Chapter 1

Relative Wages and Taxation in Endogenous Growth Models Based on Innovation: A Review of Recent Models

1.1 Introduction

The important role that technological progress, human capital, and tax policies play in the growth process has long been recognized. Recent models of endogenous growth predict that high technological progress and growth are associated with a high relative supply of skilled workers who earn constant or relatively low wages. The models of endogenous innovations, e.g., Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) take human capital as exogenous and show that higher levels of human capital lead to a higher growth rate. In Lucas (1988) and Azariadis and Drazen (1990) growth is driven by accumulation of human capital. They assume that labor of different skill levels is perfectly substitutable in production and that identical individuals accumulate the same amount of human capital at the equilibrium. The 1980s, however, are associated with high technological progress, high relative supply and increasing relative wages of skilled workers.

Neoclassical growth theory could tell us little about the effects of public policies on the long run rate of economic growth. Exogenous technological progress determined the long run growth rate so that public policies could have no long run effect on the growth rate. Recent models of endogenous growth, e.g., King and Rebelo (1990), Romer (1990), Grossman and Helpman (1991), and Rebelo (1991) allow public policies to affect the long run growth rate. For example, Rebelo (1991) and King and Rebelo (1990) show that taxation of income substantially reduces growth rates.

Section 2 of this chapter reviews the endogenous growth models of Romer (1990), Grossman and Helpman (1991, chapter 3), and Aghion and Howitt (1992). These models assume that human capital is exogenous so an increase in supply of human capital increases the amount of human capital in research, thus increasing the growth rate. Section 3 reviews the endogenous growth models of Grossman and Helpman (19991, chapter 5) with accumulation of human capital, and Eicher (1996). In Grossman and Helpman's model an increase in the productivity of education leads to an increase in relative supply of human capital, an increase in the growth rate, and a decrease in the relative wage of educated workers. Eicher (1996) shows that higher technological progress and growth can be associated with lower relative supply and higher relative wages of skilled labor. Section 4
reviews the neoclassical growth model of Solow (1956), and models by Rebelo (1991), and King and Rebelo (1991).

1.2 Models of Endogenous Technological Progress with Exogenous Human Capital

In this section we review research and development models developed by Romer (1987, 1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Labor, human capital, physical capital, and an index of technology are the four basic inputs in the model. It is assumed that the total stock of human capital is exogenous. Later, Grossman and Helpman (1991, chapter 5) endogenize the accumulation of human capital. Final output $Y$ is produced using labor $L$, human capital devoted to final output $H_Y$, and a set of intermediate inputs $x(i)$. The functional form is:

$$Y(H_Y, L, x) = H_Y^\alpha L^\beta \sum_{i=1}^{A} x(i)^{1-\alpha-\beta}$$  \hspace{1cm} (1)$$

where $x(i)$ is the amount of intermediate good $i$ used in production, and $A$ is the total number of existing designs. Long run growth is driven by technological progress that arises as a result of intentional investment decisions of profit maximizing agents. Technological change is represented by the invention of new types of designs. Differentiated goods are used as intermediate capital goods in the production of final goods. The aggregate stock of designs grows according to technology:

$$\dot{A} = \delta H_A A$$  \hspace{1cm} (2)$$

where $H_A$ is human capital devoted to research, and $\delta>0$ is a productivity parameter. The growth rate of $A$ depends on the amount of human capital devoted to research and the existing stock of designs that increases the productivity of human capital in the research sector.

Human capital is employed both in final goods production and in research and development. At the equilibrium, the wages paid to human capital in each sector must be same. Romer (1990) shows that an increase in human capital increases the amount of the human capital in the research sector and increases the growth rate.

In Aghion and Howitt (1992), there are three categories of labor in the economy: unskilled labor, skilled labor, and specialized labor. Unskilled labor is used in the production of the consumption good, skilled labor is used in research and in the production of intermediate goods, while specialized labor is used in research only. An increase in the endowment of skilled labor reduces the wage of skilled labor. As the wage of skilled labor decreases, the
marginal cost of research decreases while the marginal benefit of research increases, thus leading to an increase in the amount of labor employed in research.

1.3 Endogenous Technological Progress and Accumulation of Human Capital

1.3.1 The Grossman And Helpman Model

Grossman and Helpman (1991, chapter 5) allow accumulation of skills through education. Each agent lives for a finite period of time. For simplicity, we assume that each agent lives for two periods. Each individual must decide whether to work as an unskilled worker or to invest in education. Individuals who do not invest in education work as unskilled workers in both periods of their lives while individuals who invest in education work as skilled workers in the second period of their lives. Only individuals who plan to work as skilled workers invest in education. Individuals who invest in education acquire $A\bar{h}$ units of skilled labor, where $\bar{h}$ is the initial technology for human capital investment and $A$ is the productivity parameter in the education sector.

Let $w_H$ and $w_L$ denote the wage of one unit of skilled labor and the wage of one unit of unskilled labor, respectively. Individuals who do not invest in education supply one unit of unskilled labor input in both periods of their lives. The present value of lifetime earnings as an unskilled worker is this:

$$w_L + \beta w_L,$$  \hspace{1cm} (3)

where $\beta$ is the discount factor. Individuals who invest in education give up their earnings as unskilled workers in the first period of their lives and provide $A\bar{h}$ units of skilled labor input in the second period of their lives. The present value of lifetime earnings as a skilled worker equals:

$$\beta A\bar{h}w_H.$$ \hspace{1cm} (4)

Grossman and Helpman assume that individuals are identical, and each type of labor input is essential for production. At the equilibrium, individuals must be indifferent between investing in education and working as unskilled in both periods. The present value of lifetime income from two the paths must be equal:

$$w_L + \beta w_L = \beta A\bar{h}w_H.$$ \hspace{1cm} (5)
We can rewrite equation (5) as:

\[
\frac{w_H}{w_L} = \frac{1+\beta}{\beta A_h}.
\]  

(6)

Equation (6) says that the relative factor prices depend on the exogenously given discount factor and technology for human capital accumulation. Since individuals who invest in education give up their first period earnings as unskilled workers, the salary of a skilled worker $A_h w_H$, at the equilibrium, must exceed the salary of an unskilled worker.

As the productivity of education increases, individuals who initially were indifferent between the two paths prefer to become skilled. Individuals will remain indifferent between seeking education and working as unskilled labor only if the relative wage of unskilled labor increases. To restore equilibrium, the relative wage of skilled labor must fall sufficiently. As the relative supply of skilled workers increases, wage of the skilled decreases and wage of the unskilled increases until the equilibrium is restored. Grossman and Helpman show that extra human capital reduces research costs and increases the long run growth rate.

1.3.2 The Eicher Model

Eicher (1996) considers an overlapping generation model with individuals who live for two periods. In the first period young individuals must decide whether to enter the education sector as students or to work in production. Individuals who enter production work as unskilled labor in the first period and retire in the second period. Individuals who enter the education sector pay tuition in the first period and become skilled labor next period. Skilled workers can either work in production as engineers, or as teachers.

The education sector employs teachers, students and the newest technology to produce a new technology:

\[
\nu_{t+1} - \nu_t = \mu \nu_t \min[\gamma P_t, S_t]
\]  

(7)

where $S_t$ is the number of students who invest in education, $P_t$ is the number of skilled workers who work in the education sector, called teachers, $\gamma$ is an exogenously specified student-teacher ratio, and $\nu_t$ is the level of technology in period $t$. Students who enter the education sector pay the salary of teachers, so that the tuition cost per student depends on the student-teacher ratio and the wage of teachers.

Output is produced in two sectors, called high tech and low tech sectors. The high tech sector employs the newest technology, skilled and unskilled labor inputs:
\[ H_t = u_t F[U_t^H, E_t] \]  \hspace{1cm} (8)

where \( U_t \) is unskilled labor input and \( E_t \) is skilled labor input. The low tech sector employs old technology and only unskilled workers:

\[ L_t = u_{t-1} \delta U_t^L \]  \hspace{1cm} (9)

where \( \delta > 0 \) is the productivity of unskilled workers in the low tech sector.

Profit maximizing firms take the wage of skilled workers \( w_E \), the wage of unskilled workers in the high tech sector \( w_{UH} \) and the wage of unskilled workers in the low tech sector \( w_{UL} \), as given. Profit maximization yields:

\[ w_t^E = u_t [f \left( \frac{U_t^H}{E_t} \right) - f' \left( \frac{U_t^H}{E_t} \right) \left( \frac{U_t^H}{E_t} \right)] \]  \hspace{1cm} (10)

\[ w_t^{UH} = u_t f' \left( \frac{U_t^H}{E_t} \right) \]  \hspace{1cm} (11)

\[ w_t^{UL} = \delta u_{t-1} \]  \hspace{1cm} (12)

Factor market clearing requires:

\[ w_t^E = w_t^{UH} \]  \hspace{1cm} (13)

\[ w_t^{UL} = w_t^{UH} \]  \hspace{1cm} (14)

From equations (10) and (11), (12), (13), and (14), the relative wage of skilled workers to unskilled workers can be written as:

\[ \frac{w_t^E}{w_t^{UH}} = g \left( \frac{\delta}{1 + \mu S_{t-1}} \right) \]  \hspace{1cm} (15)

where \( g'(.) < 0 \). The relative wage of educated to uneducated workers in period \( t \) depends on investment in education in period \( t-1 \), the productivity of education \( \mu \), and the productivity of unskilled workers \( \delta \). High investment in education in period \( t-1 \) increases the demand for skilled workers in period \( t \) and increases the relative wage of educated workers.
Individuals maximize utility:

\[ W^j = \ln c_t + \beta \ln c_{t+1}, \quad (16) \]

where \( \beta > 0 \) represents the discount factor, \( j \) indexes the worker type (skilled or unskilled), and \( c_t \) represents per capita consumption in period \( t \). Individuals who invest in education borrow in the first period to pay tuition and purchase first period consumption. Individuals who choose to work in the first period receive income in the first period and save for second period consumption.

At the equilibrium total borrowing must be equal to total savings and since individuals are initially identical, they must be indifferent between the two paths. Career arbitrage, bond market clearing and utility maximization yield investment in education \( S_t \) and the stock of human capital allocated to education sector \( P_t \):

\[ S_t = \frac{\theta}{\frac{w_t^E}{\gamma w_t^U} + 1} \quad (17) \]

\[ P_t = \frac{\theta}{\frac{w_t^E}{w_t^U} + \gamma} \quad (18) \]

The marginal cost of human capital investment is strictly decreasing in \( S_t \). The higher the wage of skilled labor in period \( t \), the higher the cost of schooling and the lower \( S_t \). (The higher the wage of unskilled labor, the higher the supply of funds, and the lower the cost of schooling, and the higher the number of students.) The marginal benefit of human capital investment is increasing in \( S_{t-1} \). The higher the human capital investment in period \( t-1 \), the greater the technological progress, the higher the demand for skilled labor, and the higher the wages of skilled.

Eicher shows that an increase in the effectiveness of labor in research increases the relative demand for skilled labor, the relative wage, the rate of technological change, and long run growth, but decreases the relative supply of skilled labor.
1.4 Taxation of Income

1.4.1 The Neoclassical Model of Economic Growth

We consider first a basic neoclassical model of capital accumulation due to Cass (1965) and Koopmans (1965). The neoclassical production function is given by:

\[ Y_t = A K_t^\alpha N_t^{1-\alpha} \]  

(19)

where \( \alpha > 0 \), \( Y(t) \) is output, \( A \) is a fixed technology parameter, \( K(t) \) is the total stock of capital, and \( N(t) \) is the total labor input. In per labor units, the production function can be written as:

\[ y(t) = A k(t)^\alpha \]  

(20)

where \( y(t)=Y(t)/N(t) \) and \( k(t)=K(t)/N(t) \).

We assume that population is constant over time. The households own the labor and capital and rent them to the firms at the competitive prices. A representative, infinitely lived consumer maximizes lifetime utility:

\[ U(t) = -e^{-\rho t} \int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} \, d(t), \]  

(21)

subject to the capital accumulation constraint:

\[ \dot{k}(t) = A k(t)^\alpha - c(t), \]  

(22)

where \( c \) is per-capita consumption, \( \rho \) is the rate of time preference, \( \sigma \) is the inverse of the intertemporal elasticity of substitution, and \( \dot{k} = \frac{dk}{dt} \). The rate of capital depreciation is assumed to be zero.

The maximization of the representative household's overall utility in equation (21) implies that the growth rate of consumption per labor units at each point of time is given by

\[ f(\infty)=0, f'(0)>0, f'(\infty)<0, f(0)=0, \text{ and } f'(\infty)=0. \]

Twice differentiable, with \( f(0)=0, f'(k)>0, f''(k)<0, f(0)=\infty, \text{ and } f'(\infty)=0. \)
\[
\frac{\dot{c}(t)}{c(t)} = \sigma^{-1}[f'(k) - \rho].
\]  

(23)

For the production technology in (20), equation (23) can be rewritten as:

\[
\frac{\dot{c}(t)}{c(t)} = \sigma^{-1}[A\alpha k^{(\alpha-1)} - \rho].
\]  

(24)

The growth rate of consumption per capita depends positively on the difference between the returns to capital and the discount factor, and negatively on the inverse of the intertemporal elasticity of substitution. During the economy's transition to a steady state path, the economy may exhibit positive or negative growth rates. However, due to decreasing returns to capital, consumption, capital, and output all in per capita units grow at a zero rate in the steady state.

In the neoclassical growth model, imposition of an (unanticipated) income tax or an increase in the income tax rate lowers the rate of return on capital and generates a shift in the level of the steady state path, but does not affect the steady state growth rate. During the transition period, an increase in the income tax rate \( \tau \), \( 0 < \tau < 1 \), causes the growth rate,

\[
\frac{\dot{c}(t)}{c(t)} = \sigma^{-1}[(1 - \tau)A\alpha k^{(\alpha-1)} - \rho],
\]  

(25)

to decrease immediately, but in per labor units it eventually goes to zero in the steady state.

1.4.2 One-Sector Endogenous Growth Models with Constant Returns To Capital

Following Rebelo (1991) and Rebelo and King (1990) we start with a simple endogenous growth model to study the effects of government tax policies. Instead of assuming diminishing returns, we assume a one sector economy with a linear production technology based on Rebelo (1991):

\[
y(t) = A k(t)
\]  

(26)

where \( A > 0 \) is the constant net marginal product of capital, and \( A > \rho > A(1 - \rho) \).
For the preferences given by (21), consumption, capital, and output in per-capita units all grow at the same rate, given by the equation (23). Substituting \( f'(k) = A \) into equation (23) gives the steady state growth rate of the economy:

\[
\frac{\dot{c}(t)}{c(t)} = \sigma^{-1}[A - \rho]. \tag{27}
\]

Equation (27) says that the growth rate depends on the intertemporal elasticity of substitution and the difference between the net marginal product of output and the time preference \( \rho \). Since the social and the private rates of return to investment are the same, the undistorted equilibrium is Pareto optimal.

An unanticipated income tax, \( \tau \), reduces the real return on investment from \( f'(k) = A \) to \( f'(k) = [A(1 - \tau)] \) and lowers the long run growth rate of the economy to:

\[
\frac{\dot{c}(t)}{c(t)} = \sigma^{-1}[A(1 - \tau) - \rho]. \tag{28}
\]

Unlike basic neoclassical growth models, there are no transitional dynamics, and an increase in the income tax rate has an effect both on the short and long run growth rates. A calibrated model by Rebelo and King (1990) shows that a 10% increase in the income tax rate from 20% to 30% reduces the growth rate of the economy from 2% to 0.37%. 