

**ONE-DIMENSIONAL, FINITE-RATE MODEL
FOR GAS-TURBINE COMBUSTORS**

by

Carlos G. Rodriguez

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Walter F. O'Brien, Chairman

Uri Vandsburger

John Moore

Wing F. Ng

Balakrishnan Ganeshan

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Committee Chairman: Walter F. O'Brien

Department of Mechanical Engineering

Virginia Polytechnic Institute and State University

Abstract

An unsteady, finite-rate, one-dimensional model has been developed for the analysis for gas-turbine combustors. The basis of the model is the one-dimensional, integral form of the conservation equations for multi-species, non-equilibrium, reacting mixtures. Special procedures were devised for the flow-division of the inlet flow into primary- and annular-flows, for both straight- and reverse-flow combustors. This allows the model to handle complete combustor configurations, which at present are beyond the reach of more sophisticated CFD tools. The model was validated with a steady-state analytical solution for a basic problem, and with steady-state results from a production code applied to a production combustor. Additional calculations show the ability of the code to predict blow-out due to rich and lean mixtures, and to predict the response of a combustor to perturbations in operating and boundary conditions.

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Nomenclature

Note: In this work all units are in SI (metric).

a	Frozen speed of sound
A	Arrhenius pre-exponential factor
A	Jacobian matrix of flux vector
$A(i)$	Flow-area of face i
$A_{\text{wall}}(i)$	Wall or wetted area of CV i
C_D	Discharge coefficient
C_f	Friction coefficient
C_{Vj}	Constant-volume specific-heat of specie j
E_a	Arrhenius activation energy
E_i	i -th eigenvector of system of governing equations
e_t	Absolute total energy
\vec{F}	Vector of fluxes
f/a	Fuel/air ratio
$(h_f^0)_j$	Enthalpy of formation of specie j
F_{wall}	Axial component of wall force
h_t	Absolute total enthalpy
M	Mach number
MW_j	Molecular weight of specie j
N	Total number of CVs
N_s	Total number of species in the system
p	Static pressure

p_t	Total pressure
\vec{Q}	Vector of conservative variables
\vec{q}	Vector of primitive variables
\vec{R}	Residual vector
R	Mixture gas-constant
R_j	Gas-constant for specie j
R_u	Universal gas constant
T	Matrix of right eigenvectors of matrix A
T	Temperature
T_t	Total temperature
t	Time
Δt	Time-step
u	Axial velocity
$\Delta V(i)$	Volume of CV i
w	Mass flow-rate
\vec{W}	Vector of source terms
w_{fuel}	Fuel flow-rate
Δx	Spatial-discretization step
y_j	Mass-fraction of specie j

Greek Symbols

α_i	Wave strength associated with eigenvalue i
$\alpha_u(l)$	Fraction of total mass flow-rate into the upper (lower) annular path
$\beta_u(l)$	Ratio of central-path to upper-(lower-)path total pressure
$\gamma_u(l)$	Ratio of central-path to upper-(lower-)path total temperature
Λ	Diagonal matrix of eigenvalues of matrix A
λ_i	i -th eigenvalue of system of governing equations
ν	Stoichiometric fuel/air ratio
$\dot{\omega}_j$	Mass production rate per unit volume of specie j
$\hat{\omega}_j$	Molar production rate per unit volume of specie j
ϕ	Equivalence ratio
ρ	Total density of the mixture
ρ_j	Density of specie j

Subscripts/superscripts

$()_{\text{dil}}$	Dilution property
$()_{\text{exit}}$	Exit value
$()_{\text{inlet}}$	Inlet value
$()_{\text{L (R)}}$	Left (right) values at an interface
$()_{\text{pz}}$	Primary-zone value
$()_{\text{u (c, l)}}$	Upper-(center-, lower-)path value
$()^{\pm}$	Values associated with right-/left-running waves
$()^n$	Value at time-step n