Figure 5.13 Sensor gain matrix (Algorithm C), after 32768 time steps.

Figure 5.14 Sensor output correlation matrix (Algorithm C) after 32768 time steps.
5.4.3 Frequency-Domain Analysis

The sensor output correlation matrix resulting from the adaptive modal sensor configured with Algorithm C indicates that the correlation between different sensor outputs is very small. However, a more critical analysis of the modal sensor outputs can be done in frequency domain. Figures 5.15 through 5.24 show the magnitudes of the FRF’s from force to each sensor outputs. (This time we plot the magnitudes in log scale, so that we can see the small modal “leaks” of undesired modes through the modal filters.) These figures show very little coupling among the outputs. Each modal sensor effectively passes only one modal coordinate and filter out other modal coordinates. Signals from undesired modes is generally only around 1% of the intended modes. This leaking is worst with modes 8 and 9.

Now, to further illustrate the effectiveness of the adaptive modal filters, we redraw Figs. 5.15 through 5.24 on a linear scale. The resulting plots are shown in Figs. 5.25 through 5.34. This analysis shows that Algorithm C is indeed a powerful algorithm to separate the segment outputs into uncorrelated modal coordinates.

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Figure 5.15 Normalized magnitude response of modal filter with performance feedback: Mode 1.
Figure 5.16 Normalized magnitude response of modal filter with performance feedback:
Mode 2.

Figure 5.17 Normalized magnitude response of modal filter with performance feedback:
Mode 3.
Figure 5.18  Normalized magnitude response of modal filter with performance feedback: Mode 4.

Figure 5.19  Normalized magnitude response of modal filter with performance feedback: Mode 5.
Figure 5.20  Normalized magnitude response of modal filter with performance feedback: Mode 6.

Figure 5.21  Normalized magnitude response of modal filter with performance feedback: Mode 7.
Figure 5.22 Normalized magnitude response of modal filter with performance feedback: Mode 8.

Figure 5.23 Normalized magnitude response of modal filter with performance feedback: Mode 9.
Figure 5.24  Normalized magnitude response of modal filter with performance feedback: Mode 10.

Figure 5.25  Normalized magnitude response of modal filter with performance feedback: Mode 1.
Figure 5.26 Normalized magnitude response of modal filter with performance feedback: Mode 2.

Figure 5.27 Normalized magnitude response of modal filter with performance feedback: Mode 3.
Figure 5.28  Normalized magnitude response of modal filter with performance feedback: Mode 4.

Figure 5.29  Normalized magnitude response of modal filter with performance feedback: Mode 5.
Figure 5.30 Normalized magnitude response of modal filter with performance feedback: Mode 6.

Figure 5.31 Normalized magnitude response of modal filter with performance feedback: Mode 7.
Figure 5.32 Normalized magnitude response of modal filter with performance feedback: Mode 8.

Figure 5.33 Normalized magnitude response of modal filter with performance feedback: Mode 9.
5.5 Limitations

All three algorithms developed in this chapter are effective in computing the modal filter gain matrix $W$ without much knowledge of the modal properties of the structure. These algorithms require only the number of modes that contribute to the segment signals in the anti-aliased frequency range. These algorithms are much more powerful than, for example, the LMS-based algorithm discussed in appendix A, which requires exact knowledge of the natural frequency and damping of the structure. Likewise, these algorithms can track slow changes in the modal properties of the structure by virtue of keeping track of changes in the segment output correlation matrix. Despite the above interesting features, some of the limitations to the algorithms are as follows.

1. Spatial aliasing must not be present in the segment outputs. Spatially-aliased higher-mode coordinates will appear as lower-mode coordinates.

2. The Jacobi rotation algorithm results in a sensor gain matrix that contains eigenvectors that are arbitrarily scaled. Consequently, the outputs of the modal filter are scaled modal coordinates. The scaling factor for each mode is not likely to be the same as the scaling factors for other modes. The scaling factors may even be negative. This limitation is a result of the fact that arbitrarily scaled eigenvectors are orthogonal.
3. The ordering of the eigenvectors in the sensor gain matrix is also arbitrary. Therefore, the sensor outputs are not necessarily ordered from the lowest mode coordinate to the highest.

4. In their present forms, the new algorithms cannot solve for repeated eigenvectors. The algorithms will not work if the sensor output correlation matrix is singular.

The frequency-domain analysis also revealed that some unintended modal coordinates still leak through the modal filters. This leaking indicates that perfect modal filtering is still not achieved by the adaptive modal filtering algorithms. However, considering that the adaptive modal filters were obtained without knowing the modal properties of the structure, the adaptive algorithm is successful. Moreover, independence from exact knowledge of mode shapes gives the adaptive modal filters potential for better modal filtering effects than those shown in Fig. 3.14. The modal filters whose responses were shown in Fig. 3.14 were obtained with good knowledge of the structure’s modal properties. However, discrepancies between the predicted and the actual segment outputs result in imperfect modal filtering.

Additionally, recall that virtually all the work discussed in this dissertation relies on the assumptions that the structure and transducers are linear and that the mode shapes are orthogonal.

The newly developed algorithms are still in their primordial stage. The above limitations may determine the direction of further development. So may issues need to be addressed that it is impossible to include all of them in the scope of this dissertation. The main contribution of this research work is the conceptual creation of the novel method of modal sensing, which is a step towards the future in adaptive modal analysis.

5.6 Chapter Summary

The Jacobi rotation algorithm solves for the eigenvector matrix (modal matrix) of a symmetric real matrix $Q$ by a series of norm-preserving similarity transformations. Each transformation is done with a matrix called Jacobi rotation matrix. Each transformation makes $Q$ progressively more diagonal by annihilating one off-diagonal element of $Q$. Sweeping the upper off-diagonal terms of $Q$ a few times diagonalizes $Q$. The modal matrix of $Q$ is the product of the Jacobi rotation matrices.

As shown in chapter 4, the sensor gain matrix that transforms the segment outputs into uncorrelated sensor outputs is the transpose of the modal matrix of the segment output correlation matrix $S$. Algorithm A solves for the modal matrix of $S(k)$ at each time step $k$ by sweeping the off-diagonal upper triangular part of $S(k)$ with Jacobi rotations. This algorithm is computationally extensive because the number of Jacobi rotations for each time step equals the number of sweeps times the number of off-diagonal upper triangular elements of $S$. 
Algorithm B performs only one Jacobi rotation for each time step. The Jacobi rotation is done on an intermediate matrix $T$ which is a rotated version of the segment output correlation matrix $S$. This algorithm was developed with an assumption that the segment output correlation matrix $S$ does not vary quickly with time steps.

Algorithm C also performs one Jacobi rotation for each time step on the intermediate matrix $T$. However, this algorithm checks for correlation between different sensor outputs and uses this undesired correlation to correct the sensor gain matrix. This performance feedback scheme accelerates convergence and produces more robust steady-state gain matrices.

All of the above algorithms are effective in computing the modal filter gain matrix $W$ without any knowledge of the modal properties of the structure other than the number of modes in the given frequency range. Likewise, the algorithms can track slow changes in the modal properties of the structure. A few limitations were discussed in this chapter. Much research is required to bring the new algorithms into more practical implementation.