Appendix IX
Column Interaction Diagram of the Prestressed Girder Concrete Bridge

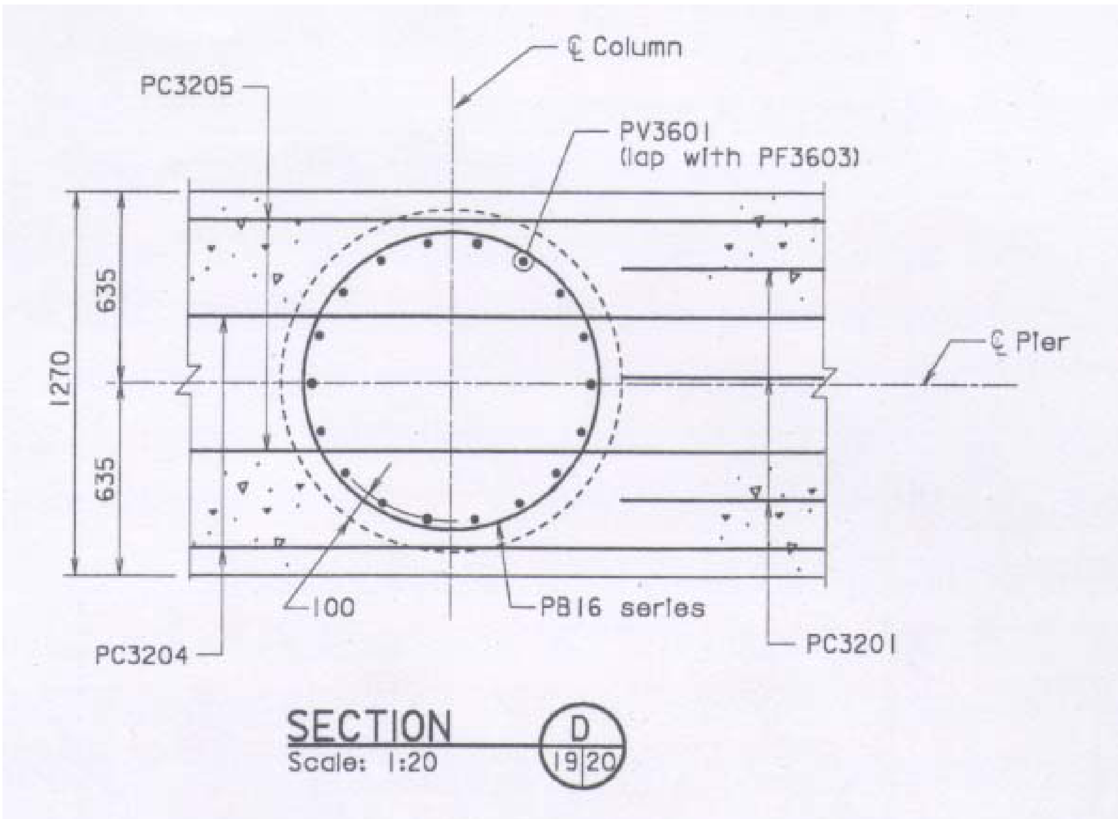


Figure IX-1. The cross section of the column, which has a diameter of 1067 mm [Maday, 2002]. All dimensions are in mm, and the 1:20 scale is no longer correct.

$f_c' = 25MPa$

$f_y = 420MPa$

$A_g = \frac{1}{4}\pi(1067mm)^2 = 894{,}000mm^2$

$A_{s1} = 2013mm^2$

$A_{s2} = 2013mm^2$

$A_{s3} = 2013mm^2$

$A_{s4} = 2013mm^2$

$A_{s5} = 2013mm^2$

$A_{s6} = 2013mm^2$

$A_{s7} = 2013mm^2$

$A_{s8} = 2013mm^2$

$A_{s9} = 2013mm^2$

$A_{st} = \sum A_{si} = 18117mm^2$

$\rho_t = \frac{A_{st}}{A_g} = \frac{18117mm^2}{894{,}000mm^2} = 0.0203$

$\varepsilon_y = \frac{f_y}{E_s} = \frac{420MPa}{200{,}000MPa} = 0.00210$

<u>Nominal Concentric Axial Load Capacity:</u>

$P_0 = (0.85 f_c')(A_g - A_{st}) + f_y A_{st}$

$= (0.85)(25N/mm^2)(894{,}167mm^2 - 18{,}117mm^2) + (420N/mm^2)(18{,}117mm^2)$

$= 26{,}200{,}000N$

$P_a = 0.133 f_c' A_g = 0.133(25N/mm^2)(894{,}000mm^2) = 2{,}970{,}000N$

$P_0 > P_a \rightarrow \phi = 0.75$

$\phi P_0 = 0.75(26{,}200{,}000N) = 19{,}700{,}000N$

$\phi P_0 = 19{,}700{,}000N \rightarrow A'$

$\phi P_{n(\max)} = 0.85\phi P_0 = 0.85(19{,}700{,}000N)$

$\phi P_{n(\max)} = 16{,}700{,}000N$

Capacity in Axial Tension

$$P_{nt} = \sum_{i=1}^{n} \left(-f_y A_{si}\right)$$

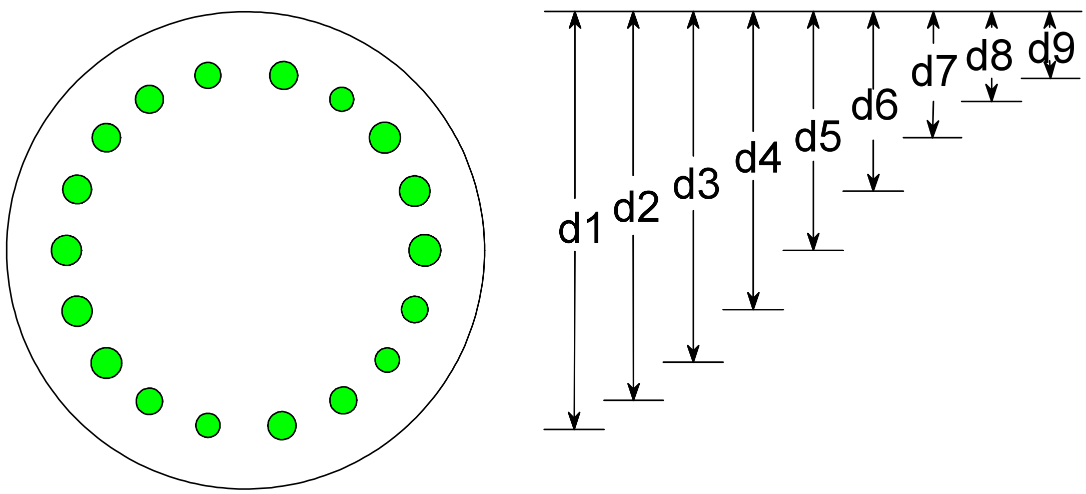
$$P_{nt} = -f_y \sum_{i=1}^{n} A_{si}$$

$$P_{nt} = -\left(420 N/mm^2\right)\left(18{,}117 mm^2\right)$$

$$P_{nt} = -7{,}610{,}000 N$$

$$\phi P_{nt} = 0.9\left(-7{,}610{,}000 N\right) = -6{,}850{,}000 N$$

$$\phi P_{nt} = -6{,}850{,}000 N \rightarrow E'$$



$$d_1 = 533.5 + \left(433.5 \sin 80^\circ\right) = 960 mm$$

$$d_2 = 533.5 + \left(433.5 \sin 60^\circ\right) = 909 mm$$

$$d_3 = 533.5 + \left(433.5 \sin 40^\circ\right) = 812 mm$$

$$d_4 = 533.5 + \left(433.5 \sin 20^\circ\right) = 682 mm$$

$$d_5 = 533.5 mm$$

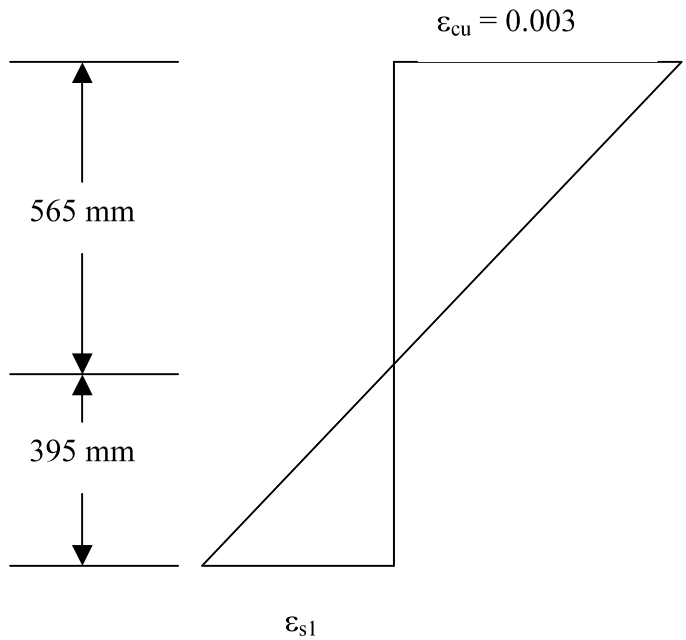
$$d_6 = 533.5 - \left(433.5 \cos 70^\circ\right) = 385 mm$$

$$d_7 = 533.5 - \left(433.5 \cos 50^\circ\right) = 255 mm$$

$$d_8 = 533.5 - \left(433.5 \cos 30^\circ\right) = 158 mm$$

$$d_9 = 533.5 - \left(433.5 \cos 10^\circ\right) = 107 mm$$

ϕP_n and ϕM_n for the balanced failure ($\varepsilon_{s1} = -\varepsilon_y$)



$$c = \frac{0.003}{0.003 - (-1 \times 0.00210)}(960mm) = 565mm$$

$$\varepsilon_{s1} = -\varepsilon_y = -0.00210$$

$$\varepsilon_{s2} = \left(\frac{909mm - 565mm}{395mm}\right)(-0.00210) = -0.00183$$

$$\varepsilon_{s3} = \left(\frac{812mm - 565mm}{395mm}\right)(-0.00210) = -0.00131$$

$$\varepsilon_{s4} = \left(\frac{682mm - 565mm}{395mm}\right)(-0.00210) = -0.000622$$

$$\varepsilon_{s5} = \left(\frac{565mm - 533.5mm}{565mm}\right)(0.003) = 0.000167$$

$$\varepsilon_{s6} = \left(\frac{565mm - 385mm}{565mm}\right)(0.003) = 0.000956$$

$$\varepsilon_{s7} = \left(\frac{565mm - 255mm}{565mm}\right)(0.003) = 0.00165$$

$$\varepsilon_{s8} = \left(\frac{565mm - 158mm}{565mm}\right)(0.003) = 0.00216$$

$$\varepsilon_{s9} = \left(\frac{565mm - 107mm}{565mm}\right)(0.003) = 0.00243$$

$$f_{s1} = \varepsilon_{s1} E_s = (-0.00210)(200{,}000 N/mm^2) = -420 N/mm^2$$
$$f_{s2} = \varepsilon_{s2} E_s = (-0.00183)(200{,}000 N/mm^2) = -366 N/mm^2$$
$$f_{s3} = \varepsilon_{s3} E_s = (-0.00131)(200{,}000 N/mm^2) = -262 N/mm^2$$
$$f_{s4} = \varepsilon_{s4} E_s = (-0.000622)(200{,}000 N/mm^2) = -124 N/mm^2$$
$$f_{s5} = \varepsilon_{s5} E_s = (0.000167)(200{,}000 N/mm^2) = 33.4 N/mm^2$$
$$f_{s6} = \varepsilon_{s6} E_s = (0.000956)(200{,}000 N/mm^2) = 191 N/mm^2$$
$$f_{s7} = \varepsilon_{s7} E_s = (0.00165)(200{,}000 N/mm^2) = 330 N/mm^2$$
$$f_{s8} = \varepsilon_{s8} E_s = (0.00216)(200{,}000 N/mm^2) = 432 N/mm^2 > 420 N/mm^2$$
$$f_{s8} = 420 N/mm^2$$
$$f_{s9} = \varepsilon_{s9} E_s = (0.00243)(200{,}000 N/mm^2) = 486 N/mm^2 > 420 N/mm^2$$
$$f_{s9} = 420 N/mm^2$$

$$a = \beta_1 c = 0.85(565mm) = 480.25mm < h = 1067mm \rightarrow OK$$
$$C_c = 0.85 f_c' h^2 \left(\frac{\theta - \sin\theta\cos\theta}{4} \right)$$
$$\theta = \cos^{-1}\left(\frac{\frac{h}{2} - a}{\frac{h}{2}} \right) = \cos^{-1}\left(\frac{533.5mm - 480.25mm}{533.5mm} \right) = 1.47 rad$$
$$C_c = 0.85(25 N/mm^2)(1067mm)^2 \left(\frac{1.47 - \sin 1.47 \cos 1.47}{4} \right) = 8{,}290{,}000 N$$
$$F_{s1} = f_{s1} A_{s1} = (-420 N/mm^2)(2013 mm^2) = -845{,}000 N$$
$$F_{s2} = f_{s2} A_{s2} = (-366 N/mm^2)(2013 mm^2) = -737{,}000 N$$
$$F_{s3} = f_{s3} A_{s3} = (-262 N/mm^2)(2013 mm^2) = -527{,}000 N$$
$$F_{s4} = f_{s4} A_{s4} = (-124 N/mm^2)(2013 mm^2) = -250{,}000 N$$
$$F_{s5} = f_{s5} A_{s5} = (33.4 N/mm^2)(2013 mm^2) = 67{,}200 N$$
$$F_{s6} = (f_{cs6} - 0.85 f_c') A_{s6} = (191 N/mm^2 - 0.85(25 N/mm^2))(2013 mm^2) = 342{,}000 N$$
$$F_{s7} = (f_{cs7} - 0.85 f_c') A_{s7} = (330 N/mm^2 - 0.85(25 N/mm^2))(2013 mm^2) = 622{,}000 N$$
$$F_{s8} = (f_{cs8} - 0.85 f_c') A_{s8} = (420 N/mm^2 - 0.85(25 N/mm^2))(2013 mm^2) = 803{,}000 N$$
$$F_{s9} = (f_{cs9} - 0.85 f_c') A_{s9} = (420 N/mm^2 - 0.85(25 N/mm^2))(2013 mm^2) = 803{,}000 N$$

$$P_n = C_c + \sum F_{si} = 8{,}560{,}000 N = P_b$$

$$M_n = C_c\left(\frac{h}{2}-\frac{a}{2}\right)+\sum_{i=1}^{9}F_{si}\left(\frac{h}{2}-d_i\right)$$
$$= (8{,}290{,}000N)(533.5mm - 240.125mm) + (-845{,}000N)(533.5mm - 960mm)$$
$$+ (-737{,}000N)(533.5mm - 909mm) + (-527{,}000N)(533.5mm - 812mm)$$
$$+ (-250{,}000N)(533.5mm - 682mm) + (67{,}200N)(533.5mm - 533.5mm)$$
$$+ (342{,}000N)(533.5mm - 385mm) + (622{,}000N)(533.5mm - 255mm)$$
$$+ (803{,}000N)(533.5mm - 158mm) + (803{,}000N)(533.5mm - 107mm)$$
$$= 4{,}120{,}000{,}000Nmm$$
$$= M_b$$
$$\varepsilon_t = \varepsilon_{s1} = -\varepsilon_y \rightarrow \phi = 0.75$$
$$\phi P_n = 0.75(8{,}560{,}000N) = 6{,}420{,}000N$$
$$\phi M_n = 0.75(4{,}120{,}000{,}000Nmm) = 3{,}090{,}000{,}000Nmm$$

The column interaction diagram is shown in Figure IX-4.

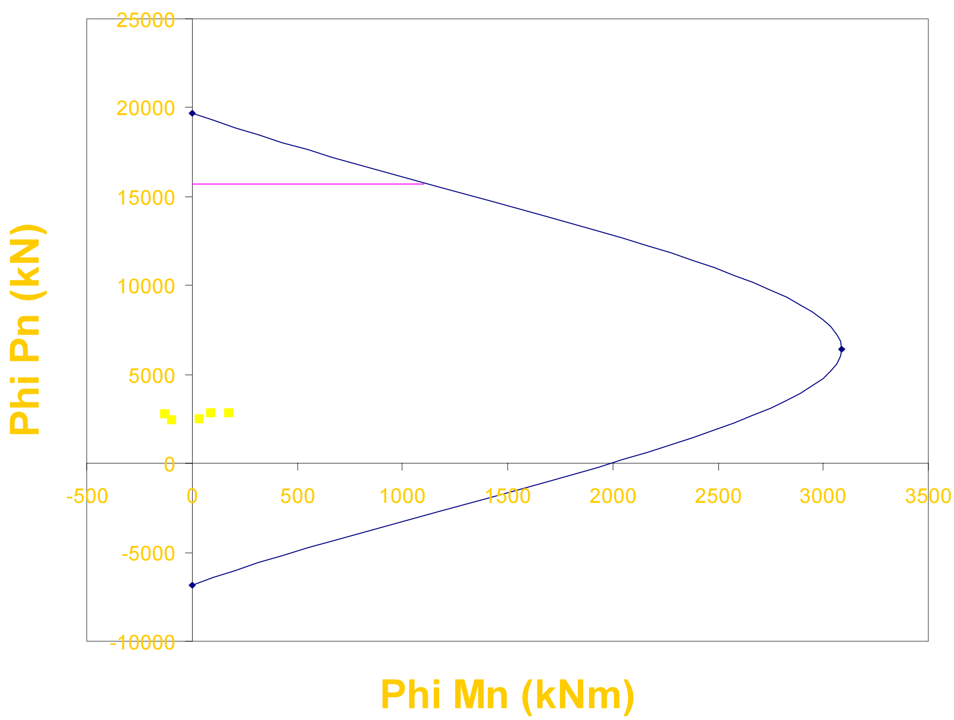


Figure IX-2. The column interaction diagram for the prestressed concrete girder bridge. Points in grey are the factored axial loads and moments in the column.