Appendix F:

Optimality of Design with Points at the Extremes for Restricted Regions for the *k*-variable Interaction and No Interaction Cases

The general proof of optimality for the k-variable D-optimal no interaction design with points at the extremes of the restricted region follows the same basic pattern as the two variable interaction case . The form of this design consists of a control point and k pure component points, each estimating its respective main effect. The form of this design is shown in Table F.1 below.

Table F.1 General form of a k-regressor no interaction (main effect) design on a restricted region.

Point	x_1	x_2	x_3	.	.	.	x_k
1	1	1	1	.	.	.	1
2	$(0.1353 + c_1)$	1	1	.	.	.	1
3	1	$(0.1353 + c_2)$	1	.	.	.	1
.	.	.	.	.			.
.	.	.	.		.		.
.	.	.	.			.	.
(k+1)	1	1	1	.	.	.	$(0.1353 + c_k)$

$$|\mathbf{I}| = Q_1 Q_2 \; . \; . \; . \; Q_k \tag{1}$$

where

$$Q_i = (0.1353 + c_i)\left[\ln(0.1353 + c_i)\right]^2 . \tag{2}$$

Since each Q_i is a decreasing function in c_i, each c_i should be as small as possible to maximize the determinant. Thus, the design points are pushed to the extremes of the region.

Similarly, for the k-variable interaction case, the design is composed of the points in Table F.2.

Table F.2 General form of the k-regressor interaction (full) model.

Number of Design Points	Description
1	Control Point
k	Main Effect Points
$\binom{k}{2}$	Two Way Interaction Points
$\binom{k}{3}$	Three Way Interaction Points
.	.
.	.
.	.
$\binom{k}{k-1}$	(k-1)-Way Interaction Points
1	k-Way Interaction Points

Each m-way interaction point has the m elements of $\mathbf{q_i}$ vector corresponding to the effects in the interaction set at 0.1353. The remaining k-m are set at 1, or the individual control for the other than those involved in the interaction.

The form of the determinant in this case is

$$|\mathbf{I}| = Q_1^k Q_2^k \ldots Q_k^k. \tag{3}$$

Since each Q_i is a decreasing function in c_i, making c_i as small as possible will maximize $|\mathbf{I}|$ for the k-variable interaction case.