CHAPTER 6
RESISTANCE FACTOR FOR
THE DESIGN OF COMPOSITE SLABS

6.1. General

Probability-based design criteria in the form of load and resistance factor design (LRFD) are now applied for most construction materials. The design requirements have to insure satisfactory performance of structures. The main advantage of the approach is the ability to achieve a uniform level of reliability for structural members, or to impose a certain level of reliability (higher or lower) of some certain parts of the structures. This gives a strong rationale to the load and resistance factors as compared to the design safety factors of the allowable stress design. Additionally, a unified design strategy as to setting up common load combinations and load factors can be obtained.

In this part of the study, resistance factors, ϕ, for the flexural design of composite slabs were evaluated based on test data of 39 full scale composite slab specimens. The tests were performed at the Structures and Materials Laboratory of Virginia Polytechnic Institute and State University, Blacksburg, Virginia. The ϕ factors evaluated correspond to the SDI-M method and direct method described in Section 3.

6.2. Review of Probabilistic Concepts of Load and Resistance Factor Design

Discussions on the probabilistic concepts of the LRFD approach are given in detail by many sources (Cornell 1969, Lind 1971, Ang and Cornell 1974, Galambos and Ravindra 1977, Ravindra and Galambos 1978, Ellingwood et al. 1980, *Load and* 1986, Hsiao et al. 1990, Geschwindner et al. 1994, *Commentary on* 1996, Barker and Puckett 1997).

In principle the following inequality applies:

$$\phi R_n \geq \sum_i \gamma_i Q_i \tag{6-1}$$

in which, R_n = nominal resistance, Q_i = load effect, ϕ = resistance factor and γ_i = load factor. The probability of failure can be expressed by:

$$p_f = 1 - \Phi(\beta) \tag{6-2}$$

where Φ is the standard normal probability function, and β is the reliability index.

6.2.1. Reliability Index

The reliability index, β, used in Eqn. (6-2), in a log-normal format, can be expressed by:

$$\beta = \frac{\lambda_R - \lambda_Q}{\sqrt{\zeta_R^2 + \zeta_Q^2}} \approx \frac{\ln\left(\frac{R_m}{Q_m}\right)}{\sqrt{V_R^2 + V_Q^2}} \tag{6-3}$$

where λ, ζ and V, respectively, denote the log-normal mean, log-normal standard deviation and the coefficient of variation. Subscript R and Q denote the resistance and the load effect, respectively. R_m and Q_m are the means of resistance and load, respectively. Introduce a linearization given by:

$$\sqrt{V_R^2 + V_Q^2} = \alpha\left(V_R + V_Q\right) \tag{6-4}$$

then Eqn. (6-3) can further be approximated as:

$$\beta = \frac{\ln\left(\frac{R_m}{Q_m}\right)}{\alpha\left(V_R + V_Q\right)} \tag{6-5}$$

where according to Lind (1971), for $1/3 \le V_Q / V_R \le 3$, α = 0.75 gives a good approximation with ±6% maximum error. Equation (6-5) forms the basis equation for the AISC and AISI load and resistance factor design specification for structural steel and cold-formed steel. From Eqn. (6-5), the central safety factor can be expressed as:

$$\theta = \frac{R_m}{Q_m} = e^{\alpha\beta\left(V_R + V_Q\right)} \tag{6-6}$$

By minimizing the error of this central safety factor, Galambos and Ravindra (1977) suggested a value of α = 0.55 which was later adopted in AISC LRFD. The reliability index, β, can be determined from Eqn. (6-5). As an illustration, the following table shows some β values and the corresponding probability of failure, p_f.

Table 6-1. β vs. p_f

β	p_f
5.0	2.9×10^{-7}
4.0	3.2×10^{-5}
3.0	1.4×10^{-3}
2.0	2.3×10^{-2}

AISC-LRFD uses the following β values:

β = 3.0, for members, under DL + LL or Snow

β = 4.5, for connections, under DL + LL or Snow

β = 2.5, for members, under DL + LL + Wind

β = 1.75, for members, under DL + Earthquake whereas the AISI-LRFD uses the following β values:

β = 2.5, for members

β = 3.5, for connections

Galambos et al. (1982) give β values for various structural members under conditions of ratio of basic specific live load to normal value of dead load equal to 1, 2 and 5. Ranges of β values were also given by Ellingwood et al. (1980). These values of β range between 1.9 - 3.5 for reinforced concrete members and 3.0 - 4.5 for steel members.

6.2.2. AISC LRFD Approach for the Resistance Factor

Using the central safety factor given in Eqn. (6-6), the following inequality can be written:

$$R_m \geq \theta\, Q_m \tag{6-7}$$

which leads to:

$$R_m\, e^{-\alpha\beta V_R} \geq Q_m\, e^{\alpha\beta V_Q} \tag{6-8}$$

or,

$$\phi\, R_n \geq \gamma\, Q_n \tag{6-9}$$

in which, R_n and Q_n are nominal values of resistance and load,

$$\phi = \frac{R_m}{R_n} e^{-\alpha\beta V_R} \tag{6-10}$$

$$\gamma = \frac{Q_m}{Q_n} e^{\alpha\beta V_Q} \tag{6-11}$$

Further, the mean resistance, R_m, can be expressed in terms of the nominal resistance and statistical parameters that represent the variability of material strength and stiffness, M, fabrication, F, and the uncertainties involved in the assumptions of the engineering design equation, P (Ravindra and Galambos 1978):

$$R_m = R_n \left(M_m F_m P_m\right) \tag{6-12}$$

where M_m, F_m and P_m are the means of M, F, and P, respectively. Accordingly, the coefficient of variation of the resistance can be approximated by using:

$$V_R \approx \sqrt{(V_M)^2 + (V_F)^2 + (V_P)^2} \tag{6-13}$$

in which, V_M, V_F and V_P are, respectively, the coefficients of variation of M, F and P. Here, Eqn. (6-13) assumes independent relations among M, F and P variables. By using Eqn. (6-12), the resistance factor given by Eqn. (6-10) can be modified to:

$$\phi = \left(M_m F_m P_m\right) e^{-\alpha\beta V_R} \tag{6-14}$$

6.2.3. AISI LRFD Approach for the Resistance Factor

The AISI specification for cold-formed steel structures follows a different approach in determining the resistance factor, ϕ. The approach is based on the research by Hsiao et al. (1990). Instead of using Eqn. (6-10), it starts by expressing the effective resistance in terms of the nominal loads and load factors multiplied by a deterministic coefficient, c, that relates the load intensities to the load effect and is given by:

$$\phi R_n = c\left(\gamma_D D_n + \gamma_L L_n\right) = \left(\gamma_D \frac{D_n}{L_n} + \gamma_L\right) c L_n \tag{6-15}$$

where γ_D and γ_L are the dead and live load factors, and D_n and L_n are the nominal values of the dead and live load. Similarly, the mean of the load effect can be expressed as:

$$Q_m = \left(1.05\,\frac{D_n}{L_n} + 1\right) c\, L_n \tag{6-16}$$

Notice that in the last equation, $D_m = 1.05\, D_n$ and $L_m = L_n$ were used (based on load statistic by Ellingwood et al. 1980). From Eqns. (6-15) and (6-16), one obtains:

$$\frac{R_m}{Q_m} = \frac{\psi}{\phi}\,\frac{R_m}{R_n} \tag{6-17}$$

with,

$$\psi = \left(\gamma_D\,\frac{D_n}{L_n} + \gamma_L\right) / \left(1.05\,\frac{D_n}{L_n} + 1\right) \tag{6-18}$$

By combining Eqns. (6-3), (6-12) and (6-17), an expression of the resistance factor can be obtained:

$$\phi = \psi\left(M_m F_m P_m\right) e^{-\beta\sqrt{V_R^2 + V_Q^2}} \tag{6-19}$$

Using this equation, determination of the α coefficient can be avoided. However, the coefficient of variation of the load has to be known.

6.3. Statistical Data

Evaluation of the resistance factor, ϕ, as given by Eqn. (6-14) or (6-19) requires statistical values of the parameters involved. These data are available from the lab tests conducted on the composite slab specimens previously mentioned. However, larger sets of database are preferred to give more representative values of means, standard deviations, and coefficients of variation of the afore-mentioned parameters. Therefore, statistical values from other sources that were based on larger sets of database were used. These values are the statistical values of the concrete compressive strength, f_c', which was based on the study by

MacGregor (1997), and steel deck yield stress, f_y, which was based on the study on cold-formed steel members by Hsiao et al. (1990). For these two parameters, the data obtained from the lab tests from the composite slab specimens were used as a comparison only.

Data obtained from the lab tests, which are not available elsewhere from larger sets of database, were used for the determination of the resistance factor. These data are the statistical data of deck thickness, t, maximum and minimum shear bond strength at the interface of steel deck - concrete, $f_{s,max}$ and $f_{s,min}$, respectively.

6.3.1. Material Factor, M

The material factor, M, represents the variability of the strength and stiffness of the material. In this case, M is affected by the variability of f_c', f_y, $f_{s,max}$ and $f_{s,min}$. Statistical data of these parameters are listed in Table 6-2.

Table 6-2. Statistical data of f_c', f_y, $f_{s,max}$ and $f_{s,min}$

	μ	σ	V
fc' (MacGregor, 1997)	3940 psi	615 psi	0.156
fc' (test)	3867 psi	878 psi	0.227
fy (Hsiao et al. 1990)	1.100 fy	0.121 fy	0.110
fy (test)	1.002 fy	0.058 fy	0.058
fs,max	0.999 fs,max	0.035 fs,max	0.035
fs,min	1.001 fs,min	0.073 fs,min	0.073

Note: μ = mean, σ = standard of deviation, V = coefficient of variation

Assuming that those parameters are statistically independent, coefficients of variation of the material factor can be approximated by:

$$V_{M,SDI} = \sqrt{V^2_{f_c'} + V^2_{f_y}} = 0.191 \tag{6-20}$$

for the SDI-M method while for the Direct method:

$$V_{M,Direct} = \sqrt{V^2_{f_c'} + V^2_{f_y} + V^2_{f_{s,max}} + V^2_{f_{s,min}}} = 0.208 \tag{6-21}$$

for the SDI-M or direct design procedure, respectively. The mean values of M for the SDI-M and direct design procedures can be evaluated from:

$$M_{m,SDI} = \left(\frac{f_c{}'_{,m}}{f_c{}'}\right)\left(\frac{f_{y,m}}{f_y}\right) = \left(\frac{\mu_{f_c'}}{f_c{}'}\right)\left(\frac{\mu_{f_y}}{f_y}\right) = 1.445 \tag{6-22}$$

$$M_{m,Direct} = \left(\frac{f_c{}'_{,m}}{f_c{}'}\right)\left(\frac{f_{y,m}}{f_y}\right)\left(\frac{\left(f_{s,max}\right)_m}{f_{s,max}}\right)\left(\frac{\left(f_{s,min}\right)_m}{f_{s,min}}\right)$$

$$M_{m,Direct} = \left(\frac{\mu_{f_c'}}{f_c{}'}\right)\left(\frac{\mu_{f_y}}{f_y}\right)\left(\frac{\mu_{f_{s,max}}}{f_{s,max}{}'}\right)\left(\frac{\mu_{f_{s,min}}}{f_{s,min}}\right) = 1.397 \tag{6-23}$$

6.3.2. Fabrication Factor, F

The fabrication factor, F, represents the variability of the manufacturing process. In this case, the variability of the steel deck thickness, t, is considered. The statistical data for this steel deck thickness are listed in Table 6-3. These data were based on the measurement conducted on the steel decks that were used for the composite slab specimen tests.

Table 6-3. Statistical data of t

μ t	σt	Vt
0.966 t	0.030 t	0.313

Based on the above statistical values, V_F and F_m can be computed as follow:

$$V_F = V_t = 0.313 \tag{6-24}$$

$$F_m = \frac{t_{,m}}{t} = \frac{\mu_t}{t} = 0.966 \tag{6-25}$$

6.3.3. Professional Factor, P

The professional factor, P, takes into account the uncertainties of the design equation. This professional factor is defined as (Ravindra and Galambos 1978, Geschwindner et al. 1994):

$$P = \frac{\text{test}}{\text{prediction}} \tag{6-25}$$

The *prediction* is the resistance of the slab as predicted by the design equation based on the measured (actual) values of its parameters. Based on the lab tests performed on the afore-mentioned full-scale composite slab specimens, the following statistical data is obtained:

Table 6-4. Statistical data of P

	μ p = Pm	σp	Vp
SDI	1.193	0.244	0.205
Direct	1.071	0.183	0.172

6.3.4. Load Statistic

Information regarding statistical data of the load in terms of the coefficient of variation, V_Q, is needed for the AISI approach as shown in Eqn. (6-19). For this reason, statistical data of dead and live loads were taken from a special publication of the National Bureau of Standards (Ellingwood et al. 1980). These data are summarized in Table 6-5. D_n and L_n denote the nominal dead and live loads.

Table 6-5. Statistical data of dead and live loads

	μ	σ	V
D	1.05 Dn	0.105 Dn	0.10
L	1.00 Ln	0.250 Ln	0.25

For the combination of the dead and live loads given by:

$$Q = \gamma_D D + \gamma_L L \tag{6-26}$$

the mean and standard of deviation of this combination can be expressed by:

$$\mu_Q = \gamma_D \mu_D + \gamma_L \mu_L \tag{6-27}$$

$$\sigma_Q = \sqrt{\mathrm{Var}(Q)} = \sqrt{\gamma_D^2\, \sigma_D^2 + \gamma_L^2\, \sigma_L^2} \tag{6-28}$$

assuming that the distribution of D and L are statistically independent. In Eqn. (6-28), Var(Q) denotes the variance of Q. By substituting values from Table 6-5 into Eqn. (6-27) and Eqn. (6-28), the coefficient of variation of Q can be obtained as:

$$V_Q = \sqrt{0.011\, \gamma_D^2 \left(\frac{D_n}{L_n}\right)^2 + 0.063\, \gamma_L^2} \Bigg/ \left(1.05\, \gamma_D \left(\frac{D_n}{L_n}\right) + \gamma_L\right) \tag{6-29}$$

6.4. The Resistance Factor

In this study, the AISI-LRFD approach is adopted. The AISC-LRFD approach is used to give a comparison. Considering the fact that composite slabs are generally used in steel framed structures, a β (reliability index) value greater than 3.0 is not considered necessary (β=3.0 for steel members). Therefore, β=3.0 is chosen as the target reliability index (AISI uses β=2.5 as the basic case). The final result of ϕ factors, however, is rounded to the closest 0.05 and hence, the actual β values used will not be exactly 3.0. A minimum limit of β=2.5 is used.

A load combination with γ_D = 1.2 and γ_L = 1.6 as given in the SDI Composite Deck Design Handbook (Heagler et al. 1997) is used as the basic load case. The combination using γ_D = 1.4 and γ_L = 1.0 is not considered because the ratio of dead to live load is typically < 3.0 for composite slabs. A range of dead to live load ratios between 0.5 (short to normal span slabs with relatively heavy live load, approximately 100 psf) and 1.5 (long span slabs up to 20 ft with relatively light live load, approximately 50 psf) is considered.

Based on the statistical data presented in section 6.3 and equations given in section 6.2, ϕ factors for several values of D/L (0.5, 1.0 and 1.5) were computed and the results are listed in Table 6-6 and Table 6-7 for the SDI-M and direct design procedures, respectively. Again, these results are based on the AISI-LRFD approach presented in section 6.2.3.

Table 6-6. Calculated ϕ factors for SDI-M method
(AISI-LRFD Approach)

	β		
D/L	3.00	2.75	2.50
0.5	0.8790	0.9558	1.0393
1.0	0.8773	0.9497	1.0282
1.5	0.8704	0.9403	1.0158

Table 6-7. Calculated ϕ factors for direct method
(AISI-LRFD Approach)

	β		
D/L	3.00	2.75	2.50
0.5	0.8112	0.8801	0.9548
1.0	0.8109	0.8757	0.9458
1.5	0.8052	0.8677	0.9350

Based on the results in Tables 6-6 and 6-7, $\phi = 0.90$ is chosen for the SDI-M method and $\phi = 0.85$ is selected for the direct method. For comparison, ϕ factors computed by using the AISC-LRFD approach are listed in Table 6-8 for the SDI-M method and Table 6-9 for the direct method for several combinations of α and β values. This later approach is not influenced by the ratio of the dead to live load (D/L). As shown in these tables, the choice of α between 0.65 to 0.75 show relatively close results to the AISI approach.

Table 6-8. Calculated ϕ factors for SDI-M method
(AISC-LRFD Approach)

	α		
β	0.55	0.65	0.75
3.00	1.046	0.961	0.883
2.75	1.087	1.006	0.931
2.50	1.130	1.053	0.982

Table 6-9. Calculated ϕ factors for direct method
(AISC-LRFD Approach)

	α		
β	0.55	0.65	0.75
3.00	0.957	0.882	0.813
2.75	0.993	0.922	0.856
2.50	1.031	0.963	0.900

6.5. Concluding Remarks

Resistance factors for the flexural design of composite slabs based on the SDI-M and direct methods have been presented. The AISI LRFD approach for evaluating the resistance factor was adopted. By this approach, the determination of the α coefficient is not necessary. A target reliability index β=3.0 and minimum limit of β=2.5 were used. This choice was based on the target reliability β=3.0 for steel members (AISC-LRFD), and the lower bound β=2.5 used in AISI-LRFD for the basic load case. The resulting resistance factors are ϕ=0.90 for the SDI-M method and ϕ=0.85 for the direct method. These ϕ values were based on a range of dead to live load ratios between 0.5 and 1.5, which is representative of typical composite slab designs.