Assessing Drought Flows For Yield Estimation

By

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Abstract

Determining safe yield of an existing water supply is a basic aspect of water supply planning. Where water is withdrawn from a river directly without any storage, the withdrawal is constrained by the worst drought flow in the river. There is no flexibility for operational adjustments other than implementing conservation measures. Where there is a storage reservoir, yields higher than the flow in the source stream can be maintained for a period of time by releasing the water in storage. The determination of safe yield in this situation requires elaborate computation.

This thesis presents a synthesis of methods of drought flow analysis and yield estimation. The yield depends on both the magnitude of the deficit and its temporal distribution. A new Markov chain analysis for assessing frequencies of annual flows is proposed. The Markov chain results compare very well with the empirical data analysis. Another advantage of the Markov chain analysis is that both high and low flows are considered simultaneously; no separate analyses for the lower and upper tails of the distribution are necessary.

The temporal distribution of drought flows is considered with the aid of the generalized bootstrap method, time series analysis, and cluster sequencing of worsening droughts called Waitt's procedure. The methods are applied to drought inflows for three different water supply reservoirs in Spotsylvania County, Virginia, and different yield estimates are obtained.

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Chapter 1: Introduction

In hydrology, the extreme events are floods and droughts. Floods are a problem when the peak flows are higher than the magnitudes that can be contained by natural water bodies or constructed infrastructure. The main components of concern for flood flow analysis are the magnitude and the frequency. Key components of drought are not only the magnitude (extent of the lack of water) and frequency (how often a particular severe drought will occur), but also the duration (the length of time this lack of water lasts). Preparing for a drought in the form of building reservoirs and other types of arrangements can be very costly. However, not preparing for drought can be even costlier. Droughts are typically associated with the following consequences: dry streambeds, fallen groundwater table, withered crops, low supply of certain commodities and water restrictions. Obviously, lack of water affects agriculture; for water-intensive industries, a shortage in the water supply could force a temporary shutdown of the industry, harming the local economy. Low flows present problems for assimilative capacity of streams, impede power generating potential of hydro units, and impair navigation. Aquatic life can be damaged during low flows, which may or may not have significant economic impacts, but definitely can present environmental problems. Finally, reduced recreational uses of streams and lakes hampered by low flows have economic impacts.

Implementation of instream flow requirements to protect environmental conditions poses special difficulties in satisfying offstream demands. For instance, suppose the federal government has regulated a minimum flow downstream of a reservoir that must always be met. As a drought worsens and the flow in the river drops below this minimum flow, the requirement will prevent withdrawal of water from the river and, therefore, hastens the depletion of the reservoir. However, if the minimum flow is not required, but just a guidance, then the uses can be prioritized against the available water supply. The authority may decide that providing potable water to its residents should have higher priority over potential environmental problems resulting from low flows harming aquatic life or waste dilution capabilities. It would also have to measure these impacts against the loss of potential shipping revenues if the flow becomes so low that it prohibits navigation in the river.

One of the supply variables is where the locality would obtain water if its water supply were emptied. If the locality holds an agreement with a neighboring jurisdiction, it might be possible to purchase water. If the neighboring area is also suffering through a drought, stringent conservation measures will have to be initiated.

The report is organized into six chapters. After the introduction, Chapter 2 contains the details of the system considered along with a system schematic and a set of data tables. Chapter 3 presents the methodology for safe yield calculation. The historical droughts are analyzed and rank ordered. The confidence levels for flows with various probabilities are obtained by using the generalized bootstrap method and the Box-Cox transformation to normality. To select drought volumes with specified return periods, a new Markov chain analysis is presented. Even though available data are limited, the method duplicates the historically observed trends well. Chapter 4 details Waitt's procedure and a synthetic flow generation procedure for flow sequencing. The yield calculation procedure is detailed in Chapter 5. Finally, Chapter 6 is a summary of the thesis.

Chapter 2: System Description

The system considered is comprised of three reservoirs in Spotsylvania County, Virginia, a fast-growing county about fifty miles south of Washington, D.C. The three reservoirs of interest are Hunting Run Reservoir, Motts Run Reservoir, and Ni River Reservoir. A schematic of the system is given in Figure 2.1. The Hunting Run reservoir is currently under construction, and the other two are already in use. This system of reservoirs provides drinking water to Spotsylvania County and the City of Fredericksburg. Hunting Run and Motts Run are offline reservoirs that receive pumped water from the Rapidan and the Rappahanock River respectively, while the Ni River Reservoir is directly fed by the Ni River.

Hunting Run Reservoir

As mentioned above, the Hunting Run Reservoir is currently under construction. It is being built to complement the other reservoirs in providing drinking water for Spotsylvania County. The design safe yield of the reservoir, as calculated by consulting engineers Hayes, Seay, Mattern, & Mattern, is 8.0 million gallons per day (mgd). The reservoir is located on Hunting Run, a tributary of the Rapidan River just above its confluence with the Rappahanock River. It has a capacity of 2196 million gallons (mg).

The reservoir is designed so that inflows will consist of natural runoff from the Hunting Run watershed, as well as water pumped from the Rapidan River when the river flow is above the specified minimum instream flows. The instream flow requirements are based on the flow in the river as well as the storage in the reservoir. The Hunting Run watershed (4,620 acres) is very small compared with the Rapidan River watershed at the pumping site (686 square miles, or approximately 439,000 acres). As a result, for the analysis, only the water available for pumping is of interest. As seen in Figure 2.1, the reservoir is on the west end of the county. Its watershed is very rural, with not much development. The pumping site on the Rapidan is approximately halfway between USGS gauging sites at Culpeper (on the Rapidan River) and Fredericksburg (on the Rappahanock). Since the pumping site is upstream of the confluence of the Rapidan and Rappahanock, the Culpeper site was used for the analysis. The drainage area at Culpeper is 472 square miles, which results in a drainage area ratio of 1.486 for the adjustment of flows measured at the Hunting Run pumping site. The maximum pumpage rate is limited to 24 mgd. The instream flow requirements are shown in Table 2.1.

Even though the analysis given in chapter 3 indicates the 1931 to be the worst drought on record, the yield analysis procedure identifies the 9-month period May 1965 – January1966 to be the critical period for Rapidan River inflows. The critical period in this case is defined as the period that results in the reservoir emptying in the yield analysis. The same critical period applies to the Motts Run reservoir as well; for the Ni River reservoir, the 9-month critical period starts in June 1980 and ends in Feb 1981. The flows are shown in Table 2.2 for the Rapidan River.

Motts Run Reservoir

Motts Run Reservoir provides drinking water to Spotsylvania County. The safe yield of the reservoir is 3.4 mgd. The reservoir is located on a tributary (Motts Run) to the Rappahanock River downstream of its confluence with the Rappahanock River, and just upstream from the City of Fredericksburg. It has a capacity of 1205 mg. Similar to Hunting Run Reservoir, the reservoir is designed so that inflows will consist of natural runoff from Motts Run, as well as water pumped from the Rappahanock River when the river is above certain minimum flows. The instream flow limits are given in Table 2.4. Tables 2.5 and 2.6 provide the inflows adjusted from the Fredericksburg gage with a multiplying factor of 0.98. Table 2.5 represents the Motts Run critical period.

The difference in the watershed areas between Motts Run and the Rappahanock is even greater than that for Hunting Run. The Motts Run watershed consists of 6,834 acres while the Rappahanock River watershed consists of 1,570 square miles (approximately 1,005,000 acres). Again, because of this magnitude of difference, for the analysis, only the water available for pumping is of interest. As can be seen in Figure 2.1, the reservoir is to the east, and downstream, of Hunting Run Reservoir. Its watershed is approximately 50% rural, 25% commercial, and 25% residential. The pumping site on the Rappahanock is closest to the USGS gauging site at

Fredericksburg. Since the pumping site is so close to the gauging site at Fredericksburg, a multiplying factor of 0.98 is used for flow adjustment between the two sites.

Ni River Reservoir

The Ni River Reservoir also provides drinking water Spotsylvania County, with a safe yield of 4.0 mgd. It has a capacity of 750 mg, and a separate treatment plant with a capacity of 6 mgd. It is somewhat different in nature than the other two reservoirs. For one, it is in a different basin, the Mattaponi River basin. The reservoir is located on the Ni River, and the only inflow to the reservoir is the flow from the river (no pumping).

As seen in Figure 2.1, the reservoir is to south of the other two reservoirs. Its watershed is approximately 60% rural, 15% residential, and 25% parkland (Fredericksburg-Spotsylvania National Historic Park). There are no gages on the Ni River. There is a gage somewhat close to the reservoir, on the Po River. The Ni River reservoir has a drainage area of 25 sq. miles and the gage on the Po River has a drainage area of 77.4 square miles. The monthly average flows at the gage are multiplied by a factor of (25/77.4), to estimate the inflows to the Ni river reservoir. However, the period of record for this gauging station is only from 1962, which misses some of the worst droughts of the 20th century. As a result, the usefulness of this data is somewhat limited. The critical drought period for the Ni River is from June 1980 through December 1981. Table 2.6 represents the Ni River critical period. Tables 2.7 and 2.8 show the inflows corresponding to the critical periods for the Motts Run/Hunting Run reservoirs and the Ni River reservoir. Table 2.9 summarizes the period of data availability and Table 2.10 contains the average monthly flows.

Table 2.1. Instream Conditions at Hunting Run Reservoir for Withdrawal from Rapidan River

Month	Emergency Volumes at Hunting Run (MG)	Provision. Storage* at Hunting Run(MG)	Rules	Instream Flow (% of MAF)	Instream Flow at Rapidan (cfs)	MG Per Month (cfs*days* 0.6463)
Jan.	990		If Storage is greater than 990 million gallons	40	306.00	6130.54
			If Storage is less than 990 million gallons	20	153.00	3065.27
Feb.	1100		If Storage is greater than 1100 million gallons	40	306.00	5537.26
			If Storage is less than 1100 million gallons	20	153.00	2768.63
Mar.	1320	2002 (91%)	If Storage is greater than 1320 million gallons but less than 2002 million gallons	60	459.00	9195.80
			If Storage is less than 1320 million gallons	40	306.00	6130.54
April	1540	2068 (94%)	If Storage is greater than 1540 million gallons but less than 2068 million gallons	60	459.00	8899.17
			If Storage is less than 1540 million gallons	40	306.00	5932.78
May	1760	2134 (97%)	If Storage is greater than 1760 million gallons but less than 2134 million gallons	60	459.00	9195.80
			If Storage is less than 1760 million gallons	40	306.00	6130.54
June	1540		If Storage is greater than 1540 million gallons	60	459.00	8899.17
			If Storage is less than 1540 million gallons	20	153.00	2966.39
July	1320		If Storage is greater than 1320 million gallons	40	306.00	6130.54
			If Storage is less than 1320 million gallons	20	153.00	3065.27

Table 2.1, continued

Month	Emergency Volumes at Hunting Run (MG)	Provision. Storage* at Hunting Run(MG)	Rules	Instream Flow (% of MAF)	Instream Flow at Rapidan (cfs)	MG Per Month (cfs*days* 0.6463)
Aug.	1100		If Storage is greater than 1100 million gallons	40	306.00	6130.54
			If Storage is less than 1100 million gallons	20	153.00	3065.27
Sept.	990		If Storage is greater than 990 million gallons	40	306.00	5932.78
			If Storage is less than 990 million gallons	20	153.00	2966.39
Oct.	990		If Storage is greater than 990 million gallons	40	306.00	6130.54
			If Storage is less than 990 million gallons	20	153.00	3065.27
Nov.	880		If Storage is greater than 880 million gallons	40	306.00	5932.78
			If Storage is less than 880 million gallons	20	153.00	2966.39
Dec.	880		If Storage is greater than 880 million gallons	40	306.00	6130.54
			If Storage is less than 880 million gallons	20	153.00	3065.27

* If storage > provisional storage, then instream requirement =764 cfs (Rapidan Mean Annual Flow)

Table 2.2 River Inflows Available for Hunting Run Reservoir (1965-66)

Month-Year	Days	Annual Monthly Rapidan Flows (cfs)	Drainage Area corrected flow = 1.486*flow (cfs)	Inflow Volume = Corrected flow*days* 0.6463 (MG per month)
May-65	31	341	506.7	10152.7
Jun-65	30	160	237.8	4610.0
Jul-65	31	104	154.5	3096.4
Aug-65	31	73	108.5	2173.4
Sep-65	30	64.3	95.5	1852.7
Oct-65	31	133	197.6	3959.8
Nov-65	30	85.2	126.6	2454.9
Dec-65	31	70.5	104.8	2099.0
Jan-66	31	93.6	139.1	2786.8
Feb-66	28	759	1127.9	20411.0
Mar-66	31	584	867.8	17387.6
Apr-66	30	375	557.3	10804.8
May-66	31	693	1029.8	20632.9
Jun-66	30	195	289.8	5618.5
Jul-66	31	70.7	105.1	2105.0
Aug-66	31	34.9	51.9	1039.1
Sep-66	30	511	759.3	14723.3
Oct-66	31	536	796.5	15958.5
Nov-66	30	346	514.2	9969.2

Table 2.3 River Inflows Available for Hunting Run Reservoir (1980-81)

Month-Year	Days	Annual Monthly Rapidan Flows (cfs)	Drainage Area corrected flow = 1.486*flow (cfs)	Inflow Volume = Corrected flow*days* 0.6463 (MG per month)
Jun-80	30	268	398.2	7721.8
Jul-80	31	196	291.3	5835.6
Aug-80	31	170	252.6	5061.5
Sep-80	30	83.4	123.9	2403.0
Oct-80	31	99.7	148.2	2968.4
Nov-80	30	161	239.2	4638.9
Dec-80	31	144	214.0	4287.3
Jan-81	31	111	164.9	3304.8
Feb-81	28	620	921.3	16673.0
Mar-81	31	248	368.5	7383.8
Apr-81	30	210	312.1	6050.7
May-81	31	293	435.4	8723.6
Jun-81	30	346	514.2	9969.2
Jul-81	31	389	578.1	11581.8
Aug-81	31	108	160.5	3215.5
Sep-81	30	145	215.5	4177.9
Oct-81	31	192	285.3	5716.5
Nov-81	30	164	243.7	4725.3
Dec-81	31	227	337.3	6758.5

Table 2.4 Instream Conditions at Motts Run Reservoir for Withdrawal from Rappahannock River

Month	Emergency Volumes at Hunting Run (MG)	Provisional Volume at Motts Run (MG)	Rules	Instream Flow (% MAF)	Instream Flow at Rappa-hannock (cfs)	MG Per Month (cfs*days* 0.6463)
Jan.	543		If Storage is greater than 543 million gallons	40	641.00	12842.07
			If Storage is less than 543 million gallons	20	321.00	6431.05
Feb.	602		If Storage is greater than 602 million gallons	40	641.00	11599.29
			If Storage is less than 602 million gallons	20	321.00	5808.69
Mar.	723	1097 (91% full)	If Storage is greater than 723 million gallons but less than 1097 million gallons	60	962.00	19273.12
			If Storage is less than 723 million gallons	40	641.00	12842.07
April	843	1133 (94% full)	If Storage is greater than 843 million gallons but less than 1133 million gallons	60	962.00	18651.41
			If Storage is less than 843 million gallons	40	641.00	12427.81
May	973	1169 (97% full)	If Storage is greater than 973 million gallons but less than 1169 million gallons	60	962.00	19273.12
			If Storage is less than 973 million gallons	40	641.00	12842.07
June	843		If Storage is greater than 843 million gallons	60	962.00	18651.41
			If Storage is less than 843 million gallons	20	321.00	6223.60

Table 2.4 , continued

Month	Emergency Volumes at Hunting Run (MG)	Provisional Volume at Motts Run (MG)	Rules	Instream Flow (% MAF)	Instream Flow at Rappa-hannock (cfs)	MG Per Month (cfs*days* 0.6463)
July	723		If Storage is greater than 723 million gallons	40	641.00	12842.07
			If Storage is less than 723 million gallons	20	321.00	6431.05
Aug.	602		If Storage is greater than 602 million gallons	40	641.00	12842.07
			If Storage is less than 602 million gallons	20	321.00	6431.05
Sept.	542		If Storage is greater than 542 million gallons	40	641.00	12427.81
			If Storage is less than 542 million gallons	20	321.00	6223.60
Oct.	542		If Storage is greater than 542 million gallons	40	641.00	12842.07
			If Storage is less than 542 million gallons	20	321.00	6431.05
Nov.	482		If Storage is greater than 482 million gallons	40	641.00	12427.81
			If Storage is less than 482 million gallons	20	321.00	6223.60
Dec.	482		If Storage is greater than 482 million gallons	40	641.00	12842.07
			If Storage is less than 482 million gallons	20	321.00	6431.05

* If storage > provisional storage, then instream requirement =1630 cfs [32656.12 MG per month] (Rappahannock Mean Annual Flow)

Table 2.5 River Inflows Available for Motts Run Reservoir, 1965-66

Month-Year	Days	Annual Monthly Rappahannock Flows (cfs)	Drainage Area corrected flow = 0.98*flow (cfs)	Inflow Volume = Corrected flow*days* 0.6463 (MG per month)
May-65	31	880	862.4	17278.9
Jun-65	30	405	396.9	7695.7
Jul-65	31	261	255.8	5124.8
Aug-65	31	192	188.2	3769.9
Sep-65	30	156	152.9	2964.3
Oct-65	31	277	271.5	5438.9
Nov-65	30	204	199.9	3876.4
Dec-65	31	205	200.9	4025.2
Jan-66	31	268	262.6	5262.2
Feb-66	28	2461	2411.8	43645.7
Mar-66	31	1611	1578.8	31632.2
Apr-66	30	1168	1144.6	22194.0
May-66	31	2049	2008.0	40232.3
Jun-66	30	508	497.8	9652.9
Jul-66	31	141	138.2	2768.6
Aug-66	31	76.4	74.9	1500.1
Sep-66	30	1965	1925.7	37338.4
Oct-66	31	1835	1798.3	36030.4
Nov-66	30	840	823.2	15961.4

Average monthly Rappahannock River flows were obtained from the USGS gage # 01668000 Rappahannock River near Fredericksburg VA, from the USGS website.

Table 2.6 River Inflows Available for Motts Run Reservoir, 1980-81

Month-Year	Days	Annual Monthly Rappahannock Flows (cfs)	Drainage Area corrected flow = 0.98*flow (cfs)	Inflow Volume = Corrected flow*days* 0.6463 (MG per month)
Jun-80	30	930	911.4	17671.6
Jul-80	31	526	515.5	10328.1
Aug-80	31	508	497.8	9974.6
Sep-80	30	187	183.3	3553.3
Oct-80	31	200	196.0	3927.0
Nov-80	30	393	385.1	7467.7
Dec-80	31	364	356.7	7147.2
Jan-81	31	297	291.1	5831.6
Feb-81	28	1751	1716.0	31053.9
Mar-81	31	620	607.6	12173.8
Apr-81	30	587	575.3	11154.0
May-81	31	809	792.8	15884.8
Jun-81	30	744	729.1	14137.3
Jul-81	31	608	595.8	11938.1
Aug-81	31	362	354.8	7107.9
Sep-81	30	373	365.5	7087.6
Oct-81	31	421	412.6	8266.4
Nov-81	30	387	379.3	7353.7
Dec-81	31	622	609.6	12213.0

Table 2.7 River Inflows to the Ni River Reservoir, 1965-66

Month-Year	Days	Monthly Averaged flows in Po River (cfs per month)	Monthly Inflows to Ni River Reservoir (After Applying Drainage Area Factor of 25/77.4 to Po River gage Data) (cfs per month)	Monthly Inflows to Ni River Reservoir (cfs*days*0.6463) (MG per month)
May-65	31	39.4	12.7	255.0
Jun-65	30	23.9	7.7	149.7
Jul-65	31	22.6	7.3	146.3
Aug-65	31	18.1	5.8	117.1
Sep-65	30	9.25	3.0	57.9
Oct-65	31	15	4.8	97.1
Nov-65	30	9.35	3.0	58.6
Dec-65	31	11.1	3.6	71.8
Jan-66	31	26.8	8.7	173.4
Feb-66	28	133	43.0	777.4
Mar-66	31	77	24.9	498.3
Apr-66	30	67	21.6	419.6
May-66	31	70.3	22.7	454.9
Jun-66	30	13	4.2	81.4
Jul-66	31	6.94	2.2	44.9
Aug-66	31	0.77	0.2	5.0
Sep-66	30	65.7	21.2	411.5
Oct-66	31	79.7	25.7	515.8
Nov-66	30	25.1	8.1	157.2

Obtained by applying a Drainage Area Factor of (25/77.4) to the Po River gage Data

Table 2.8 River Inflows to the Ni River Reservoir, 1980-81

Month-Year	Days	Monthly Averaged flows in Po River (cfs per month)	Monthly Inflows to Ni River Reservoir (After Applying Drainage Area Factor of 25/77.4 to Po River gage Data) (cfs per month)	Monthly Inflows to Ni River Reservoir (cfs*days*0.6463) (MG per month)
Jun-80	30	19.3	6.2	120.9
Jul-80	31	7.15	2.3	46.3
Aug-80	31	2.14	0.7	13.8
Sep-80	30	0.94	0.3	5.9
Oct-80	31	4.52	1.5	29.3
Nov-80	30	15.4	5.0	96.4
Dec-80	31	12.2	3.9	79.0
Jan-81	31	10.4	3.4	67.3
Feb-81	28	59.5	19.2	347.8
Mar-81	31	25.2	8.1	163.1
Apr-81	30	27.1	8.8	169.7
May-81	31	52.9	17.1	342.3
Jun-81	30	6.97	2.3	43.7
Jul-81	31	9.1	2.9	58.9
Aug-81	31	3.28	1.1	21.2
Sep-81	30	1.37	0.4	8.6
Oct-81	31	6.86	2.2	44.4
Nov-81	30	3.85	1.2	24.1
Dec-81	31	35.2	11.4	227.8

Obtained by applying a Drainage Area Factor of (25/77.4) to the Po River gage Data

Table 2.9 USGS Gauging Stations for Project

Site	River	Period of record	Drainage area (square miles)
Fredericksburg	Rappahanock	1907- present	1,596
Culpeper	Rapidan	1930- present	472
Spotsylvania	Po	1962- present	77

Table 2.10 Mean monthly flows, Culpeper and Fredericksburg

Month	Rapidan	Rappahannock	Po
	Culpeper	Fredericksburg	Spotsylvania
	1667500	1668000	1673800
	Cfs*	Cfs*	Cfs*
January	666	2197	115
February	735	2489	133
March	845	2698	154
April	773	2480	113
May	573	1891	74.7
June	482	1414	52.1
July	300	907	29
August	329	1006	24.1
September	364	948	25.5
October	421	1148	42
November	463	1308	62.7
December	549	1661	83

1Mgd = 0.646317cfs

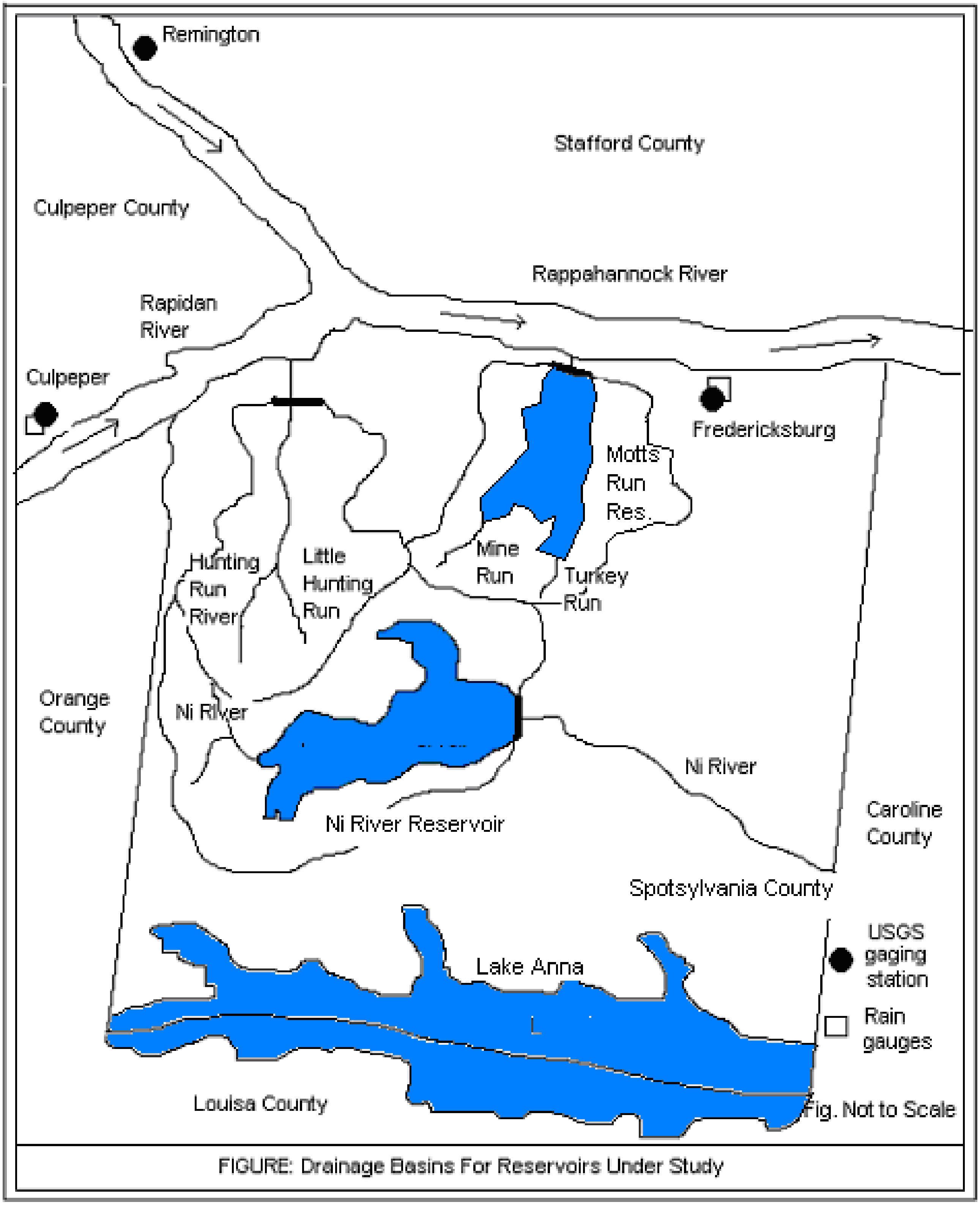

Figure 2.1 System schematic

Chapter 3: Historical Drought Analysis

Historically Worst Drought Sequence

The historically worst drought sequence involves choosing the worst drought sequence on record, in terms of lowest flow volume for a particular duration, and using the actual recorded flows as the inflow sequence. It is the most convenient method; no analysis is needed other than for evaluating the historical record and determining the worst historical drought. However, it is not reasonable to believe that a reservoir will be able to meet its demands in the future simply because it can meet the demands under past conditions. Historical periods of record are usually close to 100 years at most; typically they are 50-70 years. There is a significant chance that a drought in the next 50 years could surpass the worst drought from the past 50-100 years. The historically worst drought for the Rappahanock River basin is the 1930-1931 drought. It was, by all evaluations, an extreme drought. Table 3.1 shows the list of the ten worst annual flow volumes for two gauging stations in the Rappahanock basin, Fredericksburg and Culpeper. In examining the table, consider the grouping of volumes, such as the second through fifth droughts. Clearly, the 1931 drought is significantly worse than all other droughts. In this study, the period of May 1965- January 1966 proves to be the period of the greatest rate of depletion for the Rappahannock and Rapidan River flows.

Synthetic Generation of Drought Sequences

To assess the severity of these droughts, the historic annual flow volumes were resampled by the generalized bootstrap method suggested by Karian and Dudewicz (2000). The historical flow volumes for Culpeper gage are shown in Table 3.2. These volumes are transformed to a near normal distribution by the Box-Cox transformation given by

$$X_k = \frac{Q_k^{\lambda} - 1}{\lambda} \qquad \text{if } \lambda \neq 0 \text{ and} \tag{3.1a}$$

$$X_k = \ln(Q_k) \qquad \text{if } \lambda = 0 \tag{3.1b}$$

where Q_k are the historical flow volumes. The optimal λ value is chosen such that the skew coefficient of the dataset is zero. In this case optimal $\lambda = 0.336$. Thus transformed, 71 data values are used to fit the normal distribution with the mean of 148.61 and a standard deviation of 20.006. From this normal distribution, 500 sequences, each with 71 data points matching the historical data length are randomly generated. For each such sequence, the quantiles are obtained. The quantile averages and standard deviations are calculated using 500 such values. These values are reported in Table 3.3. The transformed quantiles are inverted to obtain the corresponding flow volumes by,

$$Q_k = (X_k + 1)^{\frac{1}{\lambda}} \tag{3.2}$$

From Table 3.1, the 1931 drought flow volume is 35,034 mg, which compares with the quantile of 35,564 mg, corresponding to a cumulative exceedance probability of 0.005. This would seem to imply that the return period is 200 years, but that value may be questionable and there is a need to consider other methods to better estimate the frequency. This is because for high volumes the cumulative probability is near 1 and it does not mean that they will occur frequently. The Markov chain analysis presented in the next section overcomes this difficulty by providing large return periods at both the extremities (high and low volumes) and small return intervals for near average volumes.

Markov Chain Analysis

In this section a new Markov chain analysis for evaluating return periods associated with various drought classes corresponding to the flow volumes is presented. In this method, the annual flow volumes are put into classes, with class 1 being the least volume range and class n being the greatest volume range. Table 3.4 shows the class designation for the flow volumes of the Culpeper gage given in Table 3.2. For example, in Table 3.4, class 1 stands for the flow range of 0 to 36,000 and class 2 represents [36,000, 60,000) and so on. For a fixed class i in period n, the number of transitions is calculated to class j in period (n+1) for n = 1,2, …, N-1 with N being the total number of periods. For the data in Table 3.2 we have N = 70 with the last period 71 being not counted because of the lack of knowledge on class j for period 72. For example, the transition was from class 1 to class 2 in year 1931 to year 1932. All such class 1 to class 2 transitions from 1931 through 2001 are added and there is just 1 transition. From class 2 there are transitions to classes 1, 6, 9, 12, and 16 for a total of 5 transitions. It is noted that we have assumed that class 1 was reached from class 2 for 1931which is not a recorded transition.

The number of transitions to class j from class i is divided by the total number of such transitions (total numbers of visits to class i) to obtain the transition probability p(i,j). The matrix thus formed is shown as Table 3.6. This matrix is called the transition probability matrix $[P]_{MxM}$ with M being the total number classes; and characteristically each row sums to 1. The steady state probability is defined as the probability with which each individual class will be realized. It is calculated as follows.

$$\lambda_{1xM} = \lambda_{1xM}[P]_{MxM} \tag{3.3}$$

where λ is the row vector of steady state also called limiting probabilities. By raising [P] matrix to large values $[P]^k$ for large *k* we can obtain a matrix Φ that has identical rows with each row representing the λ vector. From the steady class probabilities, the mean recurrence time can be calculated. This is defined as "the expected number of transitions until a Markov chain, starting in class *j*, returns to that class". It is obtained by

$$m_j = \frac{1}{\lambda_j} \tag{3.4}$$

We interpret the mean recurrence time as the return period. Table 3.7 lists the limiting probabilities λ_j for j = 1, 2, …, 20 in the second and third columns. In column 4, the empirical values evaluated from Table 3.2 data by averaging the times between occurrences of the same class are given. Columns 5 and 6 list the reciprocal values of the steady state probabilities given in columns 2 and 3. The results should be considered close considering the limited data exploited by the method.

Markov chain analysis was also performed on the Fredericksburg data as well. There were 94 years of historical data (1908-2001) for the site, but since the Markov chain analysis pertains to the effect of one year upon the next, there were only 93 of these two-year relationships. The data was grouped into twenty class intervals as seen in Table 3.8.

For reference, the 1931 drought volume was 105,375 million gallons; it was the only year in class 1. The class intervals were arranged so that the first ten class intervals were below the mean, and the next ten above the mean. The counts above only include the years 1908 to 2000; the 2001 value is only used to evaluate the transition from 2000. Since there is no value for 2002, there is no way to evaluate the transition from 2001. The historical annual volumes can be seen in Table 3.9.

The procedure discussed above was followed to obtain the Markov chain analytical steady class probabilities. The historical data in Table 3.9 was used to calculate the empirical steady class probabilities. It can be seen from Table 3.10 that the analytical values match up very well with the empirical frequencies calculated from the historical period of record. Interestingly, the Markov chain calculation raised the probability of a class 1 occurrence from the empirical frequency.

Next, the expected uninterrupted residence time was calculated from the method detailed below. The expected uninterrupted residence time is a characteristic of the duration of a particular drought class (Lohani et al, 1998). For example, the expected uninterrupted residence time of class 6 is the amount of time that the system could be expected to stay in class 6 once it reached that class. The empirical uninterrupted residence time was simply the mean residence time from the historical record. The following discussion of uninterrupted residence time calculation is based heavily on Lohani et al (1998).

A system stays in class *i* without moving to another class for *m* time periods when

$$\{X_1 = i = X_2 = ... = X_{m-1} \mid X_0 = i\} \tag{3.5}$$

Assume $P_{i,i}^{1,2}$ is the probability of moving from class i in January (month 1) to the same class in February (month 2). Then the probability of moving from one class in January to a different one in February is denoted by:

$$P[X_{feb} \neq i \mid X_{jan} = i] = P[m = 1 \mid X_{jan} = i] = 1 - P_{i,i}^{1,2} \tag{3.6}$$

Equation 3.6 is an expression for the probability of one month duration, or m=1. The probability of m=2 can be developed by:

$$P[m = 2 \mid X_{jan} = i] = P[X_{mar} \neq i, X_{feb} = i \mid X_{jan=i}] \tag{3.7}$$

$$= P[X_{feb} = i \mid X_{jan} = i] P[X_{mar} = i \mid X_{feb} = i] \tag{3.8}$$

$$= P_{i,i}^{1,2}(1 - P_{i,i}^{2,3}) \tag{3.9}$$

This process can be continued for time periods more than m=2. For instance, for twelve months, m=12, the following equation is used:

$$P[m = 12 \mid X_{jan} = i] = P_{i,i}^{1,2} P_{i,i}^{2,3} \ldots P_{i,i}^{10,11} P_{i,i}^{11,12}(1 - P_{i,i}^{12,1}) \tag{3.10}$$

This equation is used to calculate the uninterrupted residence times for all classes and all time periods. Once a probability for a certain time period is zero, then all probabilities for that class for longer than that time period are zero. The expected uninterrupted residence time for class j, $E[R_{uj}]$ is calculated by

$$E[R_{uj} \mid starting\_month] = \sum_k kP[m = k \mid starting\_month] \tag{3.11}$$

Table 3.11 implies that there is not much evidence that the process stays in a particular class interval for very long. This is partially a function of the large number of class intervals and the subsequent small class interval widths. However, even without that characteristic, the process was very mobile. Of the twenty class intervals, only three had a single instance of two consecutive years within that class interval. There was no instance of three consecutive years within a class interval. One interesting note about Table 3.11 is that the first ten class intervals (the drier class intervals) do show a greater tendency to have greater residence times. However, the general conclusion that can be drawn from this table is that despite the tendency for droughts to cluster, there is a tendency not to stay in the same class interval for long time periods. Table 3.12 shows a simple statistical count of the frequency from the historical record of the process to go to a higher class in the lower ten classes (or in other words, the frequency that the next year is wetter than the present year).

The next calculation to be done is the mean recurrence interval, which is the inverse of the steady class probabilities. The empirical recurrence interval is the mean length of time between consecutive recurrences of the same class. For this reason, no empirical recurrence interval exists for class 1 since there was only one occurrence.

Table 3.13 shows the mean recurrence interval calculation results. The analytical and empirical values match up very well, with the exception of class 3. The only four occurrences of class 3 were in 1956, 1959, 1966, and 1969. With the definition of the empirical recurrence time above, that produces a very small value, which may be uncharacteristically low. In this case, the analytical value would be more trustworthy.

As can be seen in Table 3.13, the recurrence time for class 1 is only 68.8 years, which is a lot less than the return period calculated for the 1931 drought in the synthetic time-series. Class 1 was intentionally set so that only the 1931 volume was in it. The fact that there is only one volume in class 1 could impair the accuracy of the recurrence time calculation. The recurrence time is also somewhat dependent on the number of class intervals; for instance, with just two class intervals, the recurrence time goes to 93 years (the number of values in the analysis). This number of values would seem to be the maximum recurrence time that could be calculated with the Markov chain analysis; thus there is somewhat of a ceiling to the recurrence time that can be calculated with this analysis.

Next, the mean passage times were calculated using the method detailed above, and the results can be seen in Table 3.14. Table 3.14 should be interpreted as the vertical classes are class i (year n) and the horizontal classes are class j (year n+1). For example, it would take 16.71 years to get to class 6 starting from class 1.

Analysis of Markov Chain Results

As discussed previously, the effectiveness of the Markov chain analysis is somewhat constrained by the lack of occurrences of the system going in and out of class 1 because class 1 only has one value in it. Therefore, we can assume a probability of the system moving into class 1 to get a better idea of the characteristics mentioned above.

Fredericksburg Analysis

For the Fredericksburg data, there are 93 data points with 20 classes, meaning there is an average of four to five data points per class. As a result of this limited data for each class, there are some manipulations to the raw data that can be done to examine the effect of certain assumptions. This does not imply that the raw data is invalid, but rather that there may be occurrences that are in the historical record that would be considered less likely to repeat than some artificial occurrences that can be inserted into the analysis. For instance, there is one occurrence of the system moving from class 6 to class 1. It would be intuitive to assume that this is an isolated incident, and that systems in class 6 don't typically move to class 1. Therefore, we will instead assume that there is one occurrence of the system moving from class 2 to class 1, which would seem more likely. Because it is more likely for the system to be in class 6 than in class 2, this manipulation should drive the recurrence time upward, to a more likely level. In the actual historical data, there are three other occurrences of class 2, so that we will assign a 0.25 probability to those three occurrences and the fictional occurrence we are creating. In addition, we will need to assign a 0.0 probability to the occurrence of class 6 to 1, and subsequently increase the probability of the other three occurrences of class 6 from 0.25 to 0.33. Now, proceeding with the same procedure as before, we acquire a mean recurrence time of 143 years, as opposed to 69 years before. This value is more in line with the other means of evaluation that were used, and it is a result that we can have confidence in because of the logic detailed above. The mean recurrence interval calculation results can be seen in Table 3.15, where it can be seen that the change increases the accuracy of the class 1 recurrence interval, while not changing the other values much.

Culpeper Analysis

Likewise, the same type of manipulation was performed for the Culpeper analysis. In Culpeper, there were no occurrences of the system moving into class 1, because the first data point was in class 1. Therefore, class 2, which had four historical values, was changed so that there was a 0.20 probability to go from class 2 to class 1, and a 0.20 probability for the other four occurrences. This produced a mean recurrence time of 95 years, where there was not one before because of the 0.0 steady state probability for class 1. This, too, is more in line with the values calculated with the other methods. The results can be seen in Table 3.7. The first two columns of Table 3.7 show the probability of each class occurring in a particular year. The first column is with the artificial transition mentioned above, and the second column is without that. Because there was not a historical occurrence of the system moving into class 1, then the value for that column is 0. The third column is the empirical return period of each class occurring, based on the historical record. It is merely the number of occurrences divided by the total period of record. The fourth and fifth columns are return periods corresponding to the probabilities in columns 1 and 2. According to the definitions of return period and probability, the fourth column is simply the reciprocal of the first column, and the fifth column is the reciprocal of the second column. Because the second column value for class 1 is zero, then there cannot be a return period calculated in the fifth column for class 1.

Table 3.1 Ten Worst Flow Volumes on Record

Culpepper		Fredericksburg	
Year	Volume (Mgal)	Year	Volume (Mgal)
1931	35,034	1931	101,976
1956	52,464	1981	137,811
1981	55,735	1999	152,783
1954	55,882	1954	155,741
1932	58,661	1956	185,642
1959	65,862	1959	200,435
1999	67,341	1969	209,916
1966	69,892	1966	211,979
1969	75,420	1932	215,736
1964	75,639	1911	229,595

Table 3.2 Culpeper Annual Inflow Volumes (mg)

Year	Annual volume	Transformed
1931	35,584	97.45
1932	59,517	116.38
1933	166,522	165.62
1934	101,450	139.78
1935	155,240	161.70
1936	149,133	159.49
1937	200,464	176.45
1938	133,730	153.66
1939	106,944	142.33
1940	109,671	143.57
1941	98,180	138.22
1942	94,460	136.40
1943	169,507	166.63
1944	83,569	130.79
1945	125,204	150.23
1946	120,450	148.25
1947	84,378	131.22
1948	153,863	161.21
1949	252,075	190.80
1950	121,589	148.73
1951	156,818	162.26
1952	150,670	160.05
1953	138,963	155.69
1954	56,758	114.50
1955	134,224	153.85
1956	52,815	111.69
1957	93,324	135.84
1958	136,877	154.88
1959	66,611	120.98
1960	131,039	152.59
1961	116,570	146.60
1962	128,669	151.64
1963	83,108	130.54
1964	76,177	126.69
1965	87,890	133.07
1966	69,997	123.06
1967	98,790	138.52
1968	96,134	137.23
1969	76,486	126.87
1970	103,396	140.70
1971	152,372	160.67
1972	208,778	178.92
1973	259,255	192.63

Table 3.2, continued

Year	Annual volume	Transformed
1974	129,049	151.79
1975	164,668	164.99
1976	115,155	145.99
1977	86,896	132.55
1978	162,646	164.29
1979	201,153	176.66
1980	176,295	168.88
1981	55,834	113.85
1982	120,271	148.18
1983	131,113	152.62
1984	209,344	179.08
1985	80,295	129.00
1986	127,739	151.26
1987	133,091	153.40
1988	96,874	137.59
1989	124,331	149.87
1990	134,462	153.94
1991	143,830	157.53
1992	104,692	141.30
1993	198,866	175.97
1994	148,673	159.33
1995	140,684	156.34
1996	236,423	186.67
1997	160,359	163.50
1998	235,129	186.32
1999	68,013	121.85
2000	91,256	134.80
2001	73,707	125.27
	Mean	148.61
	SD	20.00552248
	Lambda	0.335690405
	Skew	-0.0004095

Table 3.3 Quantiles and their Confidence Limits [Culpeper]

Probability	Historical Quantile	Quantile Average	Quantile Std. Dev.	95% Confidence Intervals		Flow Volume Intervals		Inverse Quantile Averages
0.001	86.79	87.23	5.74	86.73	87.74	25,420	26,278	25,846
0.005	97.08	97.43	4.92	97.00	97.87	35,111	36,022	35,564*
0.010	102.07	102.38	4.54	101.98	102.78	40,583	41,507	41,044
0.050	115.70	115.90	3.56	115.59	116.21	58,344	59,263	58,802
0.100	122.97	123.10	3.09	122.83	123.37	69,622	70,518	70,069
0.200	131.77	131.83	2.62	131.60	132.06	85,092	85,959	85,525
0.300	138.12	138.12	2.38	137.91	138.33	97,542	98,403	97,972
0.400	143.54	143.50	2.26	143.30	143.69	109,076	109,957	109,516
0.500	148.61	148.52	2.23	148.33	148.72	120,625	121,558	121,091
0.600	153.68	153.55	2.30	153.34	153.75	132,941	133,963	133,452
0.700	159.10	158.92	2.45	158.71	159.14	146,992	148,158	147,574
0.800	165.45	165.21	2.73	164.97	165.45	164,624	166,023	165,323
0.900	174.25	173.94	3.23	173.66	174.22	191,290	193,122	192,205

*1931 Culpeper annual volume = 35,584 mg

Table 3.4 Markov Chain Class Designation [Culpeper]

Class Interval	Lower Limit (Mgal)	Upper Limit (Mgal)
1	0	36,000
2	36,000	60,000
3	60,000	70,000
4	70,000	83,500
5	83,500	88,000
6	88,000	97,000
7	97,000	105,000
8	105,000	116,000
9	116,000	121,000
10	121,000	127,000
11	127,000	132,000
12	132,000	138,000
13	138,000	145,000
14	145,000	153,000
15	153,000	160,000
16	160,000	167,000
17	167,000	200,000
18	200,000	230,000
19	230,000	250,000
20	250,000	260,000

Table 3.5 Number of transitions from Class i in period n to Class j in period (n+1)

		To period n+1																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
From period n	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	1	0	0	0	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0	0
	3	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0
	4	0	0	0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
	5	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0
	6	0	0	0	2	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0
	7	0	0	0	0	0	2	0	0	0	0	0	0	0	1	1	0	1	0	0	0
	8	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0
	11	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	0	1	0	0
	12	0	1	1	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0
	13	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	2	0	0
	15	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1
	16	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	1	0
	17	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	18	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1
	19	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	20	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0

Table 3.6 Transition Probability Matrix, P [Culpeper]

		To period n+1																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
From period n	1	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.200	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.200	0.000	0.000	0.200	0.000	0.000	0.000	0.200	0.000	0.000	0.000	0.000
	3	0.000	0.000	0.000	0.000	0.000	0.333	0.333	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.000	0.000	0.000	0.250	0.250	0.000	0.250	0.000	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	5	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.250	0.250	0.000	0.000	0.000	0.000
	6	0.000	0.000	0.000	0.400	0.000	0.000	0.000	0.000	0.000	0.200	0.000	0.200	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.000
	7	0.000	0.000	0.000	0.000	0.000	0.400	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.200	0.000	0.200	0.000	0.000	0.000
	8	0.000	0.000	0.000	0.000	0.333	0.000	0.333	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	9	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.667	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.333	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000
	11	0.000	0.000	0.000	0.200	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.200	0.000	0.000	0.000	0.200	0.000	0.200	0.000	0.000
	12	0.000	0.200	0.200	0.000	0.000	0.200	0.000	0.200	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	13	0.000	0.333	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.000
	14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000	0.000	0.500	0.000	0.000
	15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.667	0.000	0.000	0.000	0.000	0.000	0.333
	16	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.250	0.000
	17	0.000	0.333	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000
	18	0.000	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.250	0.000	0.000	0.250
	19	0.000	0.000	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000	0.000
	20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 3.7 Markov Chain Results

Class	Limiting Probability		Return Period		
	w/ initial transition from Class 2 to 1	w/o initial transition from class 2 to 1	Empirical	Markov Chain w/ initial transition	Markov Chain w/o initial transition
1	0.010477	0	n/a	95	n/a
2	0.0523422	0.042772	16	19	23
3	0.0422072	0.043093	20	24	23
4	0.0747747	0.076375	10	13	13
5	0.0595409	0.060804	11	17	16
6	0.0675781	0.069014	15	15	14
7	0.0739011	0.075499	15	14	13
8	0.0404108	0.041278	19	25	24
9	0.0394916	0.04034	18	25	25
10	0.0428942	0.043822	22	23	23
11	0.0735644	0.07514	7	14	13
12	0.0673418	0.068783	13	15	15
13	0.0426166	0.043541	21	23	23
14	0.0583109	0.059538	19	17	17
15	0.0439688	0.044913	8	23	22
16	0.0539376	0.055079	21	19	18
17	0.0426394	0.043541	25	23	23
18	0.0573362	0.058593	16	17	17
19	0.0276721	0.028274	2	36	35
20	0.0289946	0.029601	24	34	34

Table 3.8: Markov Chain Class Designation, Fredericksburg

Class Interval	Lower Limit (Mgal)	Upper Limit (Mgal)	Count
1	0	110,000	1
2	110,000	180,000	3
3	180,000	220,000	4
4	220,000	260,000	4
5	260,000	280,000	4
6	280,000	300,000	6
7	300,000	325,000	6
8	325,000	350,000	6
9	350,000	370,000	7
10	370,000	400,000	8
11	400,000	415,000	7
12	415,000	455,000	7
13	455,000	477,000	6
14	477,000	500,000	5
15	500,000	550,000	3
16	550,000	580,000	4
17	580,000	625,000	3
18	625,000	680,000	4
19	680,000	730,000	3
20	730,000	750,000	2

Table 3.9: Fredericksburg historical volumes and class intervals, 1908-2001

Year	Annual volume (Mgal)	Class Interval
1908	582,040	17
1909	441,339	12
1910	319,469	7
1911	237,027	4
1912	496,573	14
1913	414,252	11
1914	305,926	7
1915	450,074	12
1916	478,202	14
1917	359,318	9
1918	476,038	13
1919	405,777	11
1920	380,915	10
1921	404,816	11
1922	350,583	9
1923	297,632	6
1924	690,186	19
1925	399,406	10
1926	310,874	7
1927	417,898	12
1928	516,548	15
1929	402,011	11
1930	270,248	5
1931	105,375	1
1932	222,927	4
1933	558,941	16
1934	301,678	7
1935	494,209	14
1936	476,860	13
1937	631,225	18
1938	418,800	12
1939	366,510	9
1940	373,502	10
1941	345,975	8
1942	289,978	6
1943	579,857	16
1944	264,354	5
1945	409,023	11
1946	453,059	12
1947	256,260	4
1948	510,077	15
1949	735,544	20

Table 3.9, continued

Year	Annual volume (Mgal)	Class Interval
1950	366,971	9
1951	490,783	14
1952	464,338	13
1953	454,701	12
1954	160,933	2
1955	377,208	10
1956	191,830	3
1957	322,434	7
1958	469,687	13
1959	207,116	3
1960	426,633	12
1961	407,360	11
1962	370,377	10
1963	268,223	5
1964	275,984	5
1965	287,154	6
1966	219,044	3
1967	325,760	8
1968	333,533	8
1969	216,913	3
1970	329,486	8
1971	477,340	14
1972	612,252	17
1973	721,540	19
1974	404,716	11
1975	550,586	16
1976	387,165	10
1977	294,105	6
1978	549,344	15
1979	660,675	18
1980	593,420	17
1981	142,405	2
1982	348,238	8
1983	455,303	13
1984	700,544	19
1985	237,248	4
1986	364,486	9
1987	340,806	8
1988	296,630	6
1989	355,050	9
1990	392,675	10
1991	390,611	10

Table 3.9, continued

Year	Annual volume (Mgal)	Class Interval
1992	308,751	7
1993	652,301	18
1994	578,434	16
1995	358,075	9
1996	740,352	20
1997	475,718	13
1998	668,868	18
1999	157,875	2
2000	290,339	6
2001	270,825	5

Table 3.10: Steady Class Probabilities (Fredericksburg)

Class Interval	Analytical	Empirical
1	0.0145	0.0108
2	0.0281	0.0323
3	0.0427	0.0430
4	0.0451	0.0430
5	0.0582	0.0430
6	0.0662	0.0645
7	0.0640	0.0645
8	0.0626	0.0645
9	0.0761	0.0753
10	0.0844	0.0860
11	0.0772	0.0753
12	0.0716	0.0753
13	0.0642	0.0645
14	0.0535	0.0538
15	0.0325	0.0323
16	0.0441	0.0430
17	0.0214	0.0323
18	0.0429	0.0430
19	0.0289	0.0323
20	0.0217	0.0215

Table 3.11: Expected Uninterrupted Residence Times (years)

Class Interval	Analytical	Empirical
1	1.00	1.00
2	1.00	1.00
3	1.00	1.00
4	1.00	1.00
5	1.33	1.20
6	1.00	1.00
7	1.00	1.00
8	1.20	1.17
9	1.00	1.00
10	1.14	1.13
11	1.00	1.00
12	1.00	1.00
13	1.00	1.00
14	1.00	1.00
15	1.00	1.00
16	1.00	1.00
17	1.00	1.00
18	1.00	1.00
19	1.00	1.00
20	1.00	1.00

Table 3.12: Frequency of Movement to a Higher Class Among Classes 1-10

Class	Historical Frequency
1	100%
2	100%
3	100%
4	100%
5	50%
6	67%
7	83%
8	33%
9	71%
10	13%

Table 3.13: Mean Recurrence Interval (years)

Class Interval	Analytical	Empirical
1	68.8	N/A
2	35.6	22.5
3	23.4	4.3
4	22.2	24.7
5	17.2	17.8
6	15.1	15.4
7	15.6	16.4
8	16.0	9.2
9	13.1	13.0
10	11.9	10.1
11	13.0	10.2
12	14.0	8.5
13	15.6	15.8
14	18.7	14.8
15	30.7	25.0
16	22.7	20.3
17	46.7	36.0
18	23.3	20.3
19	34.6	30.0
20	46.0	47.0

Table 3.14: Mean Passage Time (First Arrival)

	CLASSES																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	68.78	34.91	26.55	1.00	24.63	16.71	16.39	21.55	10.52	12.77	13.59	15.49	14.01	15.24	24.51	18.57	44.56	21.78	34.27	44.33
2	66.85	35.58	21.87	21.60	22.70	8.79	15.02	13.38	13.79	9.50	13.51	14.82	14.64	18.25	28.75	22.09	46.58	22.83	31.98	48.08
3	69.50	32.90	23.41	20.96	25.35	14.04	12.26	11.21	13.68	13.95	13.64	10.15	13.14	16.23	28.84	22.93	44.87	21.41	33.00	48.07
4	67.78	33.91	25.55	22.19	23.63	15.71	15.39	20.55	9.52	11.77	12.59	14.49	13.01	14.24	23.51	17.57	43.56	20.78	33.27	43.33
5	44.15	36.11	25.79	14.99	17.19	11.86	15.91	21.83	12.46	12.63	10.01	15.43	15.83	18.53	27.70	20.43	47.00	23.37	33.70	46.89
6	64.17	35.08	21.77	19.96	20.02	15.11	15.06	19.61	11.23	11.98	12.43	14.52	14.80	18.55	25.11	19.00	46.32	21.99	28.62	44.98
7	69.21	30.34	25.74	17.66	25.06	16.31	15.61	20.86	12.70	12.81	12.75	9.14	11.81	14.57	28.15	21.49	42.09	17.53	32.69	47.23
8	68.14	34.17	19.21	21.70	23.99	10.28	15.69	15.98	13.44	13.53	13.25	13.73	11.60	15.52	28.95	22.11	44.57	21.52	30.92	48.01
9	67.97	34.34	23.04	21.80	23.82	12.87	15.20	17.20	13.13	9.73	13.03	14.08	11.19	16.13	29.97	22.71	44.84	21.45	31.83	41.78
10	65.24	34.47	21.61	20.14	21.09	13.09	11.32	17.53	13.69	11.85	11.83	13.20	14.52	17.68	29.20	22.16	45.86	21.96	33.40	48.25
11	64.28	34.35	25.05	20.02	20.13	14.86	12.29	20.33	11.63	9.13	12.95	12.29	14.68	17.80	29.47	19.71	46.12	22.35	34.20	47.36
12	68.51	28.92	25.48	18.77	24.36	15.29	13.67	20.00	11.15	11.93	11.33	13.96	13.55	15.30	25.31	22.02	44.23	21.07	33.48	45.04
13	68.37	30.41	22.06	20.29	24.22	15.69	14.66	19.39	13.61	12.00	11.19	10.75	15.57	18.26	29.36	21.14	42.96	15.59	28.30	48.29
14	68.47	31.38	24.47	21.33	24.32	15.67	15.41	20.05	11.47	11.73	10.55	12.72	9.26	18.70	30.33	22.47	36.38	20.63	30.09	47.71
15	68.02	31.09	25.66	21.96	23.86	15.78	15.37	20.60	11.91	11.83	9.87	13.19	13.25	19.11	30.74	21.11	43.07	14.96	33.27	32.78
16	62.64	34.82	25.05	19.65	18.49	14.53	11.61	20.36	10.71	9.79	12.90	13.96	14.34	17.73	29.75	22.70	45.95	22.08	33.91	47.04
17	68.37	22.39	25.14	19.25	24.22	14.21	15.23	18.95	13.52	10.80	12.32	10.71	15.42	18.04	28.49	22.64	46.67	23.20	22.82	47.81
18	67.59	22.53	25.38	20.82	23.44	14.21	14.88	19.17	13.29	11.51	13.52	10.87	15.49	18.33	29.08	17.69	35.19	23.29	31.55	47.99
19	66.76	35.24	25.07	14.39	22.61	15.56	14.00	20.47	12.61	7.97	9.14	14.33	15.07	17.57	28.39	20.81	46.18	22.69	34.63	47.31
20	69.17	33.38	23.55	22.05	25.02	15.28	15.93	19.30	7.80	11.86	13.11	13.41	6.59	18.19	30.67	22.93	44.90	19.52	31.06	46.04

Table 3.15: Mean Recurrence Intervals, Fredericksburg

Class Interval	Raw	Manipulated
1	68.8	143.3
2	35.6	35.8
3	23.4	23.4
4	22.2	26.4
5	17.2	14.7
6	15.1	14.1
7	15.6	15.4
8	16.0	16.8
9	13.1	13.2
10	11.9	11.9
11	13.0	11.8
12	14.0	13.8
13	15.6	15.9
14	18.7	19.4
15	30.7	31.6
16	22.7	22.7
17	46.7	47.9
18	23.3	23.6
19	34.6	34.2
20	46.0	46.8

Chapter 4: Nesting of Inflow Sequences

There are several methods in which to arrange the inflow sequence for drought analysis. Among them, Waitt's procedure, U-curve, and time series analysis will be examined in this report.

Waitt's Procedure

Waitt's procedure is a method to capture the worst historical droughts for all durations. It is a very conservative method. The steps to tabulate the sequence can be found in McMahon (1978). First, incremental durations should be chosen. The incremental duration should be the smallest duration of drought that one wishes to design for. In the Rappahanock basin study, a 30-day incremental duration was used. This incremental duration will also be the duration of the *i* periods in the tabular method calculation. Next, daily data was taken and 30-day volumes computed for the entire period of record in the following manner, with *n* days of data. V_n is the 30-day volume for day n:

$$V_1 = \sum_{day=1}^{30} dailyvolume$$

$$V_2 = \sum_{day=2}^{31} dailyvolume$$

$$\vdots$$

$$V_{n-29} = \sum_{day=n-29}^{n} dailyvolume \tag{4.1}$$

Thus, there will be *n-29* 30-day volumes. The same process was performed for 60-day and all multiples of 30 days up to 720 days. There will be *n-59* 60-day volumes, *n-89* 90-day volumes, and so forth.

The maximum duration should include the longest drought period for which one wishes to design. The maximum duration of 720 days (nearly two years) was selected for the Rappahanock basin because the data shows that even when drought years are clustered together, there is still increased flow in the winter and early spring months, although those flows may be less than average. The conservative nature of Waitt's procedure allows one to feel confident that the extreme cases are being designed for. For the column of 30-day volumes, the minimum volume should be selected, for the column of 60-day volumes, the minimum volume should be selected, and likewise for all durations. An example of the Waitt's procedure calculations for the Culpeper gauging station is seen in Table 4.1.

These minimum flow volumes, denoted by fv_i, make up the flow volume column. Next the incremental volumes are calculated by the following equation:

$$iv_i = fv_i - fv_{i-1}$$

These incremental volumes now make up the inflow sequence, and the worst drought for each duration is now nested into the inflow sequence. The 1930-31 drought for the Rappahanock basin has most of the worst droughts for each duration in this particular situation. However, if the 1931 drought contained the worst volumes for longer durations, but there had been a more extreme drought for the shorter durations in another year, the Waitt's procedure would be able to capture all those extreme droughts in its inflow sequence.

The Waitt's procedure detailed above was performed for several different scenarios for the Rapidan River, and the results shown in Table 4.2, from most conservative to least conservative. There was a range of assumptions made in Table 4.2. Following is a brief description of those.

- All of the Waitt's procedure analyses were done with monthly flows, or a 30-day time increment.
- Actual flows/area factor translation. The USGS gauging station at Culpeper is upstream of the actual Hunting Run pumping site; therefore, the flows from Culpeper will be less than the actual flows at Hunting Run.

In some analyses, the actual flows from Culpeper were used to provide another layer of conservative design; in others, the Culpeper flows were multiplied by a factor proportional to the proportion of drainage areas of the two locations. This factor was obtained from the Hayes, Seay, Mattern & Mattern design report (1994).

- In all of the analyses, no flow was assumed to be from runoff of the Hunting Run drainage area. It was assumed in these scenarios that the amount of runoff would approximate the amount of evaporation from the reservoir.
- Some of the Waitt's procedure scenarios used only one cycle of flows (720 days) while others had two cycles of flows, where the 720-day flow volumes simply repeated themselves at the end of the first cycle. The 2-cycle method is typically more conservative.
- Sequencing of low flows (U-shape). As discussed above, with the strict Waitt's procedure, the most extreme drought period is at the beginning. Use of the U-shape method allows the most extreme drought periods to be in the middle, more like the actual drought hydrograph. The U-shape is also typically more conservative than the strict Waitt's procedure.
- Assuming minimum flow of 40% of MAF in the Rapidan. The Army Corps of Engineers requires that a minimum flow be maintained in the Rapidan River, usually 40-60% of mean annual flow, depending on the time of year, before water can be pumped from the river. This means that in the most extreme drought periods, no water can be added to the reservoir. Since Waitt's procedure is a theoretical flow, and does not correspond to an actual time of year, scenarios using both the 40% and 60% values were used. It should be noted that the 60% corresponds to the months of January through April, which is the wet time of year anyway, so that the most extreme drought periods should usually fall when the 40% rule is in effect. In the second scenario in Table 10, this logic was used by adopting the 40% rule for the first 240 days, 60% was used for the next

120 days, and then the cycle was repeated. In the fifth scenario in Table 10, an annual average of 47% was used as the minimum instream flow.

Analysis of Waitt's Procedure Results

Table 4.2 shows the varying results when modeling different scenarios with Waitt's Procedure. The HSMM design report, using historical flows, calculated a safe yield of 8 mgd. It is important to look at the differences made when that calculation was made compared with the Waitt's Procedure calculation.

It is the belief of the author that the second scenario in Table 4.2 most closely resembles the conditions that were assumed in the HSMM report. There are several differences that were inevitable in the conditions, however. For one, the HSMM report used daily flows, in contrast to Waitt's Procedure, which used flows in increments of thirty days. As a result, in the HSMM model, there still could be a few days even in a dry month that water could be pumped from the river, whereas in Waitt's Procedure, if the average flow for that month was under the minimum instream requirement, then no water could be pumped into the reservoir. In addition, the critical period calculated by the design report was a nine-month period in 1964. The 270-day period calculated by the Waitt's Procedure agreed substantially with this critical period. The critical periods calculated by the Waitt's Procedure are seen in Table 4.3.

However, the critical duration for Waitt's Procedure is 570 days. This can be seen from Table 4.4. The critical duration is the period of time that it takes for the reservoir to go from full (or k=0) to empty (or k=2196). The Waitt's Procedure calculation can be seen in Table 4.4.

As can be seen in Table 4.4, the reservoir becomes full (k=0) at 570 days and stays full at 600 days. At that point, the reservoir level gradually decreases, until emptying at 1170 days (k=2196). Therefore, 570 days is the critical duration for the Waitt's Procedure. It should be noted that this is a 720-day procedure; the worst 720 days are then repeated to obtain the 1440-day period. This is in effect modeling a scenario where the worst 720 days on record are then repeated for an additional 720 days. This adds an extra layer of conservatism and accounts for the remainder of the differential between the Waitt's safe yield and the design report safe yield.

U-curve effect

As can be seen in Table 4.1, the worst incremental volumes generally are at the beginning of the inflow sequence. This, however, is not how the typical drought occurs. There is usually a gradual decrease in flow, followed by a long "bottoming out", and then a gradual recovery, making a U-shaped hydrograph. We can take the inflow sequence calculated by the Waitt's procedure, and order the hydrograph values so that the worst volumes are in the middle. This serves the purpose of adding another layer of conservative design, decreasing the safe yield calculated by this method. As can be seen in Figure 4.1, the U-curve method provides an inflow hydrograph with more than a year with less than 5000 gallons per month, whereas the strict Waitt's procedure method has a little more than 200 days at that level.

Synthetic time-series

The next method that will be discussed in this report to calculate the inflow sequence is the synthetic time-series method. This method uses the characteristics of the historical dataset to produce a synthetic dataset of a thousand years or more. The benefit, obviously, is that the dataset for drawing conclusions is now a thousand years rather than 50-100 years. The synthetic time series preserves the historical parameters of the historical dataset. The method takes the following steps:

1. Transform the historical dataset using the Box-Cox transformation

$$X_{nk} = \frac{Q_{mk}{}^{\lambda} - 1}{\lambda} \qquad \text{if } \lambda \neq 0 \text{ and} \qquad (4.2a)$$

$$X_{nk} = \ln(Q_{nk}) \qquad \text{if } \lambda = 0 \qquad (4.2b)$$

where Q_{nk} are the historical flows. The λ values should be such that the skew of the dataset is zero. It should be noted that each month of data should be treated separately; therefore, the January flows are treated as a separate dataset, and could have different λ values than the February flows. Thus, there will be twelve different datasets and twelve different λ values.

2. Transform the X_{nk} values so that the mean of the dataset is zero.

$$Z_{nk} = X_{nk} - \bar{x}_k \tag{4.3}$$

Again, this should be done treating each monthly dataset separately.

3. Calculate covariance between consecutive months and variance:

$$Cov(Z_k, Z_{k-m}) = \frac{1}{N} \sum_{n=1}^{N} (Z_{n,k} * Z_{n,k-m}) \tag{4.4}$$

$$Var(Z_{k-1}) = \frac{1}{N} \sum_{n=1}^{N} (Z_{n,k-1} * Z_{n,k-1}) \tag{4.5}$$

4. Calculate Phi for each month:

$$\Phi_k = \frac{Cov(Z_k, Z_{k-m})}{Var(Z_{k-1})} \tag{4.6}$$

5. Calculate Var(a_k) for each month

$$Var(a_k) = Var(Z_k) + \Phi_k^2 Var(Z_{k-1}) - 2\Phi_k Cov(Z_{k,} Z_{k-1}) \tag{4.7}$$

for k = 1,2, …, 12.

After these statistics have been calculated for each month, the synthetic flows can be generated. First, a string of standard normal random numbers should be generated. These numbers should have a normal distribution, with a mean of zero and standard deviation of one. This string should be generated with a computer program such as Microsoft Excel in order to ensure that the mean and standard deviation are zero and one, respectively. Because the flows from one month will have an effect statistically on the flows the next month, one string of numbers should be generated, rather than twelve separate sets of numbers. In other words, for a 1000-year time series, a string of 12,000 random numbers (for each month) should be generated instead of twelve sets of 1,000 numbers. The standard normal random number will be signified by the variable u_k. Synthetic Zk values can be generated with the following equation:

$$Z_k = \Phi_k Z_{k-1} + Var(a_k)u_k$$

The above equation uses the parameters calculated from the historical data, the standard normal random string of numbers, and the synthetic value from period *i-1*to calculate the synthetic value in period *i*. These values must now be transformed back into flow values using the reverse of the process shown in steps 2 and 1.

$$X_{nk} = Z_{nk} + \bar{x}_k$$

$$Q_k = (X_{nk} + 1)^{\frac{1}{\lambda}}$$

There are now 12,000 Q values, signifying the 1,000-year time series.

Drought Severity and Yield Probability from Time series

The purpose of performing the time series analysis is to perform statistics on a larger dataset. Return periods for the historically worst drought can be calculated from the synthetic time series. First, annual volumes should be calculated for the time series. This set of 1,000 annual volumes should then be normalized using the Box-Cox transformation discussed in the previous section. The mean (should be zero) and standard deviation can be calculated from the normalized data. If the return period for a certain historical drought is desired, then it can be calculated with the following steps, where Q(t) is the actual flow volume for the historical drought.

$$Z(t) = \frac{Q(t)^{\lambda} - 1}{\lambda} - \mu \tag{4.8}$$

where μ is the mean of the data before the mean was transformed to zero. Now using the Z(t) value for the historical drought, and the mean and standard deviation, the probability can be calculated from a normal distribution. The return period, then, is simply the inverse of that probability.

A 1000-year time series was created using the process detailed above. The historical monthly means and standard deviations were preserved fairly well, but skew was higher in the synthetic time-series for all months. Refer to Figures 4.2 through 4.4 for plots of these parameters.

A normal distribution was created for the synthetic time-series using the Box-Cox transformation, and the probability of the 1931 drought volume was calculated. In the 1000-year time series, there were no annual volumes lower than the 1931 historical volume. The lowest volume in the synthetic time series was 0.10% higher than the 1931 volume. The calculated probability of the 1931 drought volume using frequency analysis on the synthetic flows was 0.0001936, corresponding to a return period of 5,165 years. While a large set of time-series was not created, several other 1000-year time series were run and resulted in calculated return periods of the same magnitude.

Figures 4.2 through 4.4 show the mean, standard deviation, and skew of the historical and synthetic data. Although the means for the historical and synthetic time series were close, the standard deviation for one month was high, and the skew was more positive for the time series than for the historical record. This means that low flows in the time series were less likely to occur than they actually would be. This can be attributed to imperfections in the time series calculation, as well as possibly the numerical stability problems involved with interaction between the twelve different Box-Cox transformations (one for each month). These shortcomings should be noted when analyzing the time series results.

There are several possible uses of the time series. A set of one hundred 1,000-year time series could be generated. The return period of the historical drought could be calculated. In addition, a certain return period, for instance, 500 years, could be selected, and then 100 different values obtained for a flow volume corresponding to that return period. A subsequent frequency analysis could then be performed on that data. Finally, the flow volumes corresponding to a certain return period could be entered in as an inflow sequence in the tabular safe yield calculations, and safe yields could be calculated for each time series. These safe yields could then have a frequency analysis performed on them, providing a return period for a certain safe yield.

Bootstrap Method

The final method that will be examined in this paper is the bootstrap method. This is a method that, similar to the synthetic time series, creates an artificial dataset. As in the case of most of the other methods, the analysis was done with annual flow volumes. This is done with the following steps:

1. Transform the raw data using the Box-Cox transformation detailed in earlier steps. For Fredericksburg, for example, there were 94 data points (for the years 1908-2001)
2. Calculate the mean and standard deviation of the transformed data.
3. Create another set of 94 data points that has the same approximate mean and standard deviation as the original transformed data, and that has a normal distribution. This can be done using the Microsoft Excel Data Analysis (random number generation) package.
4. Repeat step 3 500 times, so that there are 500 sets of 94 data points.
5. For each of the 500 datasets, calculate the quantiles for selected probabilities, using Microsoft Excel or a normal distribution table. Do the same for the historical transformed data
6. Calculate the mean and standard deviation for the 500 quantile values, for each quantile.
7. Using the mean and standard deviation, calculate the 95% confidence intervals for each quantile. This can be done using Microsoft Excel, as well.
8. Inverse transform the quantile values to obtain the cumulative quantile values for flow volumes.

Bootstrap method analysis was performed on the Fredericksburg and Culpeper data as discussed above. Quantile calculations were performed for the following probabilities: 0.001, 0.005, 0.010, 0.050, 0.100, 0.200, 0.300, 0.400, 0.500, 0.600, 0.700, 0.800, and 0.0900. The results for Fredericksburg can be seen in Table 4.5, and the results for Culpeper in Table 4.6. As can be seen in Tables 4.5 and 4.6, the probability for the 1931 drought to occur is less for Fredericksburg than for Culpeper. The difference can be attributed to the longer length of record for Fredericksburg.

Table 4.1: Waitt's procedure example

Duration (days)	Flow Volume (Million gallons)	Incremental Volume (Million gallons)
30	210	210
60	978	768
90	2,055	1076
120	4,270	2216
150	8,023	3753
180	12,499	4477
210	16,848	4349
240	20,754	3906
270	27,734	6979
300	38,845	11111
330	47,147	8302
360	51,256	4110
390	52,719	1463
420	54,121	1402
450	55,998	1876
480	64,540	8542
510	73,558	9018
540	87,691	14133
570	104,710	17019
600	125,104	20394
630	132,068	6963
660	135,989	3921
690	137,464	1475
720	138,305	841

Table 4.2: Waitt's Procedure Results

Scenario	Calculated Safe Yield (MGD)
Waitt's procedure (monthly) using actual flows from Culpepper gauging station (1931-2000), no runoff, 2-cycles, most extreme drought period is at beginning (no U-shape), 60% minimum flow in Rapidan	4.22
Waitt's procedure (monthly) using actual flows from Culpepper gauging station (1931-2000), no runoff, 2-cycles, most extreme drought period is at beginning (no U-shape), 40% minimum flow for 8 months/60% minimum flow for 4 months	4.91
Waitt's procedure (monthly) using area factor translation from Culpepper gauging station (1931-2000), no runoff, 1-cycle, most extreme drought period is in the middle (U-shape), 40% minimum flow in Rapidan	5.54
Waitt's procedure (monthly) using area factor translation from Culpepper gauging station (1931-2000), no runoff, 2-cycle, most extreme drought period is in the middle (U-shape), 40% minimum flow in Rapidan	5.54
Waitt's procedure (monthly) using area factor translation from Culpepper gauging station (1931-2000), no runoff, 2-cycle, most extreme drought period is at beginning (no U-shape), 47% minimum flow in Rapidan	5.63
Waitt's procedure (monthly) using actual flows from Culpepper gauging station (1931-2000), no runoff, 2-cycles, most extreme drought period is at beginning (no U-shape), 40% minimum flow in Rapidan	6.65
Waitt's procedure (monthly) using area factor translation from Culpepper gauging station (1931-2000), no runoff, 2-cycles, most extreme drought period is at beginning (no U-shape), 40% minimum flow in Rapidan	7.83
Waitt's procedure (monthly) using area factor translation from Culpepper gauging station (1931-2000), no runoff, 1-cycle, most extreme drought period is at beginning (no U-shape), 40% minimum flow in Rapidan	9.74

Table 4.3: Waitt's Procedure Critical Periods

Duration (days)	Critical Period
30	September 15, 1954 – October 14, 1954
60	August 16, 1954 – October 14, 1954
90	July 16, 1954 – October 14, 1954
120	July 10, 1963 – November 6, 1963
150	October 1, 1930 – February 27, 1931
180	October 1, 1930 – March 29, 1931
210	July 16, 1965 - February 10, 1966
240	June 16, 1965 – February 10, 1966
270	May 17, 1965 – February 10, 1966
300	April 17, 1965 – February 10, 1966
330	October 1, 1930 – August 26, 1931
360	October 1, 1930 – September 24, 1931
390	October 1, 1930 – October 25, 1931
420	October 1, 1930 – November 24, 1931
450	October 1, 1930 – December 24, 1931
480	October 1, 1930 – January 23, 1932
510	October 1, 1930 – February 22, 1932
540	October 1, 1930 – March 23, 1932
570	October 1, 1930 – April 22, 1932
600	June 11, 1980 – January 31, 1982
630	October 1, 1930 – June 21, 1932
660	October 1, 1930 – July 20, 1932
690	October 1, 1930 – August 20, 1932
720	October 1, 1930 – September 19, 1932

Table 4.4: Waitt's Procedure Calculation

Duration (days)	Flow Volume (Million gallons)*	Incremental Volume (Mgal)	Actual Withdrawal (Mgal)	Demand (Mgal)	Yield - Withdrawal (Mgal)	Ki-1 (Mgal)	Ki (Mgal)	Adjusted Ki (Mgal)
30	210	210.26	0.00	147.1579	147.16	0.00	147.16	147.16
60	978	768.19	0.00	147.1579	147.16	147.16	294.32	294.32
90	2,055	1076.21	0.00	147.1579	147.16	294.32	441.47	441.47
120	4,270	2215.53	0.00	147.1579	147.16	441.47	588.63	588.63
150	8,023	3752.56	0.00	147.1579	147.16	588.63	735.79	735.79
180	12,499	4476.70	0.00	147.1579	147.16	735.79	882.95	882.95
210	16,848	4348.98	0.00	147.1579	147.16	882.95	1030.11	1030.11
240	20,754	3905.73	0.00	147.1579	147.16	1030.11	1177.26	1177.26
270	27,734	6979.41	0.00	147.1579	147.16	1177.26	1324.42	1324.42
300	38,845	11111.46	600.00	147.1579	-452.84	1324.42	871.58	871.58
330	47,147	8301.66	0.00	147.1579	147.16	871.58	1018.74	1018.74
360	51,256	4109.51	0.00	147.1579	147.16	1018.74	1165.89	1165.89
390	52,719	1463.12	0.00	147.1579	147.16	1165.89	1313.05	1313.05
420	54,121	1402.08	0.00	147.1579	147.16	1313.05	1460.21	1460.21
450	55,998	1876.33	0.00	147.1579	147.16	1460.21	1607.37	1607.37
480	64,540	8542.07	600.00	147.1579	-452.84	1607.37	1154.53	1154.53
510	73,558	9018.20	600.00	147.1579	-452.84	1154.53	701.68	701.68
540	87,691	14132.55	600.00	147.1579	-452.84	701.68	248.84	248.84
570	104,710	17019.35	600.00	147.1579	-452.84	248.84	-204.00	0.00
600	125,104	20394.48	600.00	147.1579	-452.84	0.00	-452.84	0.00
630	132,068	6963.44	0.00	147.1579	147.16	0.00	147.16	147.16
660	135,989	3920.75	0.00	147.1579	147.16	147.16	294.32	294.32
690	137,464	1475.33	0.00	147.1579	147.16	294.32	441.47	441.47
720	138,305	841.44	0.00	147.1579	147.16	441.47	588.63	588.63

Table 4.4, continued

Duration (days)	Flow Volume (Million gallons)*	Incremental Volume (Mgal)	Actual Withdrawal (Mgal)	Demand (Mgal)	Yield - Withdrawal (Mgal)	Ki-1 (Mgal)	Ki (Mgal)	Adjusted Ki (Mgal)
750	138,516	210.26	0.00	147.1579	147.16	588.63	735.79	735.79
780	139,284	768.19	0.00	147.1579	147.16	735.79	882.95	882.95
810	140,360	1076.21	0.00	147.1579	147.16	882.95	1030.11	1030.11
840	142,576	2215.53	0.00	147.1579	147.16	1030.11	1177.26	1177.26
870	146,328	3752.56	0.00	147.1579	147.16	1177.26	1324.42	1324.42
900	150,805	4476.70	0.00	147.1579	147.16	1324.42	1471.58	1471.58
930	155,154	4348.98	0.00	147.1579	147.16	1471.58	1618.74	1618.74
960	159,059	3905.73	0.00	147.1579	147.16	1618.74	1765.89	1765.89
990	166,039	6979.41	0.00	147.1579	147.16	1765.89	1913.05	1913.05
1020	177,150	11111.46	600.00	147.1579	-452.84	1913.05	1460.21	1460.21
1050	185,452	8301.66	0.00	147.1579	147.16	1460.21	1607.37	1607.37
1080	189,562	4109.51	0.00	147.1579	147.16	1607.37	1754.53	1754.53
1110	191,025	1463.12	0.00	147.1579	147.16	1754.53	1901.68	1901.68
1140	192,427	1402.08	0.00	147.1579	147.16	1901.68	2048.84	2048.84
1170	194,303	1876.33	0.00	147.1579	147.16	2048.84	2196.00	2196.00
1200	202,845	8542.07	600.00	147.1579	-452.84	2196.00	1743.16	1743.16
1230	211,863	9018.20	600.00	147.1579	-452.84	1743.16	1290.32	1290.32
1260	225,996	14132.55	600.00	147.1579	-452.84	1290.32	837.47	837.47
1290	243,015	17019.35	600.00	147.1579	-452.84	837.47	384.63	384.63
1320	263,410	20394.48	600.00	147.1579	-452.84	384.63	-68.21	0.00
1350	270,373	6963.44	0.00	147.1579	147.16	0.00	147.16	147.16
1380	274,294	3920.75	0.00	147.1579	147.16	147.16	294.32	294.32
1410	275,769	1475.33	0.00	147.1579	147.16	294.32	441.47	441.47
1440	276,611	841.44	0.00	147.1579	147.16	441.47	588.63	588.63

Table 4.5: Fredericksburg Bootstrap Results

Probability	Historical Quantile Value	Bootstrap Quantile value	Lower 95% Confidence Limit	Upper 95% Confidence Limit
0.001	73,298	74,573	73,055	76,105
0.005	109,502	110,911	109,336	112,497
0.01	129,530	130,985	129,413	132,567
0.05	192,326	193,852	192,372	195,337
0.1	230,585	232,115	230,721	233,514
0.2	281,314	282,818	281,532	284,107
0.3	320,860	322,324	321,095	323,556
0.4	356,606	358,023	356,809	359,240
0.5	391,638	393,000	391,755	394,247
0.6	428,229	429,525	428,197	430,855
0.7	469,096	470,310	468,833	471,790
0.8	519,170	520,273	518,544	522,005
0.9	592,603	593,525	591,330	595,723

Table 4.6: Culpeper Bootstrap Results

Probability	Historical Quantile Value	Bootstrap Quantile value	Lower 95% Confidence Limit	Upper 95% Confidence Limit
0.001	25,468	25,846	25,420	26,278
0.005	35,189	35,564	35,111	36,022
0.01	40,680	41,044	40,583	41,507
0.05	58,513	58,802	58,344	59,263
0.1	69,847	70,069	69,622	70,518
0.2	85,415	85,525	85,092	85,959
0.3	97,965	97,972	97,542	98,403
0.4	109,613	109,516	109,076	109,957
0.5	121,299	121,091	120,625	121,558
0.6	133,785	133,452	132,941	133,963
0.7	148,058	147,574	146,992	148,158
0.8	166,005	165,323	164,624	166,023
0.9	193,205	192,205	191,290	193,122

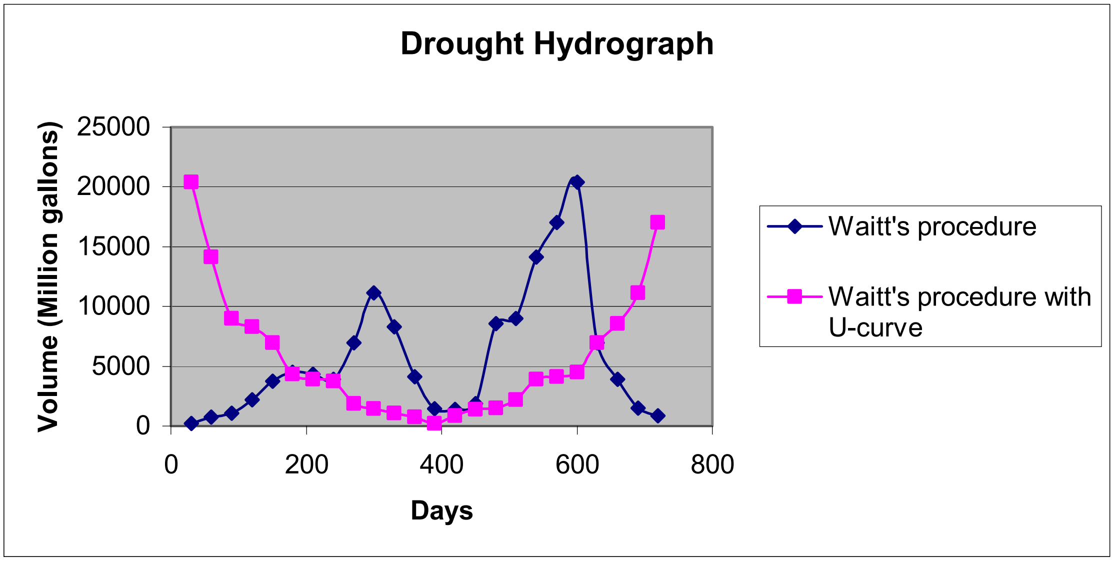

Figure 4.1: Drought Hydrograph with Strict Waitt's Procedure and U-curve

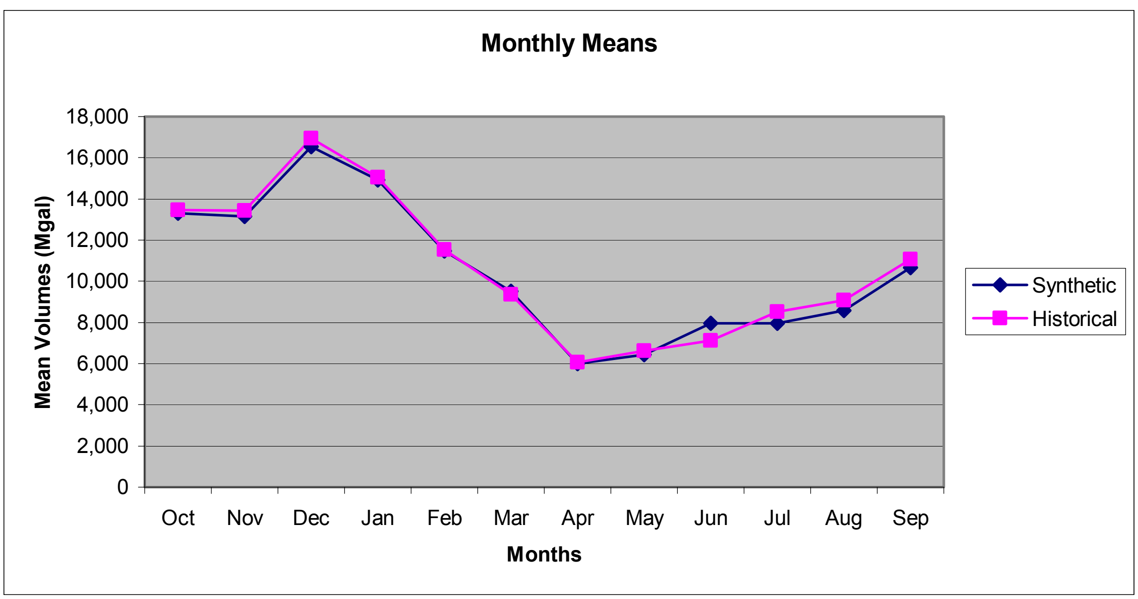

Figure 4.2: Monthly Means: Historical and Synthetic

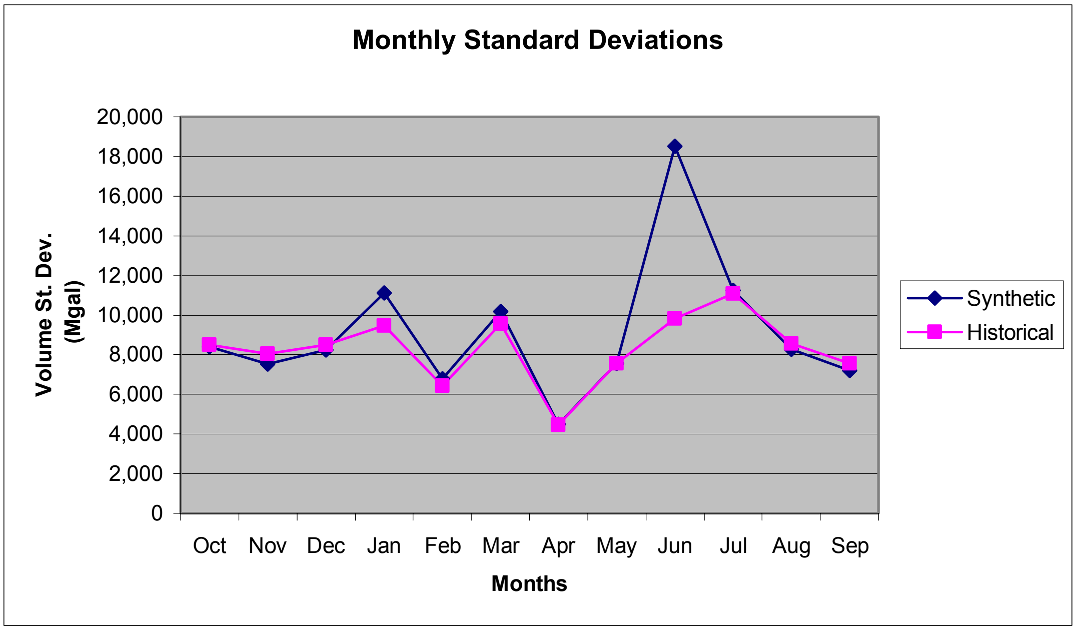

Figure 4.3: Monthly Standard Deviations: Historical and Synthetic

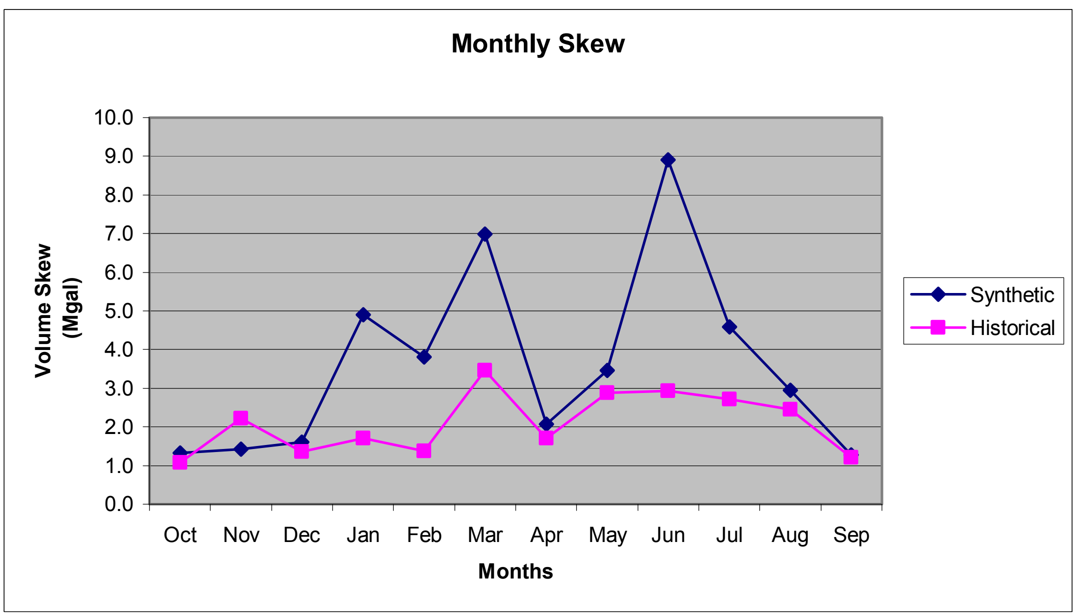

Figure 4.4: Monthly Skews: Historical and Synthetic

Chapter 5: Safe Yield Analysis

Determining the safe yield of an existing water supply is a basic aspect of water supply planning. Where water is removed from a river directly without any storage, the withdrawal is constrained by the worst drought flow in the river. There is no room for operational adjustments other than implementing conservation measures. Where there is a storage reservoir, yields higher than the flow in the source stream can be maintained for a period of time by releasing the water in storage. The calculation of safe yield in this situation requires elaborate computation, which traditionally has been based on the mass curve and a constant draft method to compute the maximum acceptable withdrawal rate as dictated by a historical drought sequence. It should be noted that this definition implies inclusion of both drought severity (magnitude of shortage) and critical duration (sequencing of flows). The *critical duration* is defined as the period from full reservoir to empty reservoir. Utilizing these concepts the yield is given by

Yield = (Reservoir Capacity + Cumulative Inflow over the Critical Drought Period) /Critical Drought Period (5.1)

When a town depends on river flow directly without a storage reservoir, it has little control option. In such a situation, the Virginia Health Department requires that the reliability be measured in terms of the 30-year 1-day flow (or a suitable threshold flow value) and the demand. However, with a storage reservoir a set of control options are possible. The Virginia State Water Control Board [SWBC] (1985) [also see McMahon and Mein(1986) for a review] has provided planning level yield estimates for municipal water supply systems.

The method involves the following steps:

1). A historically worst drought sequence is selected.
2). A constant draft (withdrawal rate) is assumed.
3). Assuming the reservoir to be full at the first period, the continuity (volume balance) equation is applied using the constant draft.
4). At the end of the drought sequence, remaining storage is determined; if it is not empty, the draft rate in step2 is modified and the steps are repeated. The resulting constant draft rate is called the yield.

This type of planning level method provides an assessment of the overall water supply potential of the system but does not give an acceptable control rule for day to day operation; no detailed system performance measures are available; and no guidance towards conservation measures is available. Therefore, each reservoir has a safe yield for a certain characteristic drought – the amount of water that the reservoir could safely be relied upon to provide during a certain characteristic drought. This safe yield would vary according to the magnitude and duration of the drought – naturally, for a more extreme or longer duration drought, the safe yield of the reservoir will be less. In terms of recurrence periods, the safe yield for the 100-year drought will be greater than the safe yield for the 500-year drought. Likewise, the assumptions made in determining the inflow sequence will also affect the safe yield. More conservative assumptions will reduce the safe yield. Another noteworthy point is that the yield computation does not consider the demands. It answers the question what is the greatest constant demand that can be satisfied while receiving inflows as specified for an initial storage of full reservoir.

A method to calculate safe yield is the analytical mass curve method (Loucks, et al., 1981). The basic variables involved are inflow (I_i) and demand(y_i) for each period i. For yield calculations we set $y_i = y$ (constant yield). The deficit for period *i* is given by $dd_i = y_i - I_i$.

The deficit is positive when the demand y_i (or y) exceeds the inflow volume, which will usually be the case in the drought situation. The cumulative deficit (draw-down) k_i, or volume between the current reservoir level and the full pool level is the sum of the cumulative deficit up to and including period i-1 plus the current deficit dd_i. Thus, when the reservoir is full, k_i is equal to zero; when it is empty, it is equal to the capacity of the reservoir. The typical assumption is that the reservoir is full at the beginning of the analysis period so that k_0 is equal to zero.

The negative deficit indicates that the inflow is greater than the demand. When the cumulative deficit turns negative, the reservoir has been filled to capacity, and the physical deficit is zero. This capacity limit is imposed by defining a new variable Adjusted k_i, given by

$$\text{Adjusted } k_i = k_i \quad \text{if } k_i > 0$$

$$\text{and} \quad = 0 \quad \text{if } k_i \leq 0 \qquad (5.2)$$

It is noted that Adjusted k_i calculates the various partial sums of deficits and typically the largest deficit corresponds to the storage. Therefore if one starts with an Adjusted k_{i1} of zero for some i = i1 and when it reaches the storage capacity for the first time at i = i2, the critical period is identified as (i2 – i1). The steps are demonstrated for an inflow sequence for the Hunting Run Reservoir in the Rappahanock River basin shown in Table 5.1. In Table 5.1, the constant demand value is adjusted until the maximum Adjusted k_i value equals the capacity of the reservoir and the corresponding demand equals the yield. The goal-seek function in Microsoft Excel can be easily used to obtain the result. For a storage capacity of 2194.16 mg, the crtical period begins in i1 = 5 and end in i2 = 19 for a critical duration of 14 periods. Applying eq.(5.1) we have yield = (2194.16 + 129.84)/14 = 166 mg that agrees with the yield shown in Table 5.1.

Table 5.1. Tabular method to calculate safe yield

Period i	Duration (days)	Inflow (Mgal)	Demand (Mgal)	Yield - Withdrawal (Mgal)	Ki-1 (Mgal)	Ki (Mgal)	Adjusted Ki (Mgal)
1	30	600.00	166	-434.00	0.00	-434.00	0.00
2	60	600.00	166	-434.00	0.00	-434.00	0.00
3	90	600.00	166	-434.00	0.00	-434.00	0.00
4	120	600.00	166	-434.00	0.00	-434.00	0.00
5	150	600.00	166	-434.00	0.00	-434.00	0.00
6	180	129.84	166	36.16	0.00	36.16	36.16
7	210	0.00	166	166.00	36.16	202.16	202.16
8	240	0.00	166	166.00	202.16	368.16	368.16
9	270	0.00	166	166.00	368.16	534.16	534.16
10	300	0.00	166	166.00	534.16	700.16	700.16
11	330	0.00	166	166.00	700.16	866.16	866.16
12	360	0.00	166	166.00	866.16	1032.16	1032.16
13	390	0.00	166	166.00	1032.16	1198.16	1198.16
14	420	0.00	166	166.00	1198.16	1364.16	1364.16
15	450	0.00	166	166.00	1364.16	1530.16	1530.16
16	480	0.00	166	166.00	1530.16	1696.16	1696.16
17	510	0.00	166	166.00	1696.16	1862.16	1862.16
18	540	0.00	166	166.00	1862.16	2028.16	2028.16
19	570	0.00	166	166.00	2028.16	2194.16	2194.16
20	600	257.84	166	-91.84	2194.16	2102.31	2102.31
21	630	600.00	166	-434.00	2102.31	1668.31	1668.31
22	660	600.00	166	-434.00	1668.31	1234.31	1234.31
23	690	600.00	166	-434.00	1234.31	800.31	800.31
24	720	600.00	166	-434.00	800.31	366.31	366.31

Chapter 6: Summary

It is envisioned that the data found in this thesis can be of use in future water resource planning in the Rappahanock River basin. The historical analysis provides a wide range of safe yield calculations. The purpose of this analysis is not to cast doubt on previous safe yield calculations but to demonstrate the sensitivity of the safe yield calculations to various parameters, and to assess the relative importance of these parameters. The Waitt's procedure analysis generally provided more conservative safe yields than previous calculations; this is typical with the procedure, though, as it nests the most extreme droughts of all durations into the inflow sequence, and it assumes that this sequence of worst droughts occurs two consecutive times. Waitt's procedure truly provides a "worst-case" scenario from a historical perspective. The use of a U-curve technique, where the worst periods are nested in the middle of the drought period, adds another layer of conservativeness. The results in chapter 4 showed a wide range of calculated safe yields, demonstrating a strong sensitivity to the assumptions made in the calculations.

It is important to realize the difference between data sequencing and data generation. Waitt's procedure takes historical data and arranges it in a more conservative manner than the historical record. The difference is in the sequencing of flows. On the other hand, time-series analysis creates flows - that is, it generates synthetic data. The created flows share the same statistical characteristics as the historical record, although each individual data point is not a data point in the historical record. The bootstrap method is a combination of sequencing and generation. A separate dataset is created, but is made up of data points from the historical record.

The time-series analysis produced some interesting results. The return periods calculated by the analysis were much higher than those calculated by other methods, suggesting that the record low flows of 1931 could be more extreme than typically thought. However, there were some concerns as to the integrity of the time series data as discussed in Chapter 4 that might cast doubt on the validity of these results.

The bootstrap method produced results that were more in line with other methods of analysis. As discussed above, the bootstrap method can be advantageous in that it produces a dataset of much longer duration than the historical record, but the dataset is made up of actual data points from the historical record, so that statistical skews such as the ones seen in the time series analysis are not a concern.

It is the author's conclusion that the Markov chain analysis provides the most accurate analysis of drought flows. The correlation between the analytical characteristics and empirical data provide confidence that this approach can reliably predict the recurrence interval of a certain flow. This analysis provides a strong theory-based calculation of the recurrence interval. The advantage to using the Markov chain analysis lies in the practice of using class intervals. This groups flows into classes, so that statistical outliers such as the 1931 flows in this study do not influence the calculations as much. This is especially useful in this project, where there is such an extreme low flow, in 1931, compared to the rest of the historical record.

There is definitely potential for more analysis on the time-series and Markov chain aspects of the project. Some of the analysis discussed in earlier sections pertaining to creating multiple sets of synthetic data and performing frequency analysis on that data could be very valuable. Likewise, for the Markov chain analysis, the more complicated monthly analysis could be undertaken for more detailed analysis.

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