Spontaneous CP-violation in Two Higgs Doublet Supersymmetric Models

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(ABSTRACT)

An alternative approach to the problem of CP-violation is presented. It is based on the possibility of spontaneous CP-breakdown in models with two Higgs doublets. General features of the phenomenon such as stability of the vacuum and the existence of a light axion are discussed. We investigate the feasibility of spontaneously broken CP in the minimal supersymmetric models - the MSSM and NMSSM. The latter is shown to be experimentally viable. The phenomenological implications of the model such as CP-violating effects in the kaon systems and a nonzero neutron electric dipole moment are studied.

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1 Introduction

As one of the fundamental symmetries, CP and its breaking have a great impact on both microscopic and macroscopic physics (for a comprehensive review see [35]). An understanding of its origin would shed light on the past (and, maybe, future) of the universe. For example, CP-violation is necessary to understand the matter-antimatter asymmetry in nature and has direct cosmological implications. The axions, hypothetical particles invoked to solve the "strong CP problem", may very well be relevant to the puzzle of dark matter and constitute the "missing mass" in the universe. Concerning the physics of small objects, CP is intimately related to the theories of interactions between elementary particles and represents a cornerstone in constructing Grand Unified and Supersymmetric Theories of nature.

The origin of CP-violation is still an open problem in particle physics. So far, CP-violating effects have been detected for certain in the $K - \bar{K}$ system and kaon decays only. The increasing precision of the measurement of the neutron electric dipole moment (NEDM) gives confidence that a nonzero NEDM value can be found in the near future. This would be another manifestation of broken CP-invariance in nature. There are also prospects of detecting CP-violation in the heavy meson systems. The complete experimental information should allow one to determine uniquely the effective low-energy sources of CP-violation.

According to the CPT theorem, the breakdown of CP-symmetry is equivalent to the violation of the time reversal invariance. This fact allows us to visualize CP-violation as the presence of complex masses and couplings in the Lagrangian: the evolution operator e^{-iHt} with a complex Hamiltonian H describes decay processes which are manifestly asymmetric with respect to the time reversal. Briefly put, CP-violation measures how irreversible the reactions are. A more rigorous definition depends on the details of the system and will be given in the text.

In this work we analyze the possibility of spontaneous CP-violation (SCPV), that is to say, assuming that all interactions at sufficiently high energy are CP-invariant, only the vacuum violates CP. This approach makes the CP phase a dynamical variable rather than a fixed parameter introduced "by hand" and allows us to restore CP-invariance at energies greater than the $SU(2) \times U(1)$ breaking scale. Unlike the case of a continuous symmetry breaking, SCPV does not lead to the existence of a Goldstone boson. However, if it is the radiative corrections that govern the CP-properties of the vacuum, then there is a severe constraint known as the Georgi-Pais theorem [36]: a pseudo-goldstone boson (massless at the tree-level) is necessary for radiative SCPV to occur. This may lead to an intolerable experimental inconsistency and rule out the model.

Spontaneous CP-violation requires extension of the Standard Model: it is possible in systems with at least two Higgs doublets. Following the Occam's razor principle, we will content ourselves with the examination of two

Higgs doublet models only, yet adding a singlet scalar field when necessary (to comply with experimental data). Moreover, we will greatly restrict the variety of applicable models by imposing supersymmetry (susy), which has good reasons of its own. Nowadays supersymmetry is considered one of the most appealing theoretical principles for extending the Standard Model of particle interactions. Besides the fact that it is closely related to space-time symmetries, it provides a solution to the naturalness problem of the Standard Model, i.e. it lets us avoid quadratically divergent radiative corrections to the Higgs mass. Among the other advantages of supersymmetry, one would emphasize its nice mathematical behavior and aesthetic appeal as a theory unifying bosons and fermions.

A two-Higgs-doublet model naturally arises as a minimal supersymmetric extension of the Standard Model (MSSM). In this work we illustrate by the example of MSSM certain general features of supersymmetric and softly broken susy theories such as, for instance, cancellation of quadratic divergences. We also derive the RGE equations for the quartic Higgs couplings and show that the renormalization group flow preserves the tree-level relations between the Higgs and gauge couplings. This fact is well known but, to my knowledge, has not been demonstrated at the Feynman diagram level in the literature. These diagrammatical considerations help us understand the interplay between different sectors of the MSSM and let us make some useful observations.

The MSSM presents a fertile ground for spontaneous CP-violation. The possibility of SCPV in the MSSM was first studied in [6] and [7]. It has been shown that the necessary conditions for SCPV can be satisfied. However, stability issues were not paid the attention they deserve. A recent work [47] showed that at low energies (below the squark threshold) the CP-violating stationary point is a local minimum. This consideration is not sufficient since it is not applicable at energies greater than 300-400 GeV and the potential may turn out to be unbounded from below.

In this dissertation, we examine the stability properties of the Higgs potential in the MSSM above the squark threshold. We show that the higgsino-gaugino contributions alone lead to an unbounded from below potential, whereas the incorporation of the soft-susy-breaking mass rectifies this problem. We obtain the finite radiative corrections to the Higgs couplings as 1-loop exact functions of the gaugino, higgsino and Higgs masses, and demonstrate that the Higgs-loop contributions stabilize the scalar potential if the Higgs are sufficiently heavy. We also point out that the stability of the Higgs potential below the squark threshold is an automatic consequence of these considerations owing to the stabilizing action of the top-quark loop corrections. Thus, it is shown that there are no theoretical obstacles for SCPV to occur in the MSSM.

As a result of the Georgi-Pais theorem, spontaneous CP-violation in the MSSM leads to the existence of a light axion. It has been suggested [6], however, that the predicted axion mass may be consistent with the experimental bound if $\tan\beta$ is large (~ 50). In this work we obtain the Higgs mass matrix elements as functions of the CP violating phase, $\tan\beta$, and the Higgs couplings, and demonstrate that the MSSM upper bound on the axion mass cannot be removed by varying $\tan\beta$. Thus, our calculations show that the bound obtained first in [7] holds true even in the large $\tan\beta$ regime. We also note that the axion field (in the lowest order approximation) can be identified with the dynamical CP-phase and that a nonzero axion VEV indicates spontaneous CP-breaking. This observation allows us to simplify the analysis of the physical boson spectrum and to study the evolution of the axion mass as a function of the Higgs couplings. However, the predicted axion mass lies well below the experimental lower bound and, therefore, SCPV in the MSSM is unrealistic.

The Next-to-Minimal Supersymmetric Standard Model (NMSSM) has been shown to be free of this problem [21] and will remain viable at least until the next round of experiments on the Higgs mass. The last chapters of this work are devoted to the observable CP-violating effects such as $K-\bar{K}$ mixing, kaon hadronic decays, and the neutron electric dipole moment. In the context of spontaneous CP-violation in the NMSSM (or similar models), these effects were first considered in [24]. It was shown that in a small region of the parametric space, SCPV can explain the above mentioned CP-violating effects. However, the estimates were based on dimensional analysis and assumed that charginos were heavy. It was unclear whether a more precise calculation would rule out the possibility of SCPV in the NMSSM completely. We carry out the 1-loop analysis of the diagrams leading to the observable CP-violating effects and obtain the CP-violating parameters as functions of the squark, gluino and chargino masses. In the limit of heavy charginos, these functions lead to the same numerical estimates as those given in [24]. We also verify that the gluino contribution to the NEDM is described by the same function of the gluino and squark masses as that in the case of explicit CP-violation [31]. Finally, we study the heavy squark limit of these expressions and point out that, in this case, the experimental bounds indeed do not conflict. Thus, SCPV in the NMSSM is shown to be experimentally viable.

2 CP-violation in Two Higgs Doublet Models

2.1 Sources of CP-violation in the Standard Model

CP-violation can be introduced in both gauge and Yukawa sectors. The former generally allows an interaction proportional to $F^a_{\mu\nu}\tilde{F}^{\mu\nu,a}$ which violates parity and time reversal invariances. The effect of this term depends on the so called θ-parameter:

$$\bar{\theta} = \theta + arg(det\ M)\ ,$$

where θ is a (properly normalized) factor in front of $F^a_{\mu\nu}\tilde{F}^{\mu\nu,a}$ in the Lagrangian and M is the original nondiagonal mass matrix for both up- and down-quarks. $\bar{\theta}$ is invariant with respect to chiral rotations and, thus, affects physical observables. Clearly, its influence is stronger in $\Delta S = 0$ processes, for example, in the neutron electric dipole moment (NEDM). The experimental value of the NEDM [28] requires $\bar{\theta}$ to be tiny, $\bar{\theta} \sim 10^{-10}$. This is known as the strong CP problem: different sectors of the Standard Model combine to give an extremely small number. Several ways to solve this puzzle have been suggested, among which the most appealing seems to be the incorporation of an extra (Peccei-Quinn [37]) symmetry allowing to eliminate the undesired $\bar{\theta}$- term.

In what follows we will mainly concentrate on the CP-violating effects in the Yukawa sector. The Higgs-quark interaction contains arbitrary mixing matrices $\Gamma_{u,d}$:

$$-\mathcal{L} = (\bar{d}_R\, \Gamma_d\, q_{L\,\alpha})\, \phi^*_\alpha + (\bar{u}_R\, \Gamma_u\, q_{L\,\alpha})\, \phi_\beta\, \epsilon_{\alpha\beta} + h.c.$$

To get the mass eigenstates after spontaneous $SU(2) \times U(1)$ breaking one needs to diagonalize the mass matrices. This can be done by the following unitary transformations:

$$d_L \rightarrow V^d_L\, d_L\ ,\ d_R \rightarrow V^d_R\, d_R\ ,$$

$$u_L \rightarrow V^u_L\, u_L\ ,\ u_R \rightarrow V^u_R\, u_R\ .$$

Note that these transformations also diagonalize the physical Higgs interactions: out of four Higgs degrees of freedom three get absorbed by the vector bosons (in the unitary gauge) and one represents a neutral Higgs boson whose interactions are flavor diagonal [38]. As a result, there are no CP-violating effects in the Higgs sector. The sole source of CP-violation is the charged weak current

$$\bar{u}^i_L\, \gamma_\mu T^a\, V^{ij}_{CKM}\, d^j_L\ ,$$

where $V_{CKM} = V^{u\dagger}_L V^d_L$. The Cabibbo-Kobayashi-Maskawa matrix [39] initially has 3×3 parameters, out of which 3 represent the $SO(3)$ angles and 6 correspond to the phases. Since all SM interactions except for the charged weak ones are invariant with respect to quark rephasing, we can absorb 5 phases into the redefinition of the quark fields. In other words, we transform the initial V_{CKM} in the following manner:

$$diag(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3})^\dagger\, V_{CKM}\, diag(e^{i\delta_4}, e^{i\delta_5}, e^{i\delta_6})\ ,$$

where only 5 combinations $\delta_i - \delta_j$ are linearly independent. Consequently, we are left with a single phase which is responsible for all CP-violating effects in the Standard Model (provided neutrinos are massless). In the Kobayashi-Maskawa parametrization the resulting mixing matrix reads

$$V_{CKM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} ,$$

with $c_i \equiv \cos\theta_i$, $s_i \equiv \sin\theta_i$. Since the quark phases are not physical, observable CP-violating effects must depend solely on the rephase-invariant quantities. For three families there is only one CP-violating invariant, which can be chosen to be (see, for example [1],[40])

$$\begin{aligned} &det\ [M_u M_u^\dagger, M_d M_d^\dagger] = -2i\ (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\ &\times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\ Im(V_{ud} V_{cs} V_{us}^* V_{cd}^*) , \end{aligned} \tag{1}$$

where $M_{u,d}$ are the mass matrices in the weak eigenstate basis. This quantity can also be written in the form

$$det\ [M_u M_u^\dagger, M_d M_d^\dagger] = \frac{1}{3} Tr\ [M_u M_u^\dagger, M_d M_d^\dagger]^3 .$$

As a consequence, all CP-violating effects in the Standard Model are proportional to

$$\delta_{KM} = Im(V_{ud} V_{cs} V_{us}^* V_{cd}^*) .$$

One also notices that if any two up- or any two down-quarks have the same masses the invariant vanishes leading to CP-conservation. This is understandable because a mass degeneracy provides an extra U(2) symmetry which allows us to eliminate the remaining phase. Another interesting observation is that one regains the CP-symmetry when certain entries of V_{CKM} vanish, for example, when $\theta_1 = 0, \pi$ or $\theta_3 = \pi/2$.

The experimental values of $|V_{ij}|$ require δ_{KM} to be very small [1]:

$$|\delta_{KM}| \leq 10^{-4} ,$$

which comes as no surprise since δ_{KM} involves off-diagonal entries of the CKM matrix. This explains the smallness of CP-violating effects in the Standard Model.

2.2 Sources of CP-violation in Two Higgs Doublet Models

We will start with the formal definition of CP-invariance. A Lagrangian $\mathcal{L}$ is CP-invariant if one can find a unitary matrix V_Φ such that $\mathcal{L}$ is invariant under the transformation [2]

$$\begin{aligned} \Phi(x) &\rightarrow V_\phi \, \Phi^*(\tilde{x}) \, , \\ W_\mu(x) \cdot \sigma &\rightarrow (-1)^{\delta_{0\mu}} W_\mu(\tilde{x}) \cdot \sigma^T \, , \\ B_\mu(x) &\rightarrow (-1)^{\delta_{0\mu}} B_\mu(\tilde{x}) \, , \\ \Psi(x) &\rightarrow C \, \Psi^*(\tilde{x}) \, , \end{aligned} \tag{2}$$

where Φ is a set of Higgs doublets, $\tilde{x} \equiv (x^0, -\mathbf{x})$ and Ψ represents fermions of the theory. If the vacuum conserves CP also, then

$$v_\alpha = (V_\Phi)_{\alpha\beta} \, v_\beta^* \, ,$$

with v_α being the vacuum expectation values of the neutral components of the Higgs doublets. Note that the unitary matrix V_Φ is the same as in the definition of the CP transformation for the Higgs doublets.

The general Yukawa interaction can be written as [3]

$$\mathcal{L} = \bar{\Psi}_L(\Gamma_1\Phi_1 + \Gamma_2\Phi_2)d_R \; + \; \bar{\Psi}_L(\Gamma_3\Phi_1^c + \Gamma_4\Phi_2^c)u_R \; + h.c.$$

Here $\Phi_i = \begin{pmatrix} \Phi_i^+ \\ \Phi_i^0 \end{pmatrix}$ and Γ_a are matrices in flavor space. After spontaneous symmetry breaking the mass matrices $\Gamma_1 v_1 + \Gamma_2 v_2$ and $\Gamma_3 v_1 + \Gamma_4 v_2$ are diagonalized in the usual way. In contrast to the Standard Model case, this does not lead to flavor diagonal neutral Higgs interactions since in the mass basis the Γ_a's do not have to be diagonal separately. As a result, we naturally encounter the FCNC (flavor changing neutral currents) problem [4]. Generally speaking, there are four distinct sources of CP-violation in such models [3]:

1. The usual CKM matrix,
2. The charged-Higgs-boson interactions,
3. The phases in FCNC,
4. Complex parameters in the Higgs potential.

So, even this simple extension of the Standard Model provides a rich set of possibilities for sources of CP-violation in addition to that from the CKM model.

The simplest way to get around the FCNC problem is to have each Higgs doublet give mass to one type of quarks (up- or down-) only [4]:

$$\mathcal{L} = \bar{\Psi}_L \, \Gamma_1 \Phi_1 \, d_R \; + \; \bar{\Psi}_L \, \Gamma_2 \Phi_2^c \, u_R \; + h.c.$$

Thus, the diagonalization of the mass matrices makes the neutral Higgs interactions flavor diagonal. Further, let us assume both $\Gamma_{1,2}$ real, so that CP is a valid symmetry of the Lagrangian. After spontaneous $SU(2) \times U(1)$ breaking the neutral Higgs fields acquire VEV's:

$$\langle \Phi_k^0 \rangle = v_k e^{i\theta_k} \; , \quad k = 1, 2.$$

Then the mass term is written as

$$\bar{d}_L \, \Gamma_1 v_1 e^{i\theta_1} \, d_R \; + \; \bar{u}_L \, \Gamma_2 v_2 e^{-i\theta_2} \, u_R \; + h.c.$$

Now let us redefine the right-handed quark fields as follows:

$$d'_{iR} = e^{i\theta_1}\, d_{iR}\ ,\ u'_{iR} = e^{-i\theta_2}\, u_{iR}\ ,\ i = \text{flavor index}\ .$$

As a result, the mass matrices become real and can be diagonalized by orthogonal transformations $O^{(u),(d)}_{L,R}$. Apparently, the CKM matrix $V_{CKM} = O^{(u)\,T}_{L} O^{(d)}_{L}$ is real in this case and there can be no CP-violation in the gauge sector of the model. This can be understood heuristically since initially there are only two phases and they can be gotten rid of by field redefinitions, in contrast to the numerous phases in the Yukawa interactions. This result was first obtained by Branco [4], who also pointed out that all CP-violating effects in this case would be due to Higgs-boson exchange.

To summarize, we see that spontaneous CP-violation in two-Higgs-doublet models with suppressed FCNC does not lead to CP-violation in the gauge sector. This statement holds for the MSSM which naturally obeys the FCNC constraint. Even though the MSSM has a larger particle content, all CP violating effects can be restricted to interactions involving Higgs bosons and higgsinos.

2.3 Spontaneous CP-violation in Two Higgs Doublet Models

In this section we will examine general features of models with spontaneous CP-violation (SCPV). The minimal Higgs content allowing for SCPV is two doublets, so we will focus on such "economical" models. The basic idea is that if the Higgs potential contains terms sensitive to the relative phase between the two doublets, the vacuum expectation value of the neutral Higgs may be complex [5]. This leads to complex masses and couplings in the Lagrangian and signifies CP-violation. The possibility of spontaneous CP-violation is closely related to the Peccei-Quinn (PQ) symmetry of the Higgs potential: it is possible only when the PQ symmetry is broken. (In this context, the "PQ-symmetry" refers to the invariance with respect to multiplication of the Higgs doublets by different phases.)

The most general renormalizable CP-conserving potential of two Higgs doublets is given by

$$\begin{aligned} V(\Phi_1,\Phi_2) &= m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 - (m_3^2\Phi_1^\dagger\Phi_2 + h.c.) \\ &+ \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ &+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}[\,\lambda_5(\Phi_1^\dagger\Phi_2)^2 + h.c.] \\ &+ \frac{1}{2}\{\Phi_1^\dagger\Phi_2\,[\,\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)] + h.c.\}, \end{aligned} \tag{3}$$

where all parameters are real. Note that the terms proportional to m_3^2, λ_{5-7}

are not invariant with respect to multiplication of the first and the second Higgs doublets by different phases and, thus, break the PQ symmetry explicitly. After spontaneous symmetry breaking, the neutral components of the Higgs fields are assumed to acquire the following vacuum expectation values

$$\langle\Phi_1^0\rangle = v_1 \ , \ \langle\Phi_2^0\rangle = v_2 e^{i\delta} \ .$$

It has been shown in Ref.[6] that the VEV of the potential for non-zero λ_5 can be rewritten as follows

$$\begin{aligned}\langle V\rangle &= m_1^2 v_1^2 + m_2^2 v_2^2 + 2\lambda_5 v_1^2 v_2^2 \,\{\, (\cos\delta - \Delta)^2 - \Delta^2 \} \\ &+ \lambda_1 v_1^4 + \lambda_2 v_2^4 + (\ \lambda_3 + \lambda_4 - \lambda_5\ ) v_1^2 v_2^2 \ , \end{aligned} \tag{4}$$

where

$$\Delta = \frac{2m_3^2 - \lambda_6 v_1^2 - \lambda_7 v_2^2}{4\lambda_5 v_1 v_2} \ . \tag{5}$$

(We deviate from the notation of [6] in the definitions of $v_{1,2}$ and λ_{5-7}.) If λ_5 is positive then the term proportional to λ_5 is minimized when $\cos\delta = \Delta$. Therefore, from equation (4) we see that in order for the spontaneous CP-breaking to occur the following inequalities must hold

$$\lambda_5 > 0 \ , \tag{6}$$

$$-1 < \Delta < 1 \ . \tag{7}$$

In this case we obtain a non-zero CP-violating phase δ given by

$$\cos\ \delta_{CP} = \frac{2m_3^2 - \lambda_6 v_1^2 - \lambda_7 v_2^2}{4\lambda_5 v_1 v_2} \ . \tag{8}$$

Note that only PQ-breaking parameters (m_3, λ_{5-7}) are involved in this formula. The requirement that our vacuum be at least a stationary point (but not necessarily the minimum point) of the potential results in the following constraints

$$m_1^2 = -2\lambda_1 v_1^2 - (\ \lambda_3 + \lambda_4 - \lambda_5\ ) v_2^2 - \lambda_6 v_1 v_2\ \cos\delta\ , \tag{9}$$

$$m_2^2 = -2\lambda_2 v_2^2 - (\ \lambda_3 + \lambda_4 - \lambda_5\ ) v_1^2 - \lambda_7 v_1 v_2\ \cos\delta\ , \tag{10}$$

$$0 = \sin\delta\ (m_3^2 - 2\lambda_5 v_1 v_2\ \cos\delta - \frac{1}{2}\lambda_6 v_1^2 - \frac{1}{2}\lambda_7 v_2^2\ )\ , \tag{11}$$

where the last condition coincides with (8) in the case of the CP-violating vacuum $(\delta \neq 0)$.

The above considerations did not take into account the stability properties of the Higgs potential. Indeed, we should first make sure that the potential is bounded from below since otherwise the whole analysis would not make any sense. To determine the necessary condition, let us consider Eq.(4) in the region $sign(\cos\delta) = sign(\Delta)$ as v_1 and v_2 go to infinity along the ray $v_1 = v_2$. Dropping dimension-2 terms along with the nonpositive third term, we require that $\langle V \rangle$ not decrease along this ray. This leads to

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 - \lambda_5 \geq 0\ . \tag{12}$$

The sufficient condition for the potential to be bounded from below is [5]

$$\frac{1}{4}\Big(\ \lambda_3 + \lambda_4 - \lambda_5 - \frac{\lambda_6\lambda_7}{4\lambda_5}\ \Big)^2 < \Big(\ \lambda_1 - \frac{\lambda_6^2}{8\lambda_5}\ \Big)\Big(\ \lambda_2 - \frac{\lambda_7^2}{8\lambda_5}\ \Big)\ . \tag{13}$$

For the direction $\cos\delta = \Delta$ it also gives the necessary condition while (12) must be satisfied for all δ's in the specified region. In the MSSM, it can be shown that the left hand side of Eq.(12) vanishes at the tree level [7]. Therefore, it is the finite radiative corrections that govern the asymptotic behaviour of the potential in the MSSM. Apparently the naive incorporation of positive corrections to λ_5 only would lead to a disaster: there would be no vacuum in such a theory; so one needs to properly account for all the radiative contributions to all the λ_i's. In the MSSM there are two sources of such corrections: gaugino masses and scalar SUSY soft-breaking masses; the trilinear soft-breaking couplings and mixing mass of the Higgs must be comparatively small to give a positive λ_5. This issue requires careful consideration and will be analysed in a subsequent chapter.

Let us now turn to the general case. The system of equations (9)-(11) has two sets of solutions: one with $\delta = 0$ and another with nonzero δ. This system can be rewritten as two independent quadratic equations with respect to v_1^2 and v_2^2, so at most there are two solutions with nonnegative v_1 and two solutions with nonnegative v_2. Besides them, there always exists another stationary point $v_{1,2} = 0$. As shown below, the CP-breaking vacuum gives the global minimum in the 3 dimensional parametric space (v_1, v_2, δ) provided Eqs.(6),(7) and (13) are satisfied.

According to Eq.(4), the VEV of the potential can be split into two terms, only one of which will involve δ. Clearly if Eqs.(6) and (7) are satisfied the vacuum is described by $\cos\delta = \Delta$ for all v_1 and v_2. Then (4) takes on the form

$$
\begin{aligned}
\langle V \rangle &= -\frac{m_3^4}{2\lambda_5} + \left(m_1^2 + \frac{\lambda_6}{2\lambda_5} m_3^2 \right) v_1^2 + \left(m_2^2 + \frac{\lambda_7}{2\lambda_5} m_3^2 \right) v_2^2 \\
&+ \left(\lambda_3 + \lambda_4 - \lambda_5 - \frac{\lambda_6\lambda_7}{4\lambda_5} \right) v_1^2 v_2^2 + \left(\lambda_1 - \frac{\lambda_6^2}{8\lambda_5} \right) v_1^4 + \left(\lambda_2 - \frac{\lambda_7^2}{8\lambda_5} \right) v_2^4 \, .
\end{aligned}
\tag{14}
$$

We can readily analyze this expression in terms of the new variables (v_1^2, v_2^2), which are equivalent to (v_1, v_2) for positive $v_{1,2}$. Eq.(14) describes a family of quadratic curves and can be cast into either elliptic or hyperbolic form (for a non-degenerate case): $\langle V \rangle = a^2 x^2 \pm y^2 b^2$ with x, y being certain linear combinations of v_1^2 and v_2^2. Obviously, the signature of the surface is controlled by (13): if (13) is upset we are dealing with a hyperbolic surface; the potential is not bounded from below and, moreover, there is no local minimum. This situation would hardly be of any physical interest. In contrast, the elliptic case is much more appealing since there would exist a unique local minimum. In order for it to be located at positive $v_{1,2}^2$ we need to impose the conditions $m_1^2 + \frac{\lambda_6}{2\lambda_5} m_3^2 < 0$ and/or $m_2^2 + \frac{\lambda_7}{2\lambda_5} m_3^2 < 0$ (for detailed treatment see [5]). It is easily seen from (14) that the minimal value of $\langle V \rangle$ is negative, whereas the stationary point $v_{1,2} = 0$ gives zero (see (4)). Therefore, the found local minimum is also the global one.

Let us now consider the implications of spontaneous CP-violation on the Higgs boson spectrum. We start with the general N-doublet system of Higgs fields. To calculate the mass spectrum of the theory one must choose appropriate coordinates. The vacuum can naturally be described by a set of polar coordinates; so, we define the new fields via

$$\Phi_k(x) = \rho_k(x) e^{i\xi_k(x)} \ , \ k = 1, ...N \ , \tag{15}$$

with $\Phi_k(x)$ being the neutral components of the doublets. Further, let us consider the behaviour of the fields near the vacuum

$$\begin{aligned} \rho_k(x) &= v_k + \eta_k(x) \ , \\ \xi_k(x) &= \bar{\delta}_k + \delta_k(x) \ . \end{aligned}$$

We would like to associate a particle with every real degree of freedom. Therefore, we should ensure that the kinetic term has the proper normalization. In terms of the new variables the kinetic term reads

$$\begin{aligned} \sum_{k=1}^{N} \partial_\mu \Phi_k^\dagger \partial^\mu \Phi_k &= \sum_{k=1}^{N} \partial_\mu \eta_k \partial^\mu \eta_k + \sum_{i=1}^{N} v_i^2 \partial_\mu \delta_i \partial^\mu \delta_i \\ &+ (\textit{higher order terms}) \\ &= \sum_{k=1}^{N} \partial_\mu \eta_k \partial^\mu \eta_k \\ &+ \sum_{a=2}^{N} \left[\partial_\mu \left(\sum_{i=1}^{a-1} (\delta_i - \delta_{i+1}) \frac{v_a (\sum_{k=1}^{i} v_k^2)}{\sqrt{(\sum_{l=1}^{a} v_l^2)(\sum_{m=1}^{a-1} v_m^2)}} \right) \right]^2 \\ &+ \left[\partial_\mu \left(\sum_{i=1}^{N} \delta_i \frac{v_i^2}{\sqrt{(\sum_{j=1}^{N} v_j^2)}} \right) \right]^2 \\ &+ (\textit{higher order terms}) \ . \end{aligned} \tag{16}$$

We have rewritten the $\delta-$dependent terms to extract the phase differences. Note that spontaneous CP-violation occurs if the VEV of $\delta_i - \delta_k$ is nonzero and not equal to $n\pi$ for some i and k [5],[8].

A general $SU(2) \times U(1)$-invariant potential cannot depend on the linear combination of all δ_k with positive coefficients since the multiplication of all the fields by the same phase factor leaves it unchanged. This observation allows one to separate out the Goldstone boson appearing due to spontaneuos U(1) breaking. It can be identified with the term

$$\gamma = \sum_{i=1}^{N} \delta_i \, \frac{v_i^2}{\sqrt{(\sum_{j=1}^{N} v_j^2)}}$$

and apparently is massless. We may assume that the real parts of Φ_i^0 are CP-even fields [9]. Then the remaining $N-1$ angular degrees of freedom, orthogonal to the Goldstone field and proportional to $\delta_i - \delta_{i+1}$, correspond to a linear combination of CP-odd axions, while η_k represent CP-even bosons (in the case of the CP-conserving vacuum). Further analysis of the general N-doublet case is extremely cumbersome, so we restrict ourselves to the 2-doublet Higgs model from this point on. In this case the Goldstone field γ and the axion field α take on the form

$$\begin{aligned}
&\alpha = \frac{v_1 v_2}{v}(\ \delta_2 - \delta_1\ ) = \frac{v_1 v_2}{v}\delta\ , \\
&\gamma = \frac{v_1^2}{v}\delta_1 + \frac{v_2^2}{v}\delta_2\ , \\
&v^2 = v_1^2 + v_2^2\ .
\end{aligned} \tag{17}$$

The mass spectrum can be found by diagonalization of the matrix of the second derivatives of the potential :

$$\begin{aligned}
&M_{ij}^2 = \frac{1}{2}\frac{\partial^2 \langle V \rangle}{\partial a_i \partial a_j}\bigg|_{vac}\ , \\
&a_i = (v_1\ , v_2\ , \alpha)\ .
\end{aligned} \tag{18}$$

In the case of relatively small λ_5 , i. e. when condition (7) cannot be fulfilled, the minimum of the potential is given by $\delta_{CP} = 0$ (note that $\langle\alpha\rangle = \frac{v_1 v_2}{v}\delta_{CP}$) and the CP-even and CP-odd states do not mix :

$$\left.\frac{\partial^2\langle V\rangle}{\partial v_{1,2}\partial\alpha}\right|_{\alpha=0} = 0$$

and when $\lambda_{5-7} = 0$ we recover the well-known result [10]

$$M_{A_0}^2 = \left.\frac{1}{2}\frac{\partial^2\langle V\rangle}{\partial\alpha^2}\right|_{\alpha=0} = \frac{2m_3^2}{\sin 2\beta} \ , \tag{19}$$

where $\tan\beta = v_2/v_1$. In the limit $m_3 \to 0$, the PQ-symmetry of the potential is restored resulting in a massless axion. The other two mass-eigenvalues correspond to h_0 and H_0 and are given, for example, in reference [10]. In general, the axion remains massive until the parameters $\lambda_{5,6,7}$ reach certain critical values, determined by $4\lambda_5 v_1 v_2 = 2m_3^2 - \lambda_6 v_1^2 - \lambda_7 v_2^2$. The curvature of the potential vanishes when this condition is satisfied

$$M_{A_0}^2 = \left.\frac{1}{2}\frac{\partial^2\langle V\rangle}{\partial\alpha^2}\right|_{\alpha=0} \propto\ 2m_3^2 - 4\lambda_5 v_1 v_2 - \lambda_6 v_1^2 - \lambda_7 v_2^2 = 0 \ , \tag{20}$$

giving rise to a massless axion. The further increase of λ_5 makes the curvature matrix flip its sign, signifying $\delta_{CP} = 0$ is not a stable stationary point any more and spontaneous CP-violation occurs (Fig.1). To get the physical boson mass spectrum we must expand all the fields around the CP-breaking vacuum. Now the axion field α acquires a VEV, as dictated by Eq.(8), even

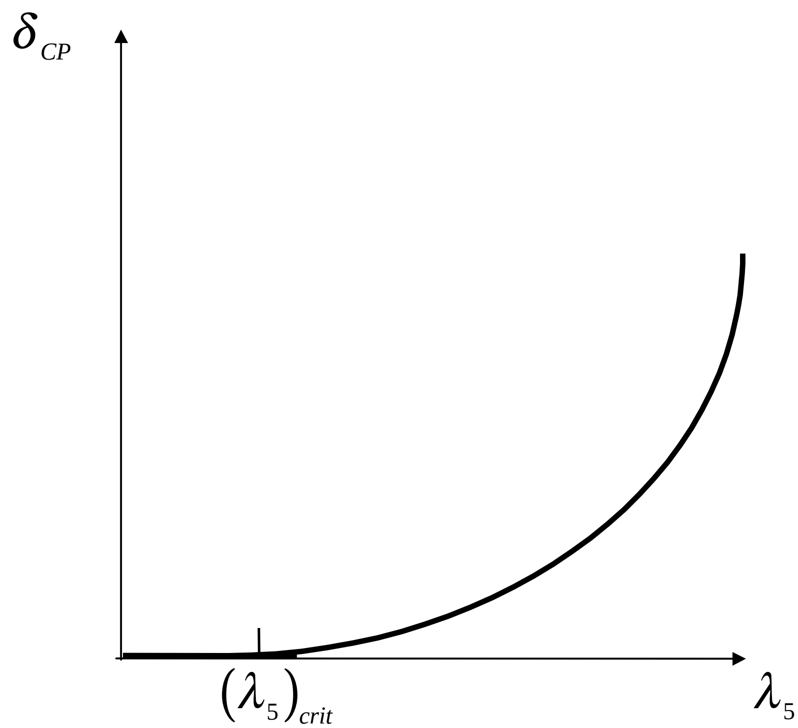


Fig.(2)-1

though it is not a mass-eigenstate any more and, generally speaking, all matrix elements of M^2_{ij} are non-zero :

$$
\begin{aligned}
& m^2_{ij} \equiv \frac{1}{v^2}\, M^2_{ij} \ , \\
& m^2_{11} = 4\lambda_1 \cos^2\beta + 2\lambda_5 \sin^2\beta\ \cos^2\delta + \lambda_6 \sin 2\beta\ \cos\delta \ , \\
& m^2_{12} = \cos\delta\ (\lambda_6 \cos^2\beta + \lambda_7 \sin^2\beta) + \sin 2\beta\ (\lambda_3 + \lambda_4 - \lambda_5\ \sin^2\delta) \ , \\
& m^2_{22} = 4\lambda_2 \sin^2\beta + 2\lambda_5 \cos^2\beta\ \cos^2\delta + \lambda_7 \sin 2\beta\ \cos\delta \ , \\
& m^2_{13} = -\sin\delta\ (2\lambda_5 \sin\beta\ \cos\delta + \lambda_6 \cos\beta) \ , \\
& m^2_{23} = -\sin\delta\ (2\lambda_5 \cos\beta\ \cos\delta + \lambda_7 \sin\beta) \ , \\
& m^2_{33} = 2\lambda_5 \sin^2\delta \ ,
\end{aligned} \tag{21}
$$

where we have used Eqs.(9) - (11) to eliminate m_{1-3} from the equations. The value of the dipole moment of the neutron shows that we should expect δ to be relatively small [6]: $\delta < 10^{-3}\cot\beta$ and $\cot\beta \leq 50$. In this case, we can estimate the axion mass by means of perturbation theory with respect to $\sin\delta$. The first order correction vanishes and at second order we find

$$
M^2_{A_0} \leq 2v^2\lambda_5 \sin^2\delta \ . \tag{22}
$$

This bound cannot be removed by varying $\lambda_{6,7}$ and $\tan\beta$. In particular, Maekawa's [6] suggestion concerning the possibility of the existence of a heavy axion in the case of large $\tan\beta$ or $\cot\beta$ does not work, at least as long as the perturbative approach is valid. In the case of the MSSM, i.e. when the Higgs coupling constants are determined by the SU(2) and U(1) gauge couplings g and g', the situation gets worse since λ_{5-7} are of the order of 1-loop corrections [7]. We can therefore consider a perturbative expansion with respect to them and to first order

$$M^2_{A_0} \approx 2v^2\lambda_5 \sin^2\delta \tag{23}$$

regardless of the value of $\sin\delta$. Numerical diagonalization of the mass matrix M^2_{ij} also shows that the axion mass never exceeds the limit (22) for any $\tan\beta$ or δ, while the other bosons stay significantly heavier. A nonzero axion mass superficially seems to contradict the Georgi-Pais theorem [36] stating that for the radiative spontaneous CP-breakdown to occur a massless-at-tree-level particle (pseudogoldstone boson) must exist. However, the axion is indeed massless at the tree level: its mass is generated by 1-loop effects, in agreement with the theorem [7]. The possibility of spontaneous CP-violation in the MSSM deserves more detailed consideration and will be examined in one of the subsequent chapters.

To summarize, we have obtained a perturbative upper bound on the axion mass in the SCPV scenario; we have also seen that one needs to impose condition (13) in addition to conditions (6) and (7) to ensure that the CP-breaking vacuum indeed represents the ground state of the theory.

3 Supersymmetric Extensions of the Standard Model

3.1 Introduction

Supersymmetry (susy) has long been of great interest to physicists (for a review see [13],[41]). Over the past twenty years it has drawn particular attention as a nicely behaved field theory rendering a boson-fermion symmetry possible and which may be relevant for a description of nature. Despite the absence of any experimental signatures, supersymmetry remains the most viable candidate for extending the Standard Model.

Supersymmetry appeals aesthetically for a few good reasons, among which one would emphasise

1. The extensions of the Space-Time symmetries must be supersymmetric,
2. Supersymmetry solves the hierarchy problem.

The first of them is a result of the "No-Go" theorem, stating that the only non-trivial extensions of the Poincaré group are those that contain anticommuting generators. The second is a consequence of the stability of supersymmetric theories with respect to quantum corrections, which is formulated in the so called "non-renormalization" theorems. The radiative corrections have even less influence on the theory as we increase the number of independent supersymmetry generators : N=2 theories are 1-loop exact and N=4 theories are finite (!) [41]. All susy models are free of quadratic divergences: bosons and fermions from the same supermultiplet contribute with opposite signs thereby cancelling out the leading term. It is this peculiar feature that solves the naturalness problem: if quadratic divergences were present, the natural mass of the scalar particles would be close to the upper limit of applicability of the theory.

So far, no experimental indication of supersymmetry has been detected. On the other hand, it has not been proven wrong either. The experimental implications of the supersymmetric extensions of the Standard Model seem to follow those of the SM very closely without any possibility to distinguish between these theories. There is one fact that we know for sure: supersymmetry must be broken badly at low energies. However, it does not stop one from expecting that it may be a valid symmetry at higher energies, presently not accessible to the accelerator experiments. So, until more powerful machines come into play, the question of supersymmetry will remain open.

3.2 The Minimal Supersymmetric Standard Model

In this subsection we will consider the "supersymmetrized" Standard Model (for a review see [43]). It is minimal in the sense that it employs the least possible number of new fields compatible with supersymmetry and the symmetries of the Standard Model.

A supersymmetric model has the same number of bosonic and fermionic degrees of freedom. The spin-1/2 fermions, as part of the superspin-1/2 multiplet, must be accompanied by the scalars carrying the same quantum numbers. The latter are called squarks and sleptons and come in two species: they can be left-handed or right-handed, referring to the helicity of their fermionic partners. Analogously, every gauge boson shares the superspin-1 supermultiplet with the corresponding spin-1/2 gaugino. Finally, we come to the question of the Higgs superpartners - the higgsinos. It is quite clear that none of the presently known particles can play the role of the higgsinos: the only plausible candidate- the neutrino- fails since the vacuum expectation value of the Higgs would break the lepton number symmetry. The incorporation of an extra fermion with the quantum numbers of the Higgs (thereby transforming nontrivially under SU(2)$\times$U(1)) renders the particle content anomalous. To cancel the anomaly an additional Higgs doublet with the opposite hypercharge has to be introduced. The inclusion of the extra doublet is also needed for generating the quark masses: in supersymmetric models, one Higgs doublet can generate either the up- or the down- quark masses, but not both. This happens due to the analytic structure of the superpotential with respect to the Grassman variables. In other words, one cannot include both a superfield and its conjugate into a supersymmetric potential.

The supersymmetric version of the Standard Model contains the vector superfields corresponding to the SU(3)×SU(2)×U(1) gauge symmetry, three families of left-handed superfields Q, L, U^c, D^c, E^c, and two Higgs chiral superfields H_1 and H_2. A supersymmetric action involves the $W^\alpha W_\alpha$ kinetic gauge multiplet terms for the SU(3)×SU(2)×U(1) fields, the chiral superfield kinetic terms including the minimal gauge coupling $\phi^* e^{2gV}\phi$, and the superpotential W for the chiral superfields. The superpotential stands for the supersymmetric version of the SM interactions which do not include the gauge couplings. One of the aesthetic problems of the MSSM is that it, as opposed to the Standard Model, does not conserve the baryon and lepton numbers automatically: various terms like QLD^c are allowed in the superpotential. To eliminate them, an extra, so called, R- symmetry [42] has to be introduced; in this paper, however, this aspect of the MSSM will not play any significant role and we can restrict ourselves to the minimal superpotential

$$W = (\lambda_E)_{ij}\ H_1 L_i E^c_j \ + \ (\lambda_D)_{ij}\ H_1 Q_i D^c_j \ + \ (\lambda_U)_{ij}\ H_2 Q_i U^c_j \ + \ \mu H_1 H_2$$

with the $\int d^4x (\int d^2\theta W + \int d^2\bar{\theta} W^*)$ contribution to the action. Here we have suppressed the SU(2) indices, the contraction of the doublets is made by means of the ϵ_{ab} tensor. The matrices λ give rise to quark masses and the mixings described by the familiar CKM-matrix. When supersymmetry is exact, all the Higgses have the same tree-level mass as dictated by the the μ-term. However, at low energies the mass term for the second doublet, driven down by the strong top-Yukawa coupling, can turn negative, leading naturally to spontaneous $SU(2)\times U(1)$ breaking. The scalar potential, arising from the supersymmetric action, is given by the sum of the F- and D-terms [11]:

$$V = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 + \sum_l \frac{g_l^2}{2} \sum_a \left|\sum_{i,j} \phi_i^* T_{l,a}^{ij} \phi_j\right|^2 ,$$

where l labels the simple symmetry groups, a enumerates the group generators, and i runs over all scalar fields of the theory. Another example of the interactions, generated by the superaction, is the fermion mass terms [34]

$$-\frac{1}{2}\frac{\partial^2 W}{\partial\phi_i \partial\phi_j}\psi_i\psi_j + h.c.$$

Concerning the other sectors of the model, the explicit expressions in the component form will be given below when necessary.

So far, the model is purely supersymmetric. Yet, there is not the slightest indication of supersymmetry at the low energies where the Standard Model processes take place. To reconcile theory with experiment one has to break supersymmetry. This can be done in two ways: the symmetry can be broken either explicitly or spontaneously. The latter, however, leads to unacceptable consequences : it predicts the existence of light susy partners. Indeed, the “supertrace” formula, valid for exact susy, holds true for the spontaneously broken case as well [13]:

$$StrM^2 \equiv \sum_J (-1)^{2J} trM_J^2 = 0$$

with J and M_J being the spin and the mass matrix for all particles with spin J, respectively. Hardly can this relation be satisfied if all the sfermions are considerably heavier than their SM partners. Even though it can be satisfied in models with heavy gauginos, this would require either fine tuning of parameters or introduction of extra superfields beyond the considered minimal content. Another way to get around the constraint is to employ radiative corrections which, in general, destroy the tree-level supertrace relation; this, however, also necessitates the introduction of additional superfields. Instead, supersymmetry in the MSSM is broken explicitly, i.e. by the incorporation of non-susy terms in the Lagrangian. Even though the model ceases to be supersymmetric in the strict sense, one would like to retain the principal feature of supersymmetry – absence of quadratic divergences. This requirement restricts the variety of the breaking terms to a few possibilities as given by the Girardello-Grisaru theorem [44]: bilinear and trilinear scalar couplings and gaugino masses; these interactions are called susy soft-breaking, referring to the fact that the theory is still free of quadratic divergences. The soft-breaking terms are usually assumed to have structures similar to that of the superpotential with the superfields replaced by the ordinary fields. In the next subsection we will demonstrate that this assumption is quite natural if one treats the MSSM as an effective theory following from supergravity. We will consider the impact of the soft-breaking terms at length later, especially in the sections devoted to the stability of the Higgs potential.

3.3 Low Energy Supergravity and The Polonyi Model

In this section we will consider a simple supergravity model leading at low energies to a MSSM-type theory (we will mostly follow the reviews [13],[16]). The most general global-susy-invariant coupling of a chiral superfield and a vector superfield can be written as follows [13]

$$Re \int d^2\theta f_{\alpha\beta}(S) W^\alpha W^\beta + \int d^4\theta \phi(\bar{S}e^{2eV}, S) + Re \int d^2\theta g(S) \ ,$$

where S and V are the chiral and vector superfields, $g(S)$ is the superpotential and f, ϕ are certain functions. The theory is renormalizable only if f is constant, g is a polynomial of degree not higher than three, and $\phi = \bar{S}e^{2eV}S$. Such a Lagrangian can be made local-susy-invariant via the Noether procedure, e.g. by considering a local transformation with a small parameter ϵ and adding to the action terms needed to cancel the ϵ-dependence up to certain order. The expression for the supergravity interactions obtained in this way is extremely cumbersome. For the time being, we will focus on the bosonic sector of the model only. The relevant part of the Lagrangian with the gravitational constant set to unity reads [13]

$$\begin{aligned}
& det^{-1}(e^m_\mu)\ \mathcal{L}_{boson} \\
& = e^{-G}(3 + G_k (G^{-1})^k_l G^l) - \frac{1}{2} e^2\ Re f^{-1}_{\alpha\beta}(G^i T^{\alpha j}_i z_j)(G^k T^{\beta l}_k z_l) \\
& -\frac{1}{4} Re f_{\alpha\beta} F^\alpha_{\mu\nu} F^{\mu\nu\beta} + \frac{i}{4} Im f_{\alpha\beta} F^\alpha_{\mu\nu} \tilde{F}^{\mu\nu\beta} + G^i_j D_\mu z_i D^\mu z^{*j} - \frac{1}{2} R \ ,
\end{aligned}$$

with z_i being the scalar field, $F^{\alpha}_{\mu\nu}$ - the vector field tensor, R - the scalar curvature, and $T_i^{\alpha j}$ - the gauge group generators. The derivative D_μ is covariant with respect to gravity and the gauge symmetries. The Kähler potential $G(z, z^*)$ is a real function of the scalar fields given by

$$G = 3 \, log(-\phi/3) - log(|g|^2) \ ,$$

and the Kähler metric G^i_j is the second derivative of the Kähler potential:

$$G^i_j = \partial^2 G / \partial z_i \partial z^{*j} \ .$$

The input functions ϕ and g enter the interactions only through the combination given by G. It is easily seen that the Kähler metric is responsible for the kinetic terms of the scalar fields which take on the canonical form when the metric is flat: $G^j_i = -\delta^j_i$. The scalar potential is given by the first two terms in $det^{-1}(e^m_\mu) \ \mathcal{L}_{boson}$:

$$V = -e^{-G}(3 + G_k (G^{-1})^k_l G^l) + \frac{1}{2} Re f^{-1}_{\alpha\beta} D^\alpha D^\beta \ ,$$

with

$$D^\alpha = e G^i T_i^{\alpha j} z_j \ .$$

The striking difference between global and local susy is that whereas the former implies vanishing vacuum energy, the latter is unbroken for $E_{vac} \leq 0$. Indeed, in the case of valid local supersymmetry the vacuum expectation values of the D-term and G_i have to vanish: $\langle G_k \rangle = \langle D^\alpha \rangle = 0$, then

$$V = -3 e^{-G} \leq 0 \ .$$

The equality is reached at the points where the superpotential vanishes. Moreover, it can be shown that supergravity can be broken even if the vacuum energy vanishes [13].

In the case of gauge singlet superfields and flat Kähler metric ($G_i^j = -\delta_i^j$) the Kähler potential expressed in terms of the input superpotential reads

$$G(z, z^*) = -zz^*/M^2 - log(|g(z)|^2/M^6) \ ,$$

where we have put back in the gravitational constant

$$k = 1/M \ , \ M \equiv M_{Plank}/\sqrt{8\pi} \ \sim 10^{18} GeV \ .$$

Such a choice of the Kähler potential corresponds to a nontrivial (exponential) kinetic function ϕ. The consequent scalar potential is given by

$$V = exp\Big(\frac{z_i z^{i*}}{M^2}\Big)\Big[\Big|g^{,i} + \frac{z^{i*}}{M^2} g\Big|^2 - \frac{3}{M^2}|g|^2\Big] \ .$$

Local supersymmetry is unbroken if [16]

$$D^i g \equiv \langle g^{,i} + \frac{z^{i*}}{M^2} g \rangle$$

and, therefore, the value of $D^i g$ sets the scale of F-type susy breaking.

Let us now consider some simple examples.

a) Constant superpotential.

Suppose that we have only one field and $W = \Delta^3 = const$. Then in terms of $x^2 \equiv zz^*/M^2$ the scalar potential reads

$$V = \frac{\Delta^6}{M^2} e^{x^2} (x^2 - 3) \ .$$

The absolute minimum is obtained for $|z| = M\sqrt{2}$ which corresponds to broken susy: $Dg \neq 0$ and large cosmological constant term $E_{vac} = -e^2\Delta^6/M^2$.

b) Polonyi potential [45].

Consider the superpotential

$$W = m^2(\ S + (2-\sqrt{3})M \ ) \ ,$$

where S is a chiral superfield and the constant in front of M is chosen so that at the minimum the vacuum energy vanishes. The absolute minimum of the scalar potential is given by $|z| = (\sqrt{3}-1)M$ with z being the scalar component of S. Note that $Dg \sim m^2$, so m^2 is the scale of susy breaking. On the other hand, it can be deduced from the supergravity Lagrangian that the gravitino mass is [13],[16]

$$m_{3/2} = \frac{m^2}{M} e^{(\sqrt{3}-1)^2/2} \ .$$

Thus we obtain the relation

$$m_{3/2} \sim m^2_{susy\ break.}/M \ ,$$

which is typical for theories with spontaneously broken susy and vanishing cosmological constant. Physically, the gravitino acquires mass due to absorption of the fermionic partner of z.

The Polonyi potential is a prototype for hidden sector models. Let us consider a system which includes observable low-energy fields y^a and "hidden sector" gauge singlets z^i:

$$W = h(z^i) + g(y^a) \ .$$

Suppose that *in the absence* of $g(y^a)$ the absolute minimum of the scalar potential is such that [16]

$$\langle V \rangle = 0 \ , \ \langle z^i \rangle = b^i M \ , \ \langle h_{,i} \rangle = a_i m M \ , \ \langle h \rangle = m M^2 \ .$$

Thus we require that the cosmologocal constant vanish and the "hidden sector" scalar fields acquire large vacuum expectation values. Further, consider the scalar potential of the full theory:

$$V = e^{(|z|^2+|y|^2)/M^2} \left(\left| h_{,i} + \frac{z_i W}{M^2} \right|^2 + \left| g_{,a} + \frac{y_a W}{M^2} \right|^2 - 3\frac{|W|^2}{M^2} \right) + \frac{1}{2} D^2 \ .$$

Assuming that the "ordinary" fields y^a do not develop large VEV's of the order of M, we can expand this potential in powers of $1/M$. In the leading order we get [16]

$$V = \langle V \rangle + V_{susy} + V_{soft} \ ,$$

where the first term is tuned to zero by the assumption:

$$\langle V \rangle = e^{|b_i|^2} (|a_i + b_i|^2 - 3) m^2 M^2 \to 0 \ ,$$

the second term consists of the F- and D-interactions of global susy:

$$V_{susy} = |\tilde{g}_{,a}|^2 + \frac{1}{2} D^2 \ ,$$

and the third term represents the universal scalar mass and the soft $\Phi^2 + \Phi^{*2}$, $\Phi^3 + \Phi^{*3}$ interactions:

$$V_{soft} = m_{3/2}^2 |y^a|^2 + m_{3/2}^2 [y^a \tilde{g}_{,a} + (A-3)\tilde{g} + h.c.] \ .$$

Here

$$m_{3/2}^2 = e^{|b_i|^2} m^2 \ ,$$

$$\tilde{g} = e^{\frac{1}{2}|b_i|^2} g \ ,$$

$$A = b_i^*(a_i + b_i) = 3 - \sqrt{3} \quad \text{for the Polonyi model.}$$

The "hidden sector" fields z^i decouple in this limit. As a result of the minimal assumption for G, only two new parameters $m_{3/2}$ and A are introduced in the visible sector.

Note that the soft breaking interaction can be rewritten as

$$V_{soft} = m_{3/2}^2 |y^a|^2 + m_{3/2} A (g^{(3)} + g^{(3)*}) + m_{3/2} B (g^{(2)} + g^{(2)*}) \ ,$$

where $g^{(3)}, g^{(2)}$ are the cubic and quadratic parts of the superpotential. For the Polonyi model we have $B = A - 1$.

The universality of the scalar mass terms is a desirable feature of this model since it may be relevant to the issue of the suppression of flavor changing neutral currents which requires squarks to be nearly degenerate in mass [22]. On the other hand, the major shortcoming of the model is that the gauginos are massless at the tree level, whereas the radiative corrections to their masses are infinite at the 2-loop level [16]. This fact manifests the non-renormalizability of the low-energy theory: the gaugino mass is a "relevant" operator which is not prohibited by any symmetry nor by the requirement of the susy breaking interactions being soft. This problem can be rectified if we consider a more general action. Going back to the supergravity Lagrangian we discover the term

$$\frac{1}{2}\exp(-\frac{G}{2})G^{,i}(G^{-1})_i^j\frac{\partial f_{\alpha\beta}}{\partial z^j}\lambda^\alpha\lambda^\beta \ ,$$

with λ^α being the gaugino fields. This term vanishes if either susy is unbroken: $\langle G^{,i}\rangle = 0$, or we take the minimal $f_{\alpha\beta} = \delta_{\alpha\beta}$. So, with general $f_{\alpha\beta}$, the hidden sector will generate the gaugino mass.

The mass terms for the matter fermions arising from the superpotential are the same as in the global theory [16], so the impact of supergravity amounts to the lifting of the scalar and gaugino masses. This effect is reflected in the generalized supertrace formula for N chiral superfields [13]:

$$STrM^2 = 2(N-1)m_{3/2}^2 \ ,$$

which is valid for F-type spontaneous supergravity breaking with minimal kinetic terms.

To summarize, we have shown that the MSSM-type models can be obtained as low-energy effective theories resulting from spontaneously broken supergravity. The hidden sector of the model induces explicit global-susy-breaking terms in the action of the low energy fields and decouples from the observable sector.

3.4 Cancellation of Quadratic Divergences in the MSSM

Absence of quadratic divergences is a general feature of supersymmetric and softly broken susy theories. For chiral supermultiplets it can readily be shown via the non-renormalization theorem of Grisaru, Rocek and Siegel [46]: the radiative corrections in the superspace formalism are represented by the integral over the entire Grassman space $\int d^4\theta$ with no terms $\int d^2\theta$ or $\int d^2\bar{\theta}$ appearing alone, i.e. quantum corrections do not affect the superpotential given by a two-dimensional integral over θ_α.

It is also important to understand quadratic divergence cancellation at the Feynman diagram level. Diagrammatic considerations help understand the interplay between the susy partners' contributions and reveal certain subtle points such as, for instance, requirement that the hypercharges of all sfermions sum up to zero (otherwise quadratic divergences would be present even in an exactly supersymmetric theory). They also serve as an important check: we use unconventional Feynman rules for two-component spinors, and as a first step toward more complicated calculations. In this subsection we will convince ourselves that the quadratic divergences are indeed absent in the 1-loop Higgs propagator.

Let us now consider the two-point function of the neutral component of the first Higgs doublet. For convenience, in the following calculations we will not adhere to a single notation: in the treatment of the self-interaction of the Higgs we use the standard convention Φ_1 and Φ_2, whereas in other cases we follow the supersymmetric convention H_1 and H_2 with

$$(\Phi_1)^j = \epsilon_{ij} H_1^{i*} \ , \ (\Phi_2)^j = H_2^j \ . \tag{24}$$

The cancellation of quadratic divergences for the interactions arising from the superpotential is trivial, so we will focus on the gauge sector of the MSSM. In particular, we are interested in the terms proportional to the SU(2) $\times$U(1) coupling strengths. It is convenient to classify different quadratically divergent contributions to the Higgs propagator as follows.

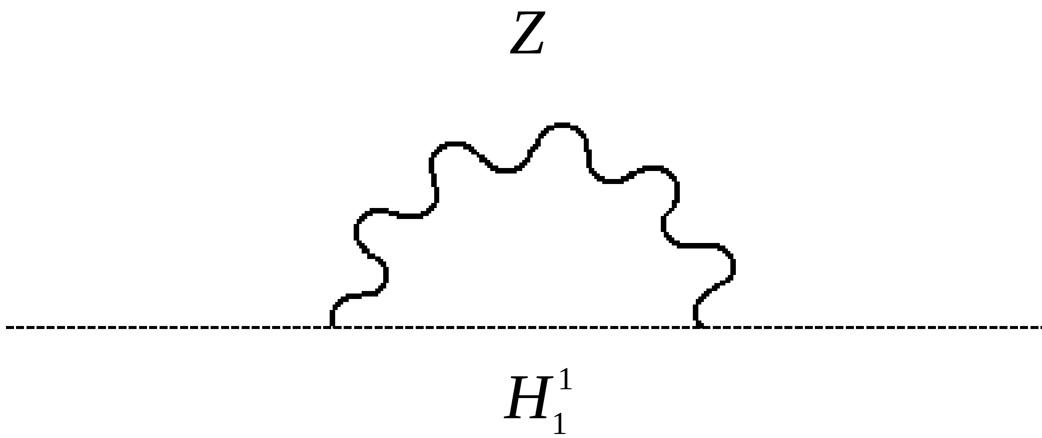
a) Higgs loops.

The complete list of MSSM (and NMSSM) interactions can be found in [34]. The Higgs self-interaction arising from the D-term is given by

$$\begin{aligned} V_H = \quad & \frac{1}{8}(g^2 + g'^2)|\Phi_1|^4 \ + \ \frac{1}{4}(g^2 - g'^2)|\Phi_1|^2|\Phi_2|^2 \ + \ (-\frac{1}{2}g^2)|\Phi_1^\dagger \Phi_2|^2 \ + \\ & mass\ term + terms\ independent\ of\ \Phi_1 \ . \end{aligned} \tag{25}$$

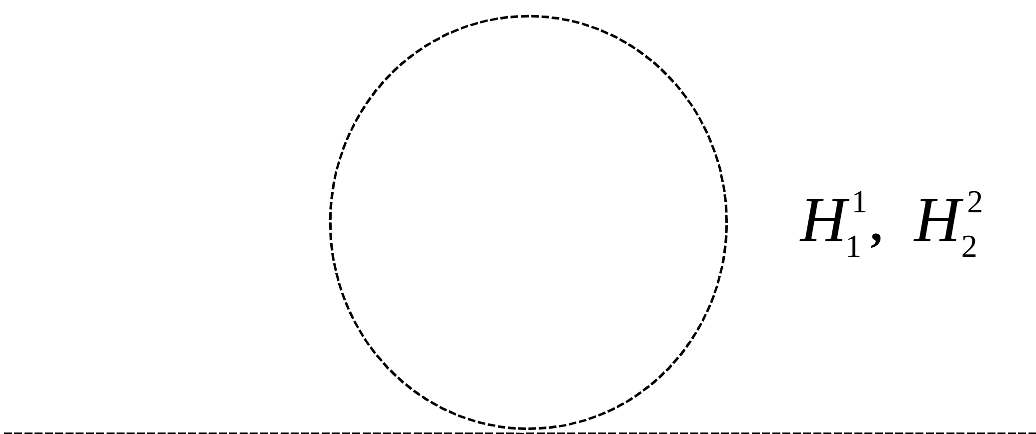
In contrast to general two-Higgs-doublet models, the Higgs couplings in the MSSM are not arbitrary and are related to the gauge couplings. The corresponding quadratically divergent diagrams are shown in Fig.1 and Fig.2. Below we list the coefficients of the diagrams which contribute to the Higgs mass; they are to be multiplied by $\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2_{H_{1,2}}}$. The masses can be ignored

a)



+

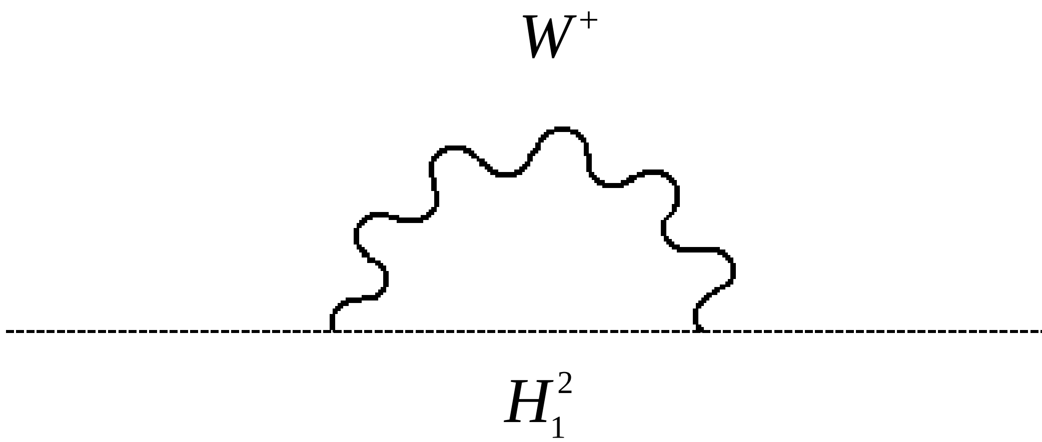
b)



$= 0$

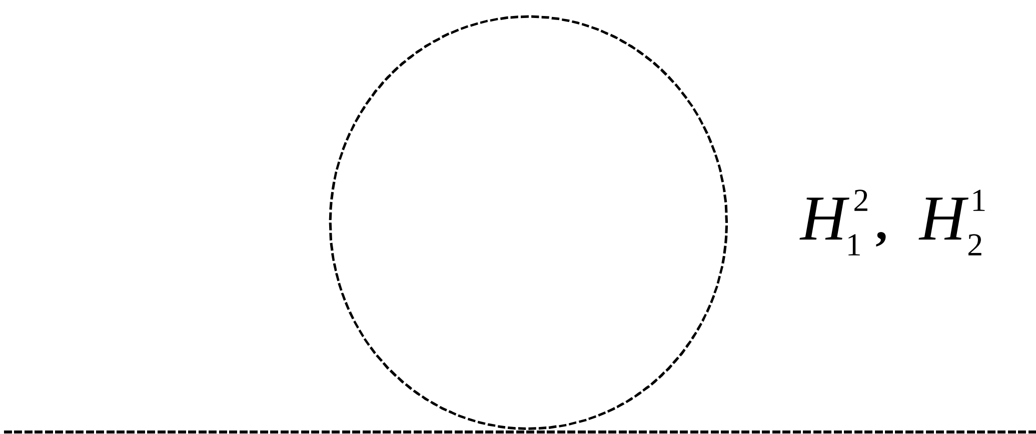
Fig.(3)-1

a)



+

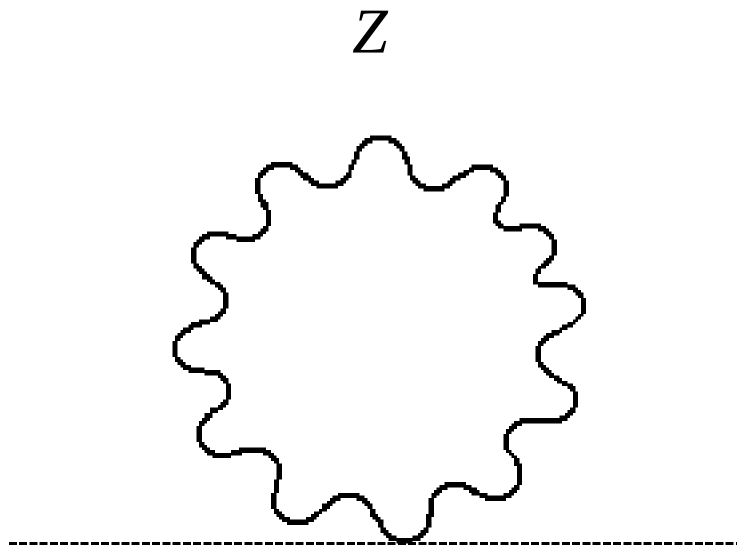
b)



= 0

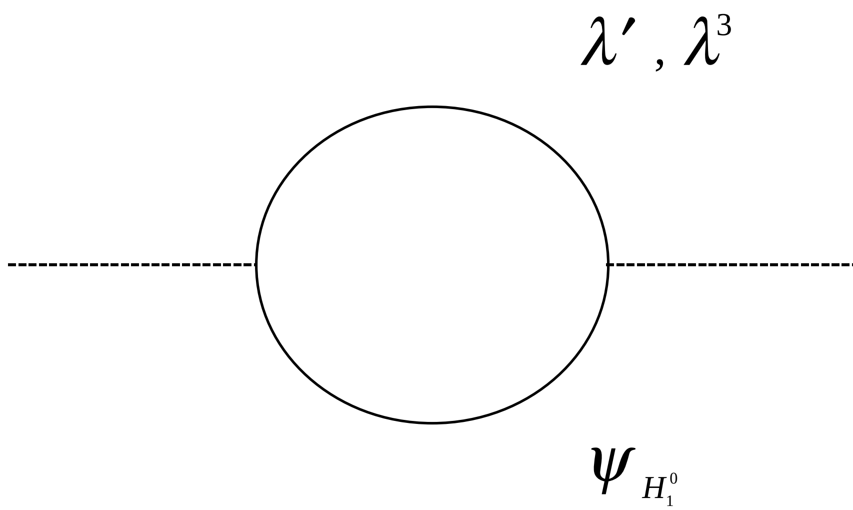
Fig.(3)-2

a)



+

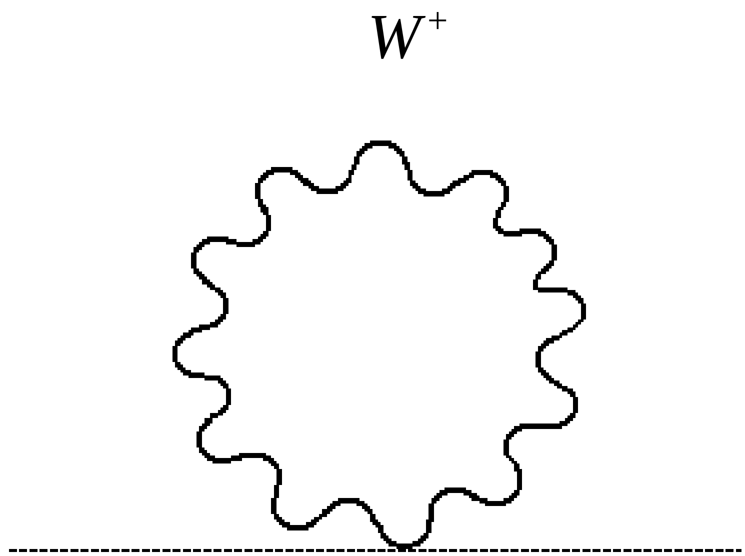
b)



$= 0$

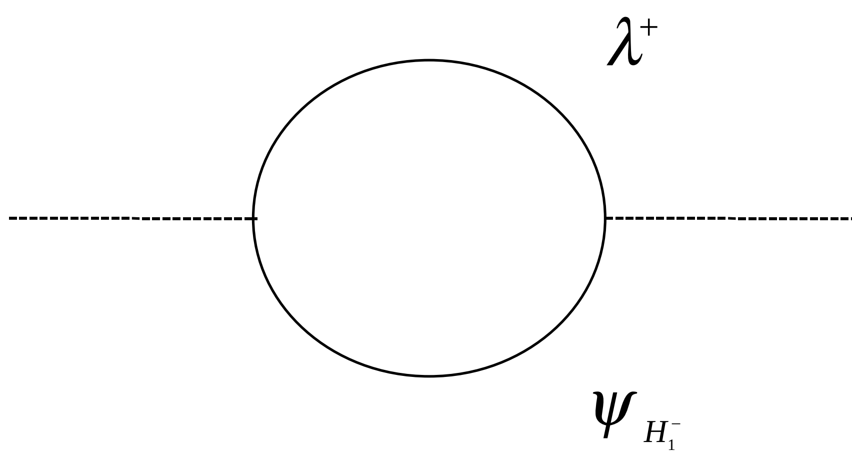
Fig.(3)-3

a)



+

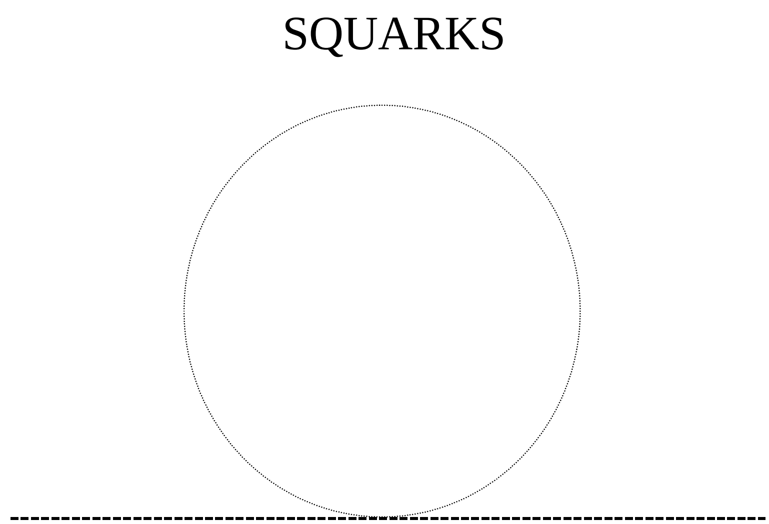
b)



= 0

Fig.(3)-4

Σ



$= 0$

Fig.(3)-5

since we are looking at the most ultraviolet piece of the integral.

$$\begin{aligned}
neutral\ Higgs\ loops\ (Fig.1) &\rightarrow 4\times\frac{1}{8}(g^2+g'^2)-\frac{1}{2}g^2+\frac{1}{4}(g^2-g'^2) \\
&= \frac{1}{4}(g^2+g'^2)\ , \\
charged\ Higgs\ loops\ (Fig.2) &\rightarrow \frac{1}{4}(g^2+g'^2)+\frac{1}{4}(g^2-g'^2)\ =\frac{1}{2}g^2\ .
\end{aligned}$$

The symmetry factors are placed in front of the vertex ones, and are followed by $\times$.

b) Higgs-vector boson loops.

At high energies the Higgs-vector boson diagrams can be calculated in terms of W^3,B or, equivalently, in terms of Z,A ; we will prefer the latter. The relevant part of the Lagrangian is

$$\mathcal{L}_{HHV}=\frac{-ig}{\sqrt{2}}W_\mu^\dagger(H_1^{1*}\overset{\leftrightarrow}{\partial^\mu}H_1^2)-\frac{ig}{2cos\theta_W}Z_\mu(H_1^{1*}\overset{\leftrightarrow}{\partial^\mu}H_1^1)+h.c.\ .$$

In this subsection we will use Feynman gauge for the gauge boson propagators. First let us compute the $Z-H_1^1$ loop in Fig.1. Noting that the vertex contains the sum of the Higgs' momenta and setting the external momentum to zero, we obtain

$$\begin{aligned}
&2\times\frac{1}{2!}\int\frac{d^4k}{(2\pi)^4}\Big(\frac{ig}{2cos\theta_W}k_\mu\Big)\frac{-ig^{\mu\nu}}{k^2}\Big(\frac{ig}{2cos\theta_W}k_\nu\Big)\frac{i}{k^2-m_H^2} \\
&=-\frac{1}{4}(g^2+g'^2)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_H^2}\ . \qquad (26)
\end{aligned}$$

Here we have used $g^2/cos^2\theta_W \ = g^2 + g'^2$. The charged loop in Fig.2 corresponds to

$$
\begin{aligned}
& 2 \times \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left(\frac{ig}{\sqrt{2}} k_\mu \right) \frac{-ig^{\mu\nu}}{k^2} \left(\frac{ig}{\sqrt{2}} k_\nu \right) \frac{i}{k^2 - m_H^2} \\
& = -\frac{1}{2!} g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_H^2} \ . \qquad (27)
\end{aligned}
$$

As a result,

$$
\begin{aligned}
& neutral\ Higgs - vector\ boson\ loop\ (Fig.1) \rightarrow -\frac{1}{4}(g^2 + g'^2) \ , \\
& charged\ Higgs - vector\ boson\ loop\ (Fig.2) \rightarrow -\frac{1}{2} g^2 \ .
\end{aligned}
$$

c) Vector boson loops.

The relevant part of the Lagrangian is

$$
\mathcal{L}_{HHVV} = \frac{1}{2} g^2 W_\mu^+ W^{\mu -} |H_1^1|^2 + \frac{g^2}{4cos^2\theta_W} Z_\mu Z^\mu |H_1^1|^2 \ . \qquad (28)
$$

The Z-loop in Fig.3 corresponds to

$$
\frac{ig^2}{4cos^2\theta_W} \int \frac{d^4k}{(2\pi)^4} \frac{-ig^{\mu\nu}}{k^2} \ g_{\mu\nu} = (g^2 + g'^2) \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \ .
$$

The expression for the W^+ loop is completely analogous. The result is

$$
\begin{aligned}
& neutral\ vector\ boson\ loop\ (Fig.3) \rightarrow g^2 + g'^2 \\
& charged\ vector\ boson\ loop\ (Fig.4) \rightarrow 2g^2 \ .
\end{aligned}
$$

d) Higgsino-gaugino loops.

The femionic analogue of the Higgs-gauge boson interaction is

$$\mathcal{L} = igH_1^{1*}\lambda^+\psi^-_{H_1} + \frac{i}{\sqrt{2}}(g\lambda^3 - g'\lambda')H_1^{1*}\psi^0_{H_1} + h.c.\ , \tag{29}$$

with λ and ψ being the two-component gaugino and higgsino, respectively. This Lagrangian generates the quadratically divergent diagrams in Fig.3 and 4. The neutral loop with the U(1) gaugino λ' is given by

$$\begin{aligned}
&-2\times\frac{1}{2!}\int\frac{d^4k}{(2\pi)^4}Tr\Big[\frac{g'}{\sqrt{2}}\,\frac{i(k_0-\bar{k}\bar{\sigma})}{k^2-m_\psi^2}\,\frac{-g'}{\sqrt{2}}\,\frac{i(k_0+\bar{k}\bar{\sigma})}{k^2-m_\lambda^2}\Big]\\
&= -g'^2\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-m_\psi^2)(k^2-m_\lambda^2)}\ .
\end{aligned}$$

Here we have used the left-left and right-right propagators for the Weyl spinors:

$$\langle\lambda'(\lambda')^\dagger\rangle \sim k\cdot\dot{\sigma}\ ,$$

$$\langle\psi^0_{H_1}(\psi^0_{H_1})^\dagger\rangle \sim k\cdot\sigma$$

and $Tr[(k\cdot\sigma)(k\cdot\dot{\sigma})] = 2k^2$. The four-component spinors are formed in the following way [34]

$$\Lambda^{',3} = \begin{pmatrix} -i\lambda^{',3} \\ i\bar{\lambda}^{',3} \end{pmatrix}\ , \Psi^0 = \begin{pmatrix} \psi^0_{H_1} \\ \bar{\psi}^0_{H_2} \end{pmatrix}\ ,$$

$$\Lambda^+ = \begin{pmatrix} -i\lambda^+ \\ i\bar{\lambda}^- \end{pmatrix}\ , \Psi^+ = \begin{pmatrix} \psi^+_{H_2} \\ \bar{\psi}^-_{H_1} \end{pmatrix}\ .$$

Such a form of the higgsino Dirac spinor is dictated by the supersymmetric higgsino mass term $\mu\psi^-_{H_1}\psi^+_{H_2} + h.c.$

The diagram with λ^3 in the loop is identical to that with λ' except for the substitution $g' \rightarrow g$, whereas the charged gaugino loop is given by

$$-2 \times \frac{1}{2!}\int \frac{d^4k}{(2\pi)^4} Tr\Big[-g\,\frac{i(k_0 + \bar{k}\bar{\sigma})}{k^2 - m_\psi^2}\; g\,\frac{i(k_0 - \bar{k}\bar{\sigma})}{k^2 - m_\lambda^2}\Big] =$$
$$-2g^2 \int \frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2 - m_\psi^2)(k^2 - m_\lambda^2)} \; .$$

Thus,

$$neutral\ higgsino - gaugino\ loop\ (Fig.3) \rightarrow -g^2 - g'^2$$
$$charged\ higgsino - gaugino\ loop\ (Fig.4) \rightarrow -2g^2 \; .$$

e) Squark loops.

The Higgs-squark interaction arises from the D-term and is given by

$$\begin{aligned} V = \quad & \frac{1}{8}g^2\ (\ 4|H_1^{i*}\tilde{Q}^i|^2 - 2(\tilde{Q}^{i*}\tilde{Q}^i)H_1^{j*}H_1^j\ ) + \\ & \frac{1}{8}g'^2\ (\ -H_1^{i*}H_1^i + Y_q\tilde{Q}^{i*}\tilde{Q}^i + Y_u\tilde{U}^*\tilde{U} + Y_d\tilde{D}^*\tilde{D}\ )^2 \; , \end{aligned} \tag{30}$$

with $Y_q = \frac{1}{3}$, $Y_u = -\frac{4}{3}$, $Y_d = \frac{2}{3}$.

As a consequence of the tracelessness of the $SU(2) \times U(1)$ generators, the total contribution of each generation to the Higgs self energy vanishes. For example, the abelian part of the contribution is proportional to

$$2Y_q + Y_u + Y_d = 0 \; .$$

Hence,

$$squark\ loops\ (Fig.5) \rightarrow 0\ .$$

The diagrams in Fig.1-5 are grouped in such a way that each set is free of quadratic divergences thereby reassuring their absence in the net result. This grouping is also instructive for the identification of superpartners. In this sense, the Higgs bosons play the role of the scalar partners of the gauge bosons; this fact can also be established for exact susy after spontaneous $SU(2) \times U(1)$ symmetry breaking : for example, the W^+, chargino, and charged Higgs form a supermultiplet with mass m_W [34].

3.5 Renormalization Group Equations for the Quartic Higgs Couplings in the MSSM

In this subsection we will examine the Renormalization Group evolution [48] of the quartic Higgs couplings in the context of exact susy and will generalize it to the case of softly broken susy as well. In particular, we are going to consider the radiative corrections to the term $\lambda_1 \Phi_1^4$ and derive the corresponding renormalization group equation for the coupling λ_1. The result of these considerations is important for the calculation of the finite radiative corrections to the Higgs couplings (see section 4): the log-divergent part needs to be separated since, as we show below, it does not modify the tree level relations between the Higgs and gauge couplings. In the following calculations the number of generations N_G is not fixed and can be arbitrary.

Again, let us first concentrate on the gauge sector of the model and put off the analysis of the Yukawa interactions until later. As in the previous subsection, we list the contributions according to the content of the loops.

a) Higgs loops.

$$\begin{aligned} V_H = \quad & \lambda_1|\Phi_1|^4 \ + \ \lambda_3|\Phi_1|^2|\Phi_2|^2 \ + \ \lambda_4|\Phi_1^\dagger \Phi_2|^2 \ + \\ & mass\ term + terms\ independent\ of\ \Phi_1 \ , \end{aligned} \tag{31}$$

with

$$\lambda_1 = \frac{1}{8}(g^2 + g'^2) \ , \lambda_3 = \frac{1}{4}(g^2 - g'^2) \ , \lambda_4 = -\frac{1}{2}g^2 \ .$$

To find a correction to $\lambda_1 \Phi_1^4$ it suffices to consider corrections to its neutral part $\lambda_1 |\Phi_1^0|^4$. Below are listed the contributions at second order in the λ_i's.

i. λ_1^2

In terms of the doublet components, $|\Phi_1|^4$ reads

$$|\Phi_1|^4 = (\Phi_1^{0*}\Phi_1^0)^2 + (\Phi_1^+\Phi_1^-)^2 + 2(\Phi_1^{0*}\Phi_1^0)(\Phi_1^+\Phi_1^-) \ .$$

The corrections to $\lambda_1 |\Phi_1^0|^4$ arising from these interactions are depicted in Fig.6. The external legs are the neutral components of the first Higgs doublet Φ_1^0. Apart from the symmetry and vertex factors, each diagram is to be multiplied by $\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_H^2)^2}$. Since we are interested in the *log*-divergent part only, it is legitimate to omit masses in the integral. As a result,

$$\begin{aligned} neutral\ H_1 -\ loops\ (Fig.6) &\to 20 \times \frac{1}{2!}\lambda_1^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \\ charged\ H_1 -\ loops\ (Fig.6) &\to 4 \times \frac{1}{2!}\lambda_1^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \ . \end{aligned}$$

ii. λ_3^2

$$\begin{aligned} |\Phi_1|^2|\Phi_2|^2 = \quad & (\Phi_1^{0*}\Phi_1^0 + \Phi_1^+\Phi_1^-)(\Phi_2^{0*}\Phi_2^0 + \Phi_2^+\Phi_2^-) \to \\ & (\Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0)^2 + (\Phi_1^{0*}\Phi_1^0\Phi_2^+\Phi_2^-)^2 \ . \end{aligned}$$

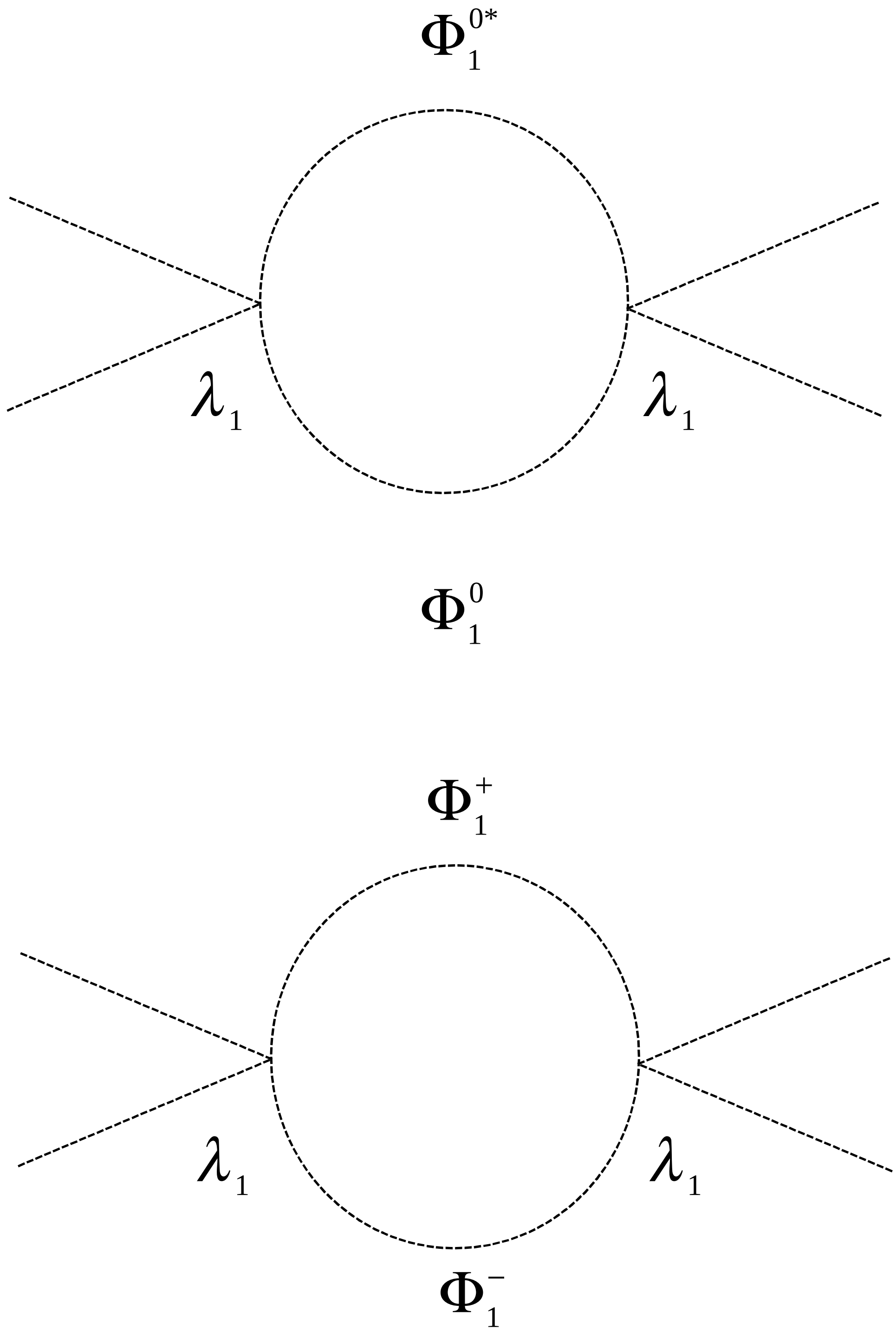


Fig.(3)-6

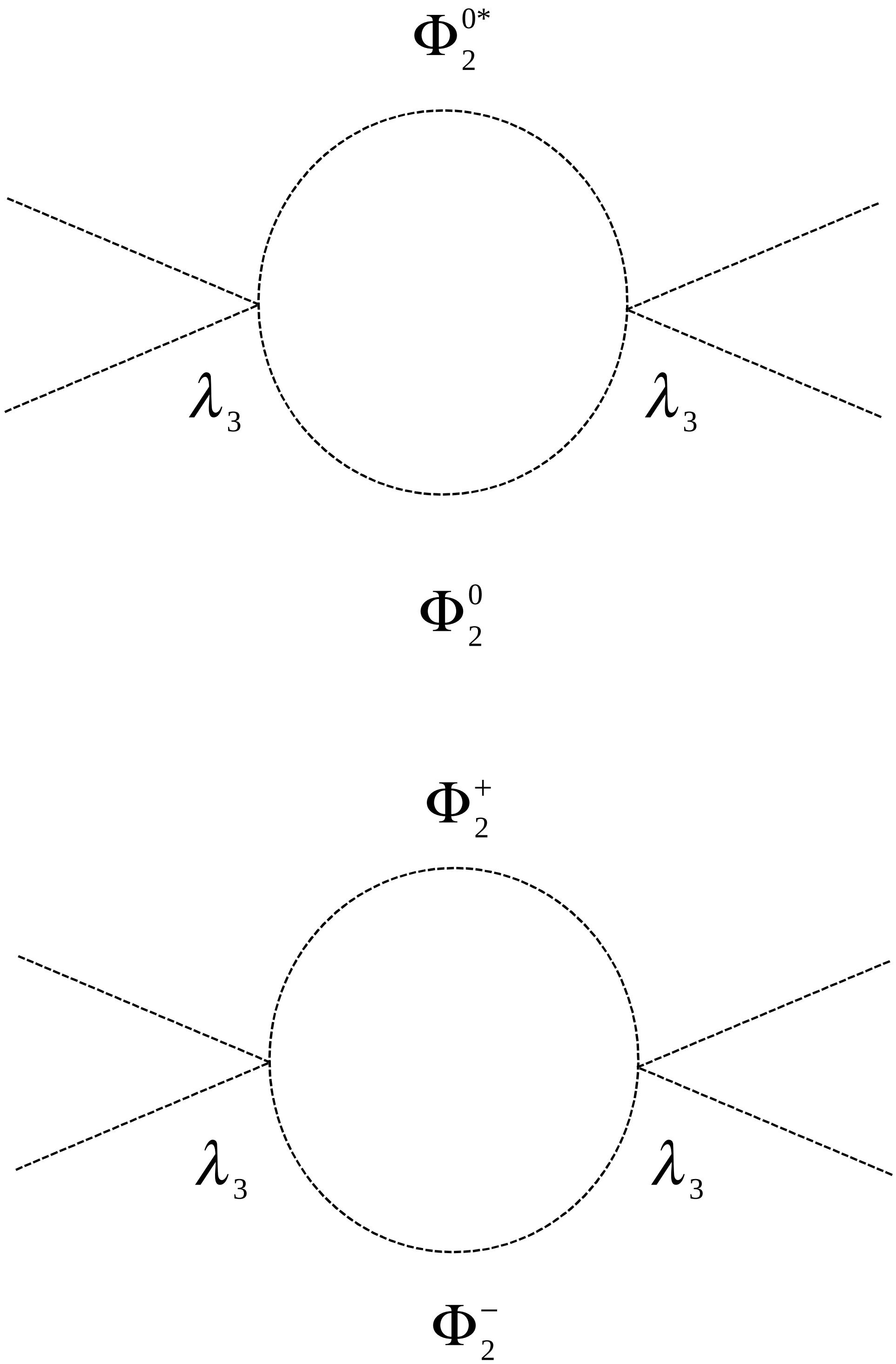


Fig.(3)-7

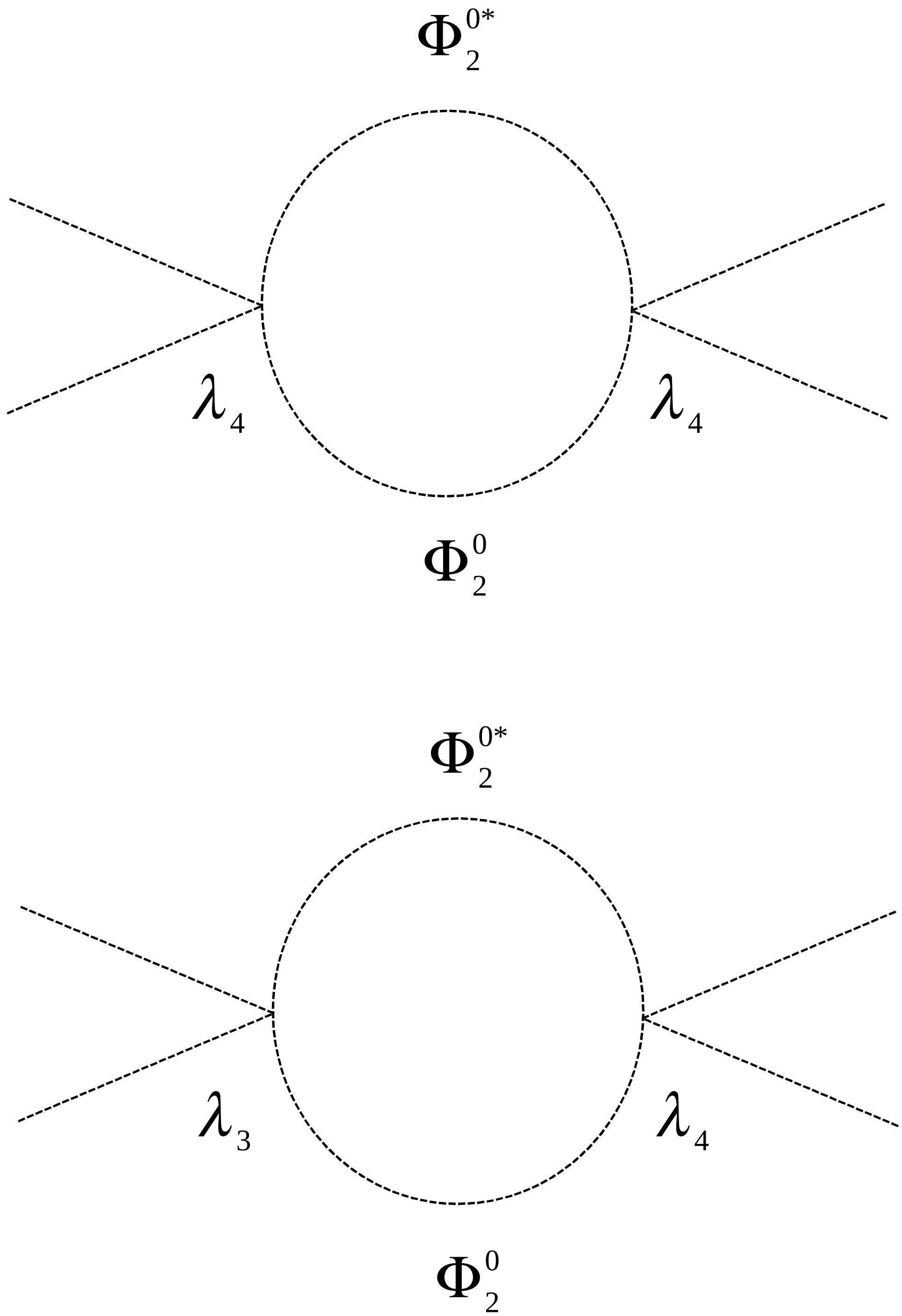


Fig.(3)-8

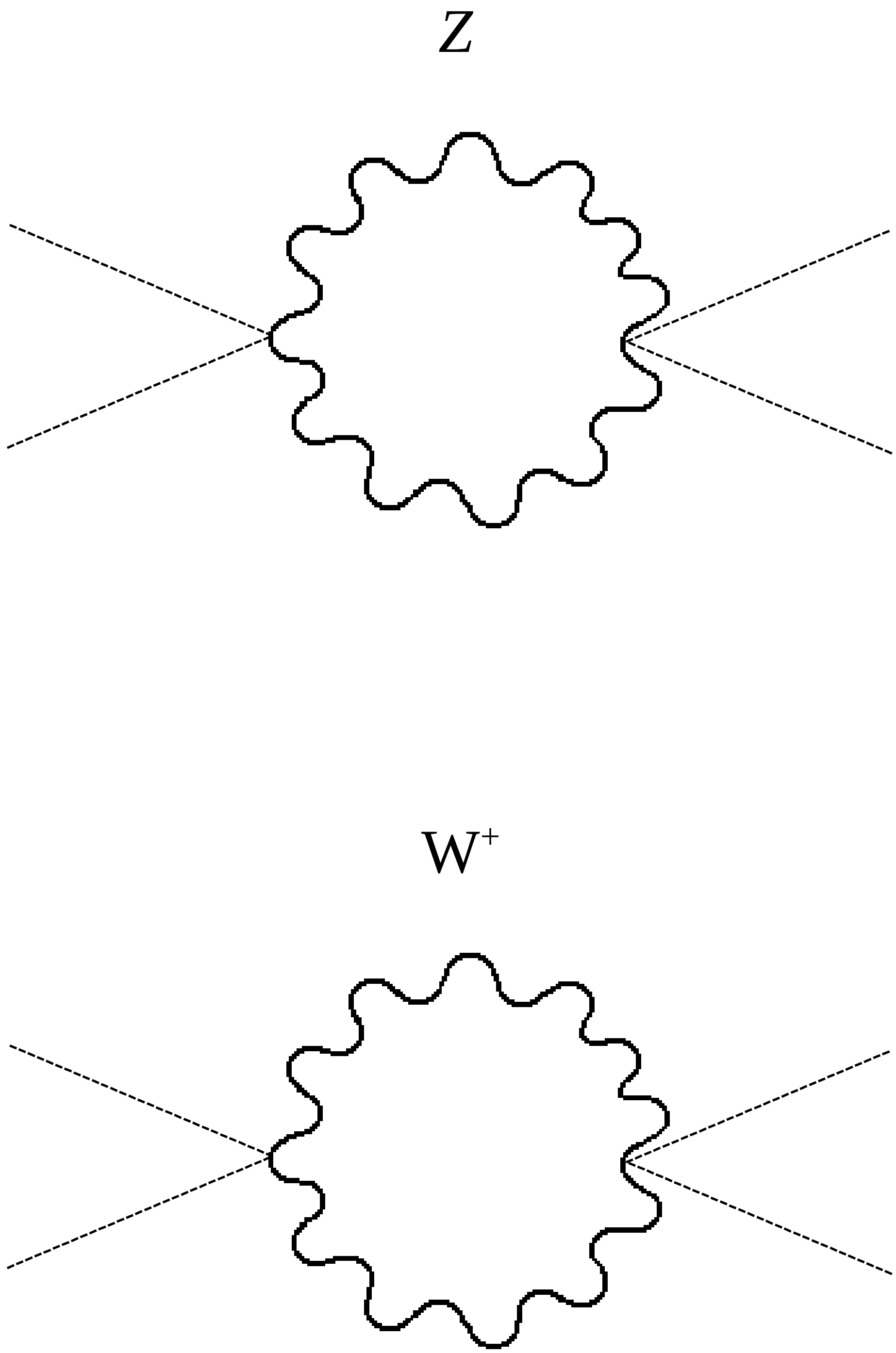


Fig.(3)-9

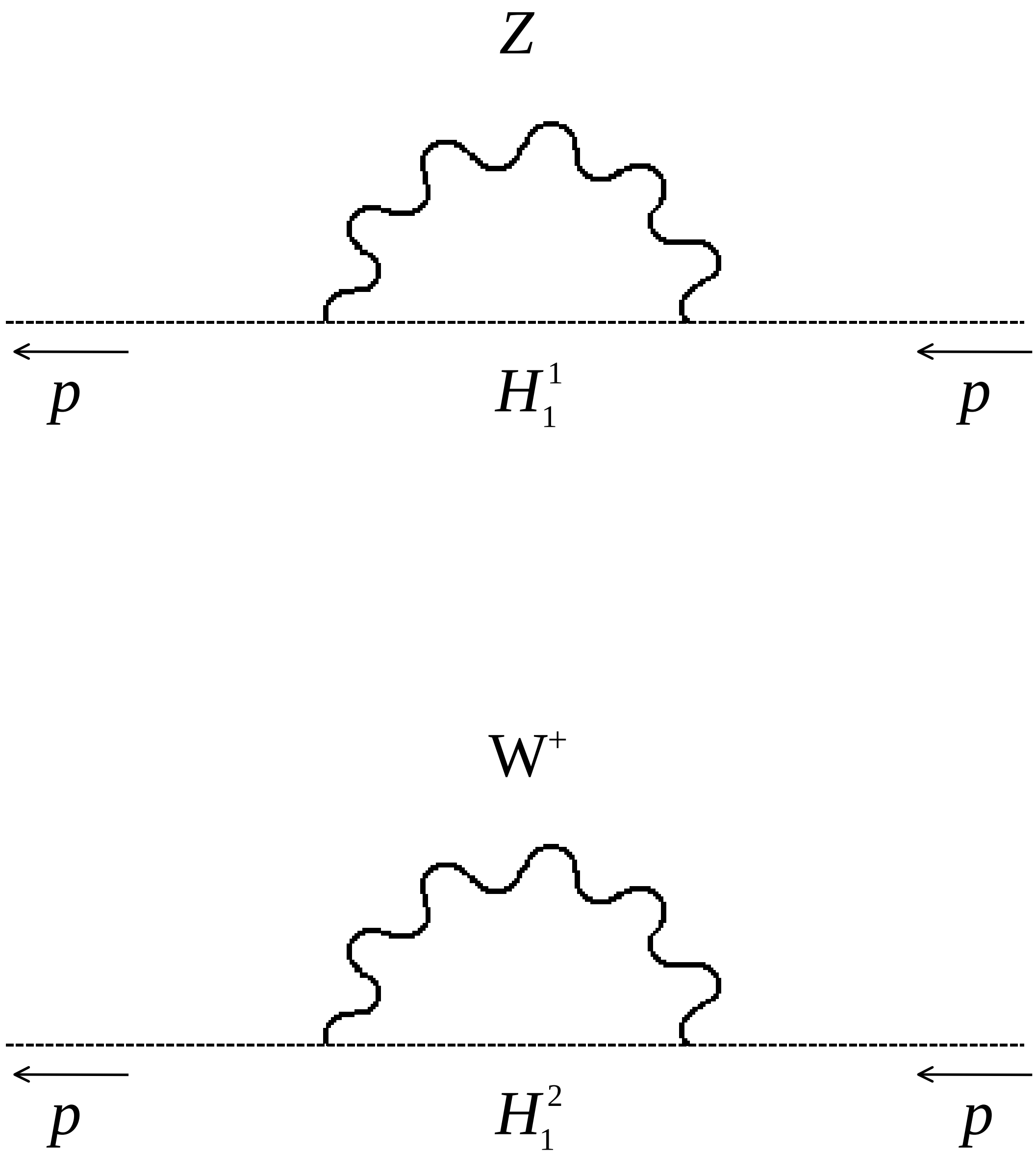


Fig.(3)-10

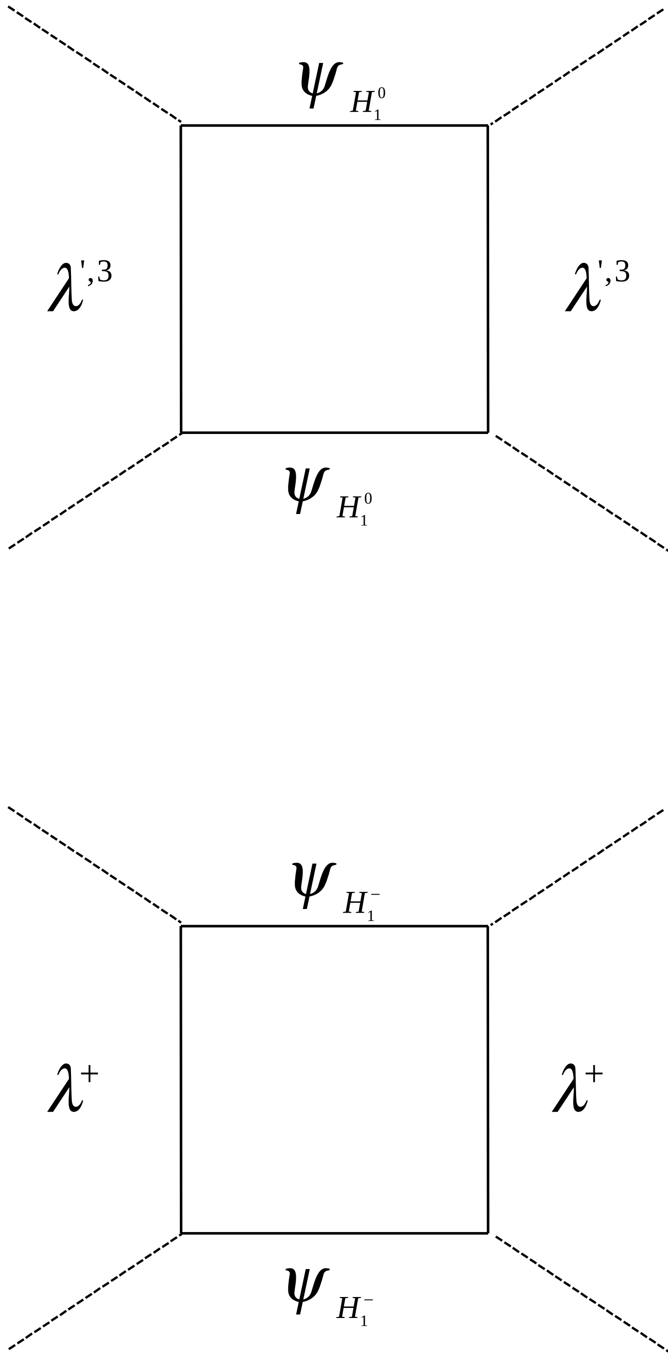


Fig.(3)-11

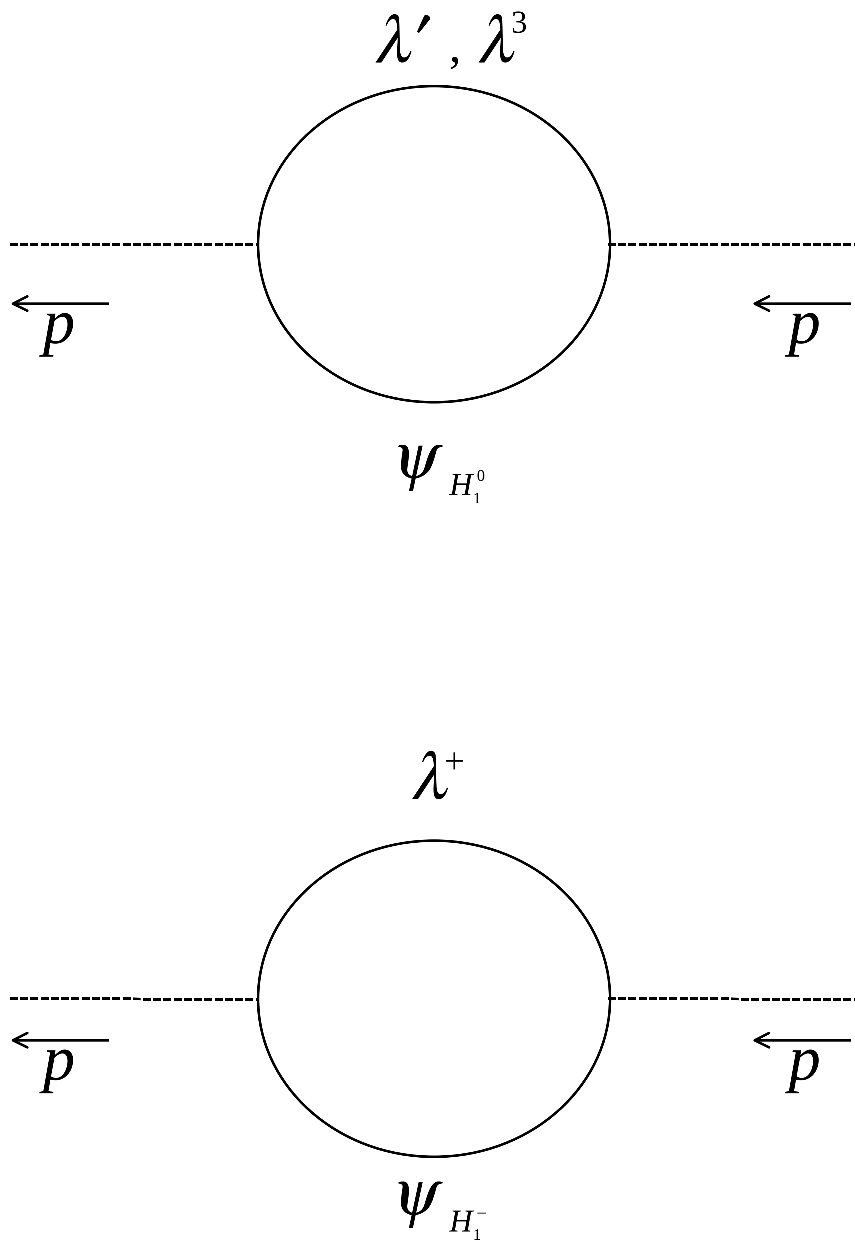


Fig.(3)-12

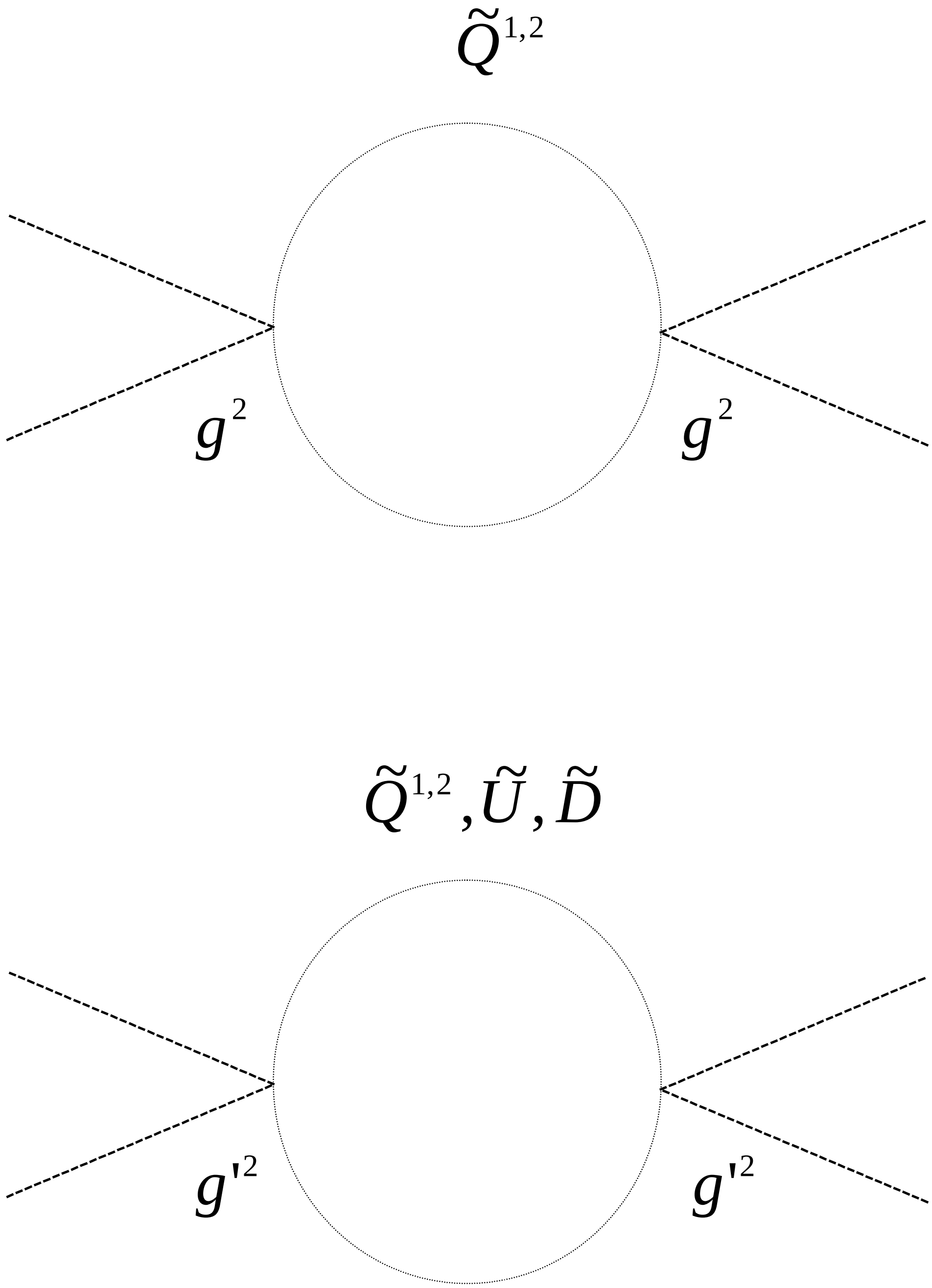


Fig.(3)-13

Here we have dropped the terms which do not contribute to $\lambda_1|\Phi_1^0|^4$ corrections. The resulting contribution is

$$
\begin{aligned}
neutral\ H_2 - \ loops\ (Fig.7) &\rightarrow \frac{1}{2!}\lambda_3^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \\
charged\ H_2 - \ loops\ (Fig.7) &\rightarrow \frac{1}{2!}\lambda_3^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \ .
\end{aligned}
$$

iii. λ_4^2

$$
|\Phi_1^\dagger \Phi_2|^2 = |\Phi_1^{0*}\Phi_2^0 + \Phi_1^+\Phi_2^-|^2 \rightarrow (\Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0)^2 \ .
$$

$$
neutral\ H_2 - \ loop\ (Fig.8) \rightarrow \frac{1}{2!}\lambda_4^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \ .
$$

iv. $\lambda_3\lambda_4$

In the second order the only contributing term has the form

$$
\frac{1}{2!}2\lambda_3\lambda_4\ \Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0\ \ \Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0 \ .
$$

This leads to

$$
neutral\ H_2 - \ loop\ (Fig.8) \rightarrow 2 \times \frac{1}{2!}\lambda_3\lambda_4 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \ .
$$

Let us sum up the above contributions of Higgs loops to the RG equation for λ_1. With the help of a cut-off regularization

$$
\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} = \frac{i}{8\pi^2}\ln\frac{\Lambda}{\mu} \ .
$$

The sum of these diagrams contributes to $-iH_{int}$ and modifies the bare vertex

$$\lambda_1 \to \lambda_1 - \frac{1}{2!}(24\lambda_1^2 + 2\lambda_3^2 + \lambda_4^2 + 2\lambda_3\lambda_4)\ \frac{i}{8\pi^2} \ln\frac{\Lambda}{\mu}\ .$$

Observing that $\partial/\partial \ln \mu = -\partial/\partial \ln \Lambda$ and introducing

$$t \equiv \ln \mu^2\ ,$$

we see that the Higgs-loop beta-function is

$$\dot{\lambda}_1|_{Higgs} = \frac{1}{32\pi^2}(24\lambda_1^2 + 2\lambda_3^2 + \lambda_4^2 + 2\lambda_3\lambda_4)\ ,$$

where the dot denotes differentiation with respect to t.

b) Vector boson loops.

$$\begin{aligned}\mathcal{L}_{HHVV} = \quad & \frac{1}{2}g^2 W_\mu^+ W^{\mu -}|H_1^1|^2 + \frac{g^2}{4cos^2\theta_W} Z_\mu Z^\mu |H_1^1|^2 \\ & -\frac{g}{\sqrt{2}}(eA^\mu - \frac{g\ sin^2\theta_W}{cos\theta_W})(H_1^{1*}H_1^2 W_\mu^+ + H_1^{2*}H_1^1 W_\mu^-)\ .\end{aligned}$$

From now on we will use Landau gauge for the gauge boson propagators:

$$D_{\mu\nu} = -i\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2}\ .$$

In this gauge, the $\mathcal{L}_{HHV}$ vertex vanishes at zero external momenta since

$$k^\mu D_{\mu\nu} = 0\ ,$$

and the only contribution to charge renormalization arises from $\mathcal{L}_{HHVV}$ (the former, however, affects the anomalous dimension).

i. *Charge renormalization corrections .*

The Z-loop in Fig.9 corresponds to

$$\begin{aligned} 2 \quad \times \quad & \frac{1}{2!}\int \frac{d^4k}{(2\pi)^4}\Big(-i\frac{g_{\mu\nu}-k_\mu k_\nu/k^2}{k^2}\Big)\frac{i}{4}(g^2+g'^2)\Big(-i\frac{g^{\mu\nu}-k^\mu k^\nu/k^2}{k^2}\Big) \\ \times \quad & \frac{i}{4}(g^2+g'^2) = \frac{3}{16}(g^2+g'^2)^2\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \ . \end{aligned}$$

The analogous W^+-loop is given by

$$\begin{aligned} & \frac{1}{2!}\int \frac{d^4k}{(2\pi)^4}\Big(-i\frac{g_{\mu\nu}-k_\mu k_\nu/k^2}{k^2}\Big)\frac{ig^2}{2}\Big(-i\frac{g^{\mu\nu}-k^\mu k^\nu/k^2}{k^2}\Big) \\ & \times\frac{ig^2}{2} = \frac{3}{8}g^4\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \ . \end{aligned}$$

ii. *Wave function renormalization corrections .*

$$\mathcal{L}_{HHV} = \frac{-ig}{\sqrt{2}}W^\dagger_\mu(H_1^{1*}\overleftrightarrow{\partial^\mu}H_1^2) - \frac{ig}{2cos\theta_W}Z_\mu(H_1^{1*}\overleftrightarrow{\partial^\mu}H_1^1) + h.c. \ .$$

The tadpole diagrams do not lead to wave function renormalization since they are independent of the external momentum. Fig.10 represents the only relevant Higgs-Vector boson contributions (p is the external momentum):

$$\begin{aligned} & 2\times\frac{1}{2!}\int \frac{d^4k}{(2\pi)^4}\Big(\frac{ig}{2cos\theta_W}(2p+k)_\mu\Big)\Big(-i\frac{g^{\mu\nu}-k^\mu k^\nu/k^2}{k^2}\Big)\times \\ & \Big(\frac{ig}{2cos\theta_W}(2p+k)_\nu\Big)\frac{i}{(k+p)^2-m^2} = \frac{-g^2}{cos\theta_W^2}\int \frac{d^4k}{(2\pi)^4}\frac{p^2-(p\cdot k)^2/k^2}{((k+p)^2-m^2)\ k^2} \\ & \to \ \frac{-g^2}{cos\theta_W^2}(p^2-\frac{1}{4}p_\mu p_\nu\ g^{\mu\nu})\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} = -\frac{3}{4}p^2\ (g^2+g'^2)\ \frac{i\ ln\Lambda/\mu}{8\pi^2} \ . \end{aligned}$$

We have omitted the convergent part of the integral and also used

$$\int k_\mu k_\nu \; f(k^2) \; d^4k = \frac{1}{4} g_{\mu\nu} \int k^2 \; f(k^2) \; d^4k \; .$$

In a similar manner, the charged Higgs-Vector boson loop leads to

$$-\frac{3}{2} p^2 \; g^2 \; \frac{i \; ln\Lambda/\mu}{8\pi^2} \; .$$

The sum of these divergent diagrams must be cancelled by the counterterm $(Z-1)p^2 \; \Phi_R^* \Phi_R$:

$$-\partial_\mu \partial^\mu \Phi^* \Phi = -\partial_\mu \partial^\mu \Phi_R^* \Phi_R - (Z-1) \partial_\mu \partial^\mu \Phi_R^* \Phi_R \; ,$$

from which it is apparent that

$$Z - 1 = \frac{3}{4} (3g^2 + g'^2) \; \frac{ln\Lambda/\mu}{8\pi^2} \; .$$

To find the corresponding contribution to the RGE for λ_1, let us observe that

$$-\lambda_1 (\Phi^* \Phi)^2 = -\lambda_{1R} (\Phi_R^* \Phi_R)^2 - \lambda_{1R} (Z_{\lambda_1} - 1)(\Phi_R^* \Phi_R)^2$$

with Z_{λ_1} given, in accordance with i), by

$$\lambda_1 (Z_{\lambda_1} - 1) = \frac{3}{16} (2g^4 + (g^2 + g'^2)^2) \frac{ln\Lambda/\mu}{8\pi^2}$$

and

$$\lambda_1 Z^2 = \lambda_{1R} Z_{\lambda_1} \; .$$

Then the evolution of the renormalized charge is determined by Z and Z_{λ_1}:

$$\frac{\partial}{\partial t} \lambda_{1R} = \frac{\partial}{\partial t} (\lambda_1 Z^2 Z_{\lambda_1}^{-1}) \; .$$

Therefore, i) and ii) result in

$$\dot{\lambda}_1|_{vector\ boson} = \frac{1}{16\pi^2}\Big[\frac{3}{16}(2g^4 + (g^2 + g'^2)^2) - \frac{3}{2}\lambda_1(3g^2 + g'^2)\Big] \ .$$

This equation agrees with the result cited in [12] for the case of a single Higgs doublet (formula (2.146)).

c) Higgsino-gaugino loops.

$$\mathcal{L} = igH_1^{1*}\lambda^+\psi_{H_1}^- + \frac{i}{\sqrt{2}}(g\lambda^3 - g'\lambda')H_1^{1*}\psi_{H_1}^0 + h.c.\ .$$

i. *Charge renormalization corrections* .

The fourth-order graph with charged fermions in Fig.11 is expressed as

$$\begin{aligned} &-12 \times \frac{1}{4!}g^4 \int \frac{d^4k}{(2\pi)^4} Tr\Big[\frac{k_0 - \bar{k}\bar{\sigma}}{k^2}\ \frac{k_0 + \bar{k}\bar{\sigma}}{k^2}\ \frac{k_0 - \bar{k}\bar{\sigma}}{k^2}\ \frac{k_0 + \bar{k}\bar{\sigma}}{k^2}\Big] \\ &= -g^4\frac{12\cdot 2}{4!}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} = \ -g^4\frac{i\ \ln\Lambda/\mu}{8\pi^2}\ , \end{aligned}$$

where we have dropped the masses of the fermions. The expression for the neutral loops is identical except for the substitution

$$12 \times g^4 \to (\frac{1}{\sqrt{2}})^4(12 \times g^4 + 12 \times g'^4 + 4! \times g^2g'^2) = 12\frac{(g^2 + g'^2)^2}{4}\ ,$$

which is due to vertex factors and a different combinatorial coefficient for the loop with distinct gauginos. The net contribution to the charge renormalization is

$$\lambda_1(Z_{\lambda_1} - 1) = -\Big(g^4 + \frac{(g^2 + g'^2)^2}{4}\Big)\frac{\ln\Lambda/\mu}{8\pi^2}\ .$$

ii. *Wave function renormalization corrections .*

The relevant graphs are shown in Fig.12. Taking into account the external momentum, one obtains

$$-2 \times \frac{1}{2!} \int \frac{d^4k}{(2\pi)^4} Tr\Big[-g \, \frac{i((p+k)_0 + \overline{(p+k)}\bar{\sigma})}{(p+k)^2} \, g \, \frac{i(k_0 - \bar{k}\bar{\sigma})}{k^2}\Big] =$$
$$-2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{(p+k)\cdot k}{(p+k)^2 \; k^2} \; .$$

Since we are interested in the p-dependent terms, it is legitimate to subtract from the integrand its $p=0$ value :

$$\frac{(p+k)\cdot k}{(p+k)^2 \; k^2} \to \frac{(p+k)\cdot k}{(p+k)^2 \; k^2} - \frac{1}{k^2} = \frac{-p^2 - p\cdot k}{(p+k)^2 \; k^2} \; .$$

Then we extract the p^2-terms and reduce the integrals to the simplest form as follows

$$\int \frac{d^4k}{(2\pi)^4} \frac{-p^2}{(p+k)^2 \; k^2} \to -p^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \; ,$$
$$\int \frac{d^4k}{(2\pi)^4} \frac{-p\cdot k}{(p+k)^2 \; k^2} \to \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \Big(1 - \frac{2p\cdot k}{k^2}\Big) \; (-p\cdot k) =$$
$$2 \int \frac{d^4k}{(2\pi)^4} \frac{(p\cdot k)^2}{k^6} = \frac{p^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \; .$$

Thus, the wave function renormalization part of the charged gaugino-higgsino loop can be written as

$$g^2 \; p^2 \frac{i \; \ln \Lambda/\mu}{8\pi^2} \; .$$

Similarly, the neutral loop results in

$$\frac{1}{2}(g^2 + g'^2) \; p^2 \frac{i \; \ln \Lambda/\mu}{8\pi^2} \; .$$

The Higgs wave function must be renormalized with the factor Z given by

$$Z-1=-\Big(g^2+\frac{g^2+g'^2}{2}\Big)\frac{\ln\Lambda/\mu}{8\pi^2}\ .$$

Consequently, one obtains the following higgsino-gaugino contribution to the Higgs coupling RGE:

$$\dot{\lambda}_1|_{higgsino-gaugino}=\frac{1}{16\pi^2}\Big[\lambda_1(3g^2+g'^2)-g^4-\frac{(g^2+g'^2)^2}{4}\Big]\ .$$

d) Sfermion loops.

$$\begin{aligned} V = \quad & \frac{1}{8}g^2\ (\ 4|H_1^{i*}\tilde{Q}^i|^2-2(\tilde{Q}^{i*}\tilde{Q}^i)H_1^{j*}H_1^j\ )+\\ & \frac{1}{8}g'^2\ (\ -H_1^{i*}H_1^i+Y_q\tilde{Q}^{i*}\tilde{Q}^i+Y_u\tilde{U}^*\tilde{U}+Y_d\tilde{D}^*\tilde{D}\ )^2\ , \end{aligned} \tag{32}$$

with $Y_q=\frac{1}{3}$, $Y_u=-\frac{4}{3}$, $Y_d=\frac{2}{3}$. For brevity, we did not show the sleptonic part explicitly, which practically duplicates the squark one and gives an identical contribution to the RGE apart from the color factor of 3. Apparently, this interaction affects charge renormalization only. First, let us calculate the corrections proportional to the $SU(2)$ coupling strength.

1. $g^4-contributions$.

The graph with the squark doublets in the loop (Fig.13) is written as

$$\frac{1}{2!}\int\frac{d^4k}{(2\pi)^4}\Big(\frac{i}{k^2-m_{\tilde{Q}}^2}\Big)^2\Big[\Big(\frac{-ig^2}{2}\Big)^2+2\times\Big(\frac{ig^2}{4}\Big)^2+2\times\Big(\frac{-ig^2}{2}\Big)\Big(\frac{ig^2}{4}\Big)\Big]$$
$$\times 3N_G\ ,$$

where a factor of three appeared due to the color and N_G is the number of generations. The sleptonic expression is identical except for the color factor.

Thus, the net contribution is

$$\left(\frac{3g^4}{16} + \frac{g^4}{16}\right) N_G \, \frac{i \, \ln \Lambda/\mu}{8\pi^2} \, .$$

2. g'^4 – *contributions* .

The squark diagrams proportional to the abelian coupling are expressed as

$$\frac{1}{2!} \int \frac{d^4k}{(2\pi)^4} \left(\frac{i}{k^2 - m_{sq}^2}\right)^2 \left[2 \times \left(\frac{ig'^2}{4} Y_q\right)^2 + \left(\frac{ig'^2}{4} Y_u\right)^2 + \left(\frac{ig'^2}{4} Y_d\right)^2\right]$$
$$\times 3 N_G \, ,$$

In fact, to incorporate the sleptons one needs to calculate

$$\sum_{all\ sfermions} Y_i^2 \, ,$$

counting each squark three times. A brute force summation yields

$$\sum_{all\ sfermions} Y_i^2 = \frac{40}{3} N_G \, .$$

Hence, the integral under consideration takes on the form

$$\frac{g'^4}{32} \, \frac{40}{3} N_G \, \frac{i \, \ln \Lambda/\mu}{8\pi^2} \, .$$

The resultant contribution to the λ_1 RGE turns out to be

$$\dot{\lambda}_1|_{sfermions} = \frac{1}{16\pi^2}\left[\frac{g^4}{4} N_G + \frac{5g'^4}{12} N_G\right] \, .$$

Now, having the complete list of beta-functions in hand, we can summarise the effect of the radiative correction on the Higgs quartic coupling. Adding up the β-functions in a)-d), one obtains the following RG equation:

$$\dot{\lambda}_1 = \frac{1}{16\pi^2}\left[\frac{1}{8} g'^4 - \frac{5}{8} g^4 + \frac{g^4}{4} N_G + \frac{5g'^4}{12} N_G\right] \, .$$

Now recall that the supersymmetric tree-level value of λ_1 is given by

$$\lambda_1 = \frac{1}{8}(g^2 + g'^2) \ .$$

A reasonable question one could ask is how this relation would be modified by the renormalization group flow. To find that out, recall that the RG evolution of the coupling constants in the MSSM is dictated by [13]

$$\frac{\partial}{\partial t} g^2 = \frac{1}{16\pi^2}(-5 + 2N_G)\ g^4 \ ,$$
$$\frac{\partial}{\partial t} g'^2 = \frac{1}{16\pi^2}(1 + \frac{10}{3}N_G)\ g'^4$$

(the second equation is usually written in terms of the "properly normalized" coupling $g_1^2 = \frac{5}{3}g'^2$ [14]).

We immediately see that the evolution of λ_1 is nothing more than the evolution of the gauge couplings, in other words, the tree-level relation is preserved by the renormalization group flow. This is another general feature characteristic to exact susy. The super Yukawa interactions do not modify this result owing to the non-renormalization theorem: no corrections to the Higgs coupling , generated by a superpotential, are allowed. All set forth here equally applies to the other Higgs quartic couplings - λ_{2-4}.

This statement survives *at the* $1-$*loop level* even in softly broken theories. Indeed, the exhaustive list of soft-breaking terms includes bi- and tri-linear scalar couplings, and the gaugino masses. All of them have an "infrared origin" : their dimensions are not higher than three and they all involve some mass parameters. Apparently, such interactions cannot contribute to the *log*-divergent parts of 1-loop diagrams - any mass-insertion would make the diagram finite. We, therefore, conclude that the 1-loop RG evolution of the Higgs quartic couplings in the MSSM is completely determined by the running of the gauge couplings.

3.6 The Next-to-Minimal Supersymmetric Standard Model

In this subsection we will briefly outline basic properties of the model closely related to the MSSM – the Next-to-Minimal Supersymmetric Standard Model (NMSSM). As we show in the subsequent chapters, spontaneous CP-violation in such a model has a good chance to be phenomenologically consistent.

The NMSSM was first introduced [49] to cure certain aesthetic shortcomings of the MSSM, in particular, the so called "$\mu-$problem". In order to break $SU(2) \times U(1)$ in a phenomenologically acceptable way, it is necessary to include the term $\mu H_1 H_2$ in the superpotential: $\mu = 0$ would lead to $v_1 \cdot v_2 = 0$ [11] and the masses for either the up- or the down-quarks would not be generated. Further, since one expects the lightest Higgs mass to be of the order of M_W, the value of μ is constrained and should be $O(M_W)$ [15]. This creates a new naturalness problem: should supersymmetry be valid up to ultra-high energies (perhaps, up to the GUT scale) it is hard to see why the mass parameter in the superpotential would be so small. The running of μ does not help: in the context of susy, masses evolve only as $log(energy)$. This problem can be solved by an introduction of an extra Higgs singlet superfield N, coupled only to the other Higgses (to satisfy phenomenological constraints). The term $\lambda N H_1 H_2$ in the superpotential can generate the desired $\mu - term$ if the singlet field N acquires a non-zero VEV. The natural value for μ would then be $\lambda\langle N\rangle \sim O(M_W)$ provided the coupling constant is of the order of one [15]. Susy models with such extended Higgs sectors appear in certain susy GUT's and superstring models.

In the simplest version of the NMSSM, the superpotential contains only two additional terms in comparison to the MSSM (one of them is a substitute for the "μ-term"):

$$W_{NMSSM} \;=\; W_{Yukawa} \;+\; \lambda N H_1 H_2 \;+\; \frac{k}{3} N^3 \; . \tag{33}$$

A nice feature of this superpotential is that it does not involve any mass parameters. The cubic term is introduced for the sake of breaking the Peccei-Quinn symmetry: otherwise a VEV of the Higgs would lead to the existence of a massless pseudo-Goldstone particle; the N^3 term generates the term $\lambda k H_1 H_2 N^{*2}$ in the scalar potential which violates the PQ-symmetry at the tree level (note that N is PQ-sterile) [15]. This is, however, not a unique way to avoid a light particle in the spectrum: the PQ symmetry can also be broken by means of the soft susy breaking terms.

In addition to the supersymmetric interactions given by the superpotential, the soft-breaking terms must be introduced. As in the MSSM, the trilinear vertices are assumed to have the same form as those in the superpotential but with the superfields replaced by their scalar components. The explicit formulas will be given in section 5 when we discuss spontaneous CP-violation in the NMSSM.

4 Spontaneous CP-violation and Stability of the Higgs Potential in the MSSM

It has been known for a long time that radiative corrections play a crucial role in MSSM (for a review see [11]). Since exact SUSY preserves tree-level relations, it forbids spontaneous CP violation. But after soft breaking terms are introduced, SCPV becomes possible through 1-loop corrections, at least superficially. The necessary condition of spontaneous CP breaking is the positivity of λ_5, which is absent at the tree level. Moreover, the necessary condition for the potential to be bounded from below

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 - \lambda_5 > 0 \tag{34}$$

is not satisfied in the strict sense: the tree level constants

$$\begin{aligned}
&\lambda_1 = \lambda_2 = \frac{1}{8}(g^2 + g^{'2}) \, , \\
&\lambda_3 = \frac{1}{4}(g^2 - g^{'2}) \, , \\
&\lambda_4 = -\frac{1}{2}g^2 \, , \\
&\lambda_5 = \lambda_6 = \lambda_7 = 0
\end{aligned} \tag{35}$$

lead to the equality sign, thereby emphasising an instability of the potential with respect to perturbations. Due to the infrared nature of the soft breaking terms (of dimension 2-3), the desired four-dimensional operators are not generated in the leading log approximation and, therefore, finite corrections must be taken into account. Then a positive λ_5 can be generated provided the gaugino-higgsino loop contribution is dominant [6,7]. The complete list of the susy soft-breaking terms is as follows [11].

$$\begin{aligned}
-\mathcal{L}_{soft} &= m^2_{\tilde{q}}|\tilde{q}_L|^2 + m^2_{\tilde{u}}|\tilde{u}^c_R|^2 + m^2_{\tilde{d}}|\tilde{d}^c_R|^2 + m^2_{\tilde{l}}|\tilde{l}_L|^2 + m^2_{\tilde{e}}|\tilde{e}^c_R|^2 \\
&+ \Big(\lambda_E A_E H_1 \tilde{l}_L \tilde{e}^c_R + \lambda_D A_D H_1 \tilde{q}_L \tilde{d}^c_R + \lambda_U A_U H_2 \tilde{q}_L \tilde{u}^c_R \\
&+ B\mu H_1 H_2 + \frac{1}{2} M' \lambda' \lambda' + \frac{1}{2} M \lambda^a \lambda^a + \frac{1}{2} M_{gluino} \tilde{g}\tilde{g} + h.c. \Big) \\
&+ m^2_{H_1}|H_1|^2 + m^2_{H_2}|H_2|^2 \ .
\end{aligned} \tag{36}$$

It is necessary for the radiative CP-breaking that the trilinear scalar couplings and the Higgs mixing mass, which give negative contributions to λ_5, be negligible [6,7]:

$$A_i \to 0 \ , \ B \to 0 \ .$$

Under this assumption there are only two sources of radiative corrections to the Higgs couplings: gaugino masses and scalar soft-breaking masses. One notices that only the former is needed to satisfy the necessary conditions (6) and (7) [6,7], since the latter does not contribute to λ_5 (it preserves the PQ-symmetry) and, at first glance, seems to be irrelevant to the discussion of CP-violation. So, first we will concentrate on the fermion-dominated case, i.g. neglecting scalar soft-breaking masses, and examine the stability properties of the consequent Higgs potential.

4.1 Higgsino-Gaugino Dominated Case

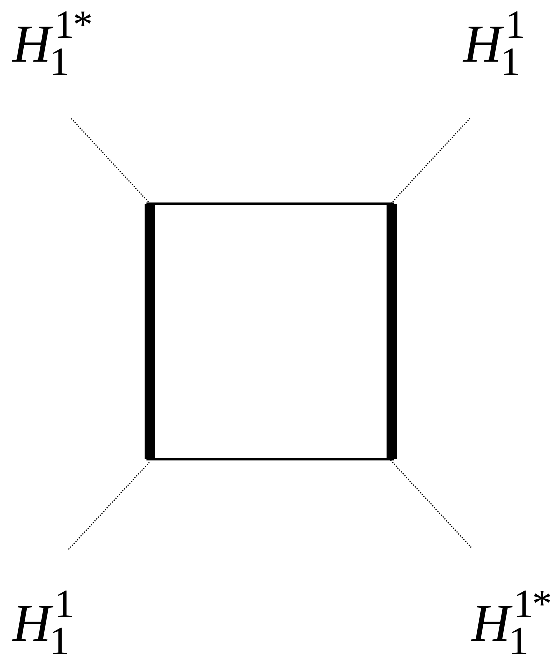
As we have seen in the previous chapter, the effects of the $leading - log$ corrections amount to the substitution of the tree level couplings by the running ones, for instance,

$$\lambda_1 = \frac{1}{8}(g^2 + g'^2) \ \rightarrow \ \lambda_{1R} = \frac{1}{8}(g_R^2 + g_R'^2) \ .$$

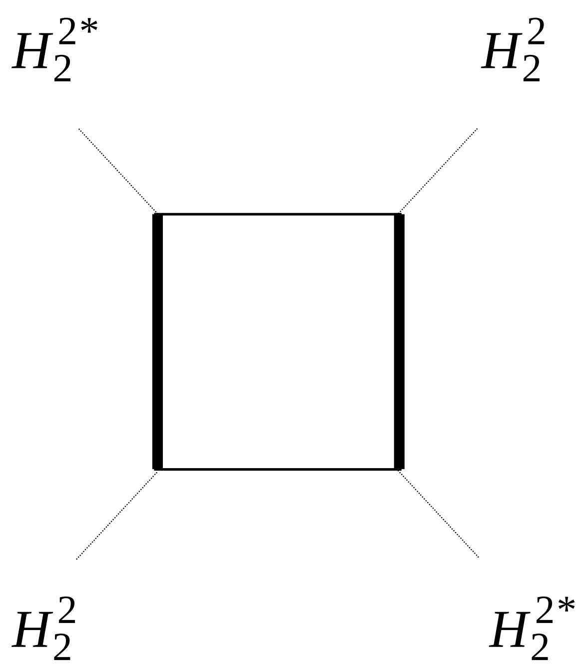
Now we will take into account the finite 1-loop corrections, which are disregarded in the derivation of the RGE.

The source of the gaugino-higgsino finite corrections is a soft SUSY breaking gaugino mass M, which we assume to be universal for $U(1)$ and $SU(2)$ gauginos [17]. The result can be expressed as a function of a single parameter $x = M^2/\mu^2$ with μ being the higgsino mass and all corrections vanishing as x approaches zero. An introduction of gaugino mass can affect a Feynman diagram with two-component spinors in two ways: it alters the left-left (right-right) propagator by an insertion of M^2 in the denominator, and it may also give rise to a new diagram involving the left-right propagator (that contains M in the numerator and M^2 in the denominator). This procedure is exact and does not require x to be small. At zero external momenta the diagrams, Fig. 1-4, can be calculated exactly for any value of x .

a) Corrections to λ_1 (Fig.1).

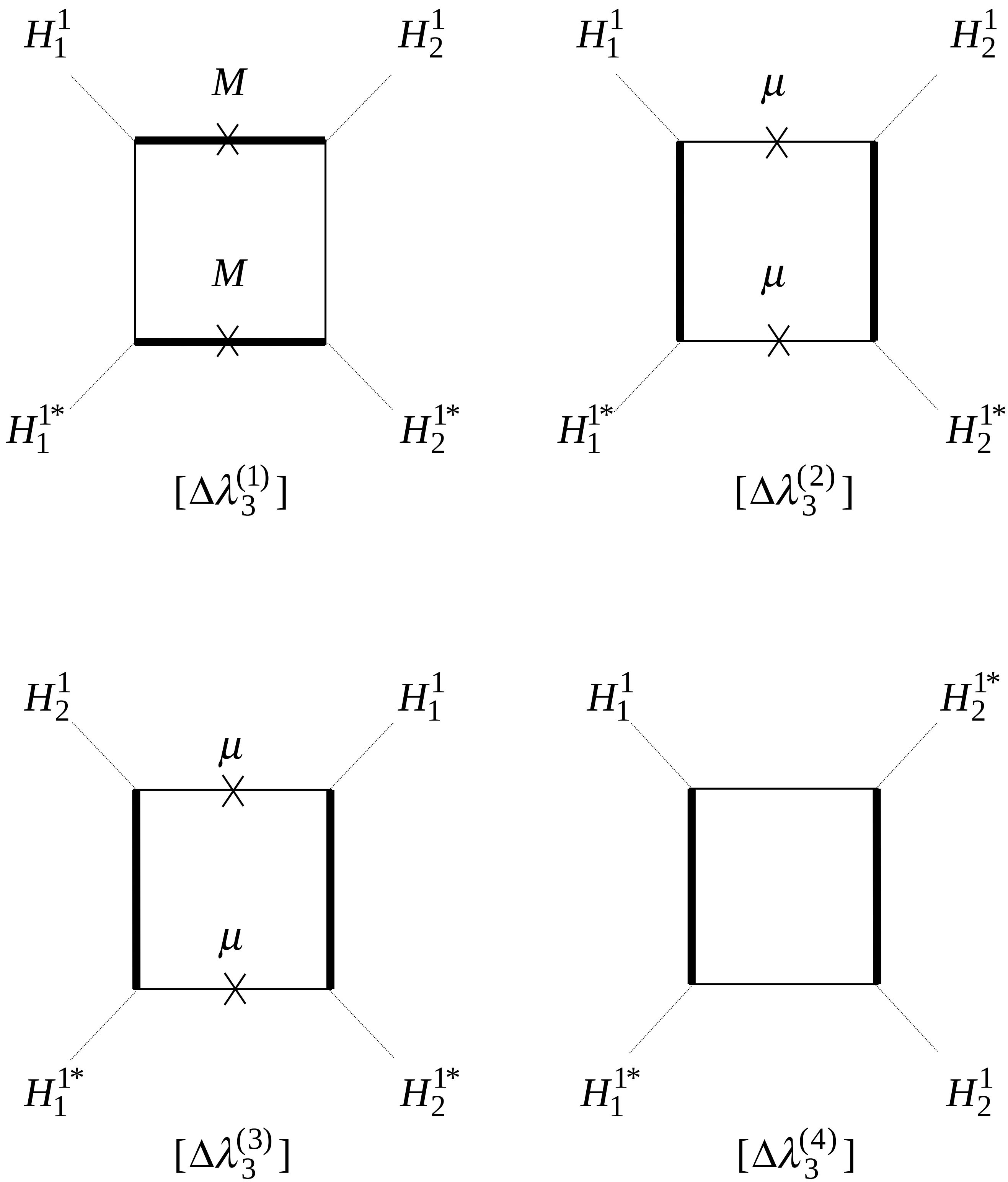


$[\Delta\lambda_1]$



$[\Delta\lambda_2]$

Fig.(4)-1. Here = higgsino, = gaugino.



H_1^1
H_2^1
M
M
H_1^{1*}
H_2^{1*}
$[\Delta\lambda_3^{(1)}]$
H_1^1
H_2^1
μ
μ
H_1^{1*}
H_2^{1*}
$[\Delta\lambda_3^{(2)}]$
H_2^1
H_1^1
μ
μ
H_1^{1*}
H_2^{1*}
$[\Delta\lambda_3^{(3)}]$
H_1^1
H_2^{1*}
H_1^{1*}
H_2^1
$[\Delta\lambda_3^{(4)}]$

Fig.(4)-2

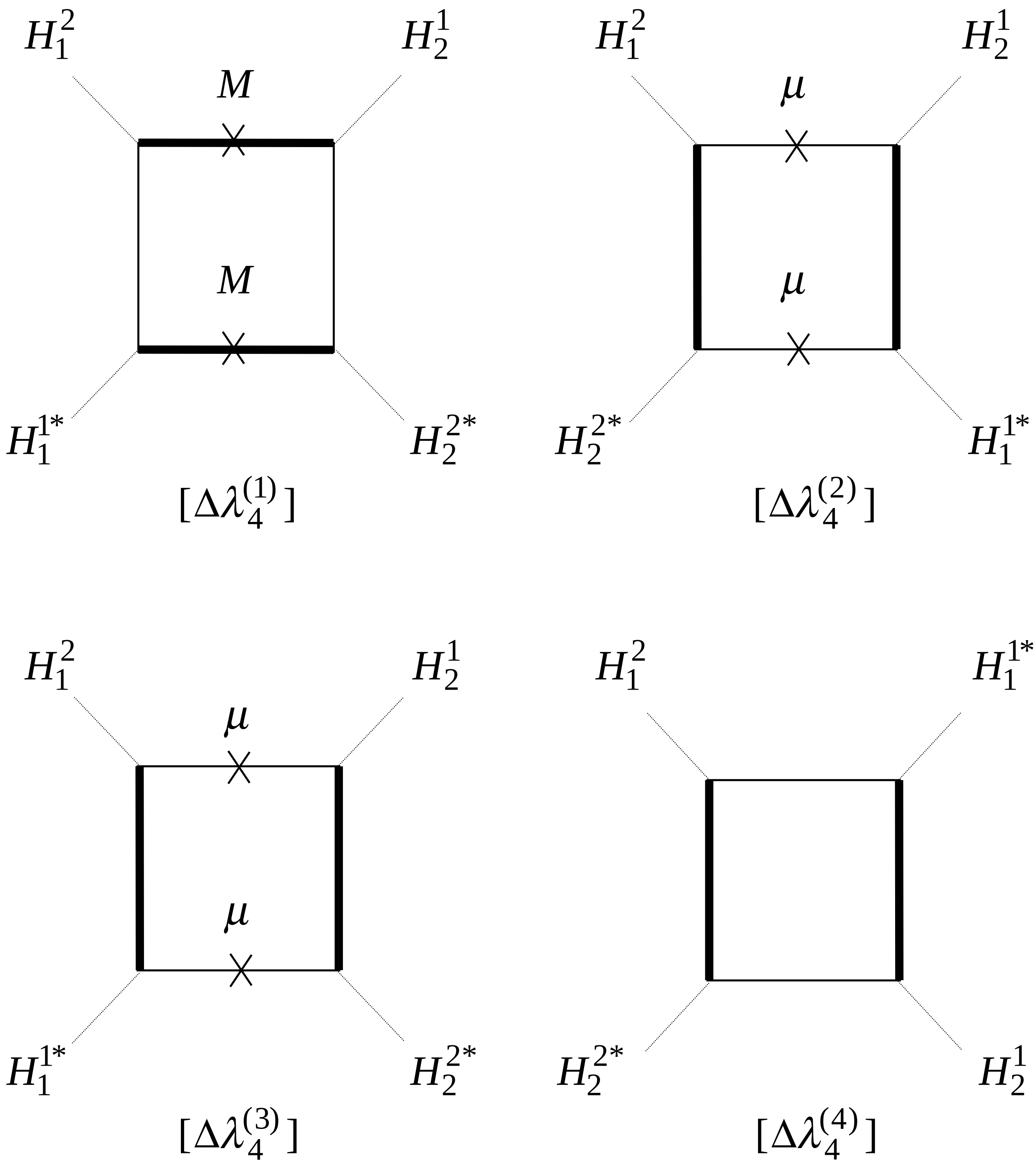
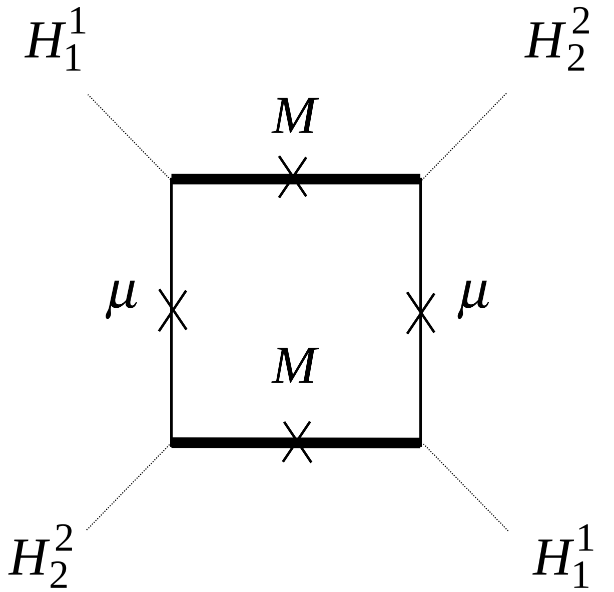
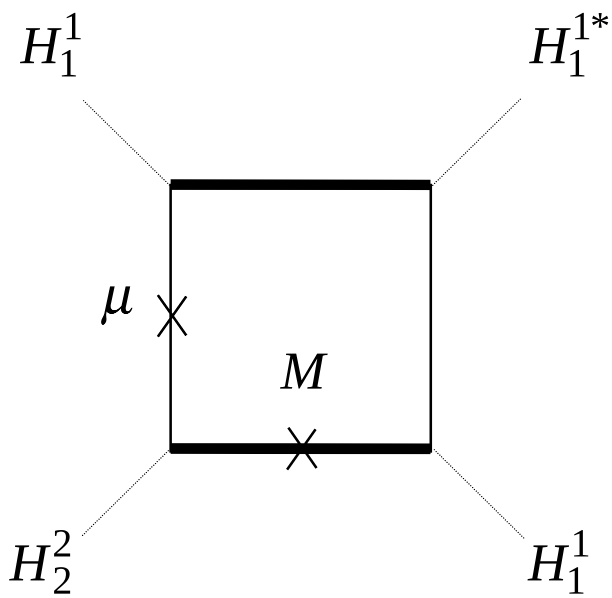
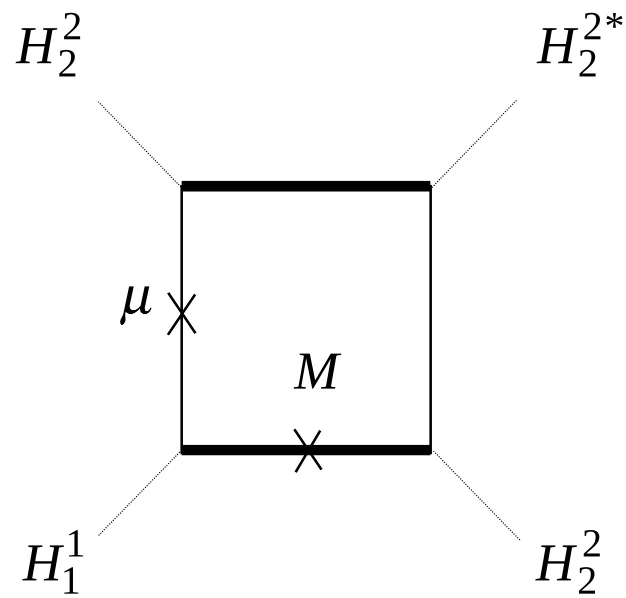

H_1^2
H_2^1
M
M
H_1^{1*}
H_2^{2*}
$[\Delta\lambda_4^{(1)}]$
H_1^2
H_2^1
μ
μ
H_2^{2*}
H_1^{1*}
$[\Delta\lambda_4^{(2)}]$
H_1^2
H_2^1
μ
μ
H_1^{1*}
H_2^{2*}
$[\Delta\lambda_4^{(3)}]$
H_1^2
H_1^{1*}
H_2^{2*}
H_2^1
$[\Delta\lambda_4^{(4)}]$

Fig.(4)-3



$[\Delta\lambda_5]$





$[\Delta\lambda_6]$

$[\Delta\lambda_7]$

Fig.(4)-4

In this case an introduction of the gaugino mass does not lead to any new diagrams: the Feynman graphs in Fig.1 were already considered in subsection 3.5. Since only left-left and right-right propagators are employed in Fig.1, the sole modification is the replacement $k^2 \to k^2 - M^2$ in the gaugino propagator. Thus, we can simply copy the factors from the previous chapter and write

$$vertex = -iV = -iV_{tree} + \Big(-g^4 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-\mu^2)^2}$$
$$-g^4 \int \frac{d^4k}{(2\pi)^4} \Big[\frac{k^4}{(k^2-M^2)^2(k^2-\mu^2)^2} - \frac{1}{(k^2-\mu^2)^2} \Big] \Big) |\Phi_1|^4 \; .$$

Having absorbed the divergent contribution into the renormalization of the gauge coupling, one obtains

$$\lambda_1 \to \lambda_1 + \int \frac{d^4k}{i(2\pi)^4} \frac{2k^2M^2 - M^4}{(k^2-M^2)^2(k^2-\mu^2)^2} \; .$$

It is convenient to rewrite the integrand in terms of the simple fractions containing the momentum k in the denominator only:

$$\frac{2k^2M^2 - M^4}{(k^2-M^2)^2(k^2-\mu^2)^2} \equiv \frac{2M^2}{(k^2-M^2)(k^2-\mu^2)^2} + \frac{M^4}{(k^2-M^2)^2(k^2-\mu^2)^2} \; .$$

Now the integral can be calculated exactly [23]:

$$\int \frac{d^4k}{i(2\pi)^4} \frac{1}{(k^2-M^2)(k^2-\mu^2)^2} = \frac{1}{16\pi^2 \, \mu^2} \, \frac{1}{x-1} \Big[1 - \frac{x}{x-1} \ln \, x \Big] \; ,$$
$$\int \frac{d^4k}{i(2\pi)^4} \frac{1}{(k^2-M^2)^2(k^2-\mu^2)^2} = \frac{1}{16\pi^2 \, \mu^4} \, \frac{1}{(x-1)^2} \Big[-2 + \frac{x+1}{x-1} \ln \, x \Big] \; ,$$

where $x \equiv M^2/\mu^2$. In order to take into account the neutral loop one needs to replace (see the box contribution to the λ_1-RGE)

$$g^4 \to g^4 + \frac{1}{4}(g^2 + g'^2)^2 \ .$$

The resulting correction to λ_1 is

$$\Delta\lambda_1 = \frac{4g^4 + (g^2 + g'^2)^2}{64\pi^2} \frac{x^2}{(x-1)^2} \Big(\frac{3-x}{x-1} \ln\, x - \frac{2}{x} \Big) \ .$$

Note that this function vanishes at $x = 0$, as it should. One gets the same result for the correction to λ_2 since the higgsino-gaugino sector does not distinguish between the first and the second Higgs doublets.

b) Corrections to $\lambda_3 + \lambda_4$ (Fig.2,3).

For our purpose it is sufficient to calculate the correction to the sum $\lambda_3 + \lambda_4$, even though the diagrams in Fig.2 and 3 represent corrections to the individual couplings. The easiest way to compute it is to find the effective $H_1^{1*} H_1^1 H_2^{2*} H_2^2$-vertex. Nevertheless, we will consider each diagram in Fig.2,3 separately. The relevant Lagrangian reads [34]

$$\begin{aligned} \mathcal{L} &= ig(\, H_1^{1*}\lambda^+\psi^-_{H_1} + \, H_1^{2*}\lambda^-\psi^0_{H_1} + \, H_2^{1*}\lambda^+\psi^0_{H_2} + \, H_2^{2*}\lambda^-\psi^+_{H_2} \,) \\ &+ \sqrt{\frac{1}{2}} i(g\lambda^3 - g'\lambda')(\, H_1^{1*}\psi^0_{H_1} - H_2^{2*}\psi^0_{H_2} \,) \\ &+ \sqrt{\frac{1}{2}} i(g\lambda^3 + g'\lambda')(\, H_2^{1*}\psi^+_{H_2} - H_1^{2*}\psi^-_{H_1} \,) + h.c. \ , \\ \mathcal{L}_{mass} &= \mu(\psi^0_{H_1}\psi^0_{H_2} - \psi^-_{H_1}\psi^+_{H_2}) - \frac{1}{2}M(\lambda'\lambda' + \lambda^a\lambda^a) + h.c. \end{aligned} \tag{37}$$

To separate corrections to λ_3 from those to λ_4, one has to consider the graphs with two charged external legs (Fig.2,3):

$$\Phi_1^\dagger \Phi_2 = \epsilon_{ij} H_1^i H_2^j \ , \ \Phi_1^\dagger \Phi_1 = H_1^{i*} H_1^i \ ,$$

$$\lambda_4 |\Phi_1^\dagger \Phi_2|^2 \ \rightarrow -H_1^{1*} H_2^{2*} H_1^2 H_2^1 \ , \ \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \ \rightarrow H_1^{1*} H_1^1 H_2^{1*} H_2^1 \ .$$

i. *Diagrams without mass insertion .*

Similarly to the previous considerations one can establish

$$\begin{aligned}
\Delta\lambda_4^{(4)} &= -2\Big(-\frac{g^2 g'^2}{2} + \frac{g^4}{2} - \frac{g^2 g'^2}{2} + \frac{g^4}{2}\Big) \times \\
&\times \int \frac{d^4k}{i(2\pi)^4} \Big[\frac{k^4}{(k^2-M^2)^2(k^2-\mu^2)^2} - \frac{1}{(k^2-\mu^2)^2}\Big] \\
&= -\frac{g^2(g^2-g'^2)}{8\pi^2} \frac{x^2}{(x-1)^2} \Big(\frac{3-x}{x-1} \ln\, x - \frac{2}{x}\Big) \ , \\
\Delta\lambda_3^{(4)} &= 2\Big(g^4 + \frac{1}{4}(g^2 - g'^2)^2\Big) \times \\
&\times \int \frac{d^4k}{i(2\pi)^4} \Big[\frac{k^4}{(k^2-M^2)^2(k^2-\mu^2)^2} - \frac{1}{(k^2-\mu^2)^2}\Big] \\
&= \frac{4g^4 + (g^2-g'^2)^2}{32\pi^2} \frac{x^2}{(x-1)^2} \Big(\frac{3-x}{x-1} \ln\, x - \frac{2}{x}\Big) \ .
\end{aligned} \tag{38}$$

One can verify that the coefficients in front of the functions of x agree with those in the corresponding beta-functions [19]: the same diagrams contribute to the RGE for λ_3 and λ_4 (for the diagrams with mass insertions this argument does not apply). The finite corrections, however, have an opposite sign compared to that of the beta-functions since the latter involve the combination $ln\Lambda/\mu$ instead of the functions of x and a differentiation with respect to the μ brings in an extra minus sign.

ii. *Diagrams with $M-insertion$.*

The diagrams with gaugino mass insertions involve left-right propagators and arise only after susy breaking. The corresponding correction to λ_4 is written as

$$\Delta\lambda_4^{(1)} = -2\Big(-\frac{g^2g'^2}{2} - \frac{g^4}{2} - \frac{g^2g'^2}{2} - \frac{g^4}{2}\Big)\int \frac{d^4k}{i(2\pi)^4}\frac{k^2M^2}{(k^2-M^2)^2(k^2-\mu^2)^2}\ .$$

Upon decomposition into simple fractions

$$\frac{k^2M^2}{(k^2-M^2)^2(k^2-\mu^2)^2} = \frac{M^2}{(k^2-M^2)(k^2-\mu^2)^2} + \frac{M^4}{(k^2-M^2)^2(k^2-\mu^2)^2}\ ,$$

the integral is easily calculated and

$$\Delta\lambda_4^{(1)} = \frac{g^4+g^2g'^2}{8\pi^2}\frac{x}{(x-1)^2}\Big(-x-1+\frac{2x}{x-1}\ln\ x\Big)\ .$$

One notices that the function involved vanishes at $x=0$ and no additional constant is required to satisfy the boundary condition. Analogously,

$$\begin{aligned}\Delta\lambda_3^{(1)} &= 2\Big(\frac{g'^4}{4} + \frac{g^4}{4} - 2\frac{g^2g'^2}{4}\Big)\int \frac{d^4k}{i(2\pi)^4}\frac{k^2M^2}{(k^2-M^2)^2(k^2-\mu^2)^2}\\ &= \frac{(g^2-g'^2)^2}{32\pi^2}\frac{x}{(x-1)^2}\Big(-x-1+\frac{2x}{x-1}\ln\ x\Big)\ .\end{aligned}$$

iii. *Diagrams with $\mu-$insertion .*

The graphs with the higgsino mass insertions are finite and present in the supersymmetric case as well, even though they do not contribute to the RGE. The introduction of the soft gaugino masses modifies them, inducing the shift

$$\begin{aligned}\Delta\lambda_4^{(2+3)} &= -2\Big(-g^4 - 2\times\frac{g^4}{4} - 2\times\frac{g'^4}{4} + 4\times\frac{g^2g'^2}{4}\Big)\\ &\times \int\frac{d^4k}{i(2\pi)^4}\frac{k^2\mu^2}{(k^2-M^2)^2(k^2-\mu^2)^2} + const\ . \end{aligned} \tag{39}$$

The constant term is chosen in accordance with the boundary condition

$$\Delta\lambda_4^{(2+3)}|_{x=0} = 0\ ,$$

which ensures that the tree-level relations between the Higgs and gauge couplings are not modified in the case of exact susy. At this stage it is important to note that the terms $\psi^0_{H_1}\psi^0_{H_2}$ and $\psi^-_{H_1}\psi^+_{H_2}$ are contained in the Lagrangian with opposite signs and, as a result, the charged and neutral higgsino mass insertions differ in sign.

Consequently,

$$\Delta\lambda_4^{(2+3)} = \frac{2g^4+(g^2-g'^2)^2}{16\pi^2}\Big(\frac{1}{(x-1)^2}\Big[-x-1+\frac{2x}{x-1}\ln\ x\Big]+1\Big)\ .$$

In a similar manner, one finds

$$\Delta\lambda_3^{(2+3)} = 2\Big(2\times(\frac{g^2g'^2}{2}+\frac{g^4}{2}) - 2\times(-\frac{g^2g'^2}{2}+\frac{g^4}{2})\Big)$$

$$
\begin{aligned}
& \times \int \frac{d^4k}{i(2\pi)^4} \frac{k^2\mu^2}{(k^2-M^2)^2(k^2-\mu^2)^2} + const \\
&= \frac{g^2g'^2}{4\pi^2}\Big(\frac{1}{(x-1)^2}\Big[-x-1+\frac{2x}{x-1}\ln\, x\Big]+1\Big)\,.
\end{aligned}
$$

c) Corrections to λ_5 (Fig.4).

The coupling λ_5 is absent at the tree level in both exact and softly broken susy. It can only be generated radiatively and turns out to be positive provided the scalar loop contributions are suppressed. Noting that the diagram in Fig.4 has two pairs of identical vertices and the internal lines can be connected in two ways (for charged fermions in the loop), one obtains[1]

$$
\begin{aligned}
\Delta\lambda_5 &= 2\cdot 2\frac{6\times 2}{4!}\Big(g^4+\frac{1}{2}(g^4+2g^2g'^2+g'^4)\Big) \\
&\times \int \frac{d^4k}{i(2\pi)^4}\frac{\mu^2M^2}{(k^2-M^2)^2(k^2-\mu^2)^2} \\
&= \frac{2g^4+(g'^2+g^2)^2}{16\pi^2}\frac{x}{(x-1)^2}\Big(\frac{x+1}{x-1}\ln\, x-2\Big)\,.
\end{aligned}
$$

d) Corrections to $\lambda_{6,7}$ (Fig.4).

Analogously, $\lambda_{6,7}$ are absent at the tree level and cannot be put in "by hand" as the soft breaking terms. Their 1-loop values read

$$
\Delta\lambda_6 = 2\cdot 2\frac{12\times 2}{4!}\Big(-g^4-2\times\frac{1}{4}(g^4+2g^2g'^2+g'^4)\Big)
$$

[1] In the limit $g'\to 0$ this result was first obtained by Maekawa [6] (note the difference in the definitions of λ_{5-7}).

$$
\begin{aligned}
&\times \int \frac{d^4k}{i(2\pi)^4} \frac{\mu M\ k^2}{(k^2-M^2)^2(k^2-\mu^2)^2} \\
&= -\frac{2g^4+(g'^2+g^2)^2}{8\pi^2}\frac{\sqrt{x}}{(x-1)^2}\left(-x-1+\frac{2x}{x-1}\ln\ x\right) .
\end{aligned}
$$

The expression for $\Delta\lambda_7$ is identical.

To summarize, we find the following radiative corrections

$$
\begin{aligned}
\Delta\lambda_1 = \Delta\lambda_2 &= \frac{4g^4+(g^2+g'^2)^2}{64\pi^2}\frac{x^2}{(x-1)^2}\left(\frac{3-x}{x-1}\ln\ x-\frac{2}{x}\right) , \\
\Delta\lambda_3+\Delta\lambda_4 &= \frac{4g^4+(g^2+g'^2)^2}{32\pi^2}\frac{x}{(x-1)^2}\left(-x-1+\frac{2x}{x-1}\ln\ x\right) \\
&+ \frac{2g^4+(g'^2+g^2)^2}{16\pi^2}\left(\frac{1}{(x-1)^2}\left[-x-1+\frac{2x}{x-1}\ln\ x\right]+1\right) \\
&+ \frac{(g^2+g'^2)^2}{32\pi^2}\frac{x^2}{(x-1)^2}\left(\frac{3-x}{x-1}\ln\ x-\frac{2}{x}\right) , \\
\Delta\lambda_5 &= \frac{2g^4+(g'^2+g^2)^2}{16\pi^2}\frac{x}{(x-1)^2}\left(\frac{x+1}{x-1}\ln\ x-2\right) , \\
\Delta\lambda_6 = \Delta\lambda_7 &= -\frac{2g^4+(g'^2+g^2)^2}{8\pi^2}\frac{\sqrt{x}}{(x-1)^2}\left(-x-1+\frac{2x}{x-1}\ln\ x\right) .
\end{aligned}
$$

Despite the seeming singularities at $x=1$, the functions appearing in these expressions are regular everywhere for $x \geq 0$. One notices that $\Delta\lambda_{1,2}$ are negative definite while $\Delta\lambda_5$ is always positive. The only positive contribution to the sum (12) comes from the diagrams with higgsino mass insertions and it can be checked that $\Delta\lambda_1+\Delta\lambda_2+\Delta\lambda_3+\Delta\lambda_4-\Delta\lambda_5$ is negative unless $x \ll 1$. Clearly for small x , $\Delta\lambda_5 \rightarrow 0$ and the higgsino-gaugino contribution to $\Delta\lambda_5$ becomes comparable to the negative one coming from sfermions and Higgs bosons [6,7]. Therefore, the $x \ll 1$ region is disfavoured by (6) and (7) (inequality (7) implies that $\Delta\lambda_5$ must be greater than a certain critical value) and we exclude it from further considerations. We see that the condition (12) is not satisfied for all acceptable x. It means that if we allow spontaneous CP violation to occur, i.g. require λ_5 to be positive, the Higgs potential becomes unbounded from below and, rigorously speaking, a vacuum state does not exist in this theory.

On the other hand, we can ask whether there is a local minimum which is relatively stable. To answer this question let us consider the behaviour of the potential with respect to the radial coordinates when a CP-breaking vacuum is picked, i.e. setting $\cos\delta = \Delta$. In this case Eq.(4) can be rewritten as

$$\begin{aligned} \langle V \rangle &= -\frac{m_3^4}{2\lambda_5} + (\, m_1^2 + \frac{\lambda_6}{2\lambda_5} m_3^2\,) v_1^2 + (\, m_2^2 + \frac{\lambda_7}{2\lambda_5} m_3^2\,) v_2^2 \\ &+ \left(\lambda_3 + \lambda_4 - \lambda_5 - \frac{\lambda_6\lambda_7}{4\lambda_5} \right) v_1^2 v_2^2 + \left(\lambda_1 - \frac{\lambda_6^2}{8\lambda_5} \right) v_1^4 + \left(\lambda_2 - \frac{\lambda_7^2}{8\lambda_5} \right) v_2^4 \,. \end{aligned} \tag{40}$$

Since we are looking for a local minimum at $v_{1,2} > 0$, it is legimate to go over to the new coordinates (v_1^2, v_2^2), in which the potential takes on a particularly simple form - it is just a family of quadratic curves. For a non-degenarate case, which we are dealing with, $\langle V \rangle$ can be cast into either elliptic or hyperbolic form by a linear transformation: $\langle V \rangle = a^2x^2 \pm b^2y^2$. It can be checked explicitly that (40) corresponds to a hyperbolic case; we can see it as well via the following observation: only a hyperbolic surface describes a potential unbounded from below (which is the one under consideration). This leads to an unambiguous conclusion: the given potential does not even have a local minimum.

The above considerations illustrate the well-known destabilizing action of the fermionic part of the Coleman-Weinberg effective potential [18]. We have seen that under the assumptions made, spontaneous CP-violation and a stable Higgs potential are incompatible.

4.2 The Stabilizing Action of the Higgs Soft-Susy-Breaking Mass

In this subsection we will examine the question whether the second sourse of the radiative corrections to λ_i - the soft scalar mass - can rectify the stability problem. To begin with, notice that the soft scalar masses contribute to λ_{1-4} only; the other couplings λ_{5-7} are not generated since in our case the scalar sector possesses PQ-symmetry at the tree level. So, the necessary conditions for spontaneous CP-violation (6) and (7) are still satisfied. On the other hand, the introduction of an additional positive mass into a 4-point function diagram with scalars running around the loop gives rise to a positive correction to the Higgs coupling. Unlike for the fermion case, the contributing diagrams are the same as the ones generating the Higgs renormalization group equations. We will express the corresponding corrections in terms of the ratios of the Higgs running masses and their supersymmetric values μ: $y \equiv m^2_{H_1}/\mu^2$, $z \equiv m^2_{H_2}/\mu^2$. First, let us compute the loop integral which we will encounter in the process of calculating the finite Higgs-loop corrections

$$
\begin{aligned}
&\int d^4k \quad \left(\frac{1}{(k^2-m_1^2)(k^2-m_2^2)} - \frac{1}{(k^2-\mu^2)^2} \right) \\
= &\int d^4k \quad \left(\frac{(m_2^2-\mu^2)^2}{m_2^2-m_1^2} \, \frac{1}{(k^2-m_2^2)(k^2-\mu^2)^2} \right. \\
&\qquad\qquad \left. - \frac{(m_1^2-\mu^2)^2}{m_2^2-m_1^2} \, \frac{1}{(k^2-m_1^2)(k^2-\mu^2)^2} \right) \\
&\qquad = \frac{i\pi^2}{z-y} \left(z - y - z \ \ln \ z + y \ \ln \ y \right) . \qquad (41)
\end{aligned}
$$

In the limit $m_2 \to m_1$ one gets

$$\int d^4k \Big(\frac{1}{(k^2 - m_1^2)^2} - \frac{1}{(k^2 - \mu^2)^2} \Big) = -i\pi^2 \ln\ y\ . \tag{42}$$

Since the relevant graphs for $\Delta\lambda_{1,2}$ are given in the previous chapter, Fig.5-7 show only the Higgs-loop graphs giving rise to $\Delta\lambda_3 + \Delta\lambda_4$. It is essential now to distinguish between the two Higgs doublets since they have different masses. The finite Higgs-loop corrections to $\lambda_{1,2}$ trivially follow from the detailed RGE considerations given in the previous chapter:

$$\begin{aligned} \Delta\lambda_1 = &\quad \frac{g^4 + g'^4}{256\pi^2} \ln z + \frac{3(g^2 + g'^2)^2}{256\pi^2} \ln y\ , \\ \Delta\lambda_2 = &\quad \frac{g^4 + g'^4}{256\pi^2} \ln y + \frac{3(g^2 + g'^2)^2}{256\pi^2} \ln z\ . \end{aligned}$$

Let us now proceed to calculating the remaining corrections $\Delta\lambda_3 + \Delta\lambda_4$. It suffices to consider the diagrams with the external legs $\Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0$. To be systematic, we will treat each channel, in which the desired operator is produced, separately.

a)$s - channel\ (Fig.5)$.

In order to get the corresponding corrections, the following expressions are to be divided by $-i(2\pi)^4$ and multiplied by the function of the mass ratios (41) or (42), depending on the loop content.

$$i. \quad (\Phi_1^{0*}\Phi_1^0)^2\ \ \Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0 \to \frac{\lambda_1\lambda_3}{2!} \times 8\ ,$$

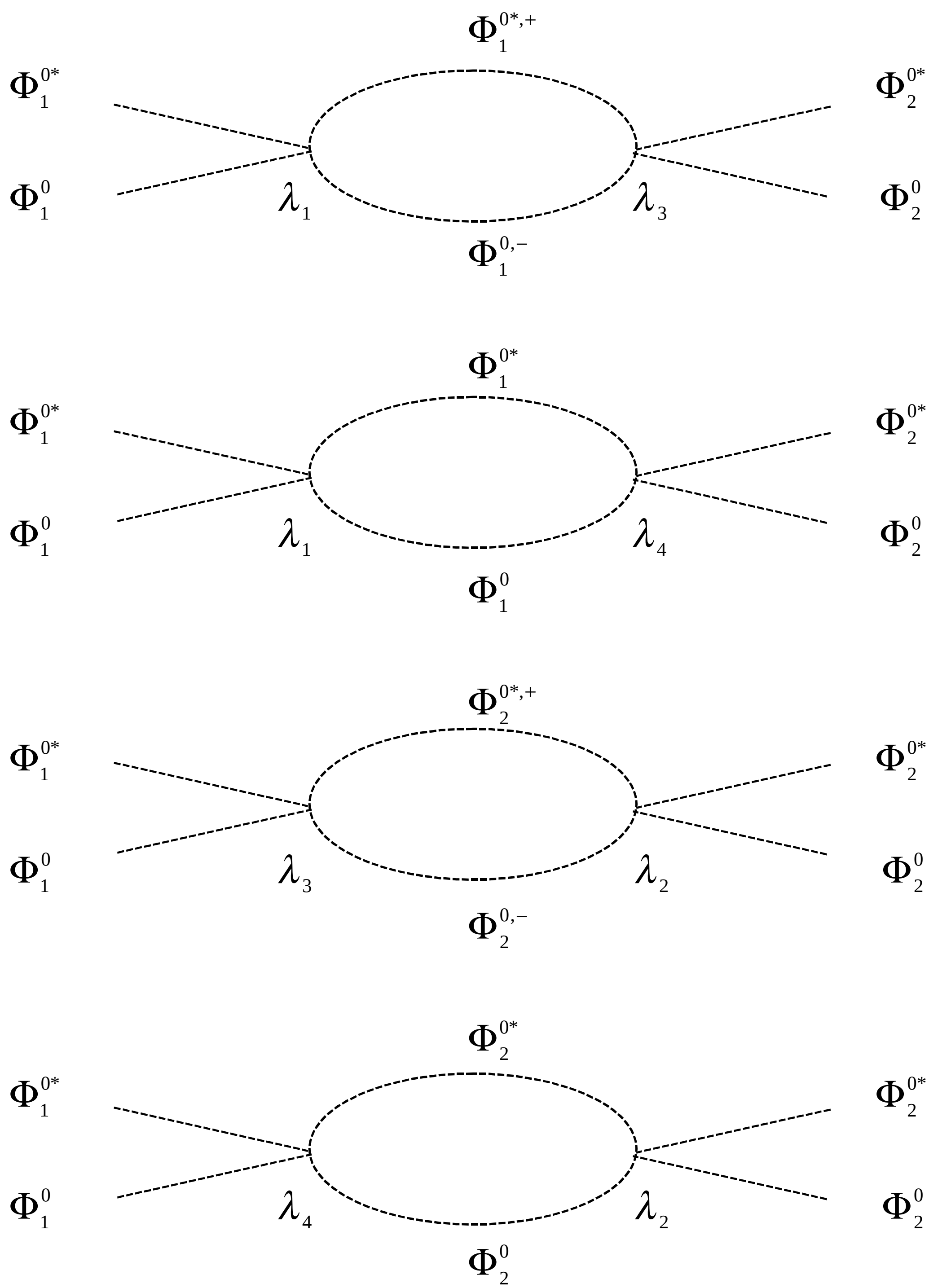


Fig.(4)-5. $\left[\Delta\lambda_3 + \Delta\lambda_4\right]^{(s)}$

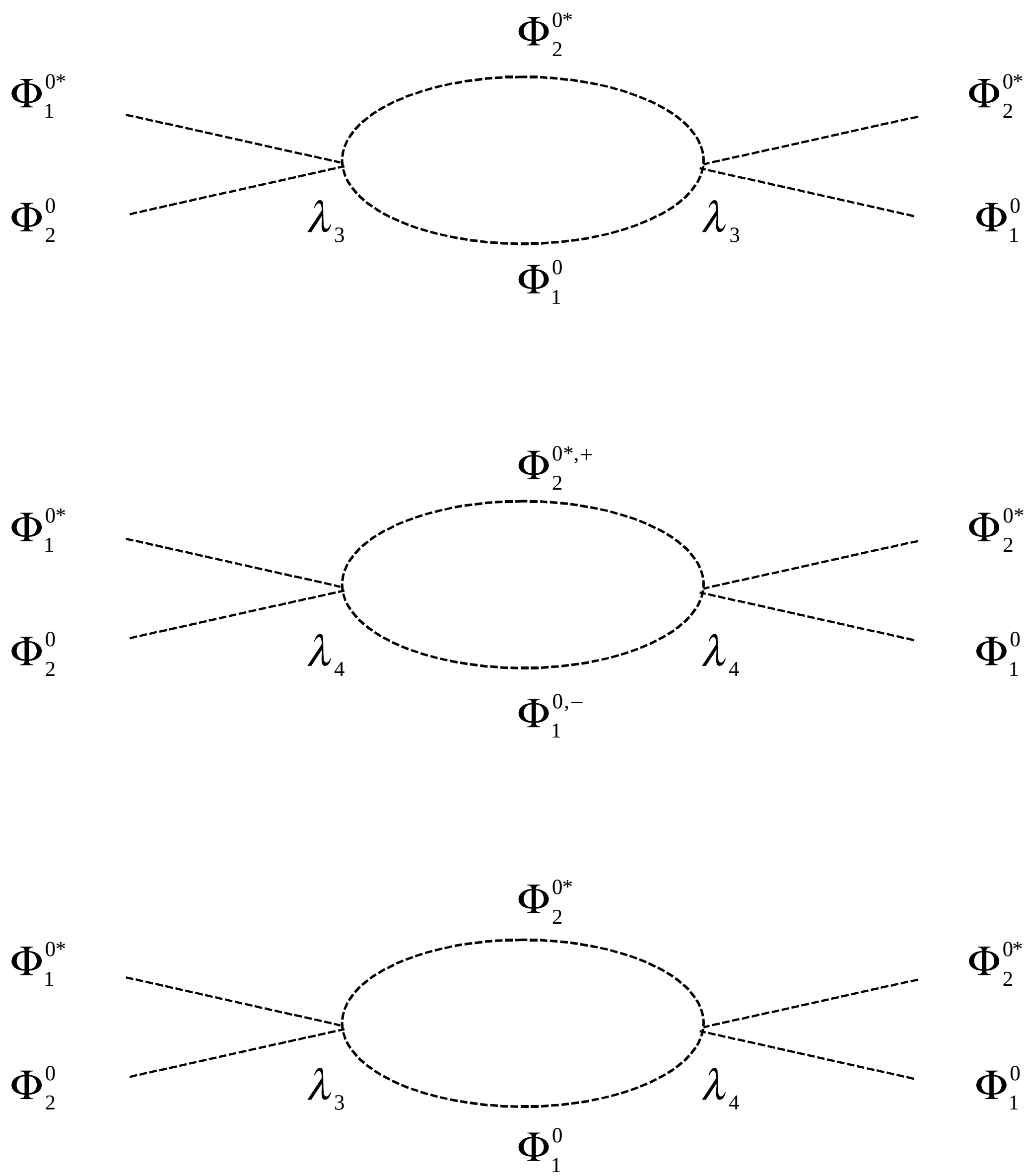


Fig.(4)-6. $\left[\Delta\lambda_3 + \Delta\lambda_4\right]^{(t)}$

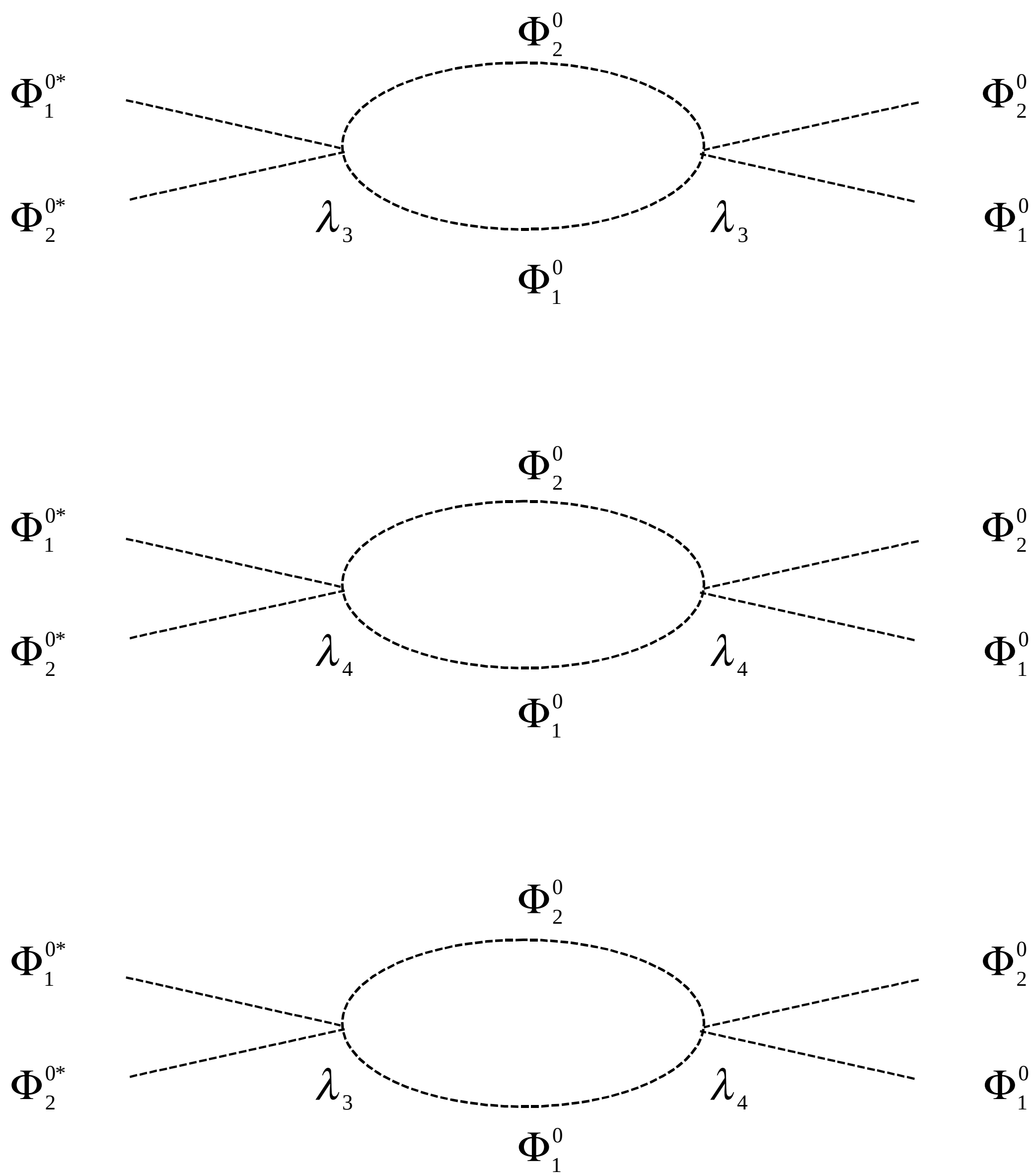


Fig.(4)-7. $\left[\Delta\lambda_3 + \Delta\lambda_4\right]^{(u)}$

$$\Phi_1^{0*}\Phi_1^0\ 2\Phi_1^+\Phi_1^- \quad \Phi_1^+\Phi_1^-\Phi_2^{0*}\Phi_2^0 \to \frac{\lambda_1\lambda_3}{2!}\times 4\ ,$$

$$ii. \quad (\Phi_1^{0*}\Phi_1^0)^2 \quad \Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0 \to \frac{\lambda_1\lambda_4}{2!}\times 8\ ,$$

$$iii. \quad (\Phi_2^{0*}\Phi_2^0)^2 \quad \Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0 \to \frac{\lambda_2\lambda_3}{2!}\times 8\ ,$$

$$\Phi_2^{0*}\Phi_2^0\ 2\Phi_2^+\Phi_2^- \quad \Phi_2^+\Phi_2^-\Phi_1^{0*}\Phi_1^0 \to \frac{\lambda_2\lambda_3}{2!}\times 4\ ,$$

$$iv. \quad (\Phi_2^{0*}\Phi_2^0)^2 \quad \Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0 \to \frac{\lambda_2\lambda_4}{2!}\times 8\ .$$

b)$t-channel\ (Fig.6)$.

$$i. \quad \Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0 \quad \Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0 \to \frac{\lambda_3^2}{2!}\times 2\ ,$$

$$ii. \quad \Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0 \quad \Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0 \to \frac{\lambda_4^2}{2!}\times 2\ ,$$

$$2\ \Phi_1^{0*}\Phi_2^0\Phi_2^+\Phi_1^- \quad (\Phi_1^{0*}\Phi_2^0\Phi_2^+\Phi_1^-)^* \to \frac{\lambda_4^2}{2!}\times 2\ ,$$

$$iii. \quad \Phi_2^{0*}\Phi_2^0\Phi_1^{0*}\Phi_1^0 \quad \Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0 \to \frac{\lambda_3\lambda_4}{2!}\times 4\ .$$

c)$u-channel\ (Fig.7)$.

$$i. \quad \Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0 \quad \Phi_1^{0*}\Phi_1^0\Phi_2^{0*}\Phi_2^0 \to \frac{\lambda_3^2}{2!}\times 2\ ,$$

$$ii. \quad \Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0 \quad \Phi_1^{0*}\Phi_2^0\Phi_2^{0*}\Phi_1^0 \to \frac{\lambda_4^2}{2!}\times 2\ ,$$

$$iii. \quad \Phi_1^{0*}\Phi_2^0\Phi_1^0\Phi_2^{0*} \quad \Phi_1^{0*}\Phi_2^0\Phi_1^0\Phi_2^{0*} \to \frac{\lambda_3\lambda_4}{2!}\times 4\ .$$

The sum of all these factors appears in the beta-function of $\lambda_3 + \lambda_4$ and, thus, can be verified[2]. Finally, recalling the tree level values of λ_i, we get

$$\begin{aligned}
\Delta\lambda_3 + \Delta\lambda_4 = & \quad -\frac{(g^2+g'^2)(g^2+3g'^2)}{256\pi^2}\ln yz \\
& \quad -\frac{(g^2+g'^2)^2+2g^4}{128\pi^2}\;\frac{z-y-z\;\ln z+y\;\ln y}{z-y}\;, \\
\Delta\lambda_{5-7} = & \quad 0.
\end{aligned} \tag{43}$$

The corresponding contribution to (12) is positive if $m_{H_1}m_{H_2} > \mu^2$.

Another possible contribution to the Higgs couplings comes from the graphs with sfermion loops. The SU(2) structure of the Lagrangian allows only sfermions with the same masses to run around the loop. Therefore these contributions can be easily inferred from the corresponding RG equations, given, for example in [19]. It comes as no surprise that the contribution of every sfermion to (12) is zero : the sfermion-loop induced beta function of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$ vanishes separately for $\tilde{Q}, \tilde{U}$ and $\tilde{D}$. The underlying reason for this is that the SUSY value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$ is zero and it will remain such under the RG flow. This argument would not work for the Higgs loops since the loops containing both doublets are not prohibited and there is also an interplay between the Higgs and the gauge sector contributions. This is in contrast to the sfermions which can be considered separately. Therefore, (43) gives an exhaustive account of the soft scalar corrections to (12). Since the Higgs-loop contribution to (12) increases logarithmically at large y and z, a

[2] The result agrees with the RGE cited in [19] (note the difference in the definitions of $\lambda_{1,2}$).

sufficiently heavy Higgs ensures that the necessary condition (12) is satisfied; for the sake of definiteness, we shall mention that (12) turns positive when $m_H^2 > 10\mu^2$ assuming that $x = 1$ and $m_{H_1} = m_{H_2}$. In a similar manner one can check the fulfilment of the sufficient condition for the boundedness of the potential

$$\frac{1}{4}\left(\lambda_3 + \lambda_4 - \lambda_5 - \frac{\lambda_6 \lambda_7}{4\lambda_5} \right)^2 < \left(\lambda_1 - \frac{\lambda_6^2}{8\lambda_5} \right)\left(\lambda_2 - \frac{\lambda_7^2}{8\lambda_5} \right) . \tag{44}$$

Indeed, the tree level MSSM yields an equality sign; then, treating the radiative corrections as a small perturbation and recalling (35) along with $\lambda_6 = \lambda_7$, we get

$$\Delta\lambda_1 + \Delta\lambda_2 + \Delta\lambda_3 + \Delta\lambda_4 - \lambda_5 - \frac{\lambda_6^2}{2\lambda_5} > 0 , \tag{45}$$

which is equivalent to (44) for the case of MSSM with $A_i = B \approx 0$. The scalar contribution to the left hand side of this inequality is a positive logarithmic function of y and z (at large y, z) and can overcome the negative fermionic contribution. Inequality (45) is satisfied for, roughly speaking, $m_H > 60\mu$ (under the above assumptions).

This completes the analysis of the stability properties of the Higgs potential at high energy scales, i.e. when the spectrum of particles is supersymmetric. As we go to lower energies, supersymmetry disappears making other sources of the radiative corrections emerge. Below the squark threshold the spectrum of particles is not supersymmetric any more and the quark loops begin to contribute. Taking into account the top-quark only we get [19]

$$16\pi^2 \, \Delta\beta_{\lambda_2} = \frac{3}{4}(g^2 + g'^2)h_t^2 - 3h_t^4 \; < 0 \; , \; t \equiv \ln s \; . \tag{46}$$

This RG running has a stabilizing effect on the Higgs potential since λ_2 increases as the energy scale decreases.

As can be seen from (40), a bounded from below potential has a unique local minimum, which coincides with a global one, and it is located at positive $v_{1,2}^2$ if we impose the standard conditions $m_1^2 + \frac{\lambda_6}{2\lambda_5} m_3^2 < 0$ and/or $m_2^2 + \frac{\lambda_7}{2\lambda_5} m_3^2 < 0$ (for detailed treatment see [5]). Apparently, spontaneous CP-violation becomes feasible in this case. A discussion on how the constraint (7) can be satisfied in radiatively corrected MSSM and some non-minimal models can be found in [20].

The above discussion, of course, does not concern itself with a certain experimental inconsistency of the described model - it predicts a very light axion:

$$M_{A_0} \approx \sqrt{2\lambda_5} v \; \sin\delta \sim 5 \; GeV \; ,$$

whereas the experimental bound [50] is $M_{A_0} > 20 \; GeV$. It is this phenomenological implication that rules out the spontaneous CP-violation scenario in the MSSM.

To summarize, we conclude that even though spontaneous CP-violation in the MSSM is possible in principle, it is inconsistent with the experimental bounds on the axion mass and, thus, is unrealistic.

5 Spontaneous CP-violation in the NMSSM

We have seen in the last chapter that spontaneous CP-violation (SCPV) in the MSSM is not prohibited by any generic mechanism. The sole reason why it is recognized as unrealistic is its prediction of a light axion. With this encouraging example in hand, we will investigate the possibility of SCPV in the closest relative to the MSSM – the NMSSM. While retaining the important features of the MSSM, this model provides a much wider range of possible axion masses and is consistent with experiment in certain regions of its parametric space. Despite the greater number of the parameters involved, spontaneous CP-violation turns out to be impossible without taking into account the radiative corrections (with the minimal superpotential).

Due to the strong Yukawa coupling, the major source of the relevant radiative corrections is the Yukawa interaction with the top-quark, so we restrict ourselves to the following superpotential (in these considerations we mostly follow Ref.[21])

$$W = \lambda H_1 H_2 N + \frac{1}{3} k N^3 + h_t Q H_2 T^c \, . \tag{47}$$

Incorporating the soft-breaking terms, we obtain the consequent scalar potential:

$$\begin{aligned} V_0 &= \frac{1}{8}(g^2+g'^2)(|H_1|^2-|H_2|^2)^2 + \frac{1}{2}g^2(|H_1|^2|H_2|^2-|H_1H_2|^2) \\ &+ \lambda^2\Big[|H_1H_2|^2+|N|^2(|H_1|^2+|H_2|^2)\Big] + k^2|N|^4 + \lambda k(H_1H_2N^{*2}+h.c.) \end{aligned}$$

$$
\begin{aligned}
&+ \quad \lambda A_\lambda (H_1 H_2 N + h.c.) + \frac{1}{3} k A_k (N^3 + h.c.) + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 \\
&+ \quad m_3^2 |N|^2 \, . \qquad (48)
\end{aligned}
$$

The leading-log top-quark radiative corrections amount to the extra piece [21]

$$
\begin{aligned}
V_{top} \quad &= \quad \frac{3}{16\pi^2} \Big[(h_t^2 |H_2|^2 + M_{sq}^2)^2 \; \ln[(h_t^2 |H_2|^2 + M_{sq}^2)/Q^2] \\
&- \quad h_t^4 |H_2|^4 \; \ln(h_t^2 |H_2|^2 / Q^2) \Big] \, ,
\end{aligned}
$$

where Q is a certain scale (it appears since a one-loop effective potential depends on an arbitrary parameter) and M_{sq} denotes the common mass for left- and right-handed top-squarks which we assume to be significantly heavier than the top-quark.

Let us now examine the vacuum of the theory. The components of the Higgs doublets are defined as

$$
H_1 = \begin{pmatrix} H_1^{0*} \\ -H_1^- \end{pmatrix} \, , H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \, .
$$

We are free to set one of the VEV's real and

$$
\arg \langle H_1^{0*} H_2^0 N \rangle = \eta \, , \; \arg \langle N^3 \rangle = \xi \, .
$$

Then the phase-dependent part of the scalar potential VEV reads

$$
\langle V \rangle |_{phase} = 2 v_1 v_2 n^2 (D \; \cos(\eta - \xi) + E \; \cos\eta + F \; \cos\xi) \, ,
$$

where v_1, v_2 and n are the magnitudes of the neutral Higgs vacuum expectation values and

$$D \equiv \lambda k \ , \ E \equiv \lambda A_\lambda / n \ , \ F \equiv \frac{1}{3} k A_k \frac{n}{v_1 v_2} \ .$$

Differentiating the VEV with respect to the phases leads to the following minimization conditions

$$\begin{aligned} -D \ \sin(\eta - \xi) - E \ \sin\eta = 0 \ , \\ D \ \sin(\eta - \xi) - F \ \sin\xi = 0 \ . \end{aligned} \tag{49}$$

This system of equations can be easily solved with the help of trigonometric identities and the solution can be written as

$$\begin{aligned} \cos\eta &= \frac{1}{2}\left(\frac{FD}{E^2} - \frac{D}{F} - \frac{F}{D}\right) \ , \\ \cos\xi &= \frac{1}{2}\left(\frac{DE}{F^2} - \frac{D}{E} - \frac{E}{D}\right) \ , \\ \cos(\eta - \xi) &= \frac{1}{2}\left(\frac{EF}{D^2} - \frac{E}{F} - \frac{F}{E}\right) \ . \end{aligned} \tag{50}$$

Such a solution has a simple geometric meaning and can be visualized via a triangle with sides $1/D, 1/E$ and $1/F$ (Fig.1): the above equations are nothing but the cosine theorem. The CP-violating phases are the external angles of this triangle. Consequently, CP-violation takes place only if the triangle can be drawn, in other words,

$$\begin{aligned} \frac{1}{F} + \frac{1}{D} > \frac{1}{E} \ , \\ \frac{1}{E} + \frac{1}{D} > \frac{1}{F} \ , \\ \frac{1}{F} + \frac{1}{E} > \frac{1}{D} \ . \end{aligned}$$

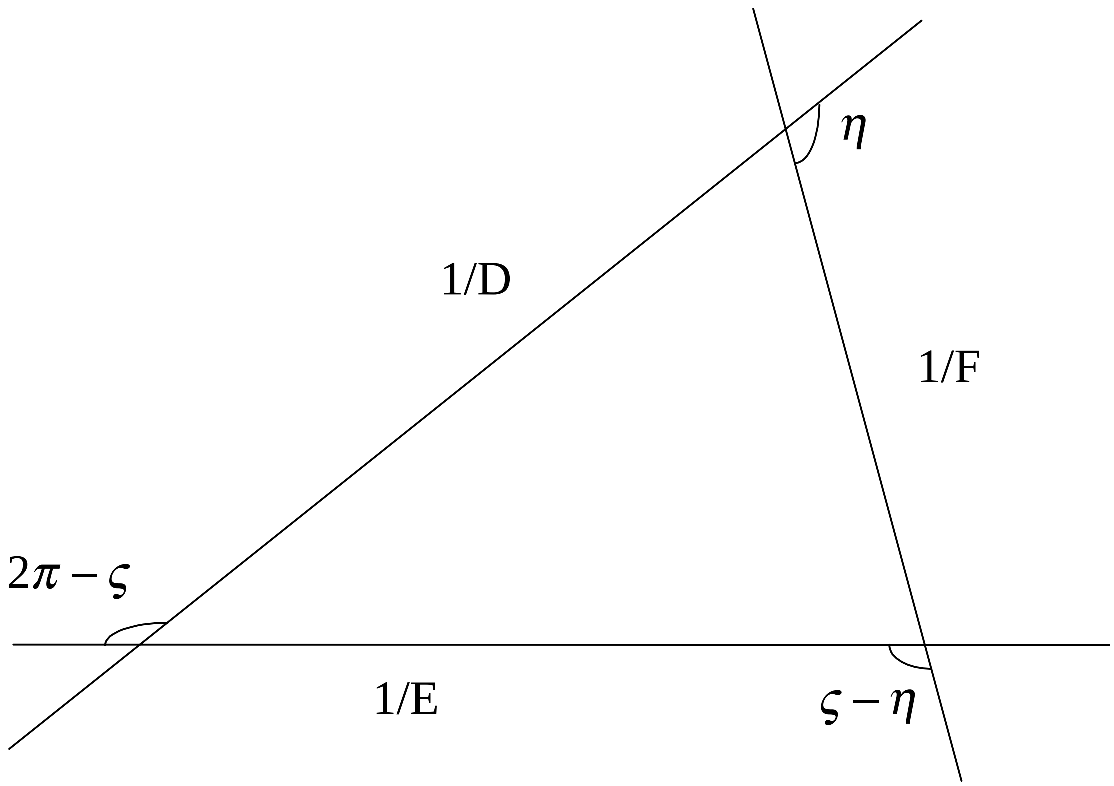


Fig.(5)-1

One may notice that the CP phases are correlated:

$$-F\ \sin\xi = E\ \sin\eta$$

and if one of them vanishes then the other has to be zero also.

So far we have considered the stationary point condition. Next we need to make sure that our vacuum is at least a local minimum. To analyze the stability properties, let us recall that we have five Higgs fields: three scalars and two pseudoscalars (which do not mix if CP is conserved); the Higgs mechanism converts the sixth pseudoscalar into a longitudinal component of the Z. Thus, one needs to analyze a 5×5 mass matrix. To be physically meaningful, the mass matrix must have all strictly positive eigenvalues. Since the eigenvalues of a general 5×5 matrix cannot be determined analytically, we will need a criterion for the positivity of a symmetric matrix. As shown in the Appendix, the necessary and sufficient condition for the positivity is that all the subdeterminants of the matrix be positive.

The mass-matrix, obtained upon expansion of the potential around the CP-violating extremum, can be analyzed, for example, in the basis suggested in Ref.[21]. Its matrix elements due to the susy and softly broken susy interactions are computed in a straightforward manner, the only subtlety is the treatment of the top-quark contribution to the effective potential V_{top} which involves an arbitrary scale parameter Q. The common approach [21] is to choose the Q in such a way that $V'_{top} = 0$ and, as a result, V_{top} does not contribute to the equations for v_1, v_2 and n. Nonetheless, the top-stop effective potential will correct the mass matrix entry responsible for the mass of the scalar component of H_2^0. This correction turns out to be essential for spontaneous CP-violation in this model: the analysis of the subdeterminants of order 3 and 4 shows that at the tree level some of them have to be negative [21]. The radiative corrections can rectify this problem in the case $\tan\beta \sim 1$. The full analysis of the general case does not seem to be possible, so we will make some simplifying assumptions. In particular, we will consider the small λ regime. From the structure of V_0 (48) it is easily seen that in the limit $\lambda \to 0$ the singlet N decouples and the mass matrix attains a block diagonal form. Since the singlet is coupled to the Higgs doublets and sfermions only, the masses of its scalar and pseudoscalar components are not constrained (or very loosely constrained) by experiment and, therefore, we effectively deal with a 3×3 mass matrix. In the basis [21] this matrix is written as

$$M^2 = \begin{pmatrix} \lambda_1 v_1^2 & -v_1 v_2[\lambda_1 + 2(3r-1)\lambda^2] & 0 \\ -v_1 v_2[\lambda_1 + 2(3r-1)\lambda^2] & \lambda_1 v_2^2(1+\Delta) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$+\frac{4r(3r-1)}{4r-1}\lambda^2 \begin{pmatrix} v_2^2\cos^2\eta & v_1 v_2\cos^2\eta & -v v_2\sin\eta\cos\eta \\ v_1 v_2\cos^2\eta & v_1^2\cos^2\eta & -v v_1\sin\eta\cos\eta \\ -v v_2\sin\eta\cos\eta & -v v_1\sin\eta\cos\eta & v^2\sin^2\eta \end{pmatrix} ,$$

where the upper-left 2×2 block represents masses and mixings of the scalar Higgs and the rest - those of the pseudoscalar; the parameter $r = A_\lambda / A_k$ is of the order of 1 and $\lambda_1 \equiv \frac{1}{2}(g^2 + g'^2) = M_Z^2/v^2 \sim 0.25$. The second matrix can be treated as a small perturbation if $\lambda_1 \gg \lambda^2$. In particular, we are interested in the lightest mass-eigenvalue, corresponding to the axion. To the first order in λ^2,

$$M^2_{axion} \approx \frac{4r(3r-1)}{4r-1}\lambda^2 v^2 \sin^2\eta \ .$$

Let us estimate the axion mass for reasonable values of the parameters. Since $\lambda_1 \sim 0.25$, the perturbative approach is valid for λ up to 0.1-0.2. Recalling that $v = 174\ GeV$ and setting

$$\sin^2\eta \sim 1/2 \ ,$$

$$r \sim 1 \ ,$$

$$\lambda \sim 0.2 \ ,$$

we get

$$M_{axion} \approx 40\ GeV \ ,$$

which is experimentally acceptable [11]. In this respect, the main difference between the MSSM and the NMSSM predictions is that in the NMSSM, the axion mass depends on the variable tree-level parameter λ, whereas in the MSSM it is strictly fixed by the radiatively generated λ_5.

A more accurate numerical analysis [21] agrees with this perturbative estimate and shows that in a certain (relatively small) region of the parametric space the lightest Higgs boson is heavier than 40 GeV. Concerning the stability of this solution, it can be verified numerically [21] that it realizes the absolute minimum of the potential for some reasonable values of the parameters. For fixed v_1, v_2 and n it can also be understood analytically from the structure of the phase-dependent part of the V. Similar numerical results have been obtained in a more recent paper [15]. It can also be shown [51] that in a more general version of the NMSSM containing a linear $const \times N$ term in the superpotential, spontaneous CP-violation is possible even at the tree level (no top-quark radiative corrections are needed in this case). This allows one to avoid the Georgi-Pais theorem [36] requiring the existence of a light axion.

To conclude, we have shown that the NMSSM is a viable model rendering spontaneous CP-violation possible. Certain tuning of the parameters is required to make it consistent with the experimental bound on the axion mass. The other phenomenological implications of this model such as observable CP-violating effects will be studied in subsequent chapters.

6 Constraints From $K - \bar{K}$ Mixing

Let us now analyze experimental implications of spontaneous CP-violation in supersymmetric models. First of them is the CP violating effects due to the imaginary part of the kaon mixing matrix. In the Standard Model, $K - \bar{K}$ mixing is successfully described by the box diargam giving rise to the $\Delta S = 2$ operator in the effective potential:

$$O_{\Delta S=2} = k \; \bar{s}\gamma^{\mu} P_L d \; \bar{s}\gamma_{\mu} P_L d. \tag{51}$$

Apparently, an imaginary part of k would induce the transition between $K - \bar{K}$ and $K + \bar{K}$, which have opposite CP-properties and, therefore, lead to CP-violation. The important experimental quantity, measuring the degree of CP-violation, is the ratio Imk/Rek. Let us now proceed to calculating the analogous effect in the simplest supersymmetric extension of the Standard Model.

6.1 Ellis-Nanopoulos Approximation for the Superbox Diagram

The supersymmetric analogue of the box diagram is obtained by replacing the internal particles with their susy partners (Fig.1) and keeping the same strengths for the interactions. However, there is one important difference. In broken susy theories the gauge fermion couplings are described by the super-CKM unitary matrices, which are, in general, different from the standard

CKM. This occurs due to the arbitrariness of the soft breaking mass terms for squarks and sleptons, which are to be diagonalized. Without loss of generality, we will assume the natural form of these super-CKM matrices to be approximately diagonal [22]:

$$V \approx \begin{pmatrix} 1 & O(\epsilon) & 10^{-2} \\ O(\epsilon) & 1 & O(\epsilon) \\ 10^{-2} & O(\epsilon) & 1 \end{pmatrix}$$

Here ϵ is of the order of Cabibbo mixing angle θ_C. Note that the entries are real since we consider spontaneous CP-violation.

First, we will estimate the $K-\bar{K}$ mixing under the simplified assumptions of Ref.22, i.e. ignoring γ-structures and assuming that the chargino is much heavier than the squarks. In this case we can replace the chargino propagator with $-i/m_{\tilde{W}}$ without spoiling the convergence of the integral. Indeed, owing to GIM cancellation on each squark line, the leading mass-independent terms in the ultraviolet region disappear. The relevant Lagrangian will be given in detail below; for now we notice that the amplitude corresponding to the superbox diagram in Fig.1 (omitting the external legs and setting the external momentum to zero) is

$$\begin{aligned} amplitude &\sim (symmetry\ factor)\frac{g^4}{m^2_{\tilde{W}}} \times \\ &\times \int \frac{d^4k}{(2\pi)^4} \sum_{ij} \frac{V_{di}V^*_{si}V_{dj}V^*_{sj}}{(k^2-m^2_{\tilde{q}_i})(k^2-m^2_{\tilde{q}_j})} \, . \end{aligned} \tag{52}$$

We can safely subtract from the integrand any quantity independent of either

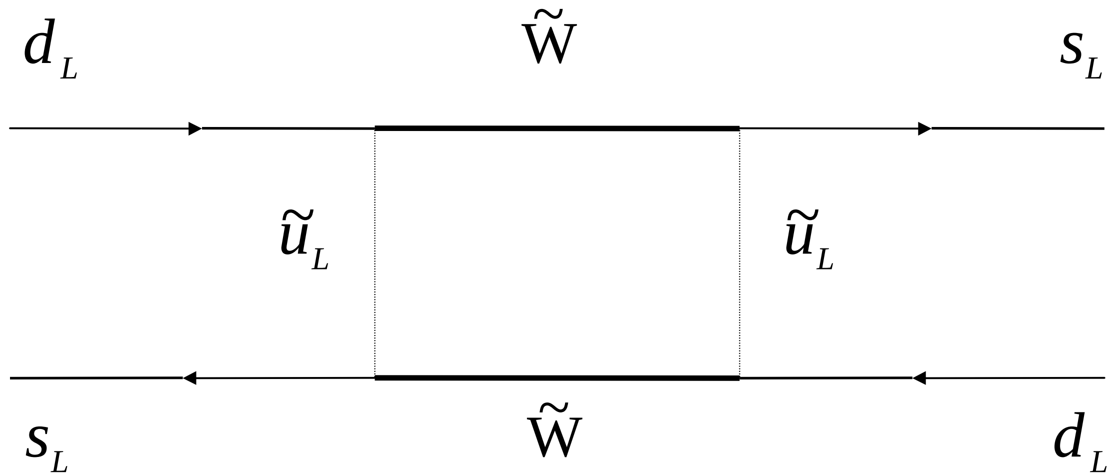


Fig.(6)-1. Superbox diagram.

summation index, since $\sum_i V_{di} V_{si}^* = 0$. Therefore

$$\int \frac{d^4k}{(2\pi)^4} \sum_{ij} \frac{V_{di} V_{si}^* V_{dj} V_{sj}^*}{(k^2 - m_{\tilde{q}_i}^2)(k^2 - m_{\tilde{q}_j}^2)} =$$
$$\int \frac{d^4k}{(2\pi)^4} \sum_{ij} V_{di} V_{si}^* V_{dj} V_{sj}^* \left(\frac{1}{(k^2 - m_{\tilde{q}_i}^2)(k^2 - m_{\tilde{q}_j}^2)} - \frac{1}{(k^2 - m_{\tilde{q}_i}^2)^2} \right) =$$
$$(m_{\tilde{q}_j}^2 - m_{\tilde{q}_i}^2) \int \frac{d^4k}{(2\pi)^4} \sum_{ij} \frac{V_{di} V_{si}^* V_{dj} V_{sj}^*}{(k^2 - m_{\tilde{q}_i}^2)^2 (k^2 - m_{\tilde{q}_j}^2)} =$$
$$\frac{i}{16\pi^2} \sum_{ij} V_{di} V_{si}^* V_{dj} V_{sj}^* \frac{m_{\tilde{q}_j}^2}{m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2} \ln \frac{m_{\tilde{q}_j}^2}{m_{\tilde{q}_i}^2} \ . \tag{53}$$

In the last step the integral is manifestly convergent and can be computed analytically [23]. Following the line of Ref.22, we can assume that the squarks are approximately degenerate in mass and put $m_{\tilde{q}_i}^2 = m_{\tilde{q}}^2 + \delta_i$, where δ_i is small compared to $m_{\tilde{q}}^2$. The first nonvanishing term in the expansion of (53) with respect to δ_i is proportional to $\delta_i \delta_j$ since the terms carrying only one index vanish:

$$\frac{m_{\tilde{q}_j}^2}{m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2} \ln \frac{m_{\tilde{q}_j}^2}{m_{\tilde{q}_i}^2} \to \frac{\delta_i \delta_j}{6 m_{\tilde{q}}^4} \ .$$

Taking into account the explicit form of the matrix V, we get

$$\sum_i \delta_i V_{di} V_{si}^* \approx \epsilon(\delta_1 - \delta_2) \equiv \epsilon \, \Delta m_{\tilde{q}}^2 \ . \tag{54}$$

The diagram contains two pairs of identical vertices and, given the vertices, can be connected in two ways. This leads to

$$(symmetry\ factor) = \frac{1}{4!} \ \times 6 \ \times 2 \ = \frac{1}{2}$$

. Combining all these factors, we obtain the following amplitude[3]

$$amplitude \sim \frac{g^4}{192\pi^2 m_{\tilde{W}}^2} \left(\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} \right)^2 \epsilon^2 \ . \tag{55}$$

Experimental constraints on $K_L - K_S$ mass difference [52] imply that ([22])

$$\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} < \frac{1}{30} \ , \tag{56}$$

justifying *a posteriori* the assumption about the degeneracy of squarks. Note, however, that this approach is of a symbolic character: left fermionic projectors would annihilate each other if we replaced the gaugino propagator by $1/m_{\tilde{W}}$, and the amplitude would vanish identically. Moreover, recent studies (see, for example [53]) favor heavy squarks over heavy gauginos. Therefore, the superbox diagram should be recalculated under more plausible assumptions. In the next subsection we will compute the superbox diagram as a function of the squark and gaugino masses, and study the limit of heavy squarks.

[3] This answer coincides with the one given in [22].

6.2 Superbox Contribution to Re k

We will work in the spinor representation of the γ-matrices :

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \dot{\sigma}_\mu & 0 \end{pmatrix}, \; C = -i\gamma_2\gamma_0 = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}.$$

$$\psi^C = C\bar{\psi}^T = \begin{pmatrix} -i\sigma_2\psi_R^* \\ i\sigma_2\psi_L^* \end{pmatrix}, \; \bar{\psi}^C = (\psi_L^T(-i\sigma_2)\;, \psi_R^T(i\sigma_2)) \;.$$

The relevant Lagrangian for one generation of fermions is [34]

$$\begin{aligned} \mathcal{L} &= -g\,\bar{\tilde{W}}^C P_L d\;\tilde{u}^* + \; h.c. \\ &= -g\,\tilde{W}_L^T(-i\sigma_2)d_L\;\tilde{u}_L^* - g\;d_L^\dagger(i\sigma_2)\tilde{W}_L^*\;\tilde{u}_L\;. \end{aligned} \tag{57}$$

It is straightforward to generalize $\mathcal{L}$ for the case of many fermion generations and include their mixings, which we will assume from now on without showing explicitly. The superbox diagram contains two identical fermion lines, so it is sufficient to consider in detail only one of them. A peculiarity of this interaction is that the vertex contains either two regular spinors or two complex congugate ones, which does not happen in the Standard Model. At second order in perturbation theory, we have

$$g^2\;s_L^\dagger(i\sigma_2)\tilde{W}_L^*\;\tilde{W}_L^T(-i\sigma_2)d_L\;\tilde{u}_L\tilde{u}_L^*\;.$$

The arising left-left gaugino propagator can be written as follows

$$\begin{aligned} &\langle(\tilde{W}_L^*)_\alpha\;(\tilde{W}_L)_\beta\rangle = -\langle(\tilde{W}_L)_\beta\;(\tilde{W}_L^*)_\alpha\rangle = \\ &-i\left(\frac{k\cdot\sigma}{k^2-m_{\tilde{W}}^2}\right)_{\beta\alpha} = -i\left(\frac{k\cdot\sigma^T}{k^2-m_{\tilde{W}}^2}\right)_{\alpha\beta}\;. \end{aligned} \tag{58}$$

The transposed σ-matrices convert into regular ones upon multiplication by σ_2:

$$\sigma_2\, k\cdot\sigma^T\, \sigma_2 = k\cdot\dot{\sigma} \tag{59}$$

The answer can be written in the four-component notation via the identity $\dot{\sigma}_\mu = \gamma_0\gamma_\mu P_L$. Recalling that the combinatorial factor is 1/2, we write the amplitude as follows

$$\begin{aligned} amplitude &= \frac{1}{2} i^4 i^4 g^4 \int \frac{d^4k}{(2\pi)^4}\left(\bar{s}\frac{-k\cdot\gamma}{k^2-m_{\tilde{W}}^2}P_L d\right)^2 \times \\ &\times \sum_{ij}\frac{V_{di}V_{si}^*V_{dj}V_{sj}^*}{(k^2-m_{\tilde{q}_i}^2)(k^2-m_{\tilde{q}_j}^2)}\,. \end{aligned} \tag{60}$$

The integral can be cast in simpler form if we note that

$$\int k_\mu k_\nu ... = \frac{1}{4}g_{\mu\nu}\int k^2 ...$$

Then the integrand is reduced to

$$\frac{k^2}{(k^2-m_{\tilde{q}_i}^2)(k^2-m_{\tilde{q}_j}^2)(k^2-m_{\tilde{W}}^2)^2}\,.$$

We can simplify this fraction further by breaking it into pieces, containing fewer powers of the momentum:

$$\begin{aligned} & k^2 \equiv k^2 - \, m_{\tilde{q}_i}^2 + \, m_{\tilde{q}_i}^2\,, \\ & \frac{k^2}{(k^2-m_{\tilde{q}_i}^2)(k^2-m_{\tilde{q}_j}^2)(k^2-m_{\tilde{W}}^2)^2} \equiv \\ & \frac{m_{\tilde{q}_i}^2}{m_{\tilde{q}_i}^2-m_{\tilde{q}_j}^2}\frac{1}{(k^2-m_{\tilde{q}_i}^2)(k^2-m_{\tilde{W}}^2)^2} - \frac{m_{\tilde{q}_j}^2}{m_{\tilde{q}_i}^2-m_{\tilde{q}_j}^2}\frac{1}{(k^2-m_{\tilde{q}_j}^2)(k^2-m_{\tilde{W}}^2)^2}\,. \end{aligned} \tag{61}$$

Now the integral of both fractions can be calculated analytically [23], so

$$\int d^4k \frac{k^2}{(k^2 - m_{\tilde{q}_i}^2)(k^2 - m_{\tilde{q}_j}^2)(k^2 - m_{\tilde{W}}^2)^2} = \tag{62}$$

$$\frac{i\pi^2}{m_{\tilde{W}}^2}\left[\frac{-1}{(x_i - 1)(x_j - 1)} + \frac{\left(\frac{x_j}{x_j - 1}\right)^2 \ln x_j - \left(\frac{x_i}{x_i - 1}\right)^2 \ln x_i}{x_i - x_j}\right],$$

where $x_i \equiv m_{\tilde{q}_i}^2 / m_{\tilde{W}}^2$. This is the *exact* answer. As in the previous subsection, we can assume that the squarks are approximately degenerate in mass: $x_i = x + \delta_i / m_{\tilde{W}}^2$ and extract the leading $\delta_i \delta_j$-term. Although this can be done exactly, we will make another simplifying assumption: $x \gg 1$. Then

$$(62) \to \frac{-i\pi^2}{3m_{\tilde{W}}^2} \frac{\delta_i \delta_j}{m_{\tilde{W}}^4 \; x^3} \; .$$

With the help of (54) we find the effective Hamiltonian to be

$$H_{\Delta S=2} = \frac{g^4}{384\pi^2 m_{\tilde{q}}^2}\left(\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}\right)^2 \epsilon^2 \; \bar{s}\gamma^\mu P_L d \; \bar{s}\gamma_\mu P_L d \; . \tag{63}$$

This result is valid for heavy squarks. Nevertheless, we note that it leads to the same order of magnitude estimate for $\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}$ as in the previous subsection if we set $m_{\tilde{q}} \sim 100 GeV$.

6.3 Superbox Contribution to Im k

In the spontaneous CP-violation scenario all CP-breaking effects are induced by complex vacuum expectation values of the Higgs. For definiteness we adopt here the NMSSM, which differs from the MSSM by the incorporation of an extra Higgs singlet. In order to keep the electromagnetic gauge symmetry exact, we will assume that only neutral Higgs components develop VEV's. We are also free to set one of them real:

$$\langle H_1^0\rangle = v_1,\ \langle H_2^0\rangle = v_2 e^{i\rho}, \langle N\rangle = n e^{i\xi}\ . \tag{64}$$

The relevant interactions are [34,24]

$$\begin{aligned}
\mathcal{L}_1 &= -g\,\tilde{W}_L^T(-i\sigma_2)d_L\,\tilde{u}_L^* - g\,d_L^\dagger(i\sigma_2)\tilde{W}_L^*\,\tilde{u}_L && (65)\\
\mathcal{L}_2 &= \frac{gm_u}{\sqrt{2}m_W sin\beta}(\,\tilde{H}_L^T(-i\sigma_2)d_L\,\tilde{u}_R^* +\ d_L^\dagger(i\sigma_2)\tilde{H}_L^*\,\tilde{u}_R\,) && (66)\\
\mathcal{L}_3 &= -g(\,v_1\tilde{H}_R^\dagger\tilde{W}_L + v_1\tilde{W}_L^\dagger\tilde{H}_R \\
&+ v_2 e^{-i\rho}\tilde{W}_R^\dagger\tilde{H}_L + v_2 e^{i\rho}\tilde{H}_L^\dagger\tilde{W}_R) && (67)\\
\mathcal{L}_4 &= e^{i\kappa}h_u m_{LR}^2\tilde{u}_R^*\tilde{u}_L + h.c. && (68)
\end{aligned}$$

Here m_u and h_u are the up-quark mass and Yukawa coupling constant, correspondingly. The phase κ is a certain function of the Higgs VEV phases ρ and ξ; $\tan\beta = v_2/v_1$.

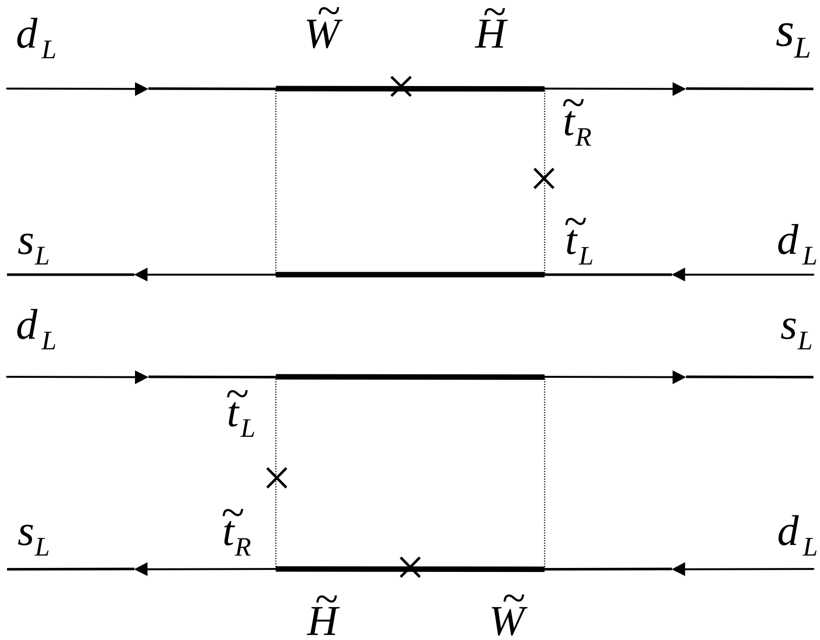
The only diagrams that contribute considerably to Im k are those involving phases in the propagators of the superpartners [24]. In order to have a complex chargino propagator, complex mixing between higgsinos and gauginos is required. Since the higgsino coupling to left handed down-quarks also involves right handed squarks, it is also necessary to have left-right squark mixing. The resulting diagrams are shown in Fig.2 .

A few reservations must be made concerning these graphs. First of all, after spontaneous symmetry breaking, strictly speaking, there are no distinct gauginos and higgsinos : they combine to produce charginos. For our purposes, however, it is admissible to treat them as different particles with the same mass, say $m_{\tilde{W}}$. Perturbation theory with respect to higgsino-gaugino mixing is justified as long as $gv_{1,2}/(k - m_{\tilde{W}}) \ll 1$; then, assuming the typical momentum in the loop to be of the order of the heaviest particle mass - $m_{\tilde{q}}$, this approach makes sense for $m_{\tilde{q}} \gg gv_{1,2}$. The same argument applies to the left-right squark mass insertion, so we assume $h_u m_{LR}^2 \ll m_{\tilde{q}}^2$.

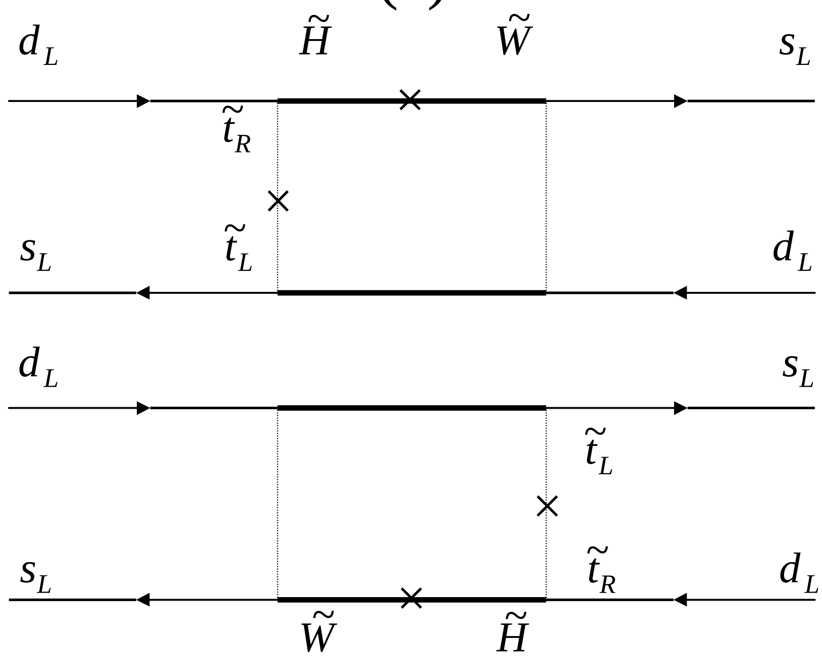
Now let us recall the super-CKM mixings at the vertices. It is natural to expect that the diagrams are dominated by t-squarks. Indeed, they are proportional to m_q^2: both left-right squark mixing and higgsino vertex involve m_q; on the other hand, the vertices contain a $V_{di}V_{si}$-factor. It is easy to see that the resulting combination $V_{di}V_{si}m_q^2$ is maximized when i=3.

The upper line in the first diagram of Fig.2,b corresponds to the following expression

$$\begin{aligned}&(-g\ s_L^\dagger(i\sigma_2)\tilde{W}_L^*\ )(-gv_1\tilde{H}_R^\dagger\tilde{W}_L - g\ v_2 e^{i\rho}\tilde{H}_L^\dagger\tilde{W}_R) \times \\ &\times\frac{gm_t}{\sqrt{2}m_W\sin\beta}(\ \tilde{H}_L^T(-i\sigma_2)d_L\ )\ \tilde{t}_R^*\tilde{u}_L\ .\end{aligned} \tag{69}$$


d_L
$\tilde{W}$
$\tilde{H}$
s_L
$\tilde{t}_R$
s_L
$\tilde{t}_L$
d_L
d_L
s_L
$\tilde{t}_L$
s_L
$\tilde{t}_R$
d_L
$\tilde{H}$
$\tilde{W}$

(a)


d_L
$\tilde{H}$
$\tilde{W}$
s_L
$\tilde{t}_R$
s_L
$\tilde{t}_L$
d_L
d_L
s_L
$\tilde{t}_L$
s_L
$\tilde{t}_R$
d_L
$\tilde{W}$
$\tilde{H}$

(b)

Fig.(6)-2

Using this as an example, let us unravel the fermionic pairings:

$$\begin{aligned}
&(\tilde{W}_L^*)_\beta(\tilde{H}_L^*)_\alpha(\tilde{W}_R)_\alpha(\tilde{H}_L)_\gamma = -(\tilde{W}_R)_\alpha(\tilde{W}_L^*)_\beta(\tilde{H}_L)_\gamma(\tilde{H}_L^*)_\alpha \\
&\rightarrow -\left(\frac{im_{\tilde{W}}}{k^2 - m_{\tilde{W}}^2}\right)_{\alpha\beta}\left(\frac{i\,k\cdot\sigma}{k^2 - m_{\tilde{W}}^2}\right)_{\gamma\alpha} = \left(\frac{m_{\tilde{W}}\;k\cdot\sigma^T}{(k^2 - m_{\tilde{W}}^2)^2}\right)_{\beta\gamma}
\end{aligned} \tag{70}$$

Then, using the identities $\sigma_2\ k\cdot\sigma^T\ \sigma_2 = k\cdot\dot{\sigma}$ and $\dot{\sigma}_\mu = \gamma_0\gamma_\mu P_L$, we can convert (69) into

$$\frac{g^3 m_t m_{\tilde{W}}(v_1 + v_2 e^{i\rho})}{\sqrt{2}m_W \sin\beta}\ \bar{s}\frac{k\cdot\gamma}{(k^2 - m_{\tilde{W}}^2)^2}P_L d\ \ \tilde{t}_R^*\tilde{u}_L\ .$$

The lower fermionic line stays unchanged and can be read off from (60); consequently

$$\begin{aligned}
amplitude \;\; &= \;\; \frac{1}{2}i^{10}\int\frac{d^4k}{(2\pi)^4}(g^2\ \bar{s}\frac{-k\cdot\gamma}{k^2 - m_{\tilde{W}}^2}P_L d)\ \frac{g^3 m_t m_{\tilde{W}}(v_1 + v_2 e^{i\rho})}{\sqrt{2}m_W \sin\beta} \\
&\times \;\; \bar{s}\frac{k\cdot\gamma}{(k^2 - m_{\tilde{W}}^2)^2}P_L d\ \ \frac{e^{-i\kappa}h_t m_{LR}^2\ V_{13}V_{23}^*}{(k^2 - m_{\tilde{t}}^2)^2}\sum_i \frac{V_{di}V_{si}^*}{(k^2 - m_{\tilde{q}_i}^2)}\ .
\end{aligned}$$

The symmetry factor for each diagram in Fig.2 is $\frac{1}{4!}\times 12\times 1 = \frac{1}{2}$, since there is one pair of identical vertices and one way of connecting the internal lines.

The integral to be calculated is

$$\int d^4k \sum_i \frac{k^2\ V_{di}V_{si}^*}{(k^2 - m_{\tilde{q}_i}^2)(k^2 - m_{\tilde{t}}^2)^2(k^2 - m_{\tilde{W}}^2)^3}$$

Rewriting k^2 as $k^2 - m_{\tilde{q}_i}^2 + m_{\tilde{q}_i}^2$ and noticing that $k^2 - m_{\tilde{q}_i}^2$ can be cancelled out to give a fraction independent of i, we see that

$$\int d^4k \sum_i \frac{k^2\ V_{di}V_{si}^*}{(k^2 - m_{\tilde{q}_i}^2)(k^2 - m_{\tilde{t}}^2)^2(k^2 - m_{\tilde{W}}^2)^3} =$$

$$\int d^4k \sum_i \frac{m^2_{\tilde{q}_i}\; V_{di} V^*_{si}}{(k^2 - m^2_{\tilde{q}_i})(k^2 - m^2_{\tilde{t}})^2 (k^2 - m^2_{\tilde{W}})^3} \approx$$
$$\epsilon \Delta m^2_{\tilde{q}} \int \frac{d^4k}{(k^2 - m^2_{\tilde{q}})^3 (k^2 - m^2_{\tilde{W}})^3}$$

At the last step we neglected higher order terms with respect to $\Delta m^2_{\tilde{q}}$. This allowed us to replace different squark masses by $m^2_{\tilde{q}}$ in the denominator. The last integral can be cast into a known form [23] as follows

$$\frac{1}{(k^2 - m^2_{\tilde{q}})^3 (k^2 - m^2_{\tilde{W}})^3} = \frac{1}{m^2_{\tilde{W}} - m^2_{\tilde{q}}} \times$$
$$\left(\frac{1}{(k^2 - m^2_{\tilde{q}})^2 (k^2 - m^2_{\tilde{W}})^3} - \frac{1}{(k^2 - m^2_{\tilde{q}})^3 (k^2 - m^2_{\tilde{W}})^2} \right),$$
$$\int \frac{d^4k}{(k^2 - m^2_{\tilde{q}})^2 (k^2 - m^2_{\tilde{W}})^3} = \frac{-i\pi^2}{2} \times$$
$$\left[-\frac{5m^2_{\tilde{W}} + m^2_{\tilde{q}}}{m^2_{\tilde{W}} (m^2_{\tilde{W}} - m^2_{\tilde{q}})^3} + \frac{2m^2_{\tilde{W}} + 4m^2_{\tilde{q}}}{(m^2_{\tilde{W}} - m^2_{\tilde{q}})^4} \; \ln \frac{m^2_{\tilde{W}}}{m^2_{\tilde{q}}} \right].$$

We are interested in the region $\frac{m^2_{\tilde{W}}}{m^2_{\tilde{q}}} \ll 1$. In this approximation,

$$\int d^4k \sum_i \frac{k^2\; V_{di} V^*_{si}}{(k^2 - m^2_{\tilde{q}_i})(k^2 - m^2_{\tilde{t}})^2 (k^2 - m^2_{\tilde{W}})^3} \approx \frac{i\pi^2}{2 m^2_{\tilde{W}} m^4_{\tilde{q}}} \; \epsilon \; \frac{\Delta m^2_{\tilde{q}}}{m^2_{\tilde{q}}} \; .$$

Combining this with the fact that $h_t = \frac{g m_t}{\sqrt{2} m_W \sin\beta}$, we get

$$\begin{aligned} amplitude \;\; &\approx \;\; i \frac{g^4}{256\pi^2} \left(\frac{g m_t}{\sqrt{2} m_W sin\beta} \right)^2 \frac{(v_1 + v_2 e^{i\rho})\; e^{-i\kappa}\; m^2_{LR}}{m_{\tilde{W}} m^4_{\tilde{q}}} \times \\ &\times \quad V_{13}\, \epsilon^2\, \frac{\Delta m^2_{\tilde{q}}}{m^2_{\tilde{q}}}\; \bar{s}\gamma^\mu P_L d\; \bar{s}\gamma_\mu P_L d \; . \end{aligned}$$

Now, we need to sum up the contrubutions of all 4 diagrams. The graphs in Fig.2,b are equivalent and contribute with opposite phases with respect to those in Fig.2,a. Hence, the imaginary part of the effective Hamiltonian may seem to vanish. However, that is the case only when the super-CKM matrices at the higgsino vertices and those at the gaugino vertices coincide exactly. Indeed, the diagrams in Fig.2,b involve $(V_{\tilde{H}})_{13}(V_{\tilde{W}})_{23}$, while ones in Fig.2,a involve $(V_{\tilde{W}})_{13}(V_{\tilde{H}})_{23}$. There's no reason to expect them to be identical since the higgsino vertex contains right squarks, as opposed to the gaugino vertex, and, assuming no left-right symmetry, a different mixing matrix. This partial cancellation naturally leads to a factor of order one [24], say $z \sim 1/2$ (unless accidental cancellation occurs).

The effective operator $O_{\Delta S=2}$ attains a complex factor with

$$|Im\ k| \approx \frac{g^4}{128\pi^2}\left(\frac{gm_t}{\sqrt{2}m_W \sin\beta}\right)^2 \frac{v\ z\ \sin\phi\ m_{LR}^2}{m_{\tilde{W}} m_{\tilde{q}}^4} V_{13}\ \epsilon^2\ \frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}\ ,$$

where the phase ϕ is a certain function of ρ, κ and β. Note that $Im\ k$ receives only one power of the suppression factor $\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}$ since the super-GIM mechanism applies to one squark line only. Using the result of the previous subsection, we get

$$\left|\frac{Im\ k}{Re\ k}\right| \approx 3\left(\frac{gm_t}{\sqrt{2}m_W \sin\beta}\right)^2 \frac{v\ m_{LR}^2 V_{13}\ z\ \sin\phi}{m_{\tilde{W}} m_{\tilde{q}}^2}\left(\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}\right)^{-1}.$$

Here we assumed that $Re\ k$ is dominated by the superbox diagram rather than the standard box. That is the case when the upper bound on $\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2}$ is saturated, namely when it approximately is $1/30$.

Let us estimate this expression for reasonable values of the parameters.

Setting

$$\tan\beta \sim 1,$$
$$m_{\tilde{W}} \sim 100\ GeV,$$
$$m_{LR}^2/m_{\tilde{q}}^2 \sim 0.1,$$
$$z \sim 1/2,$$
$$V_{13} \sim 0.01,$$

we obtain

$$\left|\frac{Im\ k}{Re\ k}\right| \leq 0.2\ \sin\phi\ .$$

The experimental value of this ratio 6×10^{-3} [24] implies that

$$\phi \geq 0.03\ . \qquad (71)$$

The lower bound corresponds to a comparatively light chargino and large left-right squark mixing. This estimate practically coincides with the one given in [24], though the latter assumed maximal higgsino-gaugino and left-right squark mixings and was based more on dimensional analysis.

The incorporation of QCD corrections amounts to a factor of the order of one, as in the SM box calculation [28]. So, we can safely neglect QCD effects.

In the above cosiderations, we have ignored a number of other possible contributions to $Re\ k$ and $Im\ k$. The neutralino and gluino diagrams can be neglected if these particles are sufficiently heavy ($\sim 200 GeV$, [24]). If that is not the case, we can duplicate the above analyses with the chargino replaced by either the gluino or the neutralino. We do not expect this to change the essential result and the estimates should be correct within an order of magnitude. The other possible contributor to $Im\ k$ is the Higgs-top-quark diagram since the Higgs interactions induce complex quark masses (so as we have only two phases initially in the theory, quark masses can be redefined to be real without gaining phases in the CKM-matrix). However this diagram is suppressed by an extra factor of V_{13} and would not alter our conclusions (note also that it does not lead to a vector$\times$vector $\Delta S = 2$ -operator).

7 Constraints From the $K \to \pi\pi$ Decays

In this section we will study the compatibility of our model with the observed CP-violating effects in the kaon decays. In the Standard Model, $K \to \pi\pi$ transitions are generated already at the tree level via W-exchange. Bose statistics and zero spin of these mesons imply that the pions in the final state can have isospin 0 or 2. The corresponding transition amplitudes ΔI=1/2, 3/2 can be derived from experimental data on $K \to \pi^0\pi^0$ and $K \to \pi^+\pi^-$ since $\pi^0\pi^0$ and $\pi^+\pi^-$ systems involve different Clebsh-Gordan coefficients upon decomposition into I=0,2 states. The tree level approach, however, does not explain the striking difference between the experimentally determined ΔI=1/2 and ΔI=3/2 amplitudes:

$$\frac{A_{3/2}}{A_{1/2}} \sim \frac{1}{20} ,$$

known as "$\Delta I = 1/2$ rule". The key to this puzzle lies in so called "Penguin" diagram, which generates the effective $\Delta S = 1$ operator [28]

$$O_{LR} = \bar{d}_L \gamma_\mu T^a s_L \, \bar{q}_R \gamma^\mu T^a q_R , \tag{72}$$

with T^a being the color generators normalized to $Tr(T^a T^b) = \delta^{ab}/2$. The susy version of this diagram is shown in Fig.1. Since the structure of the operator (72) is $(current\ I = 1/2) \times (current\ I = 0)$, it enhances $\Delta I = 1/2$ transitions only. It also has the peculiar property that it gives considerably larger hadronic matrix elements than the tree level Fermi-interaction operators because it involves right-handed particles and the strong coupling constant.

The CP-violating effects in the kaon decays can be described by the quantities [24]

$$t_i = \left| \frac{Im A_i}{Re A_i} \right| , \tag{73}$$

where A_i are the weak-decay amplitudes of the neutral kaon to two pions of isospin i. We will follow the convention $Im A_2 = 0$ [24] and need to compute t_0 only.

7.1 Superpenguin Contribution to ReA_0

The major contribution to A_0, as was mentioned in the introduction, comes from the penguin diagram [55]. In the supersymmetric case there will be two competing graphs - the SM penguin and the superpenguin, shown in Fig.1. In this subsection we will compute the latter in the context of spontaneous CP-violation. As we demonstrate below, the calculation of the superpenguin diagram basically comes down to the question of the effective quark-gluon flavour-off-diagonal vertex (its QED version is considered in the Appendices B,D of [25] and in [26]). All possible squark contributions to it at the one loop level are depicted in Fig.2. The effective vertex can be expanded in powers of the external momentum transfer $2Q$. Due to gauge invariance, this expansion starts with the first power of Q: if there were a constant term $a\gamma^\mu$, the color current would not be conserved because the incoming and outgoing quarks have different masses. The first nonvanishing term has the gauge-invariant structure $\sigma^{\mu\nu}Q_\nu$. Apparently, it does not give rise to the desired operator (72) since it flips the helicity of the quark (we can neglect the quark masses). Hence, we will need the second order term, proportional to $g_{\mu\nu}Q^2 - Q_\mu Q_\nu$. To obtain the operator (72), one extracts the $g_{\mu\nu}$-term; then Q^2 cancels out with the gluon propagator. The remaining term $Q_\mu Q_\nu$ does not lead to a product of vector currents when the quarks are on-shell and can be neglected.

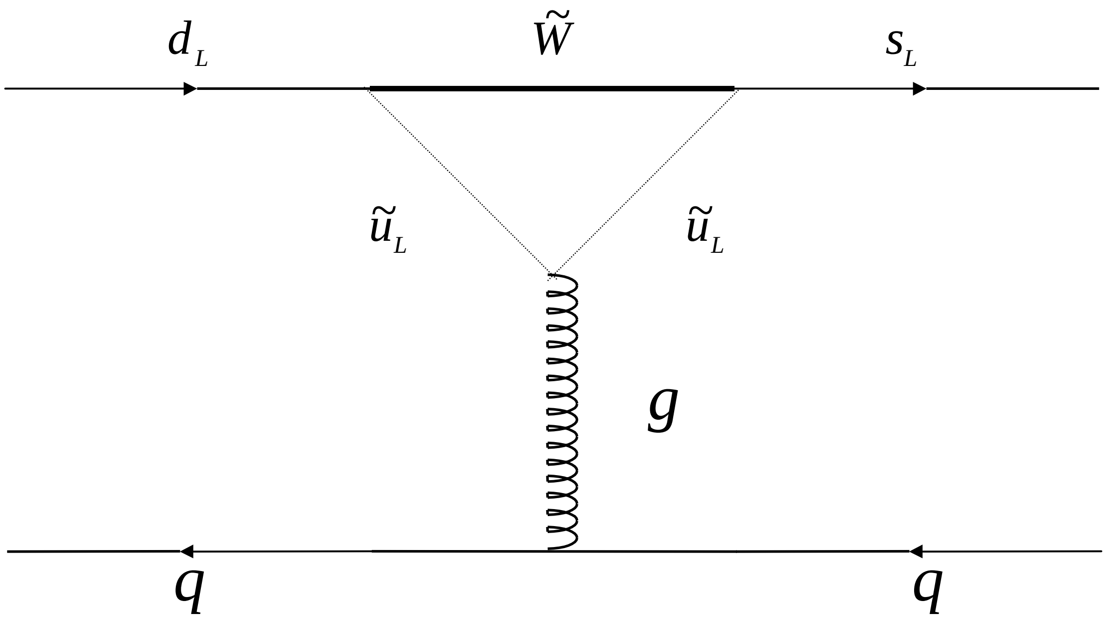


Fig.(7)-1. Superpenguin diagram.

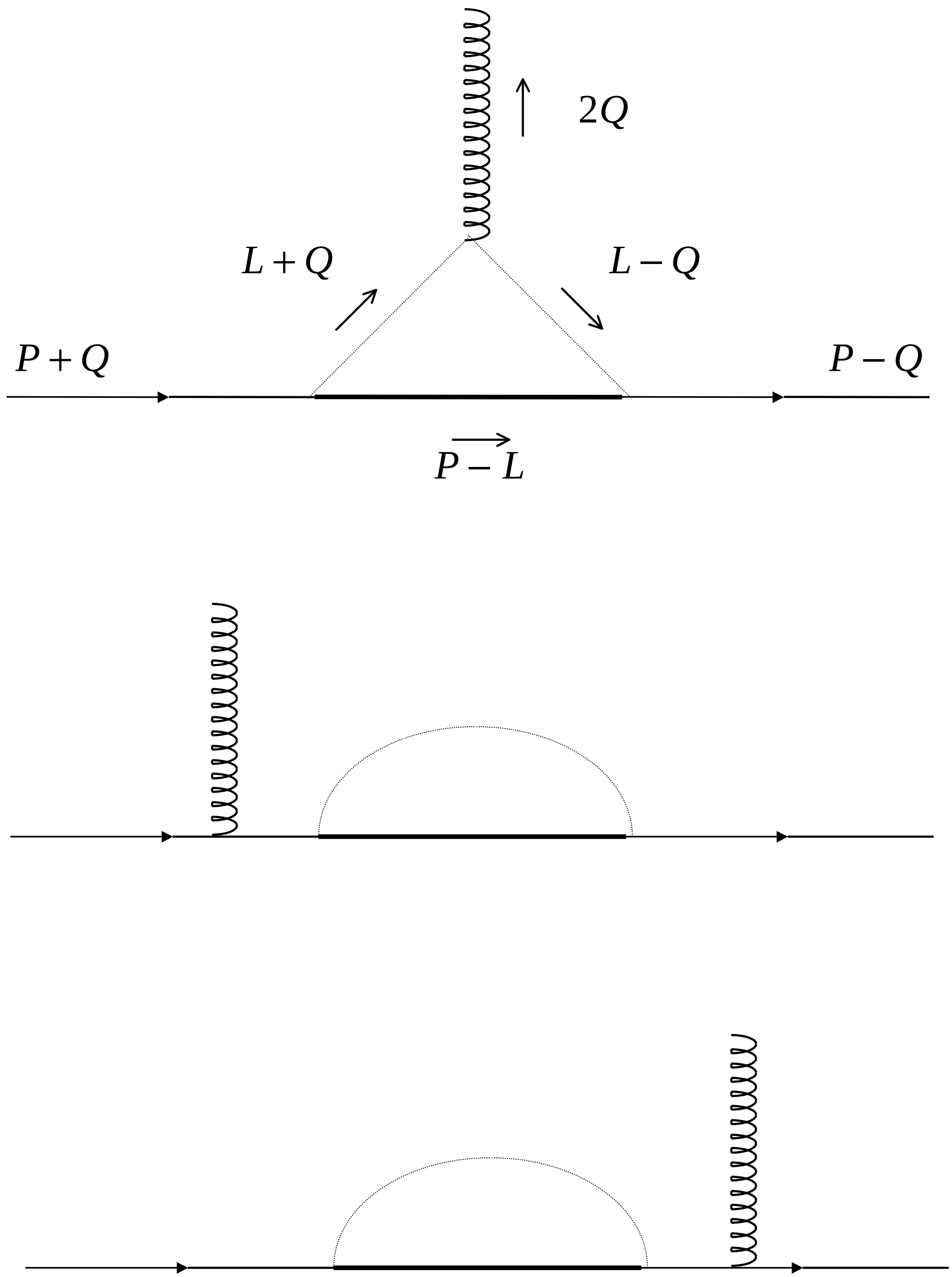


Fig.(7)-2

Now let us calculate the contribution from the upper diagram in Fig.2. The relevant interactions are as follows [34]

$$\mathcal{L}_1 = -g\; d_L^\dagger (i\sigma_2) \tilde{W}_L^* \;\tilde{u}_L \;+ h.c.\;, \tag{74}$$

$$\mathcal{L}_2 = -ig_3\; A_\mu^a\; \tilde{q}_L^{*j}\; T_{jk}^a\; \overleftrightarrow{\partial^\mu}\; \tilde{q}_L^k\;, \tag{75}$$

$$\mathcal{L}_3 = -g_3 A_\mu^a \bar{q}_j \gamma^\mu T_{jk}^a q_k\;, \tag{76}$$

with T^a being the color generators. The fermionic pairings are performed in the same way as described in the preceding section. First, let us calculate the loop integral with one squark (suppressing mixings, as usual):

$$\int \frac{d^4L}{(2\pi)^4} \frac{(2L)_\nu\; (P-L)_\mu}{((L+Q)^2-m^2)\;((L-Q)^2-m^2)\;((P-L)^2-m_{\tilde{W}}^2)}\;.$$

This expession is to be contracted with γ_μ, and ν is the external gluon index. We need to expand the denominator in Q up to the second order:

$$((L-Q)^2-m^2)^{-1} \approx (L^2-m^2)^{-1}\left(1+\frac{2L\cdot Q-Q^2}{L^2-m^2}+\frac{4(L\cdot Q)^2}{(L^2-m^2)^2}\right)\;.$$

Since we are looking for the momentum-independent color charge-radius, the external momentum P should be set to zero. This corresponds to evaluating the vertex at the point where all external momenta are of the same order, namely Q. Extracting the second order terms, we get

$$-2\int \frac{d^4L}{(2\pi)^4} \frac{L_\nu\; L_\mu}{(L^2-m^2)^3\;(L^2-m_{\tilde{W}}^2)}\left[-2Q^2+\frac{4(Q\cdot L)^2}{L^2-m^2}\right]\;.$$

To cast the integrand in a simpler form we use the following identity which follows from the Lorentz invariance and symmetry considerations:

$$\int d^4k\; k_\mu k_\nu k_\rho k_\sigma\; f(k^2) = \frac{1}{24}(g_{\mu\nu}g_{\rho\sigma}+g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\rho\nu})\int d^4k\; (k^2)^2\; f(k^2)\;.$$

Thus the integral splits into two independent functions of m and $m_{\tilde{W}}$:

$$
\begin{aligned}
g_{\mu\nu} Q^2 & \int \frac{d^4 L}{(2\pi)^4} \frac{L^2}{(L^2 - m^2)^3 \, (L^2 - m_{\tilde{W}}^2)} + \\
-\frac{1}{3}(g_{\mu\nu} Q^2 + 2 Q_\mu Q_\nu) & \int \frac{d^4 L}{(2\pi)^4} \frac{(L^2)^2}{(L^2 - m^2)^4 \, (L^2 - m_{\tilde{W}}^2)} \, .
\end{aligned}
\tag{77}
$$

It is easily seen that this combination is not gauge-invariant. One needs to incorporate all diagrams in Fig.2 to get a gauge-invariant answer. However, the other two diagrams give contributions proportional to $g_{\mu\nu} Q^2$ (for the gluon, carrying index ν, is attached to the quark line through $\gamma_\nu \times$ *Lorentz invariant structures*). Therefore, the inclusion of these two diagrams does not change the coefficient of $Q_\mu Q_\nu$ and we can read off the gauge invariant answer simply by looking at this term. It is straightforward to check this statement explicitly, expanding the loop integrals up to the third order in Q (extra $\not{Q}$ is cancelled by the quark propagator). Nevertheless, we will content ourselves with the above given argument. The integral to be calculated reduces to

$$
-\frac{2}{3}(-g_{\mu\nu} Q^2 + Q_\mu Q_\nu) \int \frac{d^4 L}{(2\pi)^4} \frac{(L^2)^2}{(L^2 - m^2)^4 \, (L^2 - m_{\tilde{W}}^2)} \, .
$$

As usual, one needs to get rid of the momentum in the numerator. For this purpose we resort to the representation

$$
\begin{aligned}
\frac{(L^2)^2}{(L^2 - m^2)^4 \, (L^2 - m_{\tilde{W}}^2)} &= \frac{1}{(L^2 - m^2)^2 \, (L^2 - m_{\tilde{W}}^2)} + \\
\frac{2m^2}{(L^2 - m^2)^3 \, (L^2 - m_{\tilde{W}}^2)} &+ \frac{m^4}{(L^2 - m^2)^4 \, (L^2 - m_{\tilde{W}}^2)} \, .
\end{aligned}
$$

Now each fraction is easily integrated and

$$\int d^4L \frac{(L^2)^2}{(L^2-m^2)^4\,(L^2-m^2_{\tilde{W}})} =$$
$$i\pi^2\Big[-\frac{2m^4-7m^2m^2_{\tilde{W}}+11m^4_{\tilde{W}}}{6(m^2-m^2_{\tilde{W}})^3}+\frac{m^6_{\tilde{W}}}{(m^2-m^2_{\tilde{W}})^4}\ln\frac{m^2}{m^2_{\tilde{W}}}\Big]\,. \qquad (78)$$

In terms of the new variable

$$x \equiv \frac{m^2}{m^2_{\tilde{W}}-m^2}$$

the answer reads

$$\textit{the loop integral} \rightarrow \frac{2}{3}(g_{\mu\nu}Q^2-Q_\mu Q_\nu)\times$$
$$\frac{i\pi^2}{(2\pi)^4(m^2_{\tilde{W}}-m^2)}\Big[x^2+\frac{5}{2}x+\frac{11}{6}-(x+1)^3\ln\frac{x+1}{x}\Big]\,. \qquad (79)$$

To get the effective operator O_{LR} one should attach the gluon line to the right-handed quarks. Recalling that the combinatorial factor is one and the gluon momentum is $2Q$, one obtains the effective Lagrangian for three generations

$$\begin{aligned}\mathcal{L}_{SP} \;\; = \;\; & -\frac{g^2g_3^2}{96\pi^2}\left(\sum_i \frac{x_i^2+\frac{5}{2}x_i+\frac{11}{6}-(x_i+1)^3\ln\frac{x_i+1}{x_i}}{m^2_{\tilde{W}}-m_i^2}V_{1i}V^*_{2i}\right)\times \\ & \bar{d}_L\gamma_\mu T^a s_L\;\bar{q}_R\gamma^\mu T^a q_R\;+h.c.\,. \end{aligned} \qquad (80)$$

This result was first obtained by P. Langacker and B. Sathiapalan [27]. A similar answer can be derived for the superpenguin gluino diagram, which we disregard following the argument given above. Due to approximate mass degeneracy of the squarks, the superpenguin contribution is suppressed as compared to that of the SM penguin unless $m_{\tilde{d}}$ is very light (30 GeV) [27]. Thus, we will assume that ReA_0 results mostly from

$$\mathcal{L}_P = -\frac{G_F}{\sqrt{2}}\frac{2\alpha_3}{3\pi}\sum_i U_{1i}U^*_{2i}\ \ln\frac{m^2_{u_i}}{M^2_W}\ O_{LR}\ + h.c.\ , \tag{81}$$

where m_{u_1} is of the order of the hadronic scale.

7.2 Superpenguin Contribution to ImA_0

The analysis of the superpenguin diagrams in Fig.3 goes along the line given in the previous section where the superbox with mass insertions was considered. In constructing the diagrams one makes use of the following interactions [34,24]

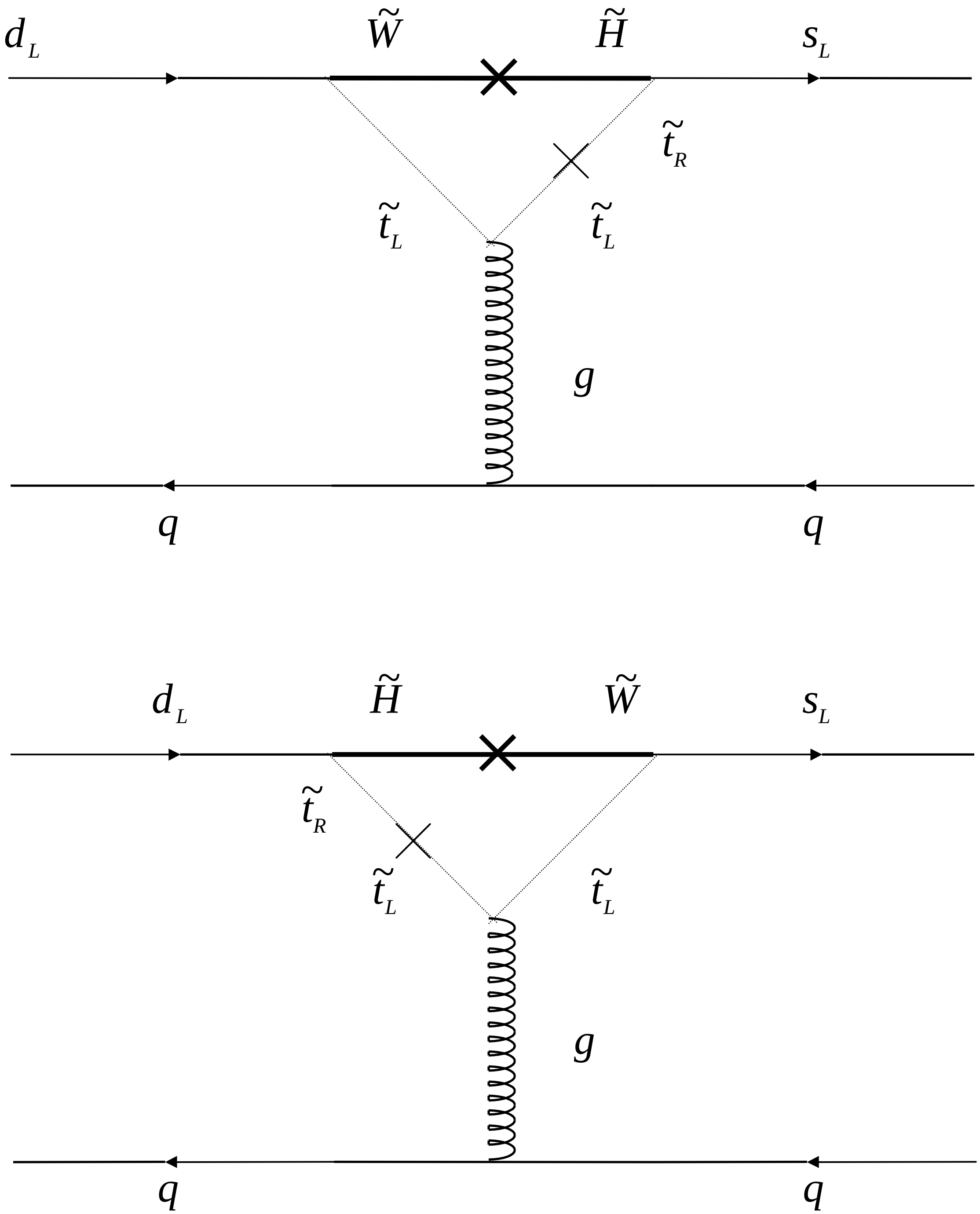
$$\begin{aligned}
\mathcal{L}_1 &= -g\,\tilde{W}_L^T(-i\sigma_2)d_L\,\tilde{u}_L^* - g\,d_L^\dagger(i\sigma_2)\tilde{W}_L^*\,\tilde{u}_L\,, && (82)\\
\mathcal{L}_2 &= \frac{gm_u}{\sqrt{2}m_W\sin\beta}(\,\tilde{H}_L^T(-i\sigma_2)d_L\,\tilde{u}_R^* + \,d_L^\dagger(i\sigma_2)\tilde{H}_L^*\,\tilde{u}_R\,)\,, && (83)\\
\mathcal{L}_3 &= -g(\,v_1\tilde{H}_R^\dagger\tilde{W}_L + v_1\tilde{W}_L^\dagger\tilde{H}_R \\
&+ \,v_2e^{-i\rho}\tilde{W}_R^\dagger\tilde{H}_L + v_2e^{i\rho}\tilde{H}_L^\dagger\tilde{W}_R)\,, && (84)\\
\mathcal{L}_4 &= e^{i\kappa}h_u m_{LR}^2\tilde{u}_R^*\tilde{u}_L + h.c.\,, && (85)\\
\mathcal{L}_5 &= -ig_3\,A_\mu^a\,\tilde{q}_L^{*j}\,T_{jk}^a\,\overleftrightarrow{\partial^\mu}\,\tilde{q}_L^k\,, && (86)\\
\mathcal{L}_6 &= -g_3A_\mu^a\bar{q}_j\gamma^\mu T_{jk}^a q_k\,. && (87)
\end{aligned}$$

The resulting loop integral is

$$\int\frac{d^4L}{(2\pi)^4}\frac{(2L)_\nu\,(P-L)_\mu}{((L+Q)^2-m^2)^2\,((L-Q)^2-m^2)\,((P-L)^2-m_{\tilde{W}}^2)^2}\,.$$

It is to be evaluated at $P=0$ and small Q. Again, owing to gauge invariance, it suffices to calculate

$$-2\int\frac{d^4L}{(2\pi)^4}\frac{L_\nu\,L_\mu}{(L^2-m^2)^3\,(L^2-m_{\tilde{W}}^2)^2}\,\frac{8(Q\cdot L)^2}{(L^2-m^2)^2} =$$
$$-\frac{2}{3}(g_{\mu\nu}Q^2+2Q_\mu Q_\nu)\int\frac{d^4L}{(2\pi)^4}\frac{(L^2)^2}{(L^2-m^2)^5\,(L^2-m_{\tilde{W}}^2)^2}\,.$$


d_L
$\tilde{W}$
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$\tilde{H}$
$\tilde{W}$
s_L
$\tilde{t}_R$
$\tilde{t}_L$
$\tilde{t}_L$
g
q
q

Fig.(7)-3

We have already computed

$$\int \frac{d^4L}{(2\pi)^4} \frac{(L^2)^2}{(L^2-m^2)^4\,(L^2-m_{\tilde{W}}^2)} \ ,$$

so we can simply differentiate it by m^2 and $m_{\tilde{W}}^2$:

$$\int \frac{d^4L}{(2\pi)^4} \frac{(L^2)^2}{(L^2-m^2)^5\,(L^2-m_{\tilde{W}}^2)^2} = \frac{1}{4}\frac{d}{dm^2}\frac{d}{dm_{\tilde{W}}^2} \int \frac{d^4L}{(2\pi)^4} \frac{(L^2)^2}{(L^2-m^2)^4\,(L^2-m_{\tilde{W}}^2)} \ .$$

Hence

$$\int \frac{d^4L}{(2\pi)^4} \frac{(L^2)^2}{(L^2-m^2)^5\,(L^2-m_{\tilde{W}}^2)^2} = \frac{i}{64\pi^2}\Big[-\frac{8m_{\tilde{W}}^6+12m^2m_{\tilde{W}}^4}{(m_{\tilde{W}}^2-m^2)^6}\times \times \ln\frac{m^2}{m_{\tilde{W}}^2} + \frac{-m^8+12m^6m_{\tilde{W}}^2+36m^4m_{\tilde{W}}^4-44m^2m_{\tilde{W}}^6-3m_{\tilde{W}}^8}{3m^2(m_{\tilde{W}}^2-m^2)^6}\Big] \ .$$

The rest of the calculations is completely analogous to that of the superbox with mass insertions except for the fact that the former has a trivial combinatorial factor of one. Finally, we obtain the CP-violating ΔS=1 effective operator

$$O_{LR} = f\ \bar{d}_L\gamma_\mu T^a s_L\ \bar{q}_R\gamma^\mu T^a q_R \ ,$$

with

$$|Im\ f| = \frac{g_3^2\ g^2}{576\pi^2}\Big(\frac{g m_t}{\sqrt{2}m_W\sin\beta}\Big)^2\ \frac{v\ z\ \sin\phi\ m_{LR}^2}{(m_{\tilde{W}}^2-m^2)^{5/2}}\ V_{13}\ \epsilon\ \times \sqrt{x+1}\Big(\ 60x^2+114x+56+\frac{3}{x}-12(x+1)^2(5x+2)\ \ln\frac{x+1}{x}\ \Big) \ ,$$

where we have introduced $x \equiv \frac{m_{\tilde{q}}^2}{m_{\tilde{W}}^2 - m_{\tilde{q}}^2}$. Let us now estimate the ratio of the imaginary and real parts of f, denoted in the introduction as t_0. The experimental limit on this ratio is

$$t_0 < 10^{-4} \ .$$

To get the most stringent constraint on $sin\phi$ we assume that masses of the chargino and squarks are comparable and of the order of 100 GeV. With this assumption

$$t_0 \sim \frac{m_W^2}{60 m_{\tilde{q}}^2} \frac{\frac{m_{LR}^2}{m_{\tilde{q}}^2} \frac{4g^2 v}{m_{\tilde{q}}} \ z \ V_{13}}{\ln \ m_c^2/m_K^2} \sin\phi \ , \tag{88}$$

where we have taken into account the dominance of the SM penguin over the superpenguin in Ref. In the case of maximal mixings

$$\begin{aligned} & \frac{m_{LR}^2}{m_{\tilde{q}}^2} \sim 1 \ , \\ & \frac{gv}{m_{\tilde{q}}} \sim 1 \ , \\ & m_{\tilde{q}} \sim 100 GeV \ , \\ & \tan\beta \sim 1 \ , \end{aligned} \tag{89}$$

one obtains, in agreement with Pomarol's estimate [24],

$$t_0 \sim 10^{-4} \sin\phi \ . \tag{90}$$

Comparing this with the experimental limit, we conclude that the CP-violating effects in the K-decays do not impose any considerable restrictions on the model.

8 Constraints From the Neutron Electric Dipole Moment

A non-zero neutron electric dipole moment (NEDM) is an important indicator of broken CP-invariance. In the nonrelativistic limit, it renders possible a spin-electric field correlation which manifestly breaks the time reversal symmetry. For a spin-1/2 particle the electric dipole interaction is described by the density [28]

$$H_{edm} = i\,\frac{d_e}{2}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}\ , \tag{91}$$

which in the nonrelativistic limit leads to the Hamiltonian

$$\mathbf{H_{edm}} = -d_e\langle\sigma\rangle \cdot E\ . \tag{92}$$

The extremely tight experimental bound on the neutron electric dipole moment

$$d_e < 10^{-25}\ e \cdot cm \tag{93}$$

imposes severe constraints on the theories of CP-violation, especially those beyond the Standard Model.

There are three distinct contributions to NEDM (sometimes overlapping) [30]:

(i) the EDM of individual valence quarks

(ii) the color dipole moment (CDM) of individual valence quarks

(iii) the hadron loop contributions to NEDM .

The CDM of a quark is induced by P and T violating quark-gluon couplings leading to the interaction density (91) with the electromagnetic field tensor replaced by the color one. The contribution (iii) is due to the collective effects of the constituent quarks and often has a semi- phenomenological character. For our purposes we will mainly concentrate on the first contribution, namely the electric dipole moment (EDM) of the valence quarks.

The neutron spin-wave function is written as the spin-1/2 part of the tensor product $\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2}$ of the individual quarks [29]:

$$|n\rangle = \frac{1}{\sqrt{6}} \left(2|d\uparrow\rangle|d\uparrow\rangle|u\downarrow\rangle - |d\downarrow\rangle|d\uparrow\rangle|u\uparrow\rangle - |d\uparrow\rangle|d\downarrow\rangle|u\uparrow\rangle\right) . \tag{94}$$

Then, assuming that the EDM of the up- and down-quarks are d_u and d_d, respectively, the neutron EDM is given by

$$d_n = \langle n| \sum_q d_q \, (\sigma_z)_q \, |n\rangle = \frac{4}{3} d_d - \frac{1}{3} d_u \ . \tag{95}$$

In the Standard Model, the EDM of individual quarks arise at the three-loop level only and the major contribution to the NEDM is given by the exchange effects among the constituent quarks (iii). The theoretical prediction is still far beyond the experimental capabilities:

$$d_e \sim 10^{-33} \div 10^{-30} \ e \cdot cm \ . \tag{96}$$

Extensions of the Standard Model often predict values of the NEDM that do not satisfy the experimental limit (93) which allows us to rule them out. It is therefore important to calculate the NEDM for the model under consideration. We will restrict ourselves to one-loop effects generating a potentially large EDM of the valence quarks.

Before we proceed to calculations, let us point out a useful identity

$$\bar{u}_2 i\sigma^{\mu\nu}\gamma_5 u_1\ k_\nu = -\bar{u}_2\gamma_5 u_1\ P^\mu\ , \tag{97}$$

where $k \equiv p_2 - p_1$ and $P \equiv p_2 + p_1$. It can easily be proven for on-shell spinors if we recall $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, $\bar{u}\,\not{p} = m\bar{u}$ and $\not{p}u = mu$.

8.1 Gluino Contribution to the NEDM

The gluino contribution to the NEDM is believed to be the largest one in most supersymmetric models owing to its strong coupling to the fermions. To construct a diagram giving rise to the effective Hamiltonian (91), notice that, due to (97), the interaction with a photon must flip the chirality of the spinor:

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\ ;\ \bar{u}_2\gamma_5 u_1 = u^\dagger_{2R}u_{1L} - u^\dagger_{2L}u_{1R}\ . \tag{98}$$

Of course, this can be done by means of mass insertions, but the masses of the u- and d-quarks are completely negligible as compared to those of the other particles involved. Therefore, we will ignore this possibility and construct "1-irreducible" diagrams with respect to the mass insertion (considering the mass insertion to be a vertex).

The only possible gluino diagrams are shown in Fig.1a and Fig.1b. Apparently, the left-right squark mixing is necessary to complete the graphs. The

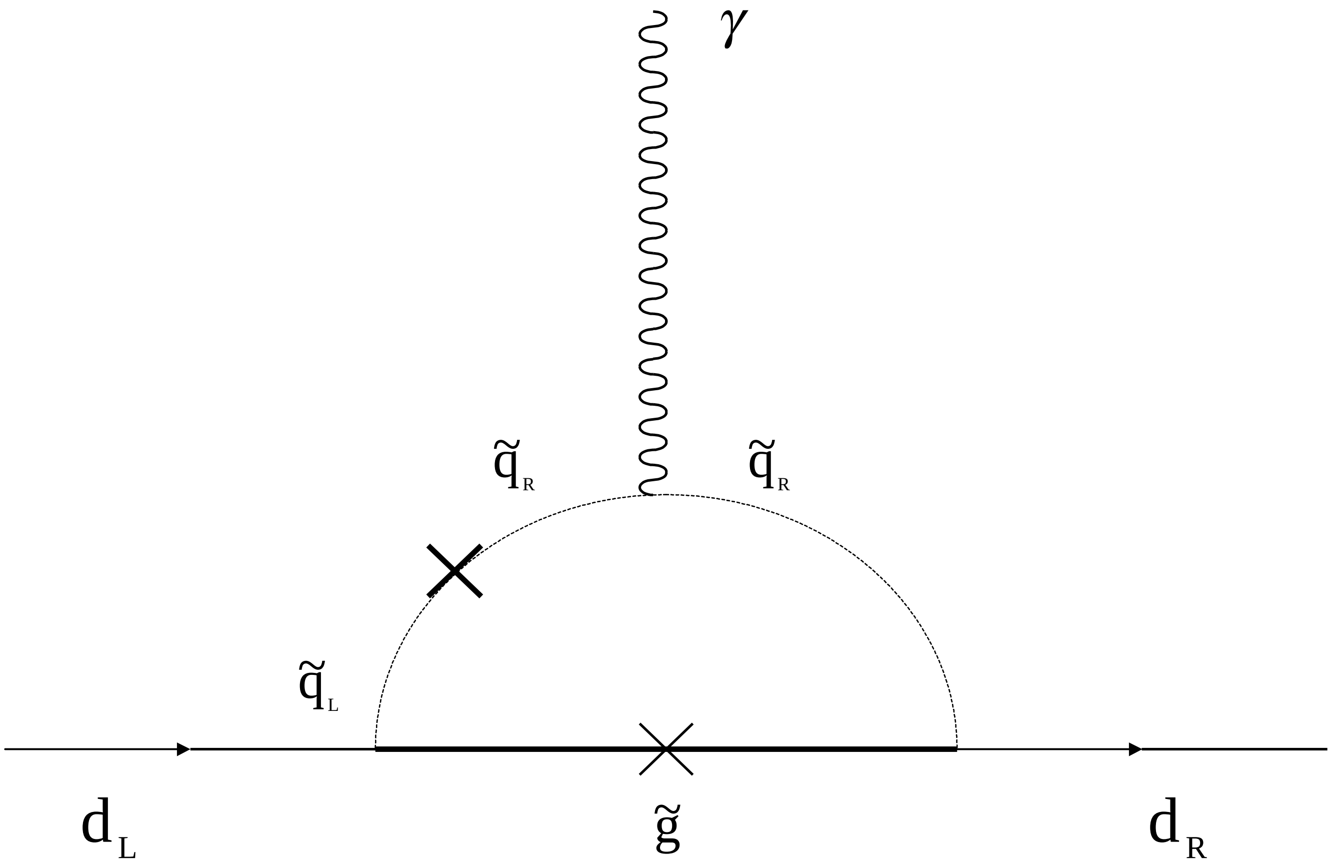

Fig.(8)-1a

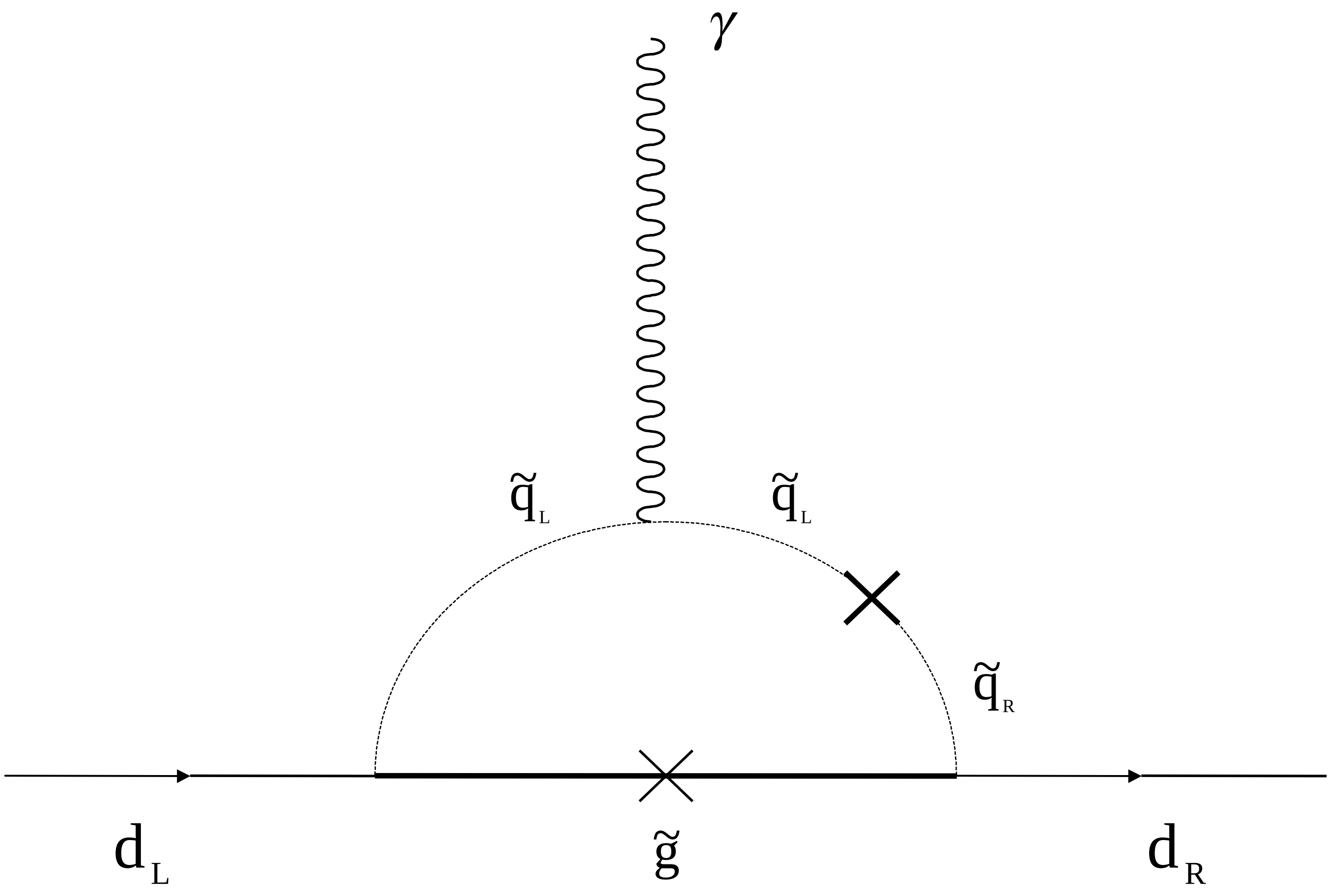

Fig.(8)-1b

relevant interactions are [34,31]

$$\mathcal{L}_1 = -\sqrt{2} g_3 T^a_{jk} \, (\, \tilde{g}^\dagger_{aR} d^k_L \, \tilde{d}^{j*}_L + d^{j\dagger}_L \tilde{g}_{aR} \, \tilde{d}^k_L - \tilde{g}^\dagger_{aL} d^k_R \, \tilde{d}^{j*}_R - d^{j\dagger}_R \tilde{g}_{aL} \, \tilde{d}^k_R \,) \, ,$$
$$\mathcal{L}_2 = -ieA_\mu \, (-1/3) \, (\tilde{d}^*_L \overleftrightarrow{\partial^\mu} \tilde{d}_L + \tilde{d}^*_R \overleftrightarrow{\partial^\mu} \tilde{d}_R \,) \, ,$$
$$\mathcal{L}_3 = e^{i\kappa} h_d m^2_{LR} \, \tilde{d}^*_R \tilde{d}_L + h.c. \, ,$$
$$\mathcal{L}_4 = -\frac{1}{2} \mu \, \bar{\tilde{g}}^a \tilde{g}^a \, . \tag{99}$$

Here T^a_{jk} are SU(3) generators; j, k and a are the color and gluon indeces correspondingly; $\tilde{g}^a$ is a gluino Majorana spinor with mass μ. Note that κ and m^2_{LR} for up-squarks and down-squarks are, generally speaking, different even though we do not show it explicitly.

The relevant momenta are defined in Fig.2. The diagrams do not possess any symmetry and have a trivial combinatorial factor of 1. In the second order with respect to $\mathcal{L}_1$ we have

$$T^{a'}_{j'k'} \, d^{j'\dagger}_R \tilde{g}_{a'L} \, \tilde{d}^{k'}_R \;\; T^a_{jk} \, \tilde{g}^\dagger_{aR} d^k_L \, \tilde{d}^{j*}_L \, .$$

Let us trace out the color indices by contraction. The gluino left-right propagator

$$\langle \tilde{g}^a_L \; \bar{\tilde{g}}^b_R \rangle = i \delta^{ab} \frac{\mu}{k^2 - \mu^2}$$

brings in $\delta_{aa'}$, whereas the squark pairing gives rise to $\delta^{jk'}$. As a result, we obtain $T^a_{j'j} T^a_{jk}$ and, since

$$T^a T^a = \frac{1}{6} \delta_{aa} = \frac{4}{3} \, ,$$

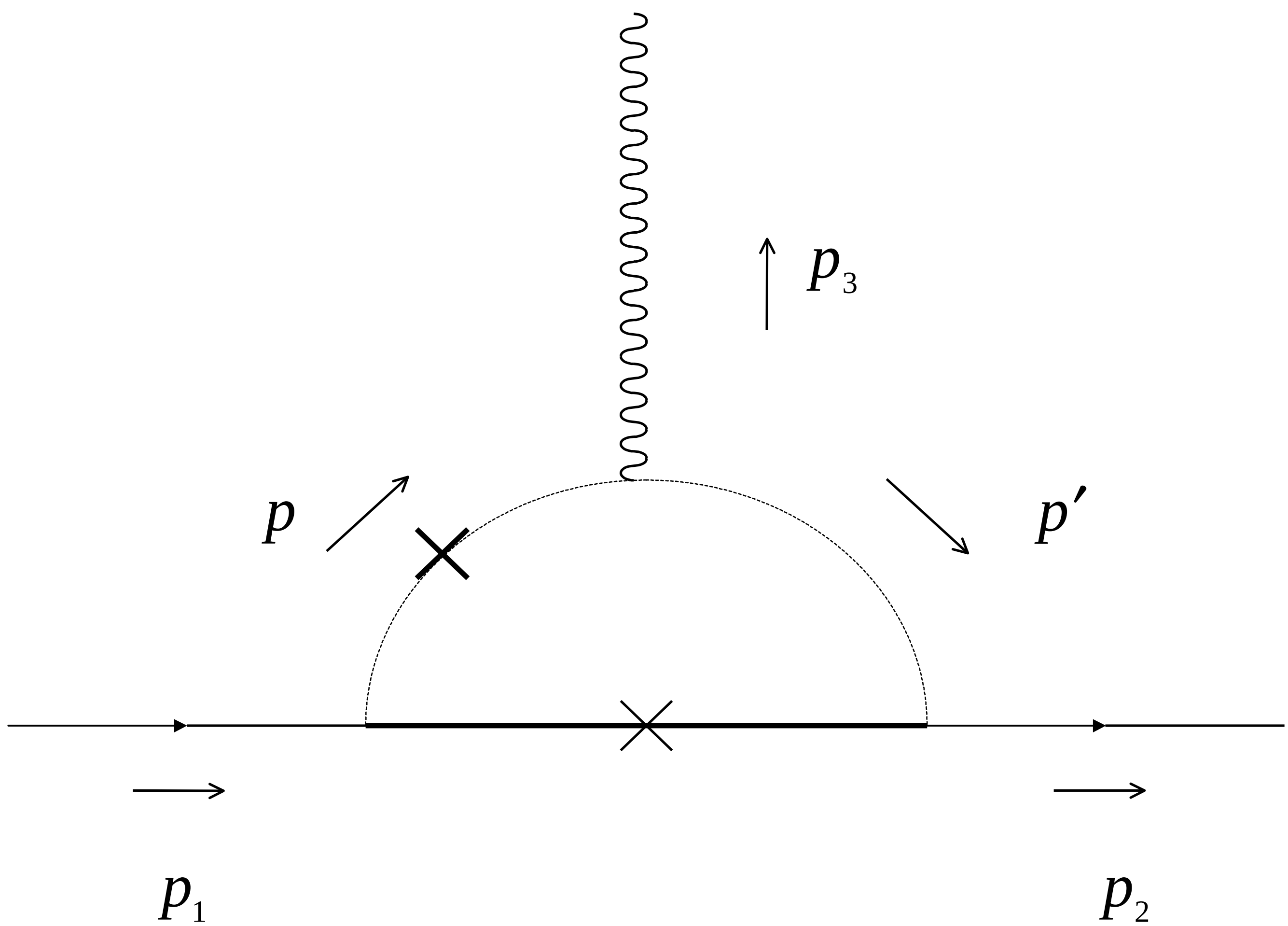

Fig.(8)-2

the consequent quark combination is $\frac{4}{3}\,\bar{d}^k_R\,d^k_L$.

Noting that the squark-photon vertex contains the sum of squark momenta $(p+p')^\mu$, we write the sum of the diagrams in Fig.1a and Fig.1b as follows

$$amplitude = i^8\ (\ \frac{2}{3}\ g_3^2\ e\ e^{i\kappa}\ h_d\ m^2_{LR}\ \mu\ )\ 2\ \frac{4}{3}\ \bar{d}^k_R d^k_L\ \times$$
$$\int \frac{d^4p}{(2\pi)^4}\frac{(2p-p_3)^\mu}{(p_1-p)^2-\mu^2}\ \frac{1}{(p^2-m^2)^2}\ \frac{1}{(p-p_3)^2-m^2}\ .$$

Since the squarks are almost degenerate in mass, we neglected contributions from $\tilde{s}$ and $\tilde{b}$, allowing only the d-squark in the loop. Its mass is denoted by m (as before, we assume a common mass for left and right squarks).

According to (97), we need to extract the term proportional to $(p_1+p_2)^\mu$. Introducing a new variable $k \equiv 2p - p_3$, let us rewrite the integral as

$$16\int \frac{d^4k}{(2\pi)^4}\frac{k^\mu}{(P-k)^2-4\mu^2}\ \frac{1}{((k+p_3)^2-4m^2)^2}\ \frac{1}{(k-p_3)^2-4m^2}\ .$$

Here $P \equiv p_1 + p_2$. We can now expand this integral in powers of P, setting other momenta to zero. The momentum-independent dipole moment is given by the linear term.

$$\frac{1}{(P-k)^2-4\mu^2} \approx \frac{1}{k^2-4\mu^2}\left(1+\frac{2P\cdot k}{k^2-4\mu^2}\right)\ .$$

Then the integral takes on the form

$$16\ \ 2P_\nu\ \frac{1}{4}g^{\mu\nu}\ \int\frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-4\mu^2)^2(k^2-4m^2)^3}\ .$$

One can get rid of k^2 in the numerator representing $k^2 = k^2 - 4\mu^2 + 4\mu^2$ and breaking the fraction into two pieces. Using [23]

$$\int \frac{d^4k}{(k^2-\mu^2)^2(k^2-m^2)^3} = \frac{-i\pi^2}{2}\times$$

$$\left[-\frac{5m^2+\mu^2}{m^2(m^2-\mu^2)^3}+\frac{2m^2+4\mu^2}{(m^2-\mu^2)^4}\ \ln\frac{m^2}{\mu^2}\right],$$

$$\int\frac{d^4k}{(k^2-\mu^2)(k^2-m^2)^3}=\frac{i\pi^2}{2}\times$$

$$\left[\frac{m^2+\mu^2}{m^2(m^2-\mu^2)^2}-\frac{2\mu^2}{(m^2-\mu^2)^3}\ \ln\frac{m^2}{\mu^2}\right],$$

we integrate each fraction and get the following result

$$integral=\frac{i\pi^2}{2}\ \frac{P^\mu}{(m^2-\mu^2)^2}\left[\frac{1}{2}+3\frac{\mu^2}{m^2-\mu^2}-\frac{\mu^2(2m^2+\mu^2)}{(m^2-\mu^2)^2}\ln\frac{m^2}{\mu^2}\right].$$

The amplitude under consideration together with its hermitean conjugate generate $-iH_{EDM+AMM}$. The EDM part is proportional to the sine of the CP-violating phase:

$$i\ \sin\kappa\ (\ \bar{d}_R d_L-\bar{d}_L d_R\ )\ .$$

According to (98), this particular combination of spinors forms $\bar{d}\gamma_5 d$. Further, we employ (97) to rewrite

$$\bar{d}\gamma_5 d\ P^\mu=\bar{d}i\sigma^{\mu\nu}\gamma_5 d\ (p_3)_\nu\ .$$

The arising combination leads to the electromagnetic field tensor:

$$(p_3)_\nu\ A_\mu=i\partial_\nu\ A_\mu\to-\frac{i}{2}F_{\mu\nu}\ .$$

Thus, we obtain the desired term

$$\bar{d}i\sigma^{\mu\nu}\gamma_5 d\ F_{\mu\nu}$$

in the effective potential. Gathering all factors and making use of (91), we find the EDM of the d-quark to be

$$|d_e|=\frac{e\ g_3^2\ \mu\ h_d\ m_{LR}^2\ \sin\kappa}{18\pi^2\ (m^2-\mu^2)^2}\left[\frac{1}{2}+3\frac{\mu^2}{m^2-\mu^2}-\frac{\mu^2(2m^2+\mu^2)}{(m^2-\mu^2)^2}\ln\frac{m^2}{\mu^2}\right].$$

This result agrees with the one obtained by Buchmuller and Wyler in [31] for the case of "hard" CP-violation up to the substitution

$$h_d \, m_{LR}^2 \, \sin\kappa \leftrightarrow v \; Im(\, V_R^\dagger m_{\tilde{d}} V_L \,)_{11}$$

which results from different notations (besides the fact that we consider "soft" CP-violation).

Let us now estimate the NEDM numerically. We can safely neglect any difference between the neutron and the d-quark EDM. Since the experimental value is given in $e \cdot cm$, it is useful to recall that 1 $(GeV)^{-1}$ can be converted into approximately $2 \cdot 10^{-14} cm$. The strictest constraint on $\sin\kappa$ is obtained for (assuming $\tan\beta \approx 1$)

$$\mu \approx m_{\tilde{q}} \sim 100 GeV \; ,$$
$$m_{LR}^2 / m_{\tilde{q}}^2 \sim 1 \; .$$

Then

$$\kappa \leq 10^{-2} \; . \tag{100}$$

This esimate agrees with that cited in [24]. In the case of smaller left-right mixing (~ 0.1) and heavy squarks ($\sim 400 GeV$) the bound relaxes to

$$\sin\kappa \leq 0.3 \; . \tag{101}$$

8.2 Chargino Contribution to the NEDM

The next most important contribution to the NEDM comes from diagrams involving charginos (Fig.3). Again, in order for the perturbation theory with respect to higgsino-gaugino mixing to work, we assume that $gv_{1,2} \ll m_{\tilde{q}}$. We employ the following Lagrangians

$$
\begin{aligned}
\mathcal{L}_1 &= -g\, \tilde{W}_L^T(-i\sigma_2)d_L\, \tilde{u}_L^*\ , && (102)\\
\mathcal{L}_2 &= \frac{gm_d}{\sqrt{2}m_W \cos\beta}\, d_R^\dagger(-i\sigma_2)\tilde{H}_R^*\, \tilde{u}_L\ , && (103)\\
\mathcal{L}_3 &= -g(v_1\tilde{W}_L^\dagger \tilde{H}_R + v_2 e^{-i\rho}\tilde{W}_R^\dagger \tilde{H}_L\ )\ , && (104)\\
\mathcal{L}_4 &= -ieA_\mu\ (2/3)\ (\tilde{u}_L^* \overleftrightarrow{\partial^\mu}\, \tilde{u}_L\ )\ . && (105)
\end{aligned}
$$

Since only complex gaugino-higgsino mixing leads to the NEDM, we can discard the v_1-term from $\mathcal{L}_3$. Consequently, the fermionic line is generated by the pairings in

$$d_R^\dagger(-i\sigma_2)\tilde{H}_R^*\ \tilde{W}_R^\dagger\tilde{H}_L\ \tilde{W}_L^T(-i\sigma_2)d_L\ .$$

The resulting left-right propagators have trivial Lorentz structure and combine into

$$-d_R^\dagger d_L\ \frac{m_{\tilde{W}}^2}{(k^2-m_{\tilde{W}}^2)^2}\ .$$

It follows that the amplitude is

$$amplitude = i^6\frac{-2e\ g^3\ m_d\ v_2\ e^{-i\rho}\ m_{\tilde{W}}^2}{3\sqrt{2}\ m_W\ \cos\beta}\ A_\mu\ d_R^\dagger d_L\ \times$$

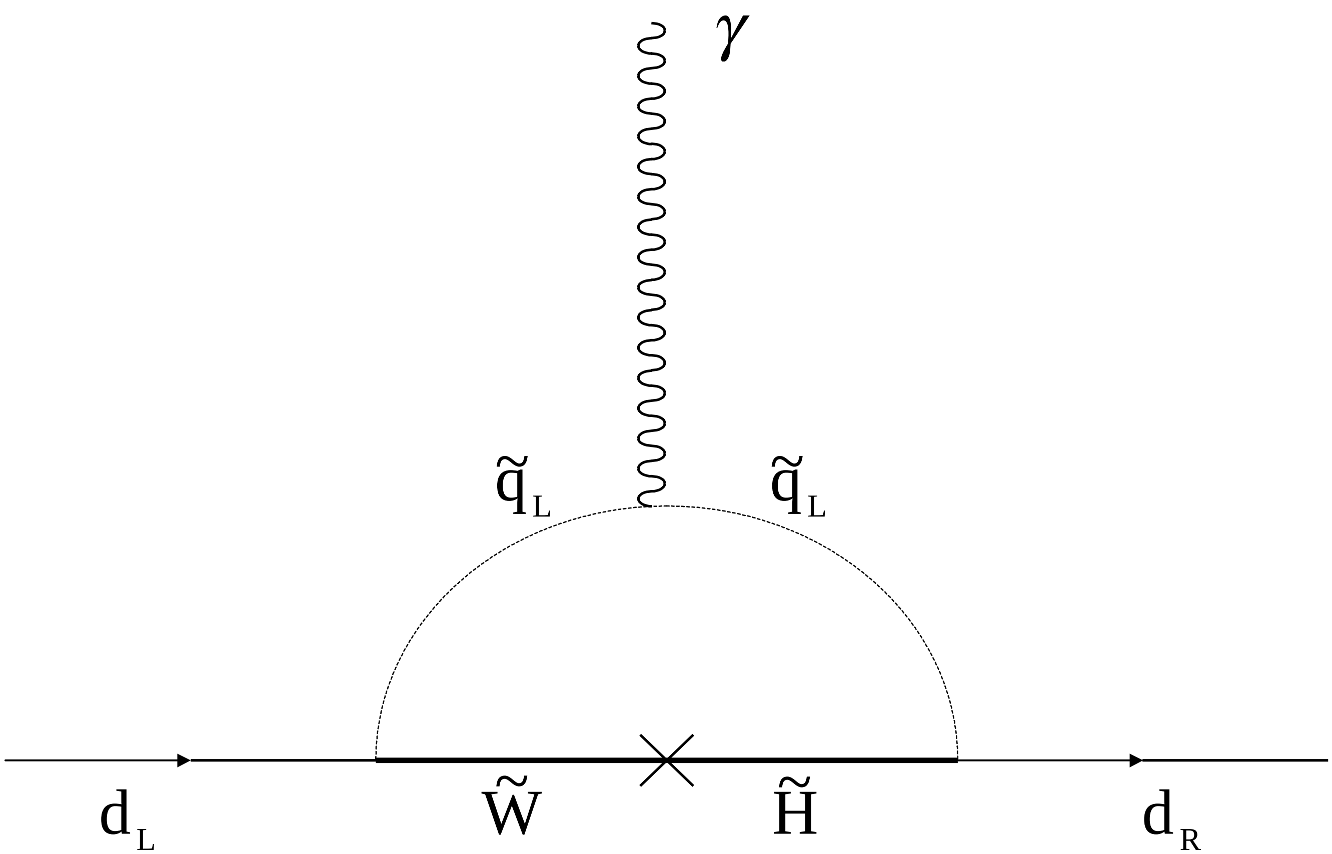

Fig.(8)-3

$$\int \frac{d^4p}{(2\pi)^4} \frac{(2p-p_3)^\mu}{((p_1-p)^2-m_{\tilde{W}}^2)^2} \frac{1}{p^2-m^2} \frac{1}{(p-p_3)^2-m^2} \,,$$

with m being the u-squark mass. The arising integral is similar to the one discussed in detail in the previous subsection,

$$\begin{aligned}
&\int \frac{d^4p}{(2\pi)^4} \frac{(2p-p_3)^\mu}{((p_1-p)^2-m_{\tilde{W}}^2)^2} \frac{1}{p^2-m^2} \frac{1}{(p-p_3)^2-m^2} \quad \rightarrow \\
&\frac{i}{16\pi^2} \frac{P^\mu}{(m^2-m_{\tilde{W}}^2)^2} \left[\frac{1}{2} + 3\frac{m^2}{m_{\tilde{W}}^2-m^2} - \frac{m^2(2m_{\tilde{W}}^2+m^2)}{(m^2-m_{\tilde{W}}^2)^2} \ln \frac{m_{\tilde{W}}^2}{m^2} \right] .
\end{aligned}$$

Repeating the same steps, we end up with

$$\begin{aligned}
|d_e| &= \frac{1}{24\pi^2} \frac{e\, g^3\, m_d\, v_2\, m_{\tilde{W}}^2\, \sin\rho}{\sqrt{2}\, m_W\, \cos\beta} \frac{m_{\tilde{W}}^2}{(m^2-m_{\tilde{W}}^2)^2} \\
&\times \left[\frac{1}{2} + 3\frac{m^2}{m_{\tilde{W}}^2-m^2} - \frac{m^2(2m_{\tilde{W}}^2+m^2)}{(m^2-m_{\tilde{W}}^2)^2} \ln \frac{m_{\tilde{W}}^2}{m^2} \right] . \qquad (106)
\end{aligned}$$

As before, the most stringent constraint on the phase is obtained in the case of maximal mixings and equal chargino and squark masses. Setting

$$gv_2/m_{\tilde{W}} \sim 1 \,,$$

$$m_{\tilde{W}} \approx m_{\tilde{q}} \sim 100 GeV \,,$$

$$\tan\beta \sim 1 \,,$$

we get

$$\rho \leq 10^{-2} \,.$$

This result complies with the bound obtained in [24]. The constraint becomes far less restrictive if the squarks are heavy:

$$m_{\tilde{W}} \sim 100 GeV \,,$$

$$m_{\tilde{q}} \sim 400 GeV$$

leads to

$$\sin \rho \leq 0.5 \ .$$

In the previous sections we have obtained bounds on a few CP-violating phases which arise in the description of physical processes. Generally speaking, they should be of the same order of magnitude. That is not the case when we encounter a special suppression mechanism, which discriminates against some phases in favor of others. For example, if $\tan \beta$ is very small the phase ρ (for its definition see (64)) will be highly suppressed in certain physical processes. In our considerations we investigated the general tendencies of SCPV in supersymmetric theories and ignored the extreme instances. We have seen that the bounds are generally compatible with one another and there is a fairly large region in the parametric space where the observed CP-violating effects can be explained by means of the spontaneous CP-breaking scenario. The favoured values of the CP-violating phases are relatively small and lie between 10^{-2} and 10^{-1}.

9 Summary and Outlook

We have reanalyzed the possibility of spontaneous CP-violation in the MSSM and presented a consistent 1-loop analysis of the radiative corrections to the Higgs couplings for arbitrary gaugino, higgsino, squark, and Higgs masses at energies above the squark threshold. We concentrated on the stability issues which were ignored elsewhere in the literature and pointed out that even though the CP-violating vacuum can appear as a local minimum, it does not necessarily mean that the Higgs potential is bounded from below. We have shown that sufficiently heavy Higgses ensure the boundedness of the potential. Due to the stabilizing action of the leading-log top-quark corrections, at energies below the squark threshold there exists a unique CP-violating local minimum and the result of Ref.[47] appears as an automatic consequence of these considerations. Thus, we conclude that spontaneous CP-violation in the MSSM is possible in principle (this would, however, be inconsistent with the experimental bounds on the axion mass).

We have demonstrated that the incorporation of the leading-log radiative corrections to the tree level relations between the Higgs and gauge couplings in the MSSM amounts to the replacement of these couplings by their running values without modifying the above mentioned relations. This allows us to separate these RG corrections from the non-leading contributions of the susy soft breaking terms which alter the tree level relations between the coupling constants. These considerations are necessary for the analysis of the stability properties of the Higgs potential and essential for the study of the possibility of spontaneous CP-violation in the MSSM.

Having observed that the axion field can be identified with the "dynamical" CP-phase and spontaneous CP-violation occurs when it acquires a vacuum expectation value, we carried out the analysis of the axion mass evolution as a function of the Higgs couplings. We have also shown that the upper bound on the axion mass (22) cannot be removed even for large $\tan\beta$ (~ 50) contrary to the recent suggestions in the literature [6]. This confirms that SCPV in the MSSM is unrealistic.

In sections 6-8 we have calculated one-loop contributions to the observable CP-violating effects in the context of spontaneous CP-violation in the Next-to-Minimal Supersymmetric Standard Model (NMSSM). Unlike the results of a recent paper [24], our calculations are valid for arbitrary chargino, gluino, and squark masses. This allows us to analyze the entire parametric space (which could not be done with dimensional analysis estimates [24]) and determine its regions which are experimentally favorable for spontaneous CP-violation. We have also shown that the problem (pointed out by Pomarol [24]) of compatibility of the bounds on the CP-phases obtained from different experiments can be rectified. In particular, we have demonstrated that sufficiently heavy squarks ($300 - 400\ GeV$) make the SCPV scenario in the NMSSM phenomenologically viable.

To summarize, we have examined the possibility of spontaneous CP-violation in the simplest supersymmetric models containing two Higgs doublets. Even though there are no theoretical obstacles for SCPV to occur in the MSSM, an extra singlet chiral superfield is required to comply with the experimental bounds on the axion mass. In this case, the observable CP-violating effects in kaon systems can be explained via the SCPV scenario, whereas, unlike most supersymmetric theories, the prediction for the neutron electric dipole moment obeys the experimental bound if the squarks are sufficiently heavy. However, this picture is only consistent in a relatively small region of the parametric space. Phases in the VEV's of the neutral Higgs bosons are the only source of CP-violation. They induce complex masses of quarks, squarks, neutralinos and charginos, while keeping the CKM matrix and the gluino mass real. As a result, $\bar{\theta} \sim 0.1 \div 0.01$ is generated at the tree level. The smallness of the CP-phases is natural according to the t'Hooft criterion [54]: setting them to zero would increase the symmetry of the theory.

One of the potential problems of spontaneous CP-violation, or any other spontaneous breakdown of a discrete symmetry, is cosmological. In the process of the electroweak phase transition, causally independent regions of space (domains) acquire "opposite" CP-properties: the vacua with opposite CP phases are allowed by the Higgs potential. This leads to the "domain wall" problem [32]: in the process of expansion the energy density corresponding to the domain walls will come to dominate that of matter and radiation, resulting in an observable anisotropy of the universe. Several ways to solve this problem have been suggested recently, see, for example, [33].

At last but not least, it is intriguing to see how supersymmetry is intertwined with CP-violation in this approach. In fact, one can consider CP-violation as a low energy supersymmetric effect. As I hope to have shown, this scenario presents a solid alternative to the standard KM approach. What really is an adequate description of CP-violation can be determined only after a scrupulous study of the heavy meson phenomenology such as, for example, B decays.

Appendix

Proposition .

A self-adjoint matrix is positive if and only if all its subdeterminants are positive.

(Here a k-subdeterminant (or a principal minor) is defined as a determinant of the matrix obtained from the initial one by deleting $i_1, ..., i_k$ rows and $i_1, ..., i_k$ (*same* !) columns. The sums of the subdeterminants of the same order are also called *Chern classes* of matrix A and constitute the complete set of unitary invariants of the matrix.)

Proof .

$\Leftarrow$). Suppose that all subdeterminants are positive. Since

$$\epsilon^{i_1 i_2 ..}_{12..} a_{1i_1} a_{2i_2} .. \ \Rightarrow \ \sum_{k,l,..} a_{kk} a_{ll} .. \epsilon^{kl \ i_1 i_2 ..}_{kl \ 12..} a_{1i_1} a_{2i_2} .. \ ,$$

the characteristic polynomial can be written as

$$det(A - \lambda I) = detA + (-\lambda) \sum_i (subdet_{(1)})_i + (-\lambda)^2 \sum_i (subdet_{(2)})_i$$
$$+ ... \equiv \sum_{k=0}^{n} (-\lambda)^k C_k = (-1)^n (\lambda - \lambda_1) \cdot ... \cdot (\lambda - \lambda_n) \ ,$$

where $C_k > 0$ and λ_k are real. In terms of $\lambda' \equiv -\lambda$ this equality reads

$$\sum_{k=0}^{n} \lambda'^k C_k = (\lambda' + \lambda_1) \cdot ... \cdot (\lambda' + \lambda_n) \ .$$

The left-hand side is positive for any positive λ'.

a) Let the eigenvalues be non-degenerate. Then all $\lambda_i > 0$. Indeed, if some of the eigenvalues are negative then the product is negative for $\lambda' \approx |\lambda_l|$ with λ_l being the "lowest" negative eigenvalue.

b) Degenerate case. The polynomial admits an even number of negative roots. Let us add an infinitesimal parameter δ to the matrix A so that the eigenvalues split up and the matrix still retains the property $subdet_{(k)} > 0$. Then the argument a) would apply and all eigenvalues would have to be positive. The described procedure is smooth (we require all subdeterminants be *strictly* positive) and this property is retained in the limit $\delta \to 0$.

$\Rightarrow$). Suppose one of the subdeterminants is negative. Without losing generality we can assume that the corresponding reduced matrix $A_{(k)}$ is obtained by deleting the k-th row and k-th column from the initial matrix A. Then there exists an $n-1$ dimensional vector $\mathbf{x}$ for which

$$\mathbf{x} \cdot A_{(k)}\mathbf{x} < 0 \ .$$

For example, it can be the eigenvector corresponding to the negative eigenvalue. Such a vector can be made n-dimensional by adding a zero in the k-th entry. Denoting the extended vector as $\mathbf{x}'$, we immediately see that

$$\mathbf{x} \cdot A_{(k)}\mathbf{x} = \mathbf{x}' \cdot A\mathbf{x}' < 0 \ ,$$

which contradicts the positivity of the matrix A. Thus, the initial assumption leads to a contradiction and all subdeterminants have to be positive.

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Vita

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