin.	0.0175	0.1647	0.0341
	Quad.	0.020	0.222				
	Cubic	0.025	0.848				
4.20b	Lin.	0.038	0.333	Loc. Lin.	0.0274	0.2329	0.0619
	Quad.	0.036	0.624				
	Cubic	0.025	0.480				

were nearly identical to s^2 for the simple linear regression fits. It appears, then, for linear crown profiles, simple linear regression performs just as well as nonparametric regression. For profiles such as the one depicted in 4.17b, where quadratic linear regression appears appropriate and in fact performs admirably, the optimal nonparametric curve performs slightly better, as indicated by $s^2_{(NP)}$ versus s^2 as well as visual inspection of the plots.

A much different result presented itself for crown profiles possessing substantial curvature, such as the profiles depicted in Figure 4.14a and 4.14c. In Figure 4.14a and other similar profiles, neither simple linear nor quadratic linear regression could capture the variation in the crown profile. Cubic linear regression can somewhat approximate such shapes, but not without odd boundary behavior at the tree top, where the estimated radii begin to increase after yielded slightly negative estimates of crown radius.

Furthermore, cubic multiple regression estimates in such cases are quite unstable as indicated by increasing PRESS values. In comparison, note the optimal nonparametric fit depicted in Figure 4.14a. It behaves very well with respect to the variation in shape. In cases of extreme curvature, then, it appears that optimal (PRESS* criterion) nonparametric regression outperforms multiple linear regression.

A final comparison was made between the nonparametric and OLS regression fits to these crown profiles. Residuals from the optimum (PRESS* criterion) nonparametric regression fits were pooled across trees and plotted against relative crown height (Figure 4.21). The same was done for residuals from the respective OLS fits. The residuals from the linear fits are shown in Figure 4.22; while those of quadratic and cubic regression appear in Figure 4.23 and 4.24 respectively. The residuals from the nonparametric fits are better behaved than those of either simple linear or quadratic regression. The cubic regression residual plot appears to be fairly well-behaved. However, the instability of the estimates (large PRESS statistics) and the questionable behavior evident in the plots (negative and/or increasing radii at tree tip) make using cubic regression a risky proposition. Mean absolute residuals were also calculated and can be found in Table 4.22. Again, nonparametric regression appears to be the best candidate once the problems with cubic regression are considered.

Table 4.22. A comparison of absolute residuals of the outer crown profile fits using optimal nonparametric regression, simple linear regression, and multiple linear (quadratic and cubic) regression.

Fit	n	Mean	Std. Dev.
Optimal nonparametric	238	0.100	0.111
Simple linear regression	238	0.152	0.144
Quadratic regression	238	0.130	0.124
Cubic regression	238	0.097	0.095

4.3.2 Summary

Based upon the analysis herein, nonparametric regression provides fits fairly equivalent to simple linear regression fits for crown profile data that are linear in nature. For crown profile data possessing some curvature, nonparametric regression provides slightly better fits than its OLS counterpart. Finally, for crown profile data with extreme curvature, nonparametric regression provides better fits than quadratic or cubic multiple linear regression are capable of providing.

4.4 Nonparametric regression versus other outer crown profile models

4.4.1 Results

The Mohren model (equation 2.1) was fit to each of the 34 trees used herein. The Baldwin model (equation 2.2) was likewise applied to each of the 34 trees. Plots of the fits and applications, respectively, can be found in Figures 4.25 through 4.33. for comparative purposes, the optimal (PRESS* criterion) nonparametric fit is also depicted for each tree.

The Mohren model is capable of representing a wide variety of shapes such as the linear shapes of Figure 4.25a and 4.26b and the curvilinear shape depicted in Figure 4.30b. However, when there is curvature present, the Mohren model sometimes fails to fully capture the curvature (Figure 4.27c and 4.29d) or ignores the curvature altogether (Figure 4.25d and 4.29b). While it behaves properly at the boundaries, the Mohren model fails to successfully capture much of the curvature present in the interior of the crown profiles.

The Baldwin model on the other hand appears to force curvature in every profile, whether it is actually present or not. Figure 4.26b is a prime example. The trend is clearly linear, yet the model forces curvature into the profile. This forced curvature often results in poor behavior in the lower portion of the crown profile. The curve tends to predict a radius much smaller than those observed. Residual plots amplify the relatively poor behavior of the Baldwin model (Figure 4.34) when compared to the optimal (PRESS* criterion) nonparametric regression residual plot (Figure 4.21). The poor behavior of the Baldwin model in the lower crown is clearly evident. The residual plot for the Mohren model (Figure 4.35) is likewise inferior to that of the nonparametric fits. Table 4.23 contains the mean absolute residuals for these fits. The nonparametric fit mean absolute residual is the superior of the three.

Table 4.23. A comparison of absolute residuals of the outer crown profile fits using optimal nonparametric regression and the Mohren model (equation 2.1), and the application of the Baldwin model (equation 2.2).

Fit	n	Mean	Std. Dev.
Optimal nonparametric	238	0.100	0.111
Mohren model	238	0.139	0.177
Baldwin model	238	0.430	0.349

4.4.2 Summary

Optimal (PRESS* criterion) nonparametric regression seems to perform better than both the Mohren model (equation 2.1) and the Baldwin model (equation 2.2) with respect to outer crown profiles. The Mohren model can assume a variety of shapes, but performed poorly in the interior of the crown profiles of these data. The Baldwin model forced an ellipsoidal shape onto each profile regardless if such a shape was actually present.

The Mohren model, once parameterized, and especially the Baldwin model, do afford one advantage over nonparametric regression in that they are mainly models describing a typical crown profile for a given species. No data are necessary to use these equations. Nonparametric regression requires data for each crown before a fit can be estimated. Therefore, if no data are available, the Mohren and/or Baldwin model will provide an approximate crown shape for a species. If data are available, though, nonparametric regression can better approximate the shapes present in the data.

4.5 Nonparametric regression versus other inner crown profile models

4.5.1 Results

The Baldwin model, equation (2.3), was the only model available for comparison with nonparametric regression fits to the inner crown profile. Nepal *et al*. (1996) mentioned stochastic frontier models could be used to fit a curve to the inner profile, but they did not attempt to fit such a model. Recall that the Baldwin model for inner crown profile was linear in nature.

Figures 4.36 through 4.44 depict the curves obtained from the Baldwin model compared to the optimal (PRESS* criterion) nonparametric regression fits to the inner crown profiles. Unlike the previous applications, the use of nonparametric regression was problematic in several instances. For example, Figures 4.36a and 4.37d depict crowns for which the nonparametric fit is practically uniform even though the data clearly are not.

In comparison, the Baldwin model performs fairly well across all crowns, and only behaves poorly when extreme curvature exists in a given profile, *i.e.* Figure 4.37a. Such must be the case because of the linear nature of the Baldwin model. The nonparametric fits, for the most part, appear superior in such instances. Residual analyses of the nonparametric fits and the Baldwin model application are shown in Figures 4.45 and 4.46 respectively. Both appear to be fairly well behaved. Mean absolute residuals are shown in Table 4.24.

Table 4.24. A comparison of absolute residuals of the inner crown profile fits using optimal nonparametric regression and the application of the Baldwin model (equation 2.2).

Fit	n	Mean	Std. Dev.
Optimal nonparametric	238	0.054	0.058
Baldwin model	238	0.089	0.103

4.5.2 Summary

As Baldwin and Peterson (1997) reported, a linear regression fit of the inner crown profile does indeed seem sufficient for most of the crowns used herein. Whereas optimal (PRESS* criterion) nonparametric regression typically captured curvature present in outer crown profiles, it failed to do so in all instances when describing inner crown profile. This might be a result of the smaller range in radii being modeled. However, because it is still capable of capturing curvature in some instances, one should consider the use of nonparametric regression to describe inner crown profile.

4.6 Optimal nonparametric models of outer and inner crown profiles

4.6.1 Results

The final analysis performed was to visually inspect the optimal (PRESS* criterion) nonparametric fits to both the outer and inner crown profiles. Figures 4.47 through 4.55 simultaneously depict these two fits for each crown. The near horizontal inner crown profile fits are again problematic because they intersect the outer crown profile (see Figure 4.47a and 4.48d) short of the tree top.

This result was highly unexpected. It may result from crossing types of nonparametric fits. That is, different forms of nonparametric regression were sometimes used when describing outer and inner profiles on the same tree. The intersections are of concern because negative values of foliated branch length (outer radius minus inner radius) result at heights above the height of intersection. Calculation of negative foliated branch lengths can be avoided if one assumes the height of the tree to be height of intersection. However, this height will always be less than the true height of the tree.

Intersections can be avoided by selecting a smaller bandwidth when fitting the inner crown profile. However, by doing so, the modeler is discarding the "protection" afforded by the use of PRESS*.

4.6.2 Summary

For the majority of the crowns (see for example Figure 4.47c and 4.51d) the shapes resulting from nonparametric fits appear much more realistic than any of those that could be obtained from geometric shapes or the equations examined in this project. Therefore, if data exist, nonparametric regression appears to be a viable method of approximating crown shapes.

4.7 Discussion and conclusions

The general shape of coniferous crowns is suggested to change with age (Mohren 1987). Cone-shaped crowns are most common prior to the onset of crown competition, while older trees often possess crowns that are more cylindric. Parabolic shapes predominate in middle aged stands. Crowns deviate and become more irregular than these generic shapes because of competition from surrounding crowns (Koop 1989) as well as site differences.

Differences in the general crown shapes of loblolly pine with differences in ages can be visually inspected with these data. For example, crowns of younger trees (less than 13 years of age) such as those depicted in Figures 4.47a, 4.47b, and 4.51a, do tend to be cone-shaped. More curvature is present in some crowns occurring in middle-aged stands (between 13 and 20 years in age), see Figures 4.52a as an example. It is during this time period that the onset of crown competition occurred for the trees in this dataset. Other crowns in this tree age class, though, remain cone-like, see Figures 4.48a and 4.48b. The oldest trees in this dataset (those greater than 20 years in age) possess a wide variety of shapes, see Figures 4.48b, 4.51b, and 4.51d as examples. The impact of years of crown competition is greatest on these trees.

Regional differences in stand and tree attributes such as yield-level (Hasanauer *et al.* 1994), site index (Amateis and Burkhart 1985) and taper (Clark and Souter 1994) do exist between the geographic regions represented in these data. However, visual inspection of the crown profile curves suggest that the tree crowns are equally variable by region.

Based upon the representations in the figures, nonparametric regression appears to be a viable method for fitting smooth curves to crown profiles, and is particularly useful when no reliable functional form is discernible. Kernel regression does possess a minor boundary point problem, exhibited in the analysis, while local polynomial regression, as expected, performs much better at the boundaries.

The use of nonparametric regression to describe crown profile is capable of capturing the tree to tree changes in crown shape whether they are caused by crown competition or other site factors. While the published models available attempt to account for such changes via the incorporation of tree attributes, visual inspection of these crown shape representations appear inferior to those obtained via nonparametric regression.

Keeping the bandwidths and the x-axis values used during nonparametric regression fitting on a 0-1 scale allows for a quick check of how the weights are distributed. Using the crown depicted in Figure 4.3c as an example (corresponding statistics are found in Table 4.4), any data point within 0.20, 0.21, or 0.25 units of the x_0

point is given substantial weight when fitting a kernel regression, local linear regression, and local quadratic regression, respectively.

For any nonparametric modeling endeavor, choice of polynomial order is based upon the amount of suspected local curvature and the amount of smoothing desired. Local linear or local quadratic fits are the forms used most often in practice. Choice of bandwidth should be made according to the minimization of a global error criterion (PRESS* recommended herein). The same considerations are made for choosing the bandwidth regardless of whether kernel, local linear, or local quadratic regression is being used.

Decisions about whether to use nonparametric techniques at all can be subjective in nature and might best be made by comparing respective plots. If an underlying functional form cannot be readily identified and a global error criterion such as PRESS* is used to determine an optimal nonparametric fit, then nonparametric regression should be considered for use in the modeling.

The use of regression splines is an alternative to the techniques herein presented. Regression splines are well discussed in Suits *et al.* (1978). The most commonly applied technique, cubic spline regression, is a form of nonparametric regression using an optimization scheme (see Reinsch [1967], Silverman [1985], Eubank [1988], or Hastie and Tibshirani [1990]). Goulding (1979) used cubic splines to estimate taper functions and thus estimate volumes. Figueiredo-Fihlo *et al.* (1996) studied the number and placement of points along a bole with respect to volume estimates from cubic spline derived taper functions. They concluded that illogical curve behavior is avoided when a minimum of eight well-distributed points are used for interpolation. Silverman (1984) showed a close relation between spline regression and a variation of kernel regression called local bandwidth regression. Kernel regression, as described herein, is a simpler and computationally more efficient form of spline regression (Mays 1995).

One major thrust of this research was to provide more realistic curves which then can be used to calculate crown volumes. While the curves are more realistic than other models currently in the literature, volumes are not as easily obtained as with the other methods described. Volumes are easily calculated when geometric shapes are used, and calculus can be used to calculate volume when equations are used. However, no functional form exists for nonparametric regression; therefore numerical techniques such as Simpson's rule (Ellis and Gulick 1986) would need to be employed to calculate volumes. The current state of computer technology easily permits the use of such numerically intensive techniques.

Relatively speaking, the field of nonparametric regression is still in its infancy. New techniques are being developed and reported. For example, Altman (1990) and Wand and Jones (1995) present asymptotic, nonparametric, large-sample results for correlated data, and Hart and Yi (1998) present another technique for bandwidth selection.

Even though the field of nonparametric regression is relatively young, nonparametric techniques have been used previously in forest research. For example, M'Hirit and Postaire (1985) applied nonparametric density estimation to taper curves; Droessler and Burk (1989, 1994) applied kernel density smoothing to diameter distributions; Kangas and Korhonen (1995) used nonparametric regression in volume estimation; Dralle and Rudemo (1996) applied kernel smoothing to derive estimates of density from aerial photos; Kangas (1996) used nonparametric techniques with respect to small-area estimators. This project, however, is the first to apply nonparametric regression to crown profile modeling, where irregular shapes and a need for smoothing exist.