Chapter 3: Extension to Multiply-Connected Domains

The theory presented so far is applicable to most simply-connected domains. This could be the case of single-can combustors, ramjets, etc. However, all current gas-turbine combustors are multiply-connected domains, that is, they consist of several separate flow-paths that interact with each other. In this chapter a general straight-flow annular gas-turbine combustor configuration is introduced first in order to extend the previous model to multi-path flows. The principal issues associated with these configurations are identified: division of an inlet single stream into several paths, interaction between these paths through dilution holes, and boundary conditions at the exits of all these paths. Finally these same issues are reconsidered for two types of reverse-flow annular combustors.

3.1. General Configuration for Multiply-Connected Domains (Straight-Flow Combustors)

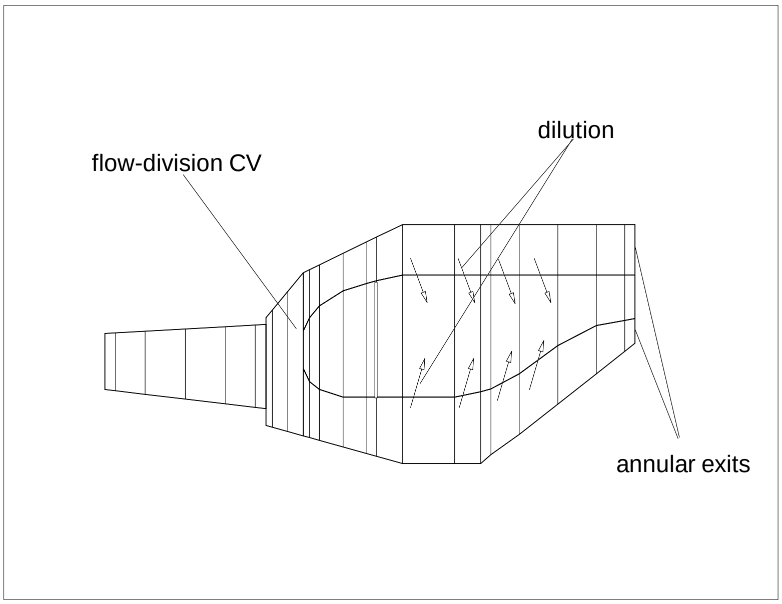

Figure 3. 1: Generic straight-flow annular combustor

Figure 3.1 shows a typical computational mesh for a straight-flow annular combustor, similar to the one presented in Fig. 1.1. It helps to identify the main issues associated with its analysis within a one-dimensional framework:

- division of the main flow into primary and annular streams,
- interaction between annular and primary flow-paths through dilution holes,
- exit boundary-conditions for annular flows.

For convenience the main flow, annular flows, and primary flow up to the dome are considered perfect-gas flows and are numerically treated with a simplified version of the model presented in Chapter 2. In particular, only three equations are needed: continuity, momentum and energy.

3.2. Flow Division

The flow division in Fig. 3.1 can not be handled in the usual one-dimensional fashion. Steady-state considerations show that a uniform state on the face would not satisfy general exit boundary conditions (Rodriguez and O'Brien, 1997). Furthermore the projection stage in the unsteady procedure will by itself give non-uniform conditions, at least for q_R.

The main problem is the determination of the vector fluxes $\vec{F}_u$, $\vec{F}_c$, $\vec{F}_l$ [see Fig 3.2, a)]. The following procedure was found to be adequate and robust enough for time-integration.

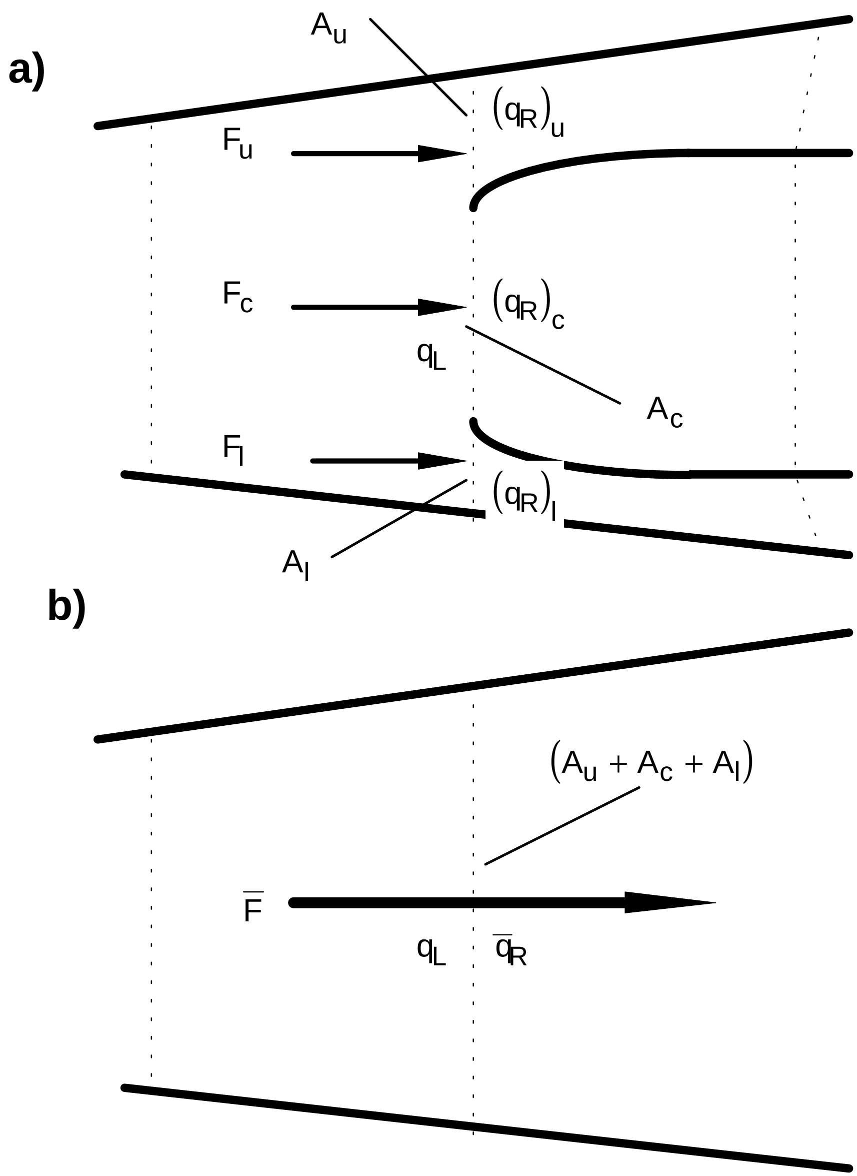

Figure 3. 2: Flow-division interface.

1. Projection of the main variables into the division interface

Starting with an initial solution the main variables are extrapolated to the flow-division interfaces with the MUSCL scheme described in Section 2.4. It was found that one-sided (left and right) interpolations ($\kappa = 1$ in equations 2.8) gave the best results from the point of view of robustness of the procedure. Therefore there will be a single value for q_L and three for q_R [Fig. 3.2, a)] (vector notation will be dropped unless otherwise specified).

2. Average of the projected values

The three right states $(q_R)_u$, $(q_R)_c$, $(q_R)_l$ are area-averaged into a single value, $\bar{q}_R$ [Fig. 3.2, b)].

3. Calculation of average flux

The pair q_L, $\bar{q}_R$ is used to calculate a single flux $\bar{F}$ using the procedure of Section 2.4. This single value can be considered an area average for the entire interface.

4. Redistribution of the average flux

If $\bar{F}$ is to be considered an area-average we can therefore write an equation for conservation of total flux

$$\vec{\bar{F}}\left(A_u + A_c + A_l\right) = \vec{F}_u A_u + \vec{F}_c A_c + \vec{F}_l A_l \qquad (3.1)$$

The above is a vector equation. Since the flow division is assumed to occur in the perfect-gas region there will be three components of fluxes. Therefore equation 3.1 represents a system of three equations in nine unknowns, the three components of each flux $\vec{F}_u$, $\vec{F}_c$, $\vec{F}_l$. As a result there is no unique solution; one-dimensional assumptions can not univocally determine the flow-division problem, which is essentially two-dimensional. As a result additional assumptions or constraints have to be brought into play in order to close the system, ensure physically-meaningful solutions, and better approximate available data.

In the present model the following constraints are used:

- Fractions of flow into the annular streams:

From q_R the flow-rates $\left(w_{u,c,l}\right)_R$ associated with right (R) (conditions can be obtained. Therefore the fractions of flow into the upper and lower annular streams

$$\alpha_{u,l} = \frac{\left(w_{u,l}\right)_R}{\left(w_u\right)_R + \left(w_c\right)_R + \left(w_l\right)_R} \tag{3.2}$$

are imposed. This imposition can be justified on the assumption that for one-dimensional subsonic flows the exit boundary-conditions (to the “right” of the interface) mainly determine the flow-fractions into the different streams (everything else being equal).

- Distribution of total pressure and temperature

It is assumed that the following total pressure and temperature ratios are known:

$$\beta_{u,l} = \frac{\left(p_t\right)_c}{\left(p_t\right)_{u,l}} \tag{3.3}$$

$$\gamma_{u,l} = \frac{\left(T_t\right)_c}{\left(T_t\right)_{u,l}} \tag{3.4}$$

Usually $\gamma_{u,l} = 1$ can be assumed. On the other hand, the ratios $\beta_{u,l}$ can have an impact in determining how much flow goes into each path. Its estimate will have to be based on available data for a given combustor, or from experience with similar combustors.

Equations 3.1-3.4 give a system of nine equations with nine unknowns. For perfect gases it can be adequately manipulated into a nonlinear system of three equations in the unknowns $\rho_{u,c,l}$.

Once the fluxes are known, the CVs on either side of the division can be handled in the usual way.

A separate issue is the implicit time-integration of this particular CV. The problem is the coupling of all the flow-paths, eliminating the tridiagonal form of the equations (see section 2.8.2). One way to handle this problem is by means of zonal boundary conditions. More details can be found in Appendix A.

3.3. Dilution Flows

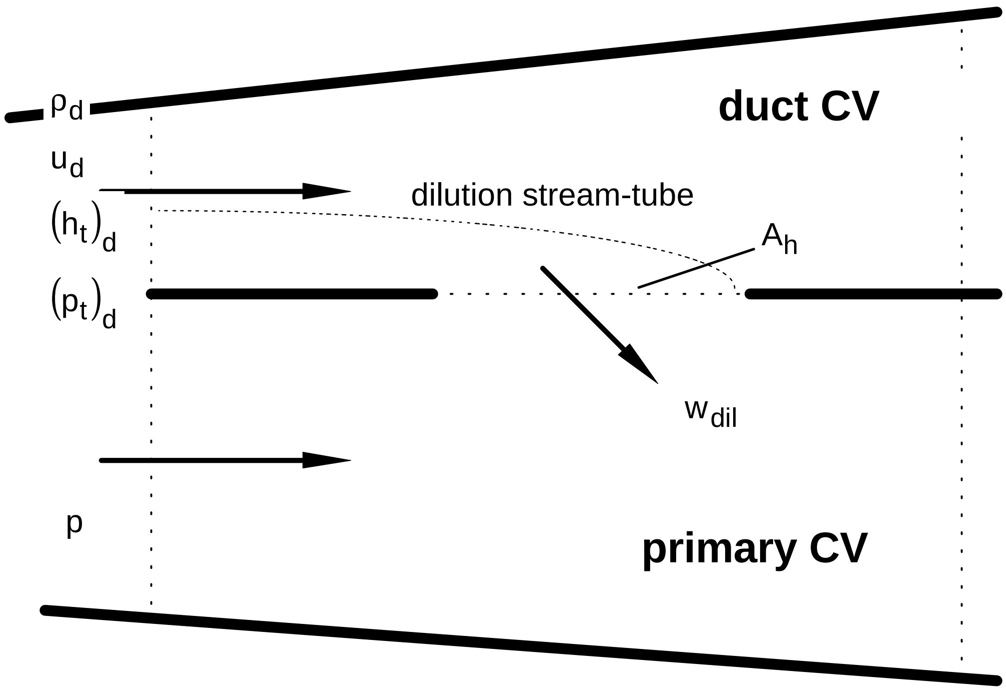

Figure 3. 3: Dilution characteristics.

Consider the situation represented in Fig. 3.3. Two superimposed CVs are connected through a dilution hole of area A_h (if it were a band of dilution holes, it would be the aggregate area). The main assumption is that flow will go from the CV with higher static pressure to the lower-pressure one. They will be called **duct** and **primary** CVs for convenience (they are not necessarily equivalent to annular and primary flow-paths).

Additional assumptions are:

- dilution flow can be modeled as quasi-steady, incompressible, perfect-gas,
- all dilution properties correspond to upstream-interface values,
- total pressure in the dilution stream-tube is that of the duct CV; static pres-

sure is that of the primary CV.

With these assumptions the dilution flow-rate will be given by (Lefebvre, 1983)

$$w_{dil} = C_D A_h \sqrt{\frac{2}{\rho_d}\left(\left(p_t\right)_d - p\right)} \qquad (3.5)$$

where C_D is the discharge coefficient, and the subindex *d* denotes duct values.

Associated with this dilution flow-rate (essentially a mass transfer) there will be transfers of axial momentum u_d and energy $\left(h_t\right)_d$ per unit mass (these properties correspond to their upstream-face values at the duct CV - see Fig. 3.3). These transfers will be subtracted from the duct CV and added to the primary CV. These transfers will appear as the terms w_K, u_K, h_K in the expressions for the source terms (equation 2.4).

3.4. Annular-Exit Boundary-Conditions

As mentioned before, the annular exits of Fig. 3.1 may be either opened or closed. If opened, the same type of exit boundary-condition is used as for primary flow (see Section 2.5). If closed, no-penetration conditions are applied:

- zero velocity
- zero derivatives of all main variables.

3.5. Reverse-Flow Annular Combustors

3.5.1. Open-inlet primary

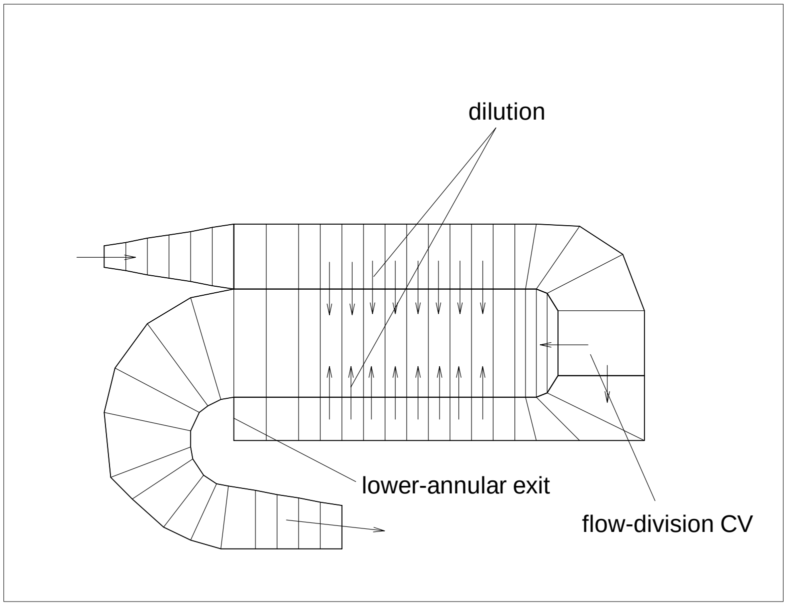

Figure 3. 4: Reverse-flow combustor configuration (open-inlet primary)

Figure 3.4 shows the computational mesh for a reverse-flow, open-inlet primary, annular combustor. Of the three main issues associated with multiply-connected domains, the interaction between the flow-paths and the annular-exit boundary conditions are treated in the same fashion as for straight-flow combustors. The flow division will be treated differently.

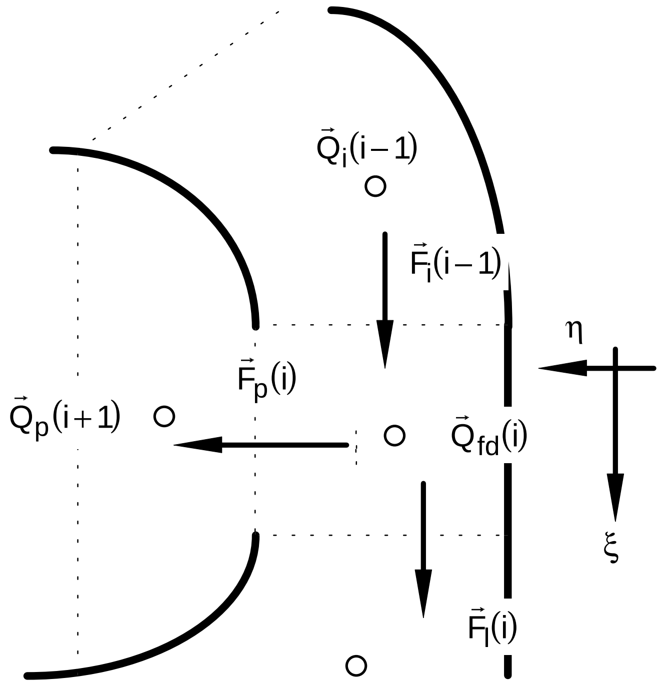

Figure 3. 5: Flow-division CV - Open-primary reverse-flow combustor

Figure 3.5 shows a detailed view of the flow-division CV (denoted with index i) for an open-primary reverse-flow combustor. It represents a boundary for the inlet, primary and lower-annular flow-paths; the variables associated with these paths are denoted by subscripts i, p, l respectively.

Assuming calorically-perfect gas the governing equations for CV i will be

$$\Delta V(i)\frac{\partial}{\partial t}\vec{Q}_{fd}(i) \quad + \quad \vec{F}_p(i)\,A_p(i) \quad + \quad \vec{F}_l(i)\,A_l(i) \quad - \quad \vec{F}_i(i-1)\,A_i(i-1) \quad = \quad \vec{W}_{fd}(i) \tag{3.6}$$

which now represents four conservation equations: continuity, momentum for each direction ξ and η (as shown in Fig 3.5), and energy. Therefore equation 3.6 corresponds essentially to a two-dimensional CV. The vector of main variables will be

$$\vec{Q}_{fd} = \begin{bmatrix} \rho \\ \rho u_\xi \\ \rho u_\eta \\ \rho e_t \end{bmatrix} \quad (3.7)$$

where u_ξ, u_η are the velocity components in each direction in the flow-division. The fluxes will be given by

$$\vec{F}_{i,l} = \begin{bmatrix} \rho u_\varepsilon \\ \rho u_\xi^2 + p \\ 0 \\ \rho u_\varepsilon h_t \end{bmatrix}_{i,l} \qquad \vec{F}_p = \begin{bmatrix} \rho u_\eta \\ 0 \\ \rho u_\eta^2 + p \\ \rho u_\eta h_t \end{bmatrix}_p . \quad (3.8)$$

Note the velocity at each face of the CV is assumed to be normal (i.e., no tangential component), as in a one-dimensional volume.

Finally the sources will be

$$\vec{W}_{fd}(i) = \begin{bmatrix} 0 \\ \overline{p}_{wall}\left[A_l(i) - A_i(i-1)\right] - w_p(i)\,u_i(i-1) \\ \overline{p}_{wall}\,A_{wall}(i) \\ 0 \end{bmatrix} \quad (3.9)$$

where

$$\overline{p}_{wall} = \frac{1}{2}\left[p_l(i) + p_i(i-1)\right] \quad (3.10)$$

is the average pressure acting on casing side of the CV (with area $A_{wall}(i)$). The second term in the second component represents a dilution-type loss of momentum in the ξ-direction (see equation 2.4 and Section 3.3).

Equation 3.6 is integrated in time with the same procedure as the rest of the combustor. If the procedure is explicit no special treatment is needed; the CV is integrated on its own anyway. If implicit, it has to be uncoupled from the rest of the combustor. This is achieved by means of zonal boundary-conditions at the three flow-areas (see Appendix A for more details on zonal boundary conditions).

3.5.2. Closed-inlet Primary

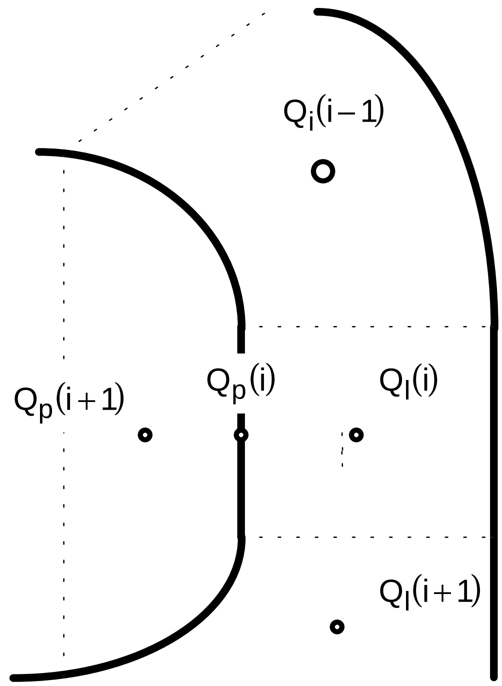

Figure 3. 6: Closed-inlet reverse-flow combustor

In the case of a closed-inlet primary (Fig. 3.6) there is no flow-division proper. All the air into the primary flows through the dilution holes only. Now the “division” CV i is treated as any other one-dimensional CV (in this case belonging to the lower-annular path). To solve the primary flow boundary conditions need to be applied at the closed inlet. In this case the variable $Q_p(i)$, assumed right at the inlet, is evaluated by means of the usual no-penetration conditions (no flow, zero-gradients for properties).