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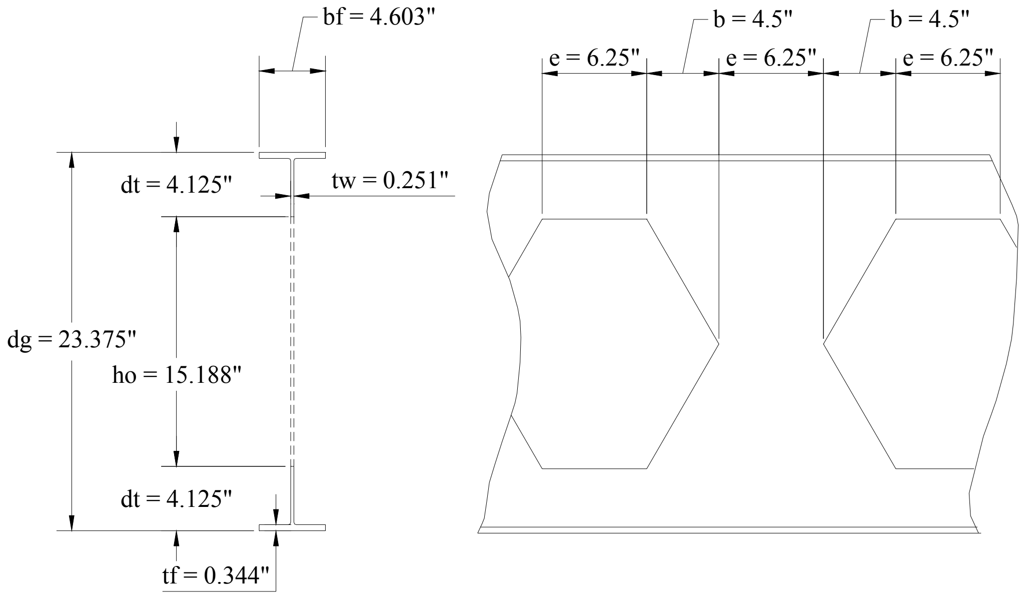
Appendix A

CB24x26 Calculations

A.1 Measured Dimensions of CB24x26 Specimen



A.2 "Tee" Section Properties

$$A_{Total} = [(b_f)(t_f) + ((d_t - t_f)(t_w))](2) = 5.75\ in.^2$$

$$C_w = \left(\frac{1}{36}\left(\frac{b_f^3\, t_f^3}{4} + h^3 t_w^3\right)\right)(2) = 0.154\ in.^6$$

$$J = \left(\frac{1}{3}\left(b_f t_f^3 + h t_w^3\right)\right)(2) = 0.194\ in.^4$$

$$I_y = \left(\frac{1}{12}(t_f)(b_f^3) + \frac{1}{12}(d_t - t_f)(t_w^3)\right)2 = 10.09\ in.^4$$

$$I_x = 683.09\ in.^4$$

$$S_x = \frac{I_x}{c} = 58.45\ in.^3$$

$$r_y = \sqrt{\frac{I_y}{A}} = 1.32\ in$$

A.2.1 Classical Lateral-Torsional Buckling Solution

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = 723.15$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = 0.00004$$

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.00325L_b^{\,2} + 0.075L_b$$

$$M_{cr} = \frac{C_b S_x X_1\sqrt{2}}{\frac{k_y L_b}{r_y}}\sqrt{1+\frac{X_1^{\,2}X_2}{2\left(\frac{k_\phi L_b}{r_y}\right)^2}} \quad \Rightarrow \quad 0.00325L_b^{\,2} + 0.075L_b = \frac{94221.89}{k_y L_b}\sqrt{1+\frac{20.28}{\left(k_\phi L_b\right)^2}}$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 25.0\,ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 25.0\,ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 27.0\,ft$$

A.2.2 Addition of Load Location Term

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.00325L_b^{\,2} + 0.075L_b$$

$$C_2 = 0.50$$

$$M_{cr} = \frac{C_b\pi\sqrt{EI_yGJ}}{k_y L_b}\left[\sqrt{1+\frac{\pi^2}{\left(k_\phi L_b\right)^2}\frac{EC_w\left(C_2^{\,2}+1\right)}{GJ}} - \frac{C_2\pi}{k_\phi L_b}\sqrt{\frac{EC_w}{GJ}}\right]$$

$$\Rightarrow 0.00325L_b^{\,2} + 0.075L_b = \frac{94221.89}{k_y L_b}\left[\sqrt{1+\frac{9.87}{\left(k_\phi L_b\right)^2}\left(2.57\right)} - \frac{2.25}{k_\phi L_b}\right]$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 24.9\,ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 24.9\,ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 26.8\,ft$$

A.2.3 Galambos Formula

$$\beta_x = \frac{A_{bottom\ chord}(d_e - \tilde{y})^3 - A_{top\ chord}\, y^3}{I_x} - 2y_o = 0$$

$$\tilde{y} = \frac{A_{bottom\ chord}\, d_e}{A_{total}} = 10.84\ in.$$

$$y_o = -y + \frac{I_{y\ bottom\ chord}\, d_e}{I_y} = 0$$

$$P^2\left[\frac{\pi^2+4}{16}\right]2 + P\left[\frac{(\pi^2+3)(\pi^2+4)wL_b}{192} - \frac{\pi^4 EI_y}{2(KL_b)^3}\left[\frac{\pi^2+4}{16}\beta_x - a\right]\right] +$$
$$\left[\frac{\pi^2+3}{24}\right]2(wL_b)^2 - wL\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^2+3}{24}\beta_x - \frac{y_o}{2}\right] -$$
$$\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^4 EC_w}{2(KL_b)^3} + \frac{\pi^2 GJ}{2KL_b}\right] = 0$$

$$156033.05 + 300\left[24.17L_b - \frac{14258225325.02}{(KL_b)^3}[0]\right] +$$
$$724.99L_b^2 - (26.0)L_b\left[\frac{14258225325.02}{(KL_b)^3}\right][0] -$$
$$\left[\frac{14258225325.02}{(KL_b)^3}\right]\left[\frac{217113568.93}{(KL_b)^3} + \frac{10707392.33}{KL_b}\right] = 0$$

When K = 1.0:

$$L_b = 20.2\,ft$$

When K = 0.8:

$$L_b = 23.4\,ft$$

When K = 0.5:

$$L_b = 32.1\,ft$$

A.3 Full Section Properties

$$A_{Total} = (b_f)(t_f)(2) + (d_g - (2 * t_f))(t_w) = 9.55\ in.^2$$

$$C_w = \frac{h^2 I_y}{4} = 1341.28\ in.^6$$

$$J = \frac{1}{3}\left(2b_f t_f^{\,3} + h t_w^{\,3}\right) = 0.273\ in.^4$$

$$I_y = \frac{1}{6}(t_f)(b_f^{\,3}) + \frac{1}{12}(h - t_f)(t_w^{\,3}) = 10.11\ in.^4$$

$$I_x = 755.46\ in.^4$$

$$S_x = \frac{I_x}{c} = 64.64\ in.^3$$

$$r_y = \sqrt{\frac{I_y}{A}} = 1.03\ in.$$

A.3.1 Classical Lateral-Torsional Buckling Solution

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = 1000.88$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = 0.24$$

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.00325L_b^{\,2} + 0.075L_b$$

$$M_{cr} = \frac{C_b S_x X_1\sqrt{2}}{\frac{k_y L_b}{r_y}}\sqrt{1+\frac{X_1^{\,2} X_2}{2\left(\frac{k_\phi L_b}{r_y}\right)^2}} \quad \Rightarrow \quad 0.00325L_b^{\,2} + 0.075L_b = \frac{112053.35}{k_y L_b}\sqrt{1+\frac{125347.16}{\left(k_\phi L_b\right)^2}}$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 29.8\,ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 33.6\,ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 35.7\,ft$$

When $k_y = 1.0$ and $L_b = 37.5$ft:

$k_\phi = 0.30$

When $k_y = 0.8$ and $L_b = 37.5$ft:

$k_\phi = 0.40$

A.3.2 Addition of Load Location Term

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.00325L_b^{\,2} + 0.075L_b$$

$$C_2 = 0.50$$

$$M_{cr} = \frac{C_b \pi \sqrt{EI_y GJ}}{k_y L_b}\left[\sqrt{1 + \frac{\pi^2}{(k_\phi L_b)^2}\frac{EC_w(C_2^2 + 1)}{GJ}} - \frac{C_2 \pi}{k_\phi L_b}\sqrt{\frac{EC_w}{GJ}}\right]$$

$$\Rightarrow 0.00325 L_b^2 + 0.075 L_b = \frac{112053.35}{k_y L_b}\left[\sqrt{1 + \frac{9.87}{(k_\phi L_b)^2}(15875.40)} - \frac{177.02}{k_\phi L_b}\right]$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 26.8\,ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 30.0\,ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 31.9\,ft$$

When $k_y = 1.0$ and $L_b = 37.5$ft:

$k_\phi = 0.18$

When $k_y = 0.8$ and $L_b = 37.5$ft:

$k_\phi = 0.23$

A.3.3 Galambos Formula

$$\beta_x = \frac{A_{bottom\ chord}(d_e - \tilde{y})^3 - A_{top\ chord}\, y^3}{I_x} - 2y_o = 0$$

$$\tilde{y} = \frac{A_{bottom\ chord}\, d_e}{A_{total}} = 8.03\ in.$$

$$y_o = -y + \frac{I_{y\ bottom\ chord}\, d_e}{I_y} = 0$$

$$P^2\left[\frac{\pi^2+4}{16}\right]2+P\left[\frac{(\pi^2+3)(\pi^2+4)wL_b}{192}-\frac{\pi^4 EI_y}{2(KL_b)^3}\left[\frac{\pi^2+4}{16}\beta_x-a\right]\right]+$$

$$\left[\frac{\pi^2+3}{24}\right]2(wL_b)^2-wL\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^2+3}{24}\beta_x-\frac{y_o}{2}\right]-$$

$$\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^4 EC_w}{2(KL_b)^3}+\frac{\pi^2 GJ}{2KL_b}\right]=0$$

$$156033.05+300\left[24.17L_b-\frac{14286376906.33}{(KL_b)^3}[0]\right]+$$

$$724.99L_b^2-(26.0)L_b\left[\frac{14286376906.33}{(KL_b)^3}\right][0]-$$

$$\left[\frac{14286376906.33}{(KL_b)^3}\right]\left[\frac{1894469871531.30}{(KL_b)^3}+\frac{15113783.67}{KL_b}\right]=0$$

When K = 1.0:

$$L_b=24.8\,ft$$

When K = 0.8:

$$L_b=29.2\,ft$$

When K = 0.5:

$$L_b=41.1\,ft$$

A.4 Weighted Average Section Properties

$$\%\,Tee=\frac{e}{2b+2e}\times 100=29\%$$

$$\%\,Solid=\frac{e}{2b+2e}\times 100=29\%$$

$$\%\,Transition=\frac{2b}{2b+2e}\times 100=42\%$$

$A_{Total} = 7.65\ in.^2$

$C_w = 670.72\ in.^6$

$J = 0.234\ in.^4$

$I_y = 10.10\ in.^4$

$I_x = 719.28\ in.^4$

$S_x = 61.54\ in.^3$

$r_y = 1.18\ in.$

A.4.1 Classical Lateral-Torsional Buckling Solution

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = 869.68$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = 0.14692$$

$$M_{cr} = \frac{wL_b^{\ 2}}{8} + \frac{PL_b}{4} = 0.00325L_b^{\ 2} + 0.075L_b$$

$$M_{cr} = \frac{C_b S_x X_1 \sqrt{2}}{\frac{k_y L_b}{r_y}}\sqrt{1 + \frac{X_1^{\ 2} X_2}{2\left(\frac{k_\phi L_b}{r_y}\right)^2}} \quad \Rightarrow \quad 0.00325L_b^{\ 2} + 0.075L_b = \frac{106009.16}{k_y L_b}\sqrt{1 + \frac{76958.01}{\left(k_\phi L_b\right)^2}}$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$L_b = 28.4\ ft$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$L_b = 31.6\ ft$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 33.6\,ft$$

A.4.2 Addition of Load Location Term

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.00325L_b^{\,2} + 0.075L_b$$

$$C_2 = 0.50$$

$$M_{cr} = \frac{C_b\pi\sqrt{EI_yGJ}}{k_yL_b}\left[\sqrt{1+\frac{\pi^2}{(k_\phi L_b)^2}\frac{EC_w\left(C_2^{\,2}+1\right)}{GJ}} - \frac{C_2\pi}{k_\phi L_b}\sqrt{\frac{EC_w}{GJ}}\right]$$

$$\Rightarrow 0.00325L_b^{\,2} + 0.075L_b = \frac{103513.55}{k_yL_b}\left[\sqrt{1+\frac{9.87}{(k_\phi L_b)^2}(9293.34)} - \frac{1435.44}{k_\phi L_b}\right]$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 25.48\,ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 27.9\,ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 29.7\,ft$$

A.4.3 Galambos Formula

$$\beta_x = \frac{A_{bottom\ chord}\,(d_e - \tilde{y})^3 - A_{top\ chord}\,y^3}{I_x} - 2y_o = 0$$

$$\tilde{y} = \frac{A_{bottom\ chord}\,d_e}{A_{total}} = 9.43\,in.$$

$$y_o = -y + \frac{I_{y\ bottom\ chord}\, d_e}{I_y} = 0$$

$$P^2\left[\frac{\pi^2+4}{16}\right]2 + P\left[\frac{(\pi^2+3)(\pi^2+4)wL_b}{192} - \frac{\pi^4 EI_y}{2(KL_b)^3}\left[\frac{\pi^2+4}{16}\beta_x - a\right]\right] +$$
$$\left[\frac{\pi^2+3}{24}\right]2(wL_b)^2 - wL\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^2+3}{24}\beta_x - \frac{y_o}{2}\right] -$$
$$\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^4 EC_w}{2(KL_b)^3} + \frac{\pi^2 GJ}{2KL_b}\right] = 0$$

$$156033.05 + 300\left[24.17L_b - \frac{114272301115.68}{(KL_b)^3}[0]\right] +$$
$$724.99L_b^2 - (26.0)L_b\left[\frac{14272301115.68}{(KL_b)^3}\right][0] -$$
$$\left[\frac{14272301115.68}{(KL_b)^3}\right]\left[\frac{947343492550.11}{(KL_b)^3} + \frac{12910588.00}{KL_b}\right] = 0$$

When K = 1.0:

$$L_b = 23.3\,ft$$

When K = 0.8:

$$L_b = 27.3\,ft$$

When K = 0.5:

$$L_b = 38.3\,ft$$

Appendix B

CB24x26 Specimen Test Data

CASTELLATED BEAM TEST SUMMARY

TEST IDENTIFICATION: CB24x26

TEST DESCRIPTION

Loading	Gravity
Point of Load Application	Mid-span
Span	37'-6"
Bracing Points	None
Number of beams	1
End Condition	Web-to-column flange double angle connection

FAILURE MODE:

Lateral-Torsional Buckling

THEORETICAL CRITICAL UNBRACED LENGTH:

(a) Classical Lateral-Torsional Buckling Solution =	35.7 ft
(b) Addition of Load Location Term =	31.9 ft
(c) Galambos Formula =	29.2 ft

EXPERIMENTAL CRITICAL UNBRACED LENGTH:

Total Applied Load =	300 lb
Unbraced Length =	37.5 ft

R-VALUE:

R(a) = Experimental Length/Theoretical Length =	1.05
R(b) = Experimental Length/Theoretical Length =	1.18
R(c) = Experimental Length/Theoretical Length =	1.28

DISCUSSION:

10 lb weights were loaded on a loading plate clamped to the top flange of the castellated beam at midspan. Catch bracing was installed to stop excessive deflections and help characterize failure.

	Concentrated Load (lb)			
Test Length / Eccentricity	48.3ft	44.8ft	41.3ft	37.5ft
e = 0	170	220	260	300
e = 1 1/2"	100	150	190	260
e = 2"	80	120	150	200

Photos of CB24x26 Testing

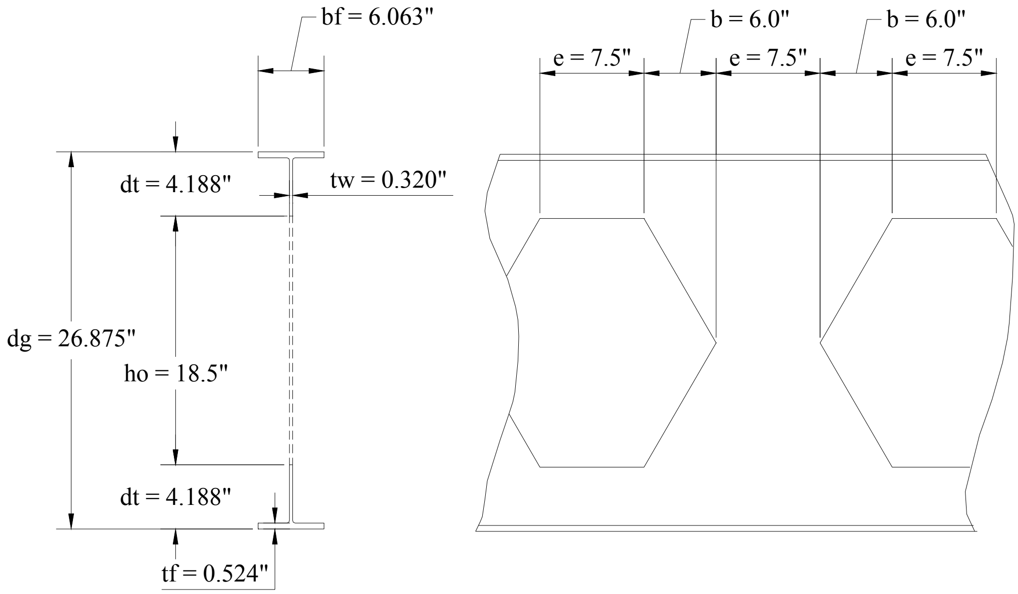
Photo of CB24x26 Entire Test Set-up

Photo of CB24x26 Specimen at failure

Appendix C

CB27x40 Calculations

C.1 Measured Dimensions of CB27x40 Specimen



C.2 "Tee" Section Properties

$$A_{Total} = \left[(b_f)(t_f) + \left((d_t - t_f)(t_w)\right)\right](2) = 8.70\ in.^2$$

$$C_w = \left(\frac{1}{36}\left(\frac{b_f^{\,3}\, t_f^{\,3}}{4} + h^3 t_w^{\,3}\right)\right)(2) = 0.555\ in.^6$$

$$J = \left(\frac{1}{3}\left(b_f t_f^{\,3} + h t_w^{\,3}\right)\right)(2) = 0.667\ in.^4$$

$$I_y = \left(\frac{1}{12}(t_f)\left(b_f^{\,3}\right) + \frac{1}{12}(d_t - t_f)\left(t_w^{\,3}\right)\right)(2) = 19.48\ in.^4$$

$$I_x = 1393.66\ in.^4$$

$$S_x = \frac{I_x}{c} = 103.71\ in.^3$$

$$r_y = \sqrt{\frac{I_y}{A}} = 1.50\ in.$$

C.2.1 Classical Lateral-Torsional Buckling Solution

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = 929.99$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = 0.00002$$

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.005L_b^{\,2} + 0.075L_b$$

$$M_{cr} = \frac{C_b S_x X_1\sqrt{2}}{\dfrac{k_y L_b}{r_y}}\sqrt{1 + \frac{X_1^{\,2}X_2}{2\left(\dfrac{k_\phi L_b}{r_y}\right)^2}} \quad \Rightarrow \quad 0.005L_b^{\,2} + 0.075L_b = \frac{242910.78}{k_y L_b}\sqrt{1 + \frac{21.27}{(k_\phi L_b)^2}}$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 30.0\,ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 30.0\,ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 32.4\,ft$$

C.2.2 Addition of Load Location Term

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.005L_b^{\,2} + 0.075L_b$$

$$C_2 = 0.50$$

$$M_{cr} = \frac{C_b\pi\sqrt{EI_yGJ}}{k_y L_b}\left[\sqrt{1 + \frac{\pi^2}{(k_\phi L_b)^2}\frac{EC_w\left(C_2^{\,2}+1\right)}{GJ}} - \frac{C_2\pi}{k_\phi L_b}\sqrt{\frac{EC_w}{GJ}}\right]$$

$$\Rightarrow 0.005L_b^{\,2} + 0.075L_b = \frac{242910.78}{k_y L_b}\left[\sqrt{1 + \frac{9.87}{(k_\phi L_b)^2}(2.69)} - \frac{2.31}{k_\phi L_b}\right]$$

When k_y = 1.0 and k_ϕ = 1.0:

$$L_b = 29.9\,ft$$

When k_y = 1.0 and k_ϕ = 0.5:

$$L_b = 29.9\,ft$$

When k_y = 0.8 and k_ϕ = 0.5:

$$L_b = 32.2\,ft$$

C.2.3 Galambos Formula

$$\beta_x = \frac{A_{bottom\ chord}\,(d_e - \tilde{y})^3 - A_{top\ chord}\,y^3}{I_x} - 2y_o = 0$$

$$\tilde{y} = \frac{A_{bottom\ chord}\,d_e}{A_{total}} = 12.61\,in.$$

$$y_o = -y + \frac{I_{y\ bottom\ chord}\,d_e}{I_y} = 0$$

$$P^2\left[\frac{\pi^2+4}{16}\right]2 + P\left[\frac{(\pi^2+3)(\pi^2+4)wL_b}{192} - \frac{\pi^4 EI_y}{2(KL_b)^3}\left[\frac{\pi^2+4}{16}\beta_x - a\right]\right] +$$
$$\left[\frac{\pi^2+3}{24}\right]2(wL_b)^2 - wL\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^2+3}{24}\beta_x - \frac{y_o}{2}\right] -$$
$$\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^4 EC_w}{2(KL_b)^3} + \frac{\pi^2 GJ}{2KL_b}\right] = 0$$

$$156033.05 + 300\left[37.19L_b - \frac{27513708959.94}{(KL_b)^3}[0]\right] +$$
$$1715.95L_b^2 - (40.0)L_b\left[\frac{27513708959.94}{(KL_b)^3}\right][0] -$$
$$\left[\frac{27513708959.94}{(KL_b)^3}\right]\left[\frac{784498707.30}{(KL_b)^3} + \frac{36879923.59}{KL_b}\right] = 0$$

When K = 1.0:

$$L_b = 24.1\,ft$$

When K = 0.8:

$$L_b = 27.9\,ft$$

When K = 0.5:

$$L_b = 38.2\,ft$$

C.3 Full Section Properties

$$A_{Total} = (b_f)(t_f)(2) + (d_g - (2 * t_f))(t_w) = 14.62\ in.^2$$

$$C_w = \frac{h^2 I_y}{4} = 3390.32\ in.^6$$

$$J = \frac{1}{3}\left(2 b_f t_f^{\,3} + h t_w^{\,3}\right) = 0.869\ in.^4$$

$$I_y = \frac{1}{6}(t_f)(b_f^{\,3}) + \frac{1}{12}(h - t_f)(t_w^{\,3}) = 19.53\ in.^4$$

$$I_x = 1562.47\ in.^4$$

$$S_x = \frac{I_x}{c} = 116.28\ in.^3$$

$$r_y = \sqrt{\frac{I_y}{A}} = 1.16\ in.$$

C.3.1 Classical Lateral-Torsional Buckling Solution

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = 1227.40$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = 0.10$$

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.005L_b^{\,2} + 0.075L_b$$

$$M_{cr} = \frac{C_b S_x X_1\sqrt{2}}{\frac{k_y L_b}{r_y}}\sqrt{1 + \frac{X_1^{\,2} X_2}{2\left(\frac{k_\phi L_b}{r_y}\right)^2}} \quad \Rightarrow \quad 0.005L_b^{\,2} + 0.075L_b = \frac{277619.67}{k_y L_b}\sqrt{1 + \frac{99663.46}{\left(k_\phi L_b\right)^2}}$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 34.0\,ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 37.7\,ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 40.1\,ft$$

When $k_y = 1.0$ and $L_b = 42.5$ft:

$k_\phi = 0.28$

When $k_y = 0.8$ and $L_b = 42.5$ft:

$k_\phi = 0.37$

C.3.2 Addition of Load Location Term

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.005L_b^{\,2} + 0.075L_b$$

$$C_2 = 0.50$$

$$M_{cr} = \frac{C_b \pi \sqrt{EI_y GJ}}{k_y L_b}\left[\sqrt{1+\frac{\pi^2}{(k_\phi L_b)^2}\frac{EC_w(C_2^2+1)}{GJ}} - \frac{C_2 \pi}{k_\phi L_b}\sqrt{\frac{EC_w}{GJ}}\right]$$

$$\Rightarrow 0.005 L_b^2 + 0.075 L_b = \frac{277619.67}{k_y L_b}\left[\sqrt{1+\frac{9.87}{(k_\phi L_b)^2}(12622.52)} - \frac{157.85}{k_\phi L_b}\right]$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 30.9\, ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 33.7\, ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 35.9\, ft$$

When $k_y = 1.0$ and $L_b = 42.5$ft:

$k_\phi = 0.16$

When $k_y = 0.8$ and $L_b = 42.5$ft:

$k_\phi = 0.231$

C.3.3 Galambos Formula

$$\beta_x = \frac{A_{bottom\ chord}(d_e - \tilde{y})^3 - A_{top\ chord}\, y^3}{I_x} - 2 y_o = 0$$

$$\tilde{y} = \frac{A_{bottom\ chord}\, d_e}{A_{total}} = 9.38\, in.$$

$$y_o = -y + \frac{I_{y\ bottom\ chord}\, d_e}{I_y} = 0$$

$$P^2\left[\frac{\pi^2+4}{16}\right]2 + P\left[\frac{(\pi^2+3)(\pi^2+4)wL_b}{192} - \frac{\pi^4 EI_y}{2(KL_b)^3}\left[\frac{\pi^2+4}{16}\beta_x - a\right]\right] +$$
$$\left[\frac{\pi^2+3}{24}\right]2(wL_b)^2 - wL\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^2+3}{24}\beta_x - \frac{y_o}{2}\right] -$$
$$\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^4 EC_w}{2(KL_b)^3} + \frac{\pi^2 GJ}{2KL_b}\right] = 0$$

$$156033.05 + 300\left[37.19L_b - \frac{27585057392.12}{(KL_b)^3}[0]\right] +$$
$$1715.95L_b^2 - (40.0)L_b\left[\frac{27585057392.12}{(KL_b)^3}\right][0] -$$
$$\left[\frac{27585057392.12}{(KL_b)^3}\right]\left[\frac{4788594942811.77}{(KL_b)^3} + \frac{48047648.43}{KL_b}\right] = 0$$

When K = 1.0:

$$L_b = 28.0\,ft$$

When K = 0.8:

$$L_b = 32.8\,ft$$

When K = 0.5:

$$L_b = 46.0\,ft$$

C.4 Weighted Average Section Properties

$$\%\,Tee = \frac{e}{2b+2e} \times 100 = 28\%$$

$$\%\,Solid = \frac{e}{2b+2e} \times 100 = 28\%$$

$$\%\,Transition = \frac{2b}{2b+2e} \times 100 = 44\%$$

$$A_{Total} = 11.66\ in.^2$$

$$C_w = 1695.44\ in.^6$$

$$J = 0.768\ in.^4$$

$$I_y = 19.50\ in.^4$$

$$I_x = 1478.07\ in.^4$$

$$S_x = 110.00\ in.^3$$

$$r_y = 1.33\ in.$$

C.4.1 Classical Lateral-Torsional Buckling Solution

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}} = 1089.31$$

$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 = 0.05681$$

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.005L_b^{\,2} + 0.075L_b$$

$$M_{cr} = \frac{C_b S_x X_1\sqrt{2}}{\dfrac{k_y L_b}{r_y}}\sqrt{1+\frac{X_1^{\,2}X_2}{2\left(\dfrac{k_\phi L_b}{r_y}\right)^2}} \quad\Rightarrow\quad 0.005L_b^{\,2} + 0.075L_b = \frac{267415.91}{k_y L_b}\sqrt{1+\frac{59281.61}{(k_\phi L_b)^2}}$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 32.7\ ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 35.7\ ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 38.0\ ft$$

C.4.2 Addition of Load Location Term

$$M_{cr} = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} = 0.005L_b^{\,2} + 0.075L_b$$

$$C_2 = 0.50$$

$$M_{cr} = \frac{C_b \pi \sqrt{EI_y GJ}}{k_y L_b}\left[\sqrt{1+\frac{\pi^2}{(k_\phi L_b)^2}\frac{EC_w\left(C_2^{\,2}+1\right)}{GJ}} - \frac{C_2 \pi}{k_\phi L_b}\sqrt{\frac{EC_w}{GJ}}\right]$$

$$\Rightarrow 0.005L_b^{\,2} + 0.075L_b = \frac{260820.97}{k_y L_b}\left[\sqrt{1+\frac{9.87}{(k_\phi L_b)^2}(7142.32)} - \frac{118.74}{k_\phi L_b}\right]$$

When $k_y = 1.0$ and $k_\phi = 1.0$:

$$L_b = 29.8\,ft$$

When $k_y = 1.0$ and $k_\phi = 0.5$:

$$L_b = 31.7\,ft$$

When $k_y = 0.8$ and $k_\phi = 0.5$:

$$L_b = 33.8\,ft$$

C.4.3 Galambos Formula

$$\beta_x = \frac{A_{bottom\ chord}\,(d_e - \tilde{y})^3 - A_{top\ chord}\ y^3}{I_x} - 2y_o = 0$$

$$\tilde{y} = \frac{A_{bottom\ chord}\ d_e}{A_{total}} = 10.99\ in.$$

$$y_o = -\,y + \frac{I_{y\ bottom\ chord}\ d_e}{I_y} = 0$$

$$P^2\left[\frac{\pi^2+4}{16}\right]2+P\left[\frac{(\pi^2+3)(\pi^2+4)wL_b}{192}-\frac{\pi^4 EI_y}{2(KL_b)^3}\left[\frac{\pi^2+4}{16}\beta_x-a\right]\right]+$$

$$\left[\frac{\pi^2+3}{24}\right]2(wL_b)^2-wL\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^2+3}{24}\beta_x-\frac{y_o}{2}\right]-$$

$$\left[\frac{\pi^4 EI_y}{2(KL_b)^3}\right]\left[\frac{\pi^4 EC_w}{2(KL_b)^3}+\frac{\pi^2 GJ}{2KL_b}\right]=0$$

$$156033.05+300\left[37.19L_b-\frac{27549383176.03}{(KL_b)^3}[0]\right]+$$

$$1715.95L_b^2-(40.0)L_b\left[\frac{27549383176.03}{(KL_b)^3}\right][0]-$$

$$\left[\frac{27549383176.03}{(KL_b)^3}\right]\left[\frac{2394689720759.53}{(KL_b)^3}+\frac{42463786.01}{KL_b}\right]=0$$

When K = 1.0:

$$L_b = 26.5\,ft$$

When K = 0.8:

$$L_b = 31.0\,ft$$

When K = 0.5:

$$L_b = 43.3\,ft$$

Appendix D

CB24x26 Specimen Test Data

CASTELLATED BEAM TEST SUMMARY

TEST IDENTIFICATION: CB27x40

TEST DESCRIPTION

Loading	Gravity
Point of Load Application	Mid-span
Span	42.5"
Bracing Points	None
Number of beams	1
End Condition	Web to column flange double angle connection

FAILURE MODE:

Lateral Torsional Buckling

THEORETICAL CRITICAL UNBRACED LENGTH:

(a) Classical Lateral-Torsional Buckling Solution = 40.1 ft
(b) Addition of Load Location Term = 35.9 ft
(c) Galambos Formula = 32.8 ft

EXPERIMENTAL CRITICAL UNBRACED LENGTH:

Total Applied Load = 300 lb
Unbraced Length = 42.5 ft

R-VALUE:

R(a) = Experimental Length/Theoretical Length = 1.06
R(b) = Experimental Length/Theoretical Length = 1.18
R(c) = Experimental Length/Theoretical Length = 1.30

DISCUSSION:

10 lb weights were loaded on a loading plate clamped to the top flange of the castellated beam at midspan. Catch bracing was installed to stop excessive deflections and help characterize failure.

	Concentrated Load (lb)			
Eccentricity \ Test Length	51.8ft	47.3ft	44.5ft	42.5ft
e = 0	self wt.	120	270	300
e = 1 1/2"	self wt.	60	210	250
e = 2"	self wt.	40	160	190

Photos of CB27x40 Testing

Photo of CB27x40 Entire Test Set-up

Photo of CB27x40 Specimen at failure

VITA

T. Patrick Bradley was born on July 14, 1977 in Clemmons, North Carolina. He graduated from West Forsyth high school in Lewisville, North Carolina. He received his Associate in Applied Science in Architectural Technology from Guilford Technical Community College in May 1998. He received his Bachelor of Science in Civil Engineering from North Carolina Agricultural and Technical State University in Greensboro, North Carolina in May of 2001. He enrolled in the graduate program at Virginia Tech in the fall of 2001 and plans to work for a metal building manufacturer in North Carolina after completion.

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