3. Spherical Helices

Since the invention of cylindrical helix by Kraus, many other helical geometries have been proposed and studied. These helices are, however, largely modified forms of the conventional cylindrical helix that were introduced to achieve improved radiation properties. The conical helix has also been investigated [17], but the unique helical antenna of spherical geometry had received little attention until recent years. An exhaustive search of the literature points to one radiator of spherical geometry proposed in 60's for use in an early generation of satellites [18]. The spherical helix studied here was first proposed in 1992, and this thesis is devoted to further development and investigation of this antenna.

Spherical helices studied here are wire antennas made of helical windings over spherical surfaces, as shown in Figure 3.1a. The winding is made such that a fixed spacing is maintained between the turns. This allows the spherical helix to be more easily modeled and constructed.

3.1 Geometry and Parameters of the Spherical Helix

With a constant spacing between the turns, the parameters required to describe the geometry of a spherical helix are only the number of turns (N) and the radius of the sphere on which the helix is wound (a). N is the number of turns if the sphere is wound fully, pole to pole, as shown in Figure 3.1a. The actual number of turns (n) is less than N if the helix is a truncated one as that illustrated in Figure 3.1b. Both

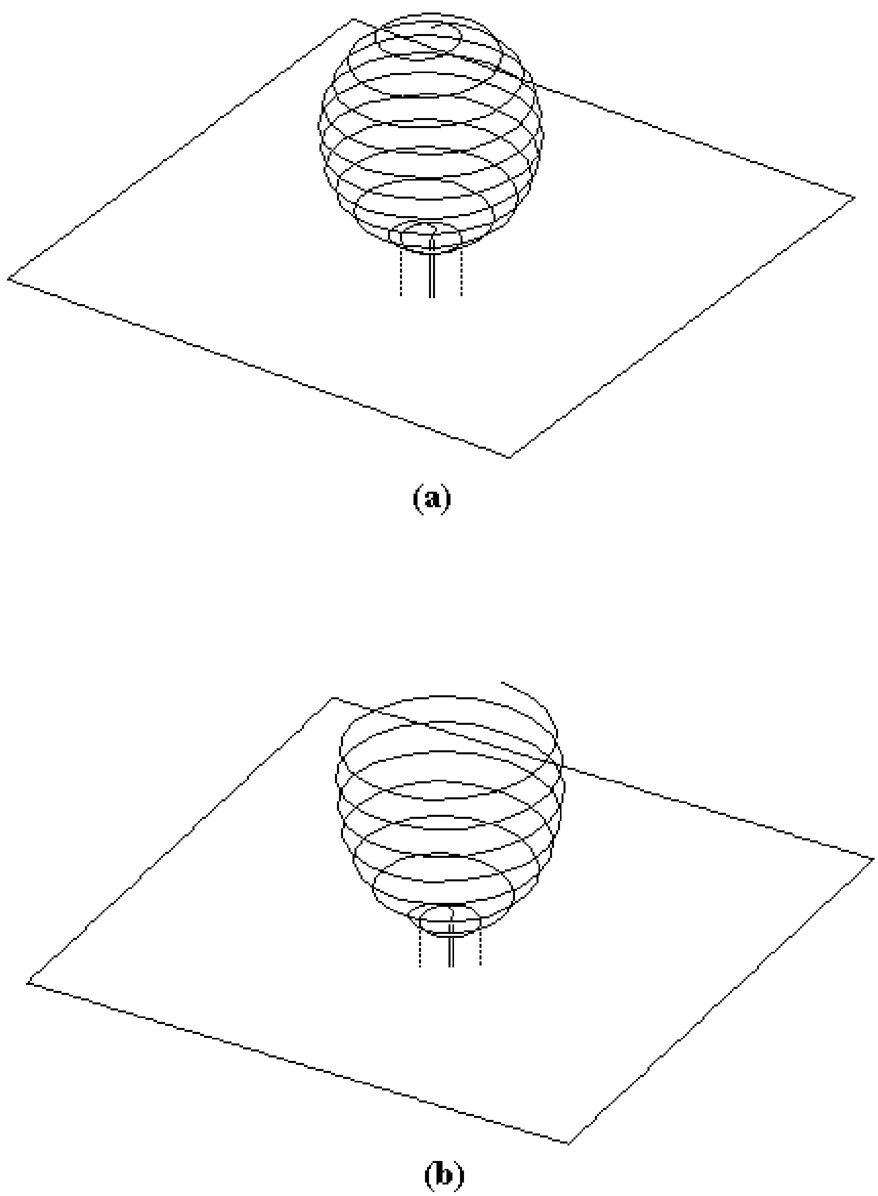

Figure 3.1 **Geometry of spherical helix: (a) Fully wound 10-turn helix, (b) Truncated 7-turn helix**

spherical coordinates (r, θ, ϕ) and cylindrical coordinates (ρ, ϕ, z) are used to derive the expressions describing the spherical helix. This derivation is detailed in the next section.

3.1.1 Full Spherical and Truncated Geometries

With a constant spacing between the turns, there is a linear relationship between the z and ϕ_a coordinates over the entire length of the spherical helix. Accordingly, for $0 \le \phi_a \le 2\pi N$, z is related to ϕ_a as

$$z = m\phi_a + b, \tag{3.1}$$

where m and b are constant coefficients. By choosing two known points on the sphere, $\phi_a = 0,\ z = -a$ and $\phi_a = 2\pi$, $z = -a + \dfrac{2a}{N}$, m and b are easily determined. Here, it is assumed that the center of the sphere coincides with the origin of the coordinate system. Substituting the coordinates of the above two points, we obtain $m = a/N\pi$ and $b = -a$. Then, using the values of m and b in (3.1), the z-ϕ_a relationship becomes

$$z = a\left(\frac{\phi_a}{N\pi} - 1\right). \tag{3.2}$$

A second equation is required to uniquely describe the geometry of the spherical helix. This equation is simply $r = a$, where r is the spherical radial coordinate. In numerical modeling of the spherical helix, it is more convenient to use Cartesian coordinates. These coordinates in terms of the spherical coordinates are stated as

$$x = r\sin\theta_a \cos\phi_a, \tag{3.3a}$$

$$y = r\sin\theta_a \sin\phi_a, \tag{3.3b}$$

$$z = r\cos\theta_a. \tag{3.3c}$$

Using $r = a$ and (3.2) in (3.3c), θ_a is obtained as

$$\theta_a = \cos^{-1}\left(\frac{\phi_a}{N\pi} - 1\right). \tag{3.4}$$

Approximating the spherical winding as I straight segments (with the beginning of the i^{th} segment connected to the end of the $(i-1)^{th}$ segment), the coordinates for the end point of an i^{th} segment are summarized as

$$z_i = -a + i\Delta z\,, \tag{3.5a}$$

$$x_i = a\sin\left[\cos^{-1}\left(z_i \big/ a \right)\right]\cos\left(\frac{Npz_i}{a}+1\right), \tag{3.5b}$$

$$y_i = a\sin\left[\cos^{-1}\left(z_i \big/ a \right)\right]\sin\left(\frac{Npz_i}{a}+1\right), \tag{3.5c}$$

where $i = 1,2,3,\ldots,I$ and $\Delta z = 2a \big/ I$. The value of I is chosen such that the segmented winding models the spherical helix with sufficient accuracy. As a rule of thumb, I is chosen so that the length of a segment in less than $0.1l$. If the spherical helix is truncated as in Figure 3.1b, all the above equations remain valid, but f_a varies in the range $0 \le f_a \le 2np$ and $\Delta z = 2na \big/ NI$, where n is the actual number of turns of the helix $(n \le N)$. Equations (3.5a) to (3.5c) are utilized in the subroutine WGEOM to furnish input data for the ESP code.

3.1.2 Hemispherical Helix

The hemispherical helix is an important variation of the spherical helix to which much of the investigations in this thesis are devoted. This antenna, as shown in Figure 3.2, consists of a winding over the upper half of a spherical surface and is fed by a coaxial cable like the spherical helix. The feed point for the hemispherical helix is at the center of the diameter plain and is connected to the winding by means of a straight wire as illustrated in Figure 3.2. The expressions describing the geometry of the hemispherical helix can be obtained in the same manner as in the case of spherical helix. Substituting the coordinates of two known points $(f_a = 0, z = 0)$ and $(f_a = 2p, z = \frac{2a}{N})$ in (3.1), the relationship between z and f_a coordinates for the hemispherical helix is obtained as

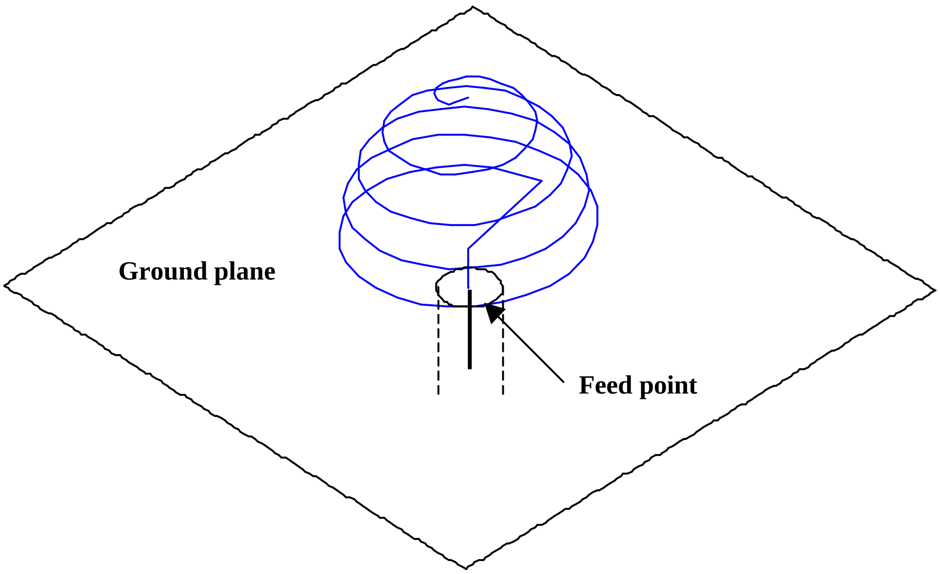

Figure 3.2 Hemispherical helical antenna with 4.5 turns

$$z = \frac{a}{pN} f_a \,, \tag{3.6}$$

where $0 \le f_a \le Np$. The other equation is obviously $r = a$. Now, using $z = r\cos q_a$ yields

$$q_a = \cos^{-1} \frac{f_a}{pN} \,. \tag{3.7}$$

From (3.6) and (3.7) and using the relationships between the Cartesian and spherical coordinates, the coordinates for the end point of an i^{th} segment are given as

$$z_i = i\Delta z \,, \tag{3.8a}$$

$$x_i = a\sin\left[\cos^{-1}\left({z_i}/{a} \right)\right]\cos\left(\frac{Npz_i}{a} \right), \tag{3.8b}$$

$$y_i = a\sin\left[\cos^{-1}\left({z_i}/{a} \right)\right]\sin\left(\frac{Npz_i}{a} \right), \tag{3.8c}$$

where $i =$ 1,2,…, I and $\Delta z = {a}/{I}$.

3.2 Basic Radiation Properties of the Spherical Helix

3.2.1 Radiation Pattern

The study by Cardoso [2] revealed that spherical helices between 3 and 10 turns have almost the same radiation patterns. The major lobe of the patterns occupies the entire regions, $0^\circ \le q_a \le 90^\circ$. The 3-dB and 10-dB beamwidths are about 60° and 120°, respectively. The first sidelobe level is about 20 dB below the major lobe. Additionally, the magnitudes of the q and f components of the fields in the major lobe are nearly equal over the beamwidth.

3.2.2 Polarization

Generally, spherical helices are elliptically polarized. But, under certain circumstances, these antennas become circularly polarized. Circular polarization, when exists, is maintained over a narrow bandwidth. The sense of the polarization of the spherical helix is determined by the sense of the winding over the sphere. If it is wound in a right- (left-) handed direction, the polarization will be right- (left-) hand.

3.2.3 Bandwidth

The bandwidth of a spherical helix providing circular polarization varies depending upon the number of turns wound around the sphere. For example, a 4-turn spherical helix provides 800 MHz bandwidth, while a 10-turn spherical helix has a bandwidth of only 50 MHz [4]. The bandwidth is a small fraction of the operating frequency, thus the spherical helix is categorized as a narrow-band antenna.

3.2.4 Input Impedance

The input impedance of a spherical helix operating in the axial mode is complex. Both resistance and reactance vary with frequency in an oscillatory manner. The oscillatory nature of the input impedance is attributed to the fact that the spherical helix behaves more like a standing-wave antenna [4].