Dedication

To my wife, Alvera.

Acknowledgments

First and foremost, I would like to thank the thesis committee for their encouragement and guidance. Their comments helped me to narrow my topic in order to produce a more compressive paper. Dr. Lutton's continuous assistance with the econometrics aspects of this paper was very much appreciated, particularly because he managed to make the process into a challenging learning experience. Secondly, I would like to thank my classmates, co-workers, friends, and family for their encouragement and support.

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I. Introduction

Are labor inputs in U.S. manufacturing industries perfectly substitutable?

The traditional capital-labor substitution framework assumes that labor inputs are perfectly substitutable; therefore, they can be combined into an aggregate labor index without losing economic information.[1] This assumption allows one the luxury of using two-factor production and cost functions to estimate the various combinations of capital and labor needed to produce a given level of output and to determine the optimal combination of capital and labor for a given fixed dollar expenditure. While it is true that little is lost by aggregating perfectly substitutable labor inputs, it does not follow that little is lost when labor inputs are highly but imperfectly substitutable.

If labor inputs are not perfectly substitutable with capital, then it is possible to test the capital-skill complementarity hypothesis. This hypothesis states that the more skills acquired by workers, the more likely they will complement capital in the production process (Griffin, 1992). To provide support for this hypothesis, the elasticity of substitution estimates between labor inputs and capital must vary based on skill differences between labor inputs.

This paper consists of six sections. The first section examines the four issues that arise when using three-factor functions. The second section outlines the translog model and describes how it is used to calculate elasticity of substitution estimates. The third section discusses three data issues associated with the translog model that should be addressed before calculating elasticity of substitution estimates. The fourth section displays elasticity of substitution estimates from past studies. The fifth section presents elasticity estimates using an updated data set that is comparable to the ones used in past studies. The

[1] Ernst R. Berndt and Laurits R. Christensen, “Testing for the Existence of a Consistent Aggregate Index of Labor Inputs”, *The American Economic Review*, 1974

sixth section provides additional results using various industry breakouts designed to subdivide the data set.

II. Three-factor functions

Much of the empirical literature from the mid-1970s involved testing whether three-factor production and cost functions were a complexity worth considering and whether their theoretical implications made empirical sense.[2] The desire to perform this research stemmed from the oil price shock experienced in the mid-1970s. This price shock caused an unexpected period of high inflation, coupled with high unemployment. Given the sharp rise in the cost of capital associated with this price shock, researchers began using three-factor functions in an attempt to understand the relationship between capital and labor demand.

By the late 1970s, economists recognized that empirical estimates of elasticities of substitution were useful for predicting the effects of labor market policy changes.[3] This realization led to using three-factor functions to explore the effects of labor policies ranging from employment tax credits to employment training programs. These functions were deemed advantageous because they provided elasticity of substitution estimates between multiple inputs. For example, the benefits derived from a training program aimed at low-skill workers may be influenced by the degree of substitutability between these workers and high-skill workers in the production process.[4] If this is true, then calculating elasticity of substitution estimates can help researchers better understand the effects associated with the training program.

Four important issues, which do not arise in a two-factor framework, must be addressed when working with three-factor functions. The first issue is choosing

[2] Daniel S. Hamermesh and James Grant, “Econometric Studies of Labor-Labor Substitution and Their Implications for Policy”, *The Journal of Human Resources*, 1979
[3] Ibid.
[4] George Johnson, "The Potential Efficacy of Labor Market Policy", *University of Michigan*, 1977

whether to estimate a cost or a production function. The second issue is choosing the functional form to estimate. The third issue is selecting the criteria to disaggregate the labor force. The fourth issue is deciding the criteria to disaggregate the economy.

Microeconomic theory has clearly shown that duality exists between cost and production functions—so which approach should be chosen? The use of a cost function assumes that firms minimize costs, while the use of a production function requires the more stringent assumption of profit maximization.[5] This allows for elasticities of substitution estimates between factors to be calculated from one equation using a cost minimization framework (see the Allen Elasticity of Substitution equation on page 10).[6] Using a production function to do the same calculation would require inverting a matrix of estimated coefficients.[7] For this reason, a cost minimization framework has been chosen for this paper.

The next issue is choosing the functional form. The first generation of studies involved a variation of the constant elasticity of substitution function (CES), while more recent studies have used the transcendental logarithmic function, a second-order approximation to a generalized cost function.[8] Although the CES form is simpler to estimate, it constrains the substitution elasticities between all pairs of factors to be equal (Hamermesh et al., 1979). This restriction is not imposed by the translog function. Unfortunately, it is difficult to estimate a dynamic version of the translog function because it is usually estimated indirectly by assuming a competitive equilibrium. This may be a problematic issue for time-series estimation and for cases where adjustment costs are likely to be substantial.[9] However, the translog's flexibility in calculating the elasticity of

[5] Ernst Berndt and David Wood, "Engineering and Economic Approaches to Industrial Energy Conservation and Capital Formation", *MIT Energy Laboratory*, 1977

[6] Daniel S. Hamermesh and James Grant, "Econometric Studies of Labor-Labor Substitution and Their Implications for Policy", *The Journal of Human Resources*, 1979

[7] Ibid.

[8] Ibid.

[9] Ibid.

substitution between input factors makes it the preferred functional form for empirical research.[10]

The first of the two disaggregation issues is how to divide the labor force. The most common breakdowns are by age group, educational attainment, race, and occupation. For example, Berger (1983) separates different labor groups according to gender and education levels in order to study the effects of changes in the composition of the labor force on the earnings of different classes of workers. Similarly, Griffin (1991) sorts workers into race and gender classes to examine the interactions between these labor market groups. While intuitively appealing, such studies are not as empirically valid as studies utilizing labor groups defined by occupation.[11] Since the aggregation of heterogeneous workers into groups can be problematic, the seemingly more valid labor group classifications are those for which all workers within each labor group are homogeneous in their productive capabilities.[12] In general, the more homogeneous the groupings, the more meaningful the empirical results. This can be best achieved by dividing the data by occupation (Griffin, 1992).

The second disaggregation issue involves the criteria used to disaggregate the economy. Past studies have used industries, States, Metropolitan Statistical Areas (MSA), or combinations of industries and States.[13] While much of the available data on wage and employment are by State, the State is a political rather than an economic unit. To the extent that labor markets overlap States, estimates based on State data are difficult to interpret. Using MSA data minimizes this problem, since it constitutes a closer approximation of local labor markets. The use of industry data poses conceptual problems in interpreting estimates of substitution parameters, since speaking about a labor market within an industry is conceptually difficult.

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[10] Ibid.
[11] Peter Griffin, "The substitutability of Occupational Groups", *Economic Letters*, 1992
[12] Ibid.

To recapitulate, three-factor production and cost functions are useful for addressing elasticity of substitution concerns between input factors. The use of a cost minimization framework is advantageous because it simplifies elasticity calculations.   The use of a translog functional form is preferable because it permits flexible elasticity calculations. The use of occupational and industry breakouts address the question regarding the degree of substitutability between labor inputs in U.S. manufacturing industries and allows for a comparison to research done in the 1970s.

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[13] Daniel S. Hamermesh and James Grant, "Econometric Studies of Labor-Labor Substitution and Their Implications for Policy", *The Journal of Human Resources*, 1979

III.Translog model

Many studies of production are done in the context of a flexible functional form. Flexible functional forms are used in econometrics because they allow analysts to model second-order effects such as elasticities of substitution, which are functions of the second derivatives of production, cost, or utility functions.[14] The linear model restricts these to equal zero, whereas the log-linear model (e.g., Cobb-Douglas model) restricts the elasticities to the values of -1 or +1.[15] Among the most frequently used flexible functional forms in empirical work is the translog function. It is derived from a Taylor Series expansion and is often interpreted as a second-order approximation to an unknown functional form.[16] As shown below, the translog functional form of a Cobb-Douglas function mimics a large amount of curvature that allows elasticity values to vary.

Suppose that production is characterized by a Cobb-Douglas production function of the (implicit) form:

$$Q = f(X_n, X_p, X_k)$$

where X_n is the flow of non-production worker services, X_p is the flow of production worker services, X_k is the flow of capital services. A cost minimization approach yields the minimum cost of producing a specific output level for any input prices. Given the input prices, the approach provides the optimal levels of labor and capital to employ by solving:

$$\underset{X_n,\, x_p,\, x_k}{\text{Min}}\ C = P_nX_n + P_pX_p + P_kX_k \quad \text{s.t. } Q = f(X_n, X_p, X_k)$$

[14] William H. Greene, “Econometric Analysis”, third edition, *Prentice Hall*, 1997
[15] Ibid.
[16] Ibid.

where P_n is the average annual wage for the firm's non-production labor X_n, P_p is the average annual wage for the firm's production labor X_p, and P_k is the rental rate for the firm's capital X_k.

By taking the log of the Cobb-Douglas cost function and expanding "lnC" in a second-order Taylor Series, the function obtains the translog form:

$$\ln C = \ln Q + \ln\delta_0 + \sum\delta_i \ln P_i + 1/2\sum\sum\delta_{ij}\ln P_i \ln P_j + \mu$$

where the technology parameters are δ_0, δ_i, and δ_{ij}. C and P_i represent the total cost and input prices, respectively. Furthermore, μ represents the stochastic error inherent in the approximation of an unknown function. If δ_{ij} equals zero, the translog reduces to the standard Cobb-Douglas function.

By using a cost minimization approach and assuming input markets are competitive, Shephard's Lemma demonstrates that the partial logarithmic derivatives of the Cobb-Douglas translog function are:

$$\partial \ln C/\partial \ln P_i = (X_i P_i)/C = \delta_i + \sum\delta_{ij}\ln P_i = S_i + \varepsilon_i$$

where:

$X_i = Q_n^*, Q_p^*, Q_k^*$—optimal level of usage of i_{th} factor

δ_i, δ_{ij} = technology parameters

$S_i = S_n, S_p, S_k$—the cost share of input X_i in the total cost of producing Q

ε_i = stochastic error

The stochastic errors arise because the cost share equations approximate the underlying cost structure.[17] The three cost shares in terms of non-production workers (S_n), production workers (S_p), and capital (S_k) are:

[17] Raymond Kopp and Kerry Smith, "Measuring the Prospects for Resource Substitution under Input and Technology Aggregations", *The MIT Press*, 1979

$$S_n = Q_n(P_n)/C = \partial \ln C/\partial \ln P_n = \delta_n + \delta_{nn}(\ln P_n) + \delta_{np}(\ln P_p) + \delta_{nk}(\ln P_k) + \varepsilon_n$$

$$S_p = Q_p(P_p)/C = \partial \ln C/\partial \ln P_p = \delta_p + \delta_{np}(\ln P_n) + \delta_{pp}(\ln P_p) + \delta_{pk}(\ln P_k) + \varepsilon_p$$

$$S_k = Q_k(P_k)/C = \partial \ln C/\partial \ln P_k = \delta_k + \delta_{nk}(\ln P_n) + \delta_{pk}(\ln P_p) + \delta_{kk}(\ln P_k) + \varepsilon_k$$

The cost share S_n is the total cost of employing non-production workers (n) to produce at the optimal level of output. The cost share S_p is the total cost of employing production workers (p) to produce at the optimal level of output. Likewise, the cost share S_k is the total cost of employing capital (k) to produce at the optimal level of output.

In order for the translog cost function to be homogeneous in prices, the cost shares must sum to one. This requires that the following three constraints be imposed:

1. $\delta_n + \delta_p + \delta_k = 1$
2. $\delta_{nn} + \delta_{np} + \delta_{nk} = 0$
3. $\delta_{nk} = \delta_{kn}$ (symmetry)

There are two standard approaches to econometrically estimate the translog function. The first approach is to estimate the translog equation directly and then solve for the cost shares. The second approach is to estimate two of the cost share equations simultaneously and then impose the constraints to solve for the third cost share equation.

By using the second approach, the cost share equations will provide a seemingly unrelated regression model that can be used to estimate the parameters of the model. To make the model operational, one must impose the constraints and solve the problem of singularity of the disturbance covariance matrix of the share equations.[18] This can be done by eliminating the last term in each row and column of the parameter matrix and by dropping one of the cost share equations.

It is possible to substitute the constraint $\{\delta_{nn} = -\delta_{np} + -\delta_{nk}\}$ into the first cost share equation:

$$\hat{S}_n = \delta_n + \delta_{np}(\ln P_p - \ln P_n) + \delta_{nk}(\ln P_k - \ln P_n).$$

Likewise, substituting in the fact that $\{\delta_{pp} = -\delta_{pn} + -\delta_{pk}\}$ into the second cost share yields:

$$\hat{S}_p = \delta_p + \delta_{np}(\ln P_n - \ln P_p) + \delta_{pk}(\ln P_k - \ln P_p).$$

By estimating two of the three cost shares using the seemingly unrelated regression technique and using the fact that $\{S_k = 1 - S_n - S_p\}$, it is possible to solve for the third cost share.

Once the three cost shares are estimated, the elasticity of substitution (Allen Elasticity of Substitution) between pairs of factors can be calculated.[19] For the translog cost specification, the elasticity of substitution between factors is:

$$\sigma_{ij} = (\delta_{ij} + \hat{S}_i\hat{S}_j)/\hat{S}_i\hat{S}_j \qquad i \neq j$$

where:

$\sigma_{ij} > 0 \rightarrow$ the factors are substitutes

$\sigma_{ij} < 0 \rightarrow$ the factors are complements

$\sigma_{ij} = 0 \rightarrow$ the factors have no relationship

[18] William H. Greene, “Econometric Analysis”, third edition, *Prentice Hall*, 1997
[19] Daniel S. Hamermesh and James Grant, “Econometric Studies of Labor-Labor Substitution and Their Implications for Policy”, *The Journal of Human Resources*, 1979

For further details on how to derive the translog model for two-factor and three-factor functions, refer to the mathematical appendix beginning on page 29.

IV. Data issues

There are three important data issues that should be discussed before interpreting the translog model's estimates of elasticity of substitution between factors. The first issue involves the scale effect; the second issue involves the degree of technological change; and the third issue involves measurement problems.

Scale effects can arise from dividing sample data into narrowly defined industries. For example, an increase in the demand for capital can reflect an industry-wide expansion induced by relative declines in the prices of factors that are complements to capital, as well as substitution within an industry towards cheaper factors. Such scale effects may induce an upward bias in substitution estimates based on industry data; however, studies have shown that the use of time-series data will minimize this effect.[20] For this reason, time-series data may produce elasticity of substitution estimates that are less than those produced by studies using cross-sectional data.

Although the use of time-series data may minimize scale effects, estimates derived from time-series data tend to be sensitive to non-neutral technical changes that are correlated with factor price changes but not induced by them.[21] The effect of technological innovations will be incorrectly attributed to factor price changes if the following two conditions hold: (1) the correlation implies increasing use of factors that are declining in relative price and (2) the estimates are derived using the assumption of Hicks-neutral technical change.[22] Hence, time-series elasticity of substitution estimates may be biased due to technical changes.

[20] Ibid.
[21] Ibid.
[22] Ibid.

In addition to scale effects and non-neutral technical changes, measurement problems are a potential source of bias. The most common measurement problem involves not accounting for the types of training or education that different labor groups receive. If part of the relative wage difference between college and high school graduates results from the higher quality of college education, the appropriate wage differential between labor quantities will be less than the measured wage difference.[23] By not holding these differences constant estimates may be biased, if wages reflect differences in the quality of education.

Like past studies that divide data by occupation and by industry, this study does not directly address these three potential data issues (see section IV). This study attempts to account for the scale effect by using time-series data; however, non-neutral technological changes, and measurement problems remain a possible source of bias.

[23] Ibid.

V. Research results from past studies

Many three-factor models use two types of labor inputs to divide labor by occupation—with the majority using a breakdown between production and non-production workers.[24] Undoubtedly, this is largely due to the availability of data from government sources that separate labor by occupation.

Although three-factor models have been used for a wide variety of issues, they are well suited for estimating the partial elasticities among inputs. The elasticity of substitution results from six studies done in the 1970s are presented in table one. The subscript ‘p’ denotes production workers, ‘n’ denotes non-production workers, and ‘k’ denotes capital. The elasticities of substitution between pairs of factors are shown using the convention that $\sigma_{ij} > 0$ denotes the factors are substitutes.

Table one: Elasticities of substitution—production and non-production workers[25]

Study	Data and Method	σ_{pk}	σ_{nk}	σ_{pn}
Berndt-White	Manufacturing, 1941-71, translog	0.91	1.09	3.70
Clark-Freeman	Manufacturing, 1950-76, translog	2.10	-1.98	0.91
Denny-Fuss	Manufacturing, 1929-68, translog	1.50	-0.91	2.06
Dennis-Smith	Manufacturing, 1952-73, translog	0.14	0.38	-0.05
Kesselman	Manufacturing, 1962-71, translog	1.28	-0.48	0.49
Berndt-Christensen	Manufacturing, 1929-68, translog	2.92	-1.94	5.51
	Average	1.48	-0.64	2.10

Five of the six studies in table one find that production and non-production workers are substitutes. All six studies find that production workers and capital

[24] Ibid.

are substitutes. The results are less clear-cut between non-production workers and capital; four of six studies find that they are complements.

Calculating the average of each of the elasticity columns in table one indicates that economic information may be lost when combining labor into an aggregate index, since non-production workers and production workers substitute differently with capital. The average elasticity of substitution estimate between non-production workers and capital is –0.64 according to the table. The average elasticity of substitution estimate between production workers and capital is 1.48. This suggests that non-production workers and capital are complements, while production workers and capital are substitutes.

VI. Research results

The next two sections provide research results from a translog model designed to estimate the elasticity of substitution between capital, non-production workers, and production workers. This section provides elasticity estimates for U.S. manufacturing that are comparable to the results from the studies shown in table one on page 13. The next section provides the elasticity of substitution estimates for four manufacturing industry breakouts. The breakouts include labor-intensive, capital-intensive, non-durable goods, and durable goods industries.

It will be shown that the elasticity estimates provide support to the Capital-skill complementarity hypothesis (Griffin, 1992). The hypothesis states that the more skills a group of workers have, the more likely they will complement capital in the production process. Using the translog model described above, support for this hypothesis requires that the elasticity of substitution estimates between non-production workers and capital are less than they are for production workers and

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[25] For a complete table that includes cross-sectional data results see Hamermesh and Grant.

capital. The preferred elasticity estimates will show capital to be a substitute for production workers, but a complement to non-production workers.

Data description

The data set used in this study was developed using 2-digit Standard Industrial Classification (SIC) manufacturing data for the time period 1988 to 1997. Data elements were obtained from the Bureau of Labor Statistics (BLS), the Bureau of Economic Analysis (BEA), and the Federal Reserve Board (FRB). Although the data set is unique, it was designed to mimic the data elements used in past studies.

The data set relies on the Bureau of Labor Statistics' definitions for production and non-production workers used in their various data series. Non-production workers are defined as employees who direct, supervise, or plan the work of others. This category includes workers in occupations that generally require skills obtained from education or training. Production workers are defined as employees who are not primarily employed to direct, supervise, or plan the work of others. These definitions increase the accuracy of elasticity calculations by decreasing the amount of earnings overlap between production and non-production workers and by dividing workers by skill levels.

The capital data consists of the total dollar expenditures on equipment and structures for each 2-digit manufacturing SIC industry group. Since the capital data obtained from BEA did not distinguish between the quantity and price of capital, an implicit rental rate (price) of capital was derived using an equation developed by Hall and Jorgenson.[26] The equation is described in detail in the statistical appendix beginning on page 34.

Research method

[26] Hall and Jorgenson, “Tax Policy and Investment Behavior”, *American Economic Review*, 1967

Unlike the studies done in the 1970s that utilized two-stage least squares, this study uses the seemingly unrelated regression approach to estimate the elasticity of substitution between factors. This approach is advantageous for estimating three-factor functions because it provides a straightforward method to impose the constraints, to stack the data, and to estimate the cost shares simultaneously.

The seemingly unrelated regression technique requires a three-step process. The first step is to stack the data from two cost shares. The second step is to use ordinary least squares (OLS) to estimate the share equations simultaneously. The third step is to use generalized least squares (GLS) to allow the variances to change between the shares.

The following two cost shares were estimated using the three-step process:

$$\hat{S}_n = \delta_n + \delta_{np}(\ln P_p - \ln P_n) + \delta_{nk}(\ln P_k - \ln P_n)$$

$$\hat{S}_p = \delta_p + \delta_{np}(\ln P_n - \ln P_p) + \delta_{pk}(\ln P_k - \ln P_p)$$

The cost share S_n represents the total cost of employing non-production workers (n) to produce at the optimal level of output. Similarly, the cost share S_p represents the total cost of employing production workers (p) to produce at the optimal level of output. The technology parameters (δ_i and δ_{ij}) are estimated by the model using the assumption that they are Hicks-neutral (technology advances that are caused by external factors do not change the relative price between factors).

Regression results

Table two: Regression results for manufacturing industries for the period 1988 to 1997

R-square	0.528
Adjusted R-sq.	0.521
Standard error	0.982
Observations	200

Variable	Parameter estimate	Standard error	T-statistic
δ_n	-8.028	1.327	-6.048
δ_{np}	0.069	0.011	6.214
δ_{nk}	-0.056	0.006	-8.611
δ_p	8.426	1.480	5.694
δ_{pk}	0.015	0.006	2.278

Table three: Elasticity of substitution estimates for manufacturing industries for the period 1988 to 1997

Variable	Elasticity of Substitution	Standard error[27]	T-statistic
σ_{pn}	1.41	0.278	5.080
σ_{pk}	1.18	0.264	4.473
σ_{nk}	0.06	0.326	0.184

The elasticity of substitution estimates presented in table three are calculated using the formula:

$$\sigma_{ij} = (\delta_{ij} + \hat{S}_i\hat{S}_j)/\hat{S}_i\hat{S}_j \qquad i \neq j$$

[27] See statistical appendix on page 35

The elasticity of substitution estimates from this study indicate that production workers and capital are substitutes (σ_{pk} = 1.18), as well as production workers and non-production workers (σ_{np} = 1.41). However, the elasticity of substitution estimate between non-production workers and capital (σ_{nk} = 0.06) is inconclusive since the estimate does not differ significantly from zero. It is not possible to conclude if non-production workers and capital are complements or if they are substitutes. The results do show, however, that the degree of substitutability between production workers and capital is greater than it is for non-production workers and capital.

The elasticity of substitution estimates from this study are consistent with the results from the six studies shown in table one on page 13. The elasticity of substitution estimate between production workers and capital calculated from this study (σ_{pk} = 1.18) compares favorably with the average elasticity calculated from the past studies (σ_{pk} = 1.48). These estimates suggest that production workers and capital are substitutes for one another in the production process. Similarly, the degree of substitutability between non-production workers and production workers derived from this study (σ_{np} = 1.41) compare favorably with the average elasticity derived from the past studies (σ_{np} = 2.10). Both of these estimates suggest that non-production workers and production workers are substitutes for each other in the production process.

VII. Research results from disaggregated data

Since the results using the entire manufacturing data set are inconclusive regarding the degree of substitutability between non-production workers and capital, elasticity estimates for four industry breakouts are presented below. The four industry breakouts are labor-intensive, capital-intensive, non-durable goods, and durable goods manufacturing industries. These breakouts were chosen to investigate the relationship between non-production workers and capital, and to test the capital-skill complementarity hypothesis.

The labor and capital intensive industry breakouts are derived directly from the data, based on the ratio between the average annual capital expenditures to total wage bill, for each 2-digit manufacturing industry group. Labor-intensive industries are defined as those industries that have a capital-to-labor ratio of .20 or less. Similarly, capital-intensive industries are defined as those industries that have a capital-to-labor ratio of .21 or more. Chart one below shows the average capital-to-labor ratio for each of the 2-digit manufacturing industry groups between 1988 and 1997.

The eight capital-intensive industries are:

- Petroleum refining and related industries (SIC 29)
- Chemicals and allied industries (SIC 28)
- Paper and allied industries (SIC 26)
- Tobacco products (SIC 21)
- Food and kindred products (SIC 20)
- Electronic and other electrical equipment and components (SIC 36)
- Primary metal industries (SIC 33)
- Rubber and miscellaneous plastics products (SIC 30)

Chart one: The average annual capital expenditures to total wage bill for U.S. manufacturing industries (1988 to 1997)

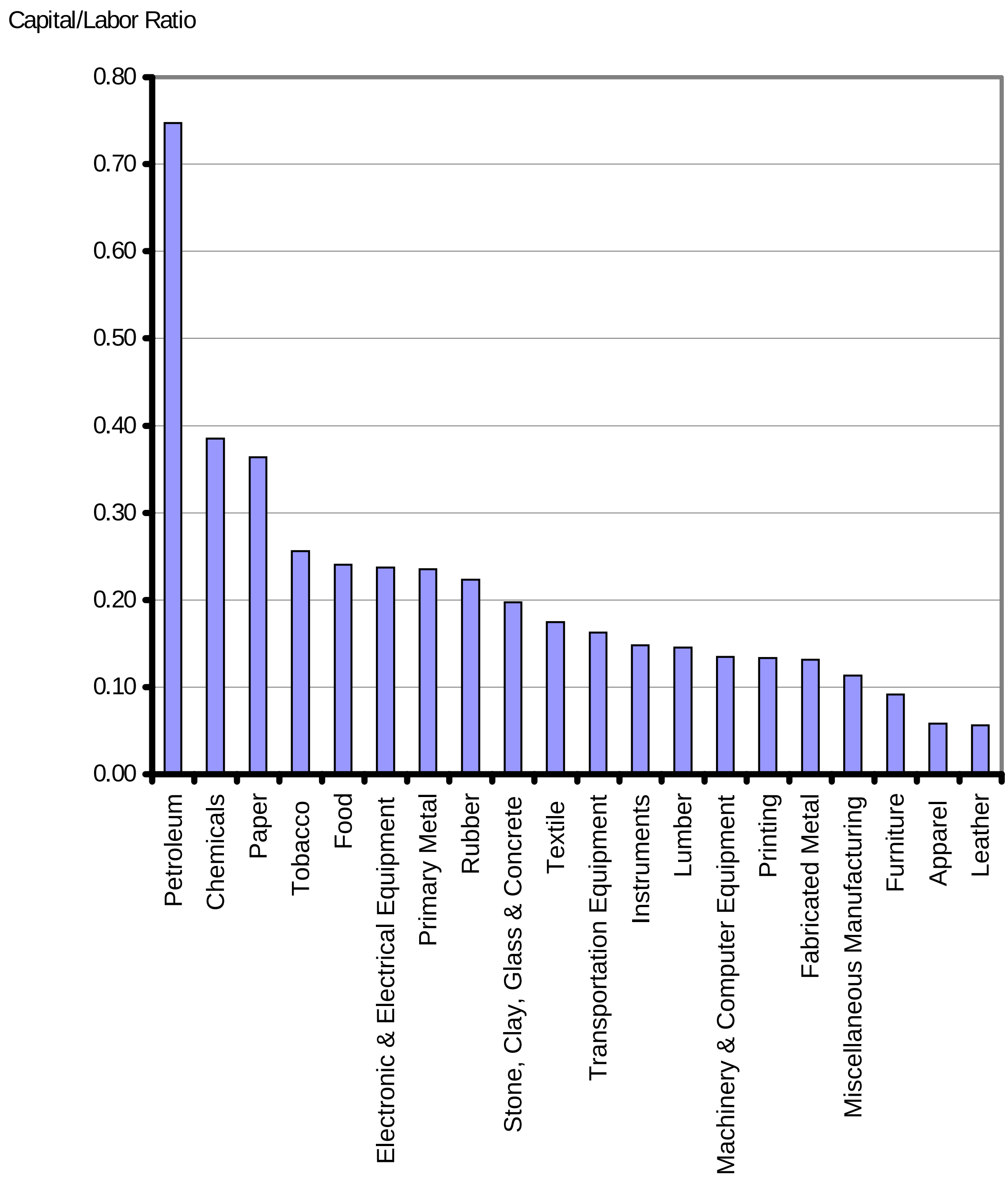


Table four: Regression results for capital-intensive manufacturing industries for the period 1988 to 1997

R-square	0.552
Adjusted R-sq.	0.534
Standard error	0.954
Observations	80

Variable	Parameter estimate	Standard error	T-statistic
δn	-5.830	1.877	-3.105
δnp	0.085	0.017	4.843
δnk	-0.040	0.008	-5.023
δp	5.995	2.029	2.955
δpk	0.002	0.007	0.331

Table five: Elasticity of substitution estimates for capital-intensive manufacturing industries for the period 1988 to 1997

Variable	Elasticity of Substitution	Standard error	T-statistic
σ_{pn}	1.60	0.580	2.757
σ_{pk}	1.02	0.271	5.897
σ_{nk}	0.49	0.321	1.528

The elasticity of substitution estimates for capital-intensive manufacturing industries are shown in table five. The elasticity of substitution between production workers and non-production workers is 1.60. The elasticity of substitution between production workers and capital is 1.02. The elasticity of substitution between non-production workers and capital is 0.49 (90% significance level). This suggests that for capital-intensive industries, production workers, non-production workers, and capital are substitutes for one another.

The twelve labor-intensive industries are:

- Stone, clay, glass, and concrete products (SIC 32)
- Textile mill products (SIC 22)
- Transportation equipment (SIC 37)
- Measuring, analyzing, and controlling instruments (SIC 38)
- Lumber and wood products (SIC 24)
- Industrial and commercial machinery and computer equipment (SIC 35)
- Printing, publishing, and allied industries (SIC 27)
- Fabricated metal products (SIC 34)
- Miscellaneous manufacturing industries (SIC 39)
- Furniture and fixtures (SIC 25)
- Apparel and other textile products (SIC 23)
- Leather and leather products (SIC 31)

Table six: Regression results for labor-intensive manufacturing industries for the period 1988 to 1997

R-square	0.675
Adjusted R-sq.	0.665
Standard error	0.984
Observations	120

Variable	Parameter estimate	Standard error	T-statistic
δ_n	-10.848	1.813	-5.984
δ_{np}	0.073	0.013	5.669
δ_{nk}	-0.071	0.009	-7.898
δ_p	13.270	2.118	6.266
δ_{pk}	0.031	0.009	3.554

Table seven: Elasticity of substitution estimates for labor-intensive manufacturing industries for the period 1988 to 1997

Variable	Elasticity of Substitution	Standard error	T-statistic
σ_{pn}	1.38	0.276	4.992
σ_{pk}	1.54	0.514	2.994
σ_{nk}	-0.74	0.642	-1.153

The elasticity of substitution estimates for labor-intensive manufacturing industries are shown in table seven. They indicate that production workers and non-production workers are substitutes (σ_{pn} = 1.38), as well as production workers and capital (σ_{pk} = 1.54). However, the elasticity of substitution estimate for non-production workers and capital is -0.74, which suggests that they are complements (85% significance level). This result differs from the capital-intensive industry's elasticity estimate of 0.49 between non-production workers and capital.

The elasticity of substitution estimates shown in tables nine and eleven below are based on a disaggregation of the data into non-durable and durable goods industries. This breakout divides the data into two industry groups consisting of ten manufacturing industries each, as defined by the *1987 Standard Industrial Classification (SIC) Manual*. Broadly defined, durable goods industries manufacture products designed to last over three years, while non-durable goods industries manufacture products designed to last less than three years.

The ten non-durable goods industries are:

- Food and kindred products (SIC 20)
- Tobacco products (SIC 21)
- Textile mill products (SIC 22)
- Apparel and other textile products (SIC 23)
- Paper and allied industries (SIC 26)

- Printing, publishing, and allied industries (SIC 27)
- Chemicals and allied industries (SIC 28)
- Petroleum refining and related industries (SIC 29)
- Rubber and miscellaneous plastics products (SIC 30)
- Leather and leather products (SIC 31)

Table eight: Regression results for non-durable goods manufacturing industries for the period 1988 to 1997

R-square	0.404
Adjusted R-sq.	0.386
Standard error	1.009
Observations	100

Variable	Parameter estimate	Standard error	T-statistic
δ_n	-2.442	1.731	-1.411
δ_{np}	0.058	0.018	3.340
δ_{nk}	-0.027	0.008	-3.418
δ_p	7.843	1.832	4.281
δ_{pk}	0.013	0.008	1.618

Table nine: Elasticity of substitution estimates for non-durable goods manufacturing industries for the period 1988 to 1997

Variable	Elasticity of Substitution	Standard error	T-statistic
σ_{pn}	1.37	0.463	2.956
σ_{pk}	1.14	0.281	4.053
σ_{nk}	0.57	0.409	1.395

The elasticity of substitution estimates for non-durable goods industries are shown in table nine. The elasticity of substitution between production workers and non-production workers is 1.37. The elasticity of substitution between production workers and capital is 1.14. The elasticity of substitution between non-production workers and capital is 0.57 (90% significance level). This suggests that for non-durable goods industries, production workers, non-production workers, and capital are substitutes for one another.

The ten durable goods industries are:

- Lumber and wood products (SIC 24)
- Furniture and fixtures (SIC 25)
- Stone, clay, glass, and concrete products (SIC 32)
- Primary metal industries (SIC 33)
- Fabricated metal products (SIC 34)
- Industrial and commercial machinery and computer equipment (SIC 35)
- Electronic and other electrical equipment and components (SIC 36)
- Transportation equipment (SIC 37)
- Measuring, analyzing, and controlling instruments (SIC 38)
- Miscellaneous manufacturing industries (SIC 39)

Table ten: Regression results for durable goods manufacturing industries for the period 1988 to 1997

R-square	0.760
Adjusted R-sq.	0.750
Standard error	0.970
Observations	100

Variable	Parameter estimate	Standard error	T-statistic
δ_n	-21.524	2.388	-9.013
δ_{np}	0.111	0.013	8.680
δ_{nk}	-0.118	0.011	-10.551
δ_p	15.624	3.133	4.987
δ_{pk}	0.032	0.011	2.990

Table eleven: Elasticity of substitution estimates for durable goods manufacturing industries for the period 1988 to 1997

Variable	Elasticity of Substitution	Standard error	T-statistic
σ_{pn}	1.61	0.293	5.498
σ_{pk}	1.47	0.581	2.529
σ_{nk}	-1.28	0.635	-2.017

The elasticity of substitution estimates for durable goods industries are shown in table eleven. They indicate that production workers and non-production workers are substitutes (σ_{pn} = 1.61), as well as production workers and capital (σ_{pk} = 1.47). Unlike the case for non-durable goods industries, the elasticity of substitution estimate for non-production workers and capital is -1.28. This suggests that non-production workers and capital are complements.

Table twelve: Summary of elasticity results

Data	σ_{pn}	σ_{pk}	σ_{nk}
Capital-intensive	1.60	1.02	0.49
Labor-intensive	1.38	1.54	-0.74
Non-durable goods	1.37	1.14	0.57
Durable goods	1.61	1.47	-1.28

Instead of helping to clarifying the relationship between non-production workers and capital, the industry breakouts provide a mixed message. The durable goods and labor-intensive industry estimates suggest that non-production workers and capital are complements, while the non-durable goods and capital-intensive industry estimates suggest that they are substitutes.

So what is the relationship between non-production workers and capital? Averaging the elasticity estimate between the labor and capital intensive industry breakouts yields an elasticity estimate of approximately -0.12. Doing the same calculation using the non-durable and durable goods industry breakouts yields an elasticity estimate of approximately -0.35. This suggests that, on average, non-production workers and capital are complements in the production process. However, it is possible that the elasticity of substitution between non-production workers and capital is simply different between the various industry groupings and that it is not valid to generalize one way or the other for all manufacturing industries.

Unlike the elasticity estimates between non-production workers and capital, the results are consistent for the other two elasticity estimates. All four of the industry breakouts suggest that production workers and capital are substitutes in the production process. Similarly, each breakout indicates that production workers and non-production workers are substitutes.

To provide conclusive support for the capital-skill complementarity hypothesis, the elasticity results should demonstrate that non-production workers and capital are complements and production workers and capital are substitutes. This is synonymous with the belief that production workers have fewer skills and can be viewed as cost-effective substitutes for capital inputs, such as machines designed to do monotonous, routine work. Non-production workers possess additional skills that can complement capital inputs, such as computers; however, only the elasticity estimates for the labor-intensive and durable goods industry breakouts demonstrate this relationship.

Although the industry breakout results do not provide conclusive support for the hypotheses, they do show that the elasticity of substitution between production workers and capital is greater than the elasticity between non-production workers and capital. This suggests that skill differences may influence elasticity estimates between labor inputs and capital.

VIII. Conclusion

This paper presents a translog model designed to estimate the elasticity of substitution between capital, non-production workers, and production workers using U.S. manufacturing data from 1988 to 1997. The elasticity of substitution estimates derived from the translog model suggests that production workers and capital are substitutes. They also suggest that production and non-production workers are substitutes. However, they do not provide conclusive evidence regarding the degree of substitutability between non-production workers and capital.

The elasticity of substitution estimates shown in this paper demonstrate that dividing labor inputs by their skill differences indicate that different types of labor are not perfectly substitutable with capital. The estimates uniformly suggest that the degree of substitutability between non-production workers and capital is less than it is for production workers and capital. This supports the capital-skill complementarity hypothesis, which states that as workers gain more skills they are more likely to complement capital in the production process. The elasticity estimates do not ultimately support the hypothesis' stronger assertion that non-production workers and capital are complements in the production process for all manufacturing industries.

Mathematical Appendix

A. Two-Factor Cost Minimization

(1) $C \equiv WL + RK + \delta M$

where W is the total annual wage bill for the firm's labor L, R is the rental rate of capital K, and δ is the cost of materials M, such as energy, inventory, and working capital. If we assume that M is separable { $C = f(L, K, g(M))$ } then the cost function reduces to

(2) $C^* = WL + RK$

where C* is equal to variable costs.

If it is assumed that the production function facing the average firm is Cobb-Douglas,

(3) $Q = AK_{\alpha}L_{\gamma}$

when K_{α} is the flow of capital services and L_{γ} is the flow of labor services. Substituting (3) into (2) and minimizing cost yields the first-order conditions

(4) $\partial C^*/\partial L = W + \lambda(-\gamma A\ K_{\alpha}L_{\gamma}^{-1}) = 0$
(5) $\partial C^*/\partial K = R + \lambda(-\alpha A\ K_{\alpha}^{-1}L_{\gamma}) = 0$
(6) $\partial C^*/\partial \lambda = Q - AK_{\alpha}L_{\gamma} = 0$

Note: second-order conditions hold for Cobb-Douglas production functions.

Substituting $Q = AK_{\alpha}L_{\gamma}$ into (4) and (5) yields

(7) $W = \lambda\gamma Q/L$
(8) $R = \lambda\alpha Q/K$

Dividing (7) by (8) and transforming the result into log form provides

(9) $\ln W - \ln R = \ln\gamma - \ln\alpha + \ln K - \ln L$

From the production function (in log form) we know that

(10) $\ln K = [\ln Q - \ln A - \gamma \ln L]/\alpha$

Substituting (10) into (9) will result in the derived demand for labor

(11) $\ln L^* = \phi_0 + \phi_1 \ln Q + \phi_2 \ln A + \phi_3 \ln(R/W)$

where $\phi_0 = \alpha/(\alpha+\gamma)\ln(\gamma/\alpha)$, $\phi_1 = 1/(\alpha+\gamma)$, $\phi_2 = -1/(\alpha+\gamma)$, and $\phi_3 = \alpha/(\alpha+\gamma)$, respectively.

By symmetry, the derived demand for capital is simply

(12) $\ln K^* = \varphi_0 + \varphi_1 \ln Q + \varphi_2 \ln A + \varphi_3 \ln(W/R)$

where $\varphi_0 = \gamma/(\alpha+\gamma)\ln(\alpha/\gamma)$,$\varphi_1 = 1/(\alpha+\gamma)$, $\varphi_2 = -1/(\alpha+\gamma)$, and $\varphi_3 = \gamma/(\alpha+\gamma)$, respectively.

Cost minimization approach yields the minimum cost of producing a given output level for any input prices. Given the input prices (W and R), the approach provides the optimal levels of labor and capital to employ, that is
$C^* = WL^* + RK^*$.

B. Two-factor Translog Function

A translog function is derived from a Taylor Series expansion and is a flexible functional form used to relax the unitary constraint inherent in Cobb-Douglas functions.

The Cobb-Douglas cost function $C = AW_{\delta}^{L}R_{\delta}^{K}$ in log terms is

(14) $\ln C = \ln A + \delta_L \ln W + \delta_K \ln R$

A Taylor series expansion of (14) to the second moment is

(15) $\ln C = \ln A + \delta_L \ln W + \delta_K \ln R + 1/2\delta_{LL}(\ln W)^2 + 1/2\delta_{LK}(\ln W)(\ln R) + 1/2\delta_{KK}(\ln R)^2 + 1/2\delta_{KL}(\ln R)(\ln W)$

Assuming symmetry ($\delta_{LK} = \delta_{KL}$), equation (15) takes the form

(16) $\ln C = \ln A + \delta_L \ln W + \delta_K \ln R + 1/2\delta_{LL}(\ln W)^2 + \delta_{LK}(\ln W)(\ln R) + 1/2\delta_{KK}(\ln R)^2$

Equation (16) is the translog functional form of a two-factor Cobb-Douglas cost function.

By using a cost minimization approach and assuming input markets are competitive, Shephard's Lemma demonstrates that

(17) $\partial \ln C/\partial \ln P_i = (X_i P_i)/C = S_i$

where $X_i = L^*$ or K^*, $P_i = W$ or P, and S_i is the cost share of the input in the total cost of producing Q.

In general, Shephard's Lemma is defined as the derivative of the expenditure function with respect to the price of a good that gives the Hicksian demand for that good.

Taking partial logarithmic derivatives and equating them with the cost shares from the cost function (16), we have

(18) $S_L = \partial \ln C/\partial \ln W = \delta_L + \delta_{LL}\ln W + \delta_{LK}\ln R$
(19) $S_K = \partial \ln C/\partial \ln R = \delta_K + \delta_{LK}\ln W + \delta_{KK}\ln R$

For the translog cost specification

(20) $\sigma_{ij} = (\delta_{ij} + S_iS_j)/S_iS_j \qquad i \neq j$

where σ_{ij} is the elasticity of substitution (Allen Elasticity of Substitution) between pairs of factors.

$\sigma_{ij} > 0 \rightarrow$ the factors are substitutes
$\sigma_{ij} < 0 \rightarrow$ the factors are complements
$\sigma_{ij} = 0 \rightarrow$ the factors have no relationship

C. Three-factor Translog Function

Expanding the Translog model from two factors to three factors requires the cost and production functions to change from two to three-input functions.

(21) $C = Qg(P_n, P_p, P_k)$
(22) $Q = f(X_n, X_p, X_k)$

where P_n is the average annual wage for the firm's non-production labor X_n, P_p is the average annual wage for the firm's production labor X_p, and P_k is the rental rate for the firm's capital X_k.

As shown for the two-factor model, the Cobb-Douglas cost function (13) has the translog form

(23) $\ln C = \ln Q + \ln\delta_0 + \sum\delta_i \ln P_i + 1/2\sum\sum\delta_{ij}\ln P_i \ln P_j$

where $\delta_{ij} = \delta_{ji}$, technology parameters are $\delta_0\ \delta_i\ \delta_{ij}$,and C and P_i represent the total cost and input prices, respectively.
Once again, Shephard's Lemma demonstrates

(24) $\partial \ln C/\partial \ln P_i = (X_iP_i)/C = S_i$

where $X_i = X_n^*, X_p^*, X_k^*$ and S_i is the cost share of input X_i in the total cost of producing Q.

Taking partial logarithmic derivatives and equating them with the cost shares for the cost function, we have

(25) $S_n = \delta_n + \delta_{nn}\ln P_n + \delta_{np}\ln P_p + \delta_{nk}\ln P_k$
(26) $S_p = \delta_p + \delta_{np}\ln P_n + \delta_{pp}\ln P_p + \delta_{pk}\ln P_k$
(27) $S_k = \delta_k + \delta_{nk}\ln P_n + \delta_{pk}\ln P_p + \delta_{kk}\ln P_k$

Similar to the two-factor translog cost specification, σ_{ij} is the elasticity of substitution (Allen Elasticity of Substitution) between pairs of factors

(28) $\sigma_{ij} = (\delta_{ij} + S_iS_j)/S_iS_j \qquad i \neq j$

where:
- $\sigma_{ij} > 0 \rightarrow$ the factors are substitutes
- $\sigma_{ij} < 0 \rightarrow$ the factors are complements
- $\sigma_{ij} = 0 \rightarrow$ the factors have no relationship

Statistical Appendix

Variable Cost (C)

Variable Cost is equal to the annual wage bill (WL) multiplied by the total annual capital expenditures (K) for each 2-digit manufacturing Standard Industrial Classification (SIC) industry group.

- WL = the total annual cost of labor in dollars
- K = annual investment in equipment and structure in dollars

Production Worker Employment (X_p)

The Bureau of Labor Statistics (BLS) defines production workers in goods producing industries as employees who are not primarily employed to direct, supervise, or plan the work of others. This definition means that the production worker category excludes employees who are not directly involved in production.

Production Workers' Total Annual Wage Bill (P_p)

The total wage bill for production workers in dollars for each 2-digit manufacturing SIC industry group. This data element was derived from estimates published by the Bureau of Labor Statistics' Employment and Wage (ES-202) and Current Employment Statistics (CES) programs.

Non-production Worker Employment (X_n)

The category of non-production workers includes occupations that generally require more skill and training than the occupations included in the category of production workers.

Non-production Workers' Total Annual Wage Bill (P_n)

The total wage bill for non-production workers in dollars for each 2-digit manufacturing SIC industry group. This data element was derived from estimates published by the Bureau of Labor Statistics' Employment and Wage (ES-202) and Current Employment Statistics (CES) programs.

Rental Price of Capital (P_k)

The following is the implicit rental price of capital in the absence of an investment tax credit from Hall and Jorgenson:

(A) $C = q[(r+\delta)/(1-u)](1-uZ)$

where:

- C = implicit rental price of capital
- q = purchase price of the capital good (variable)
- r = discount rate
- δ = rate of replacement - physical rate of depreciation (constant)
- u = corporate income tax rate (variable)
- Z = present value of the depreciation deduction on a one dollar investment

(B) $Z = \lambda/(r+\lambda)$

where:

- λ = rate of depreciation allowed for income tax purposes (constant)
- r = discount rate

Substituting (B) into (A)

$C = q[(r+\lambda(1-u))/(r+\lambda)]\ [(r+\delta)/(1-u)]$

q = divide current dollar gross fixed capital formation in the manufacturing sector by the corresponding constant dollar figure for gross fixed capital formation
r = annual yield on long-term government bonds

Variables:
q = Producer Price Index
r = 10 year T-bill annual yield
u = Corporate income tax rate

Constants:
δ = (capital consumption allowances)/(net capital stock)
λ = Weighted average of the rate of depreciation for buildings and the rate of depreciation for machinery and equipment

This data element was derived using estimates produced by the Bureau of Economic Analysis (BEA) and the Federal Reserve Board (FRB).

Testing the Elasticity of Substitution Estimates:

According to Kopp and Smith, "calculating the standard error for the Allen elasticity of substitution estimates is complicated because the elasticity estimates from a translog cost function are nonlinear functions of the parameter estimates and the cost shares of the factors involved. Conventional practice calls for the use of the asymptotic variance."

$$\text{asymptotic variance} = \text{var}(\hat{\delta}_{ij}) / (\hat{S}_i \times \hat{S}_j)^2 \quad \text{where } i \neq j$$

Once the asymptotic variance is calculated, a standard t-test can be used to measure the statistical significance of the elasticity of substitution estimates.

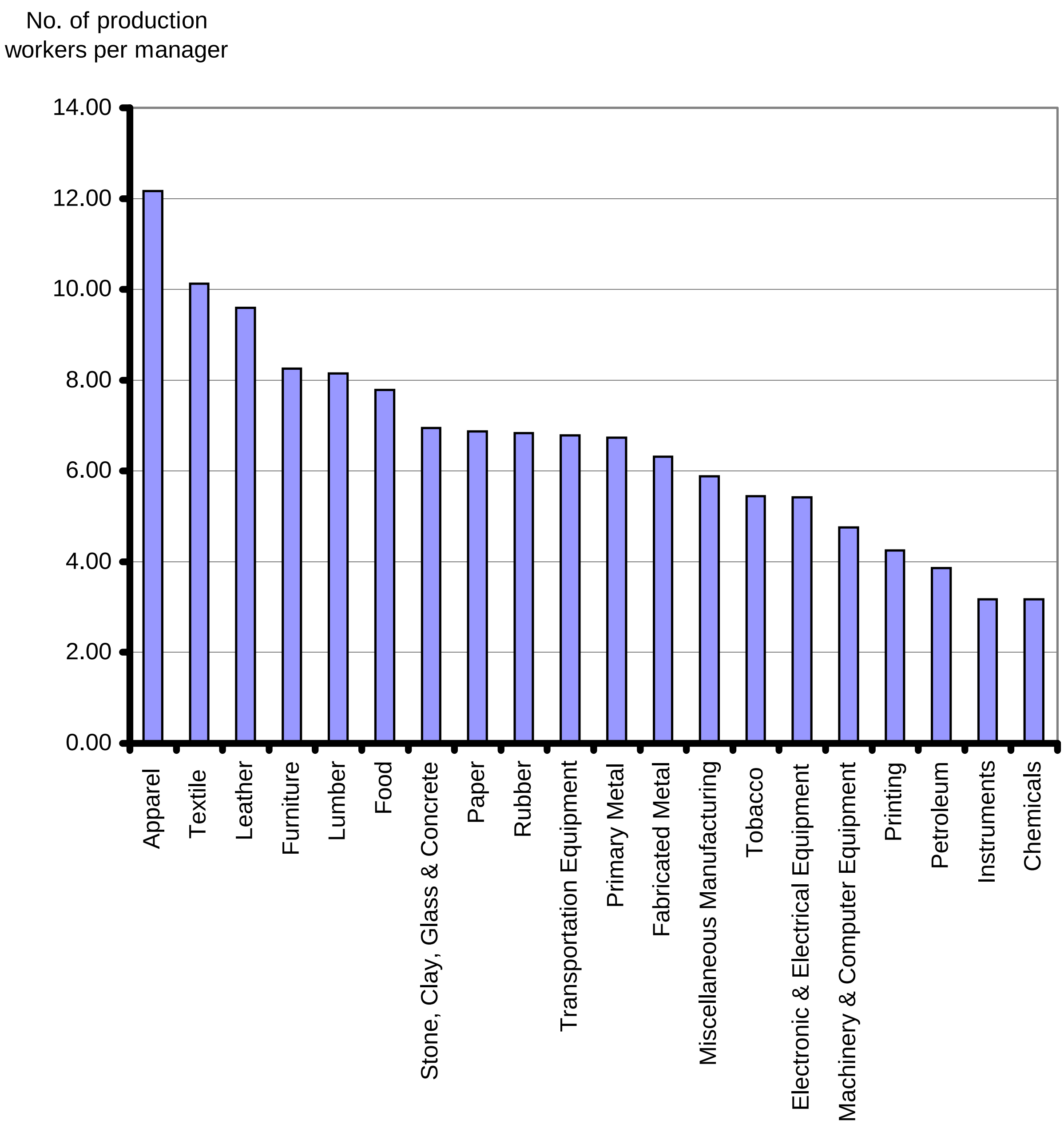
$$\text{t-statistic} = (\hat{\delta}_{ij} - Ho) / \sigma$$

where:

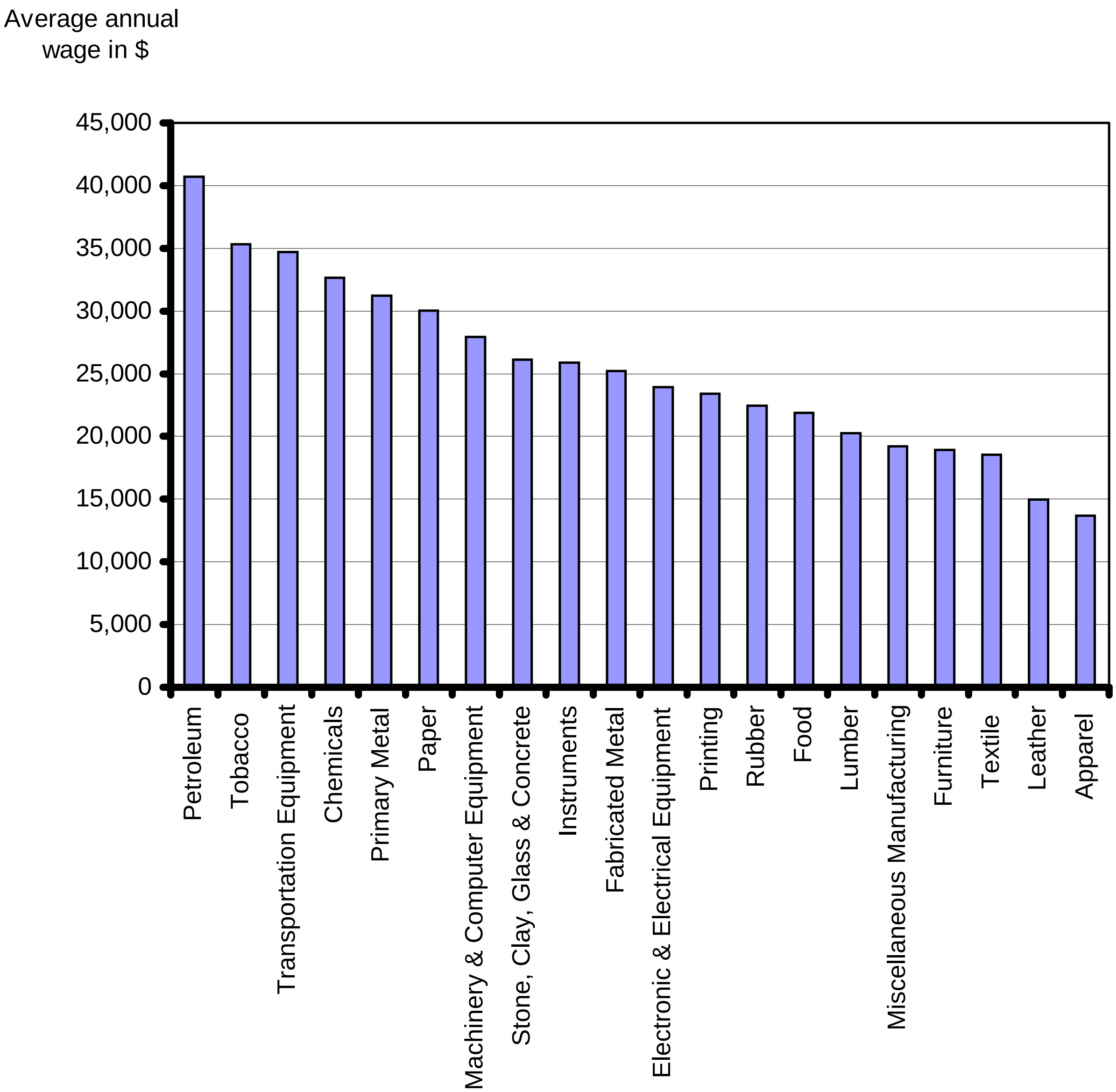
Ho = the null hypothesis that elasticity of substitution between the factors is zero (no relationship)

σ = the square root of the asymptotic variance

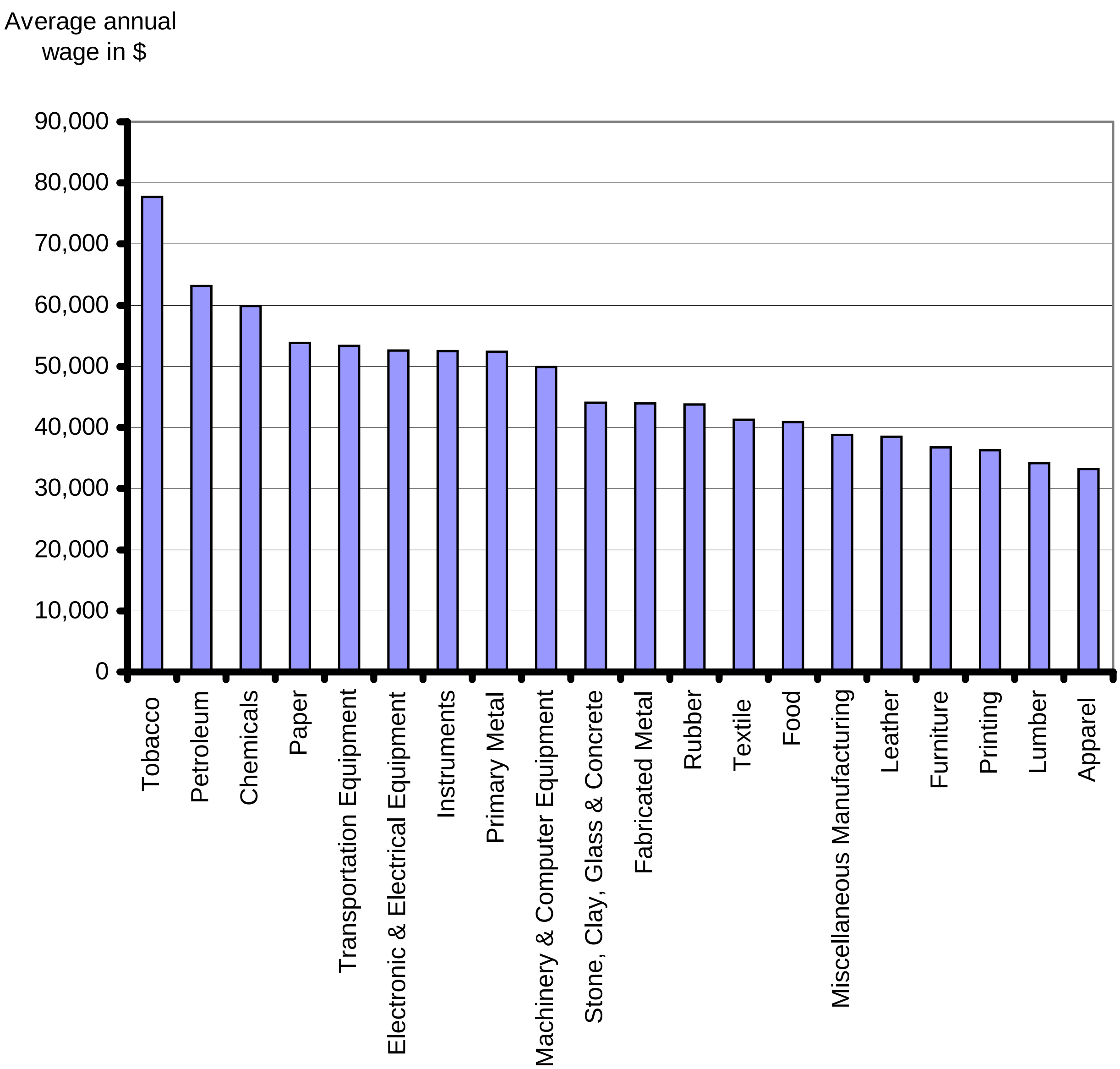
The average number of production workers per manager for U.S. manufacturing industries (1989 to 1995)



The average annual wage for production workers in U.S. manufacturing industries (1989 to 1995)



The average annual wage for non-prduction workers in U.S. manufacturing industries (1989 to 1995)



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Vita

Gregory A. Wilson obtained a Bachelor's degree in Economics from the University of Wisconsin - Madison and a Master's in Economics from Virginia Tech in Falls Church, Virginia. Mr. Wilson's course concentration was in Labor Economics. Currently, he works as an Economist for the United States' Department of Labor, Bureau of Labor Statistics in Washington, D. C.