Chapter II

Review of Lateral-Torsional Buckling Design Provisions

2.1 Introduction

When designing with hot-rolled sections and open-web steel joists, there are design procedures that may be used to check for lateral-torsional buckling. For hot-rolled sections the American Institute of Steel Construction (AISC 1999) design specification is used, and for open-webbed steel joists the Steel Joist Institute (SJI 1994) has a published approach. However, there is not a prescribed method or design procedure to use when designing structures utilizing castellated beams. In this chapter the AISC and SJI provisions are examined and modifications to allow their use when designing castellated beams are suggested.

2.2 Cross-Sectional Properties

In the following, certain material and section properties are in common. The modulus of elasticity of steel (E) is taken as 29000 ksi and the shear modulus (G) is taken as 11200 ksi. The flexural effective length factor (k_y) is taken as 1.0, and the torsional effective length factor (k_ϕ) is assumed to be 1.0.

The loading on the castellated beams is a combination of a concentrated load at midspan, weight of an erector, and a uniformly distributed load over the length of the beam, self-weight. Therefore, the construction stage design moment is

$$M = \frac{wL_b^{\,2}}{8} + \frac{PL_b}{4} \tag{2.1}$$

where w is the weight of the beam, lb per linear ft, and P is the weight of the erector and tools, which will be taken as 300 lb.

The weak axis (out-of-plane bending) moment of inertia (I_y), the warping constant (C_w), and the torsional constant (J) all are directly associated with the lateral stiffness and stability and are dependent on the geometry of the section they represent. Due to the unique geometry and nature of castellated beams, the cross-sectional properties that affect lateral stiffness and stability have been calculated using different approaches. The three approaches to calculating these cross-sectional properties are examined in the following sections: Section 2.2.1, the “Tee” Sections assumption; Section 2.2.2, the Full Section assumption; and Section 2.2.3, the Weighted Average Section assumption.

The results of the three assumptions were used in various lateral-buckling solutions to determine the critical unbraced length of the castellated beams used in this study. These analytical solutions were then compared to the results of the experimental testing to determine which approach is best.

2.2.1 “Tee” Sections Assumption

If it is assumed that the web posts do not contribute to the lateral stiffness and stability of the castellated beam, the cross-section is assumed to be two “tee” sections. This assumption is the most conservative approach. The top “tee” consists of the top flange and joining web material down to the top of the hole in the web, as seen in Figure 2.1. Likewise, the bottom “tee” consists of the bottom flange and joining web material extending to the bottom of the web hole. Therefore in this assumption the cross-sectional properties are calculated through a castellation (hole location) of the beam.

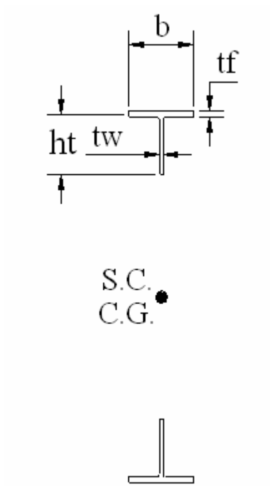

Figure 2.1 Tee Section Assumption

The center of gravity (C.G.) and the shear center (S.C.) are in the same location for the specimens used in this study. However, castellated beams are available where the top half and bottom half are cut from different size root beams. This type of castellated beam is fabricated to provide more efficient composite construction. The section properties required for lateral-torsional buckling analyses are determined, for each tee section.

$$C_w = \frac{1}{36}\left(\frac{b^3 t_f^{\,3}}{4} + h_t^{\,3} t_w^{\,3}\right) \tag{2.2}$$

$$J = \frac{1}{3}\left(b t_f^{\,3} + h_t t_w^{\,3}\right) \tag{2.3}$$

$$I_y = \frac{1}{12}(t_f)(b^3) + \frac{1}{12}\left(h_t - \left(\frac{t_f}{2}\right)\right)(t_w^{\,3}) \tag{2.4}$$

The variables in the equations are defined in Figure 2.1. The calculated values must be doubled to include both top and bottom tees in the calculations.

2.2.2 Full Section Properties Assumption

The full section assumption is that the cross-section of the castellated beam is a solid section. The section is analyzed as if it had no holes in the web. This approach is the least conservative approach. In this approach, the same equations are used as when calculating the cross-sectional properties of a solid hot-rolled section, that is Equations 2.5, 2.6, and 2.7 with the variables defined in Figure 2.2.

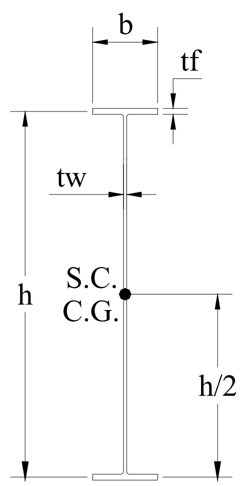

Figure 2.2 Full Section Assumption

$$C_w = \frac{h^2 I_y}{4} \tag{2.5}$$

$$J = \frac{1}{3}\left(2bt_f{}^3 + ht_w{}^3\right) \tag{2.6}$$

$$I_y = \frac{1}{6}(t_f)\left(b^3\right) + \frac{1}{12}(h - t_f)\left(t_w{}^3\right) \tag{2.7}$$

2.2.3 Weighted Section Properties Assumption

In this approach, the segment of the beam that is solid, the transition segment, and the castellated segment are all taken into account when the cross-sectional properties are calculated. The geometry of the hole is used to compute a percentage that is associated with the different geometrical sections of the beam as shown in Figure 2.3. Equation 2.8 is used to calculate the percentage of the beam that is considered as the "Tee" section. Equation 2.9 is the percentage of the beam that is considered as solid, and Equation 2.10 is the percentage of the beam that is considered as the transition segment. The properties used for the transition segment are an average of the full section properties and the "tee" section properties.

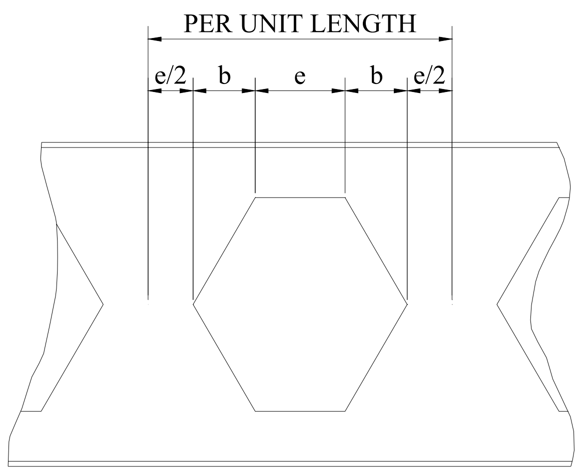

Figure 2.3 Percentage and Geometry Description

$$\%\,Tee = \frac{e}{2b + 2e} \times 100 \tag{2.8}$$

$$\%\,Solid = \frac{e}{2b + 2e} \times 100 \tag{2.9}$$

$$\% \, Transition = \frac{2b}{2b + 2e} \times 100 \tag{2.10}$$

2.2.4 Comparison of Calculated Properties

The cross-sectional properties were calculated for the CB24x26 and CB27x40 castellated sections, see Appendix A and Appendix C. The castellated beams in this research were each cut from one root beam and therefore have identical top and bottom flanges. Due to this, the properties can be calculated for one "tee" section and then simply doubled to determine the value for the entire section.

The cross-sectional properties for the specimens used in this study using the measured specimen dimensions are listed in Table 2.1. From the magnitude of the warping and torsional properties it is evident that the consideration of the hole geometry largely affects these terms. When the web posts are neglected, the resistance of the sections to warping and torsion is drastically reduced. A large warping constant, C_w, means the cross-section is less likely to fail due to lateral buckling since the section is laterally stiff. The larger the torsional constant, J, the less likely a twisting or torsion failure will occur.

Table 2.1 Comparison of Cross-sectional Calculation Assumptions

	Tee Section Properties			Full Section Properties			Weighted Section Properties		
	Iy [in^4]	Cw [in^6]	J [in^4]	Iy [in^4]	Cw [in^6]	J [in^4]	Iy [in^4]	Cw [in^6]	J [in^4]
CB24x26	10.09	0.154	0.194	10.11	1341	0.273	10.10	671	0.234
CB27x40	19.48	0.555	0.667	19.53	3390	0.869	19.50	1695	0.768

2.3 Classical Lateral-Torsional Buckling Solution

Salmon and Johnson (1996) review the derivation of the classical solution for calculating elastic lateral-torsional buckling strength of beams. The derivation involves the summation of the resistance to lateral movement (Equation 2.11) and the resistance to twist (Equation 2.12), in addition to standard flexure theory. The derivation assumes the application of the load passes through the centroid of the beam.

$$M_y = EI_y \frac{d^2 x}{dy^2} \tag{2.11}$$

$$T_{Total} = GJ\phi' - EC_w\phi''' \quad \text{with} \quad T_{St.Venant} = GJ\phi' \quad \text{and} \quad T_{Warping} = -EC_w\phi''' \tag{2.12}$$

The use of this classical solution is dependent on the beam being in the elastic range. The limiting elastic range parameter (L_r) can be calculated using the AISC

Specification (1999). For the elastic solution to apply, the unbraced length (L_b) must be greater than the limiting laterally unbraced length (L_r) given by

$$L_r = \frac{r_y X_1}{F_L}\sqrt{1+\sqrt{1+X_2 F_L^2}} \tag{2.13}$$

$$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}}$$
$$X_2 = 4\frac{C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2 \tag{2.14}$$

where

r_y = radius of gyration about the y-axis,
S_x = section modulus with respect to the x-axis,
A = area of section, and
F_L = the minimum of the yield stress of the flange minus the residual stress

If the beam properties and the design parameters satisfy the above, the classical solution as presented in the AISC specification, equation F1-13, applies, that is

$$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \tag{2.15}$$

In addition to the variables defined earlier, the classical solution contains C_b, which accounts for the moment gradient and is presented in the AISC Specification (1999). All the moments in the calculation of C_b are the absolute values.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \tag{2.16}$$

where

M_{max} = maximum moment in unbraced segment

M_A = moment at ¼ point of unbraced segment

M_B = moment at midpoint of unbraced segment

M_C = moment at ¾ point of unbraced segment

For the beam loading evaluated in this research, Figure 2.4 shows the moment diagram including the M_A, M_B, and M_C terms. After analyzing this diagram, the C_b coefficient was taken as 1.19 for both specimens.

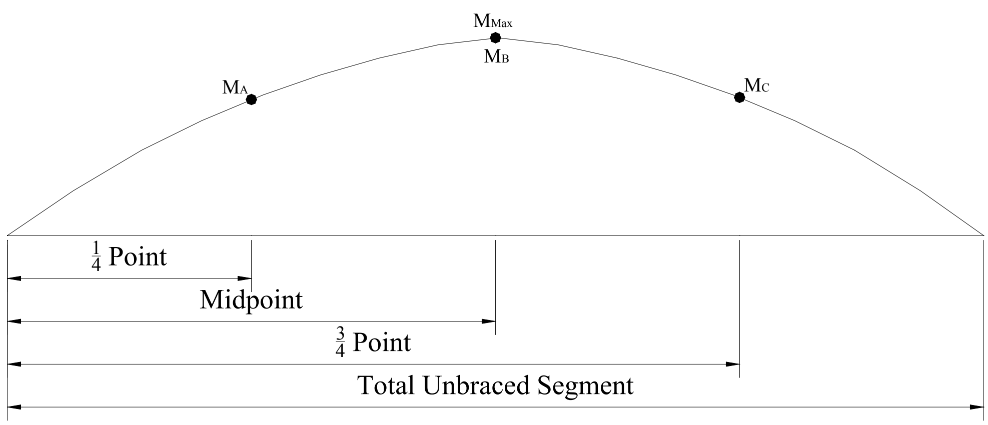

Figure 2.4 Moment Diagram and C_b Terms

The calculated critical unbraced lengths using the classical lateral-torsional buckling solution for the specimens used in this research are presented in Table 2.2. Calculations are in Appendices A.2.1, A.3.1, A.4.1, B.2.1, B.3.1, and B.4.1.

Table 2.2 Evaluation of Classical Lateral-Torsional Buckling Solution

	Critical Unbraced Length (ft)		
Specimen	Tee Section Properties	Full Section Properties	Weighted Section Properties
CB24x26	25.0	29.8	28.4
CB27x40	30.0	34.0	32.7

2.4 Classical Lateral-Torsional Buckling Solution With Load Location Term

Considerable research has been done to evaluate the effects of the location of load application on the lateral-torsional stability of beams. Through testing, as well as analytical research, the load location has been found to contribute significantly to the effects of lateral-torsional buckling. The effect on lateral-torsional buckling strength because of load location is presented in Figure 2.5.

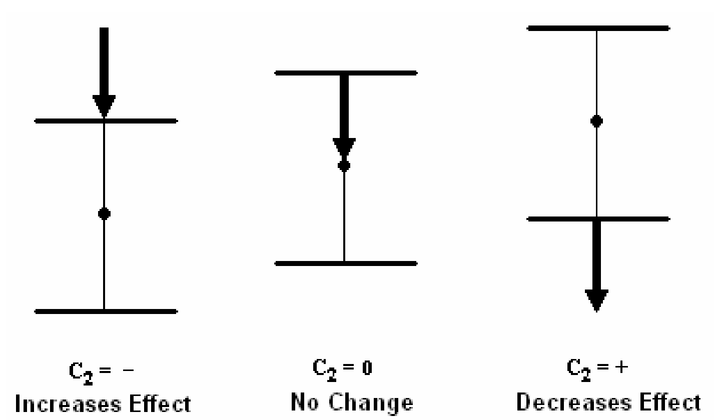

Figure 2.5 Load Location Effects

During construction, the load is applied to the top flange of the castellated beam. The research has shown that this load location increases the probability of lateral-torsional buckling occurring, with the probability of lateral-torsional buckling increasing as the unbraced length decreases. The classical lateral-torsional buckling solution with the addition of the load location term is presented in “Lateral Buckling of Beams” by Clark and Hill (1960), and by Equation 2.17.

$$M_{cr} = \frac{C_b \pi \sqrt{E I_y G J}}{k_y L_b} \left[\sqrt{1 + \frac{\pi^2}{(k_\phi L_b)^2} \frac{E C_w (C_2{}^2 + 1)}{G J}} \pm \frac{C_2 \pi}{k_\phi L_b} \sqrt{\frac{E C_w}{G J}} \right] \qquad (2.17)$$

The sign of the additional term is determined by the location of the load application as prescribed in Figure 2.5. The variables in the equation have been defined earlier in this thesis, however the C_2 has not been explained. This term is dependent on support conditions and the type of load that is being applied, as shown in Figure 2.6. For the loading used in this study, C_2 is between 0.45 (simple span, uniform load) and 0.55 (simple span, midspan concentrated load). The parameter C_2 was taken as 0.50 for the critical length calculations.

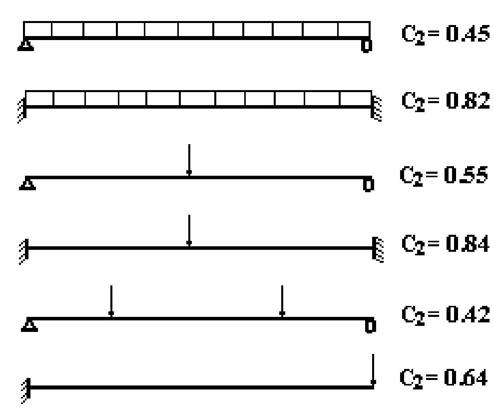

Figure 2.6 C_2 Value Description

The calculated critical unbraced lengths when the classical lateral-torsional buckling solution with the load location term was evaluated are presented in Table 2.3.

Table 2.3 Evaluation of Classical Solution with Load Location Term

	Critical Unbraced Length (ft)		
Specimen	Tee Section Properties	Full Section Properties	Weighted Section Properties
CB24x26	24.9	26.8	25.5
CB27x40	29.9	30.9	29.8

2.5 Galambos Formulation

Galambos presented the background of the Steel Joist Institute's method for determining critical unbraced length and spacing of lateral bracing of open-web steel joists in *Bracing of Trussed Beams* (Galambos 1993). He recognized the importance of examining the structural member under the unique loading of self-weight plus an erector's weight. His formula was derived using the Rayleigh-Ritz method assuming sinusoidal lateral and torsional deformations:

$$\begin{aligned} &P^2\left[\frac{\pi^2+4}{16}\right]2 + P\left[\frac{(\pi^2+3)(\pi^2+4)wL}{192} - \frac{\pi^4 EI_y}{2(KL)^3}\left[\frac{\pi^2+4}{16}\beta_x - a\right]\right] + \\ &\left[\frac{\pi^2+3}{24}\right]2(wL)^2 - wL\left[\frac{\pi^4 EI_y}{2(KL)^3}\right]\left[\frac{\pi^2+3}{24}\beta_x - \frac{y_o}{2}\right] - \\ &\left[\frac{\pi^4 EI_y}{2(KL)^3}\right]\left[\frac{\pi^4 EC_w}{2(KL)^3} + \frac{\pi^2 GJ}{2KL}\right] = 0 \end{aligned} \tag{2.18}$$

with cross-sectional properties

$$\widetilde{y} = \frac{A_{bottom\ chord}\, d_e}{A_{total}} \quad (2.19)$$

$$\beta_x = \frac{A_{bottom\ chord}\,(d_e - \widetilde{y})^3 - A_{top\ chord}\, y^3}{I_x} - 2y_o \quad (2.20)$$

$$y_o = -y + \frac{I_{y\ bottom\ chord}\, d_e}{I_y} \quad (2.21)$$

where

y_o = distance between shear center and centroid,

a = distance between shear center and point of load application (negative if above the shear center),

d_e = distance between the centroid of the top chord and centroid of the bottom chord,

y = distance between centroid of top chord and centroid of cross section

In this formulation, the top and bottom chords of an open-web steel joist are considered to be the only components of the joist that contribute to the lateral stability of the joist. The castellated beams were analyzed as two "tees" to fit the joist model. However, the size of the "tees" was modified to allow the different cross-sectional property approaches to be applied. This is illustrated in Figure 2.7. The results of the evaluation of the Galambos formulation for determining critical unbraced lengths for the specimens used in this research are presented in Table 2.4.

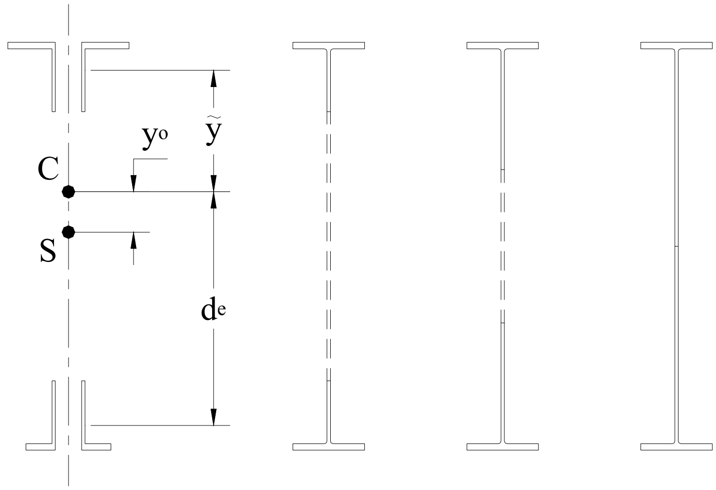

Figure 2.7 "Tee" Truss Analogy

Table 2.4 Evaluation of Galambos Formula

	Critical Unbraced Length (ft)		
Specimen	Tee Section Properties	Full Section Properties	Weighted Section Properties
CB24x26	20.2	24.8	23.3
CB27x40	24.1	28.0	26.5

2.6 Discussion of Results

The calculated unbraced lengths for the specimens used in this research, when the classical lateral-torsional buckling solution, the addition of the load location term, and the Galambos formulation are evaluated with the "Tee" section property assumption, are presented in Table 2.5, for the full section property assumption in Table 2.6, and the weight section property assumption in Table 2.7.

Table 2.5 Comparison of Methods ("Tee" Properties)

	Critical Unbraced Length (ft)		
	Classical Solution	Load Location Term	Galambos
CB24x26	25.0	24.9	20.2
CB27x40	30.0	29.9	24.1

Table 2.6 Comparison of Methods (Full Properties)

	Critical Unbraced Length (ft)		
	Classical Solution	Load Location Term	Galambos
CB24x26	29.8	26.8	24.8
CB27x40	34.0	30.9	28.0

Table 2.7 Comparison of Methods (Weighted Properties)

	Critical Unbraced Length (ft)		
	Classical Solution	Load Location Term	Galambos
CB24x26	28.4	25.5	23.3
CB27x40	32.7	29.8	26.5

The classical lateral-torsional buckling solution using the Full Section properties results in the largest unbraced length in all the calculations. The addition of the load location term to the classical solution lowers the unbraced length nearly 10% and the Galambos solution lowers the value nearly 17%. When the lateral-torsional buckling solutions were evaluated using the Tee Section Assumption, the unbraced lengths decreased. However, the margin of difference between the different solutions decreased. Lastly, when the lateral-torsional buckling solutions were evaluated using the Weighted Section Assumption, the unbraced lengths fall between the previous two approaches as expected.