Appendix XVI
Cracked Section Properties of the Pier Cap Beams of the Steel Girder Bridge using the Moment Curvature Method and ACI Equation

West Bound Pier Cap Beam

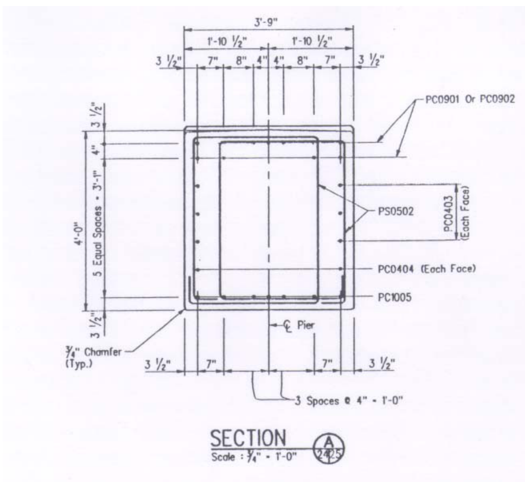

Figure XVI-1. The actual pier cap beam cross section [Brown, 1993]. The ¾” = 1’-0” scale is no longer correct.

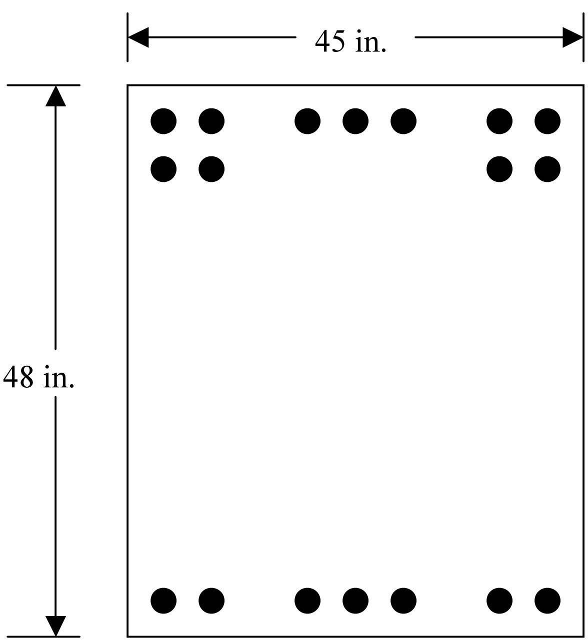

Figure XVI-2. The simplified pier cap beam cross section. This figure was not drawn to scale.

Moment Curvature Method

$$T = C_C + C_S$$

$$A_S f_y = b \int_0^{kd} f_c dy + A_s' f_s'$$

$$A_s f_y - A_s' f_s' = b \int_0^{kd} f_c dy$$

$$d = \frac{(4)(1.00in.^2)(40.5in.^2) + (7)(1.00in.^2)(44.5in.)}{(4)(1.00in.^2) + (7)(1.00in.^2)}$$
$$= 43.05in.$$

$$A_s = (11)(1.00in.^2) = 11.0in.^2$$
$$A_s' = (7)(1.27in.^2) = 8.89in.^2$$

Assume the compression steel will not yield:

$$\frac{\varepsilon_c}{kd} = \frac{\varepsilon_y}{d - kd} \Leftrightarrow \varepsilon_c = \varepsilon_y \frac{kd}{d - kd}$$
$$\frac{\varepsilon_s'}{kd - d'} = \frac{\varepsilon_y}{d - kd} \Leftrightarrow \varepsilon_s' = \varepsilon_y \frac{kd - d'}{d - kd}$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60ksi}{29{,}000ksi} = 0.00207$$
$$\varepsilon_s' = 0.00207\left(\frac{kd - 3.5in.}{43.05in. - kd}\right)$$
$$\varepsilon_0 = \frac{2f_c'}{E_c} = \frac{2(3ksi)}{57\sqrt{3000}ksi} = 0.00192$$
$$A_s f_y - A_s' E_s \varepsilon_s' = bf_c'\left[\frac{\varepsilon_y}{\varepsilon_0}\frac{(kd)^2}{d - kd} - \frac{1}{3}\left(\frac{\varepsilon_y}{\varepsilon_0}\right)^2 \frac{(kd)^3}{(d - kd)^2}\right]$$

By plugging in these following values to the equation above,

$A_s = 11.0 in.^2 (11 - \#9 bars)$
$f_y = 60 ksi$
$A_s' = 8.89 in.^2 (7 - \#10 bars)$
$E_s = 29{,}000 ksi$
$\varepsilon_s' = 0.00207 \left(\frac{kd - 3.5 in.}{43.05 in. - kd} \right)$
$b = 45 in.$
$f_c' = 3 ksi$
$\varepsilon_y = 0.00207$
$\varepsilon_0 = 0.00192$
$d = 43.05 in.$

then a third-degree equation in terms of kd is obtained. The solutions of the third-degree equation are

kd = 32.10 in. or kd = 11.45in. or kd = -17.92in.

kd = -17.92in. is obviously ruled out, and kd = 32.10in. is ruled out because the reinforcing area at the bottom is less than that at the top, therefore kd is supposed to be less than half of 43.05 in., which is equal to 21.525in. Thus kd = 11.45in. is the only possible solution.
Now the initial assumption that the compression steel will not yield must be checked:

$$\varepsilon_s' = \varepsilon_y \frac{kd - d'}{d - kd} = 0.00207 \left(\frac{11.45 in. - 3.5 in.}{43.05 in. - 11.45 in.} \right) = 0.000521 < \varepsilon_y = 0.00207$$

Thus the initial assumption is correct.
Now $\varepsilon_{c\,max}$ must be checked to see if it is less than ε_0. If it is, then the stress block will have a parabolic shape.

$$\varepsilon_{c\max} = \varepsilon_y \frac{kd}{d - kd} = 0.00207 \left(\frac{11.45 in.}{43.05 in. - 11.45 in.} \right) = 0.000750 < \varepsilon_0 = 0.00192$$

Thus the stress block will be parabolic. The curvature at the yield point is

$$\phi_y = \frac{\varepsilon_{c\max}}{kd} = \frac{0.000750}{11.45in.} = 6.55 \times 10^{-5} \ / in.$$

Now the moment at yield can be obtained using Figure V-4.

$$\bar{y} = \frac{\frac{2}{3}\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)(kd)^2 - \frac{1}{4}\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)^2 (kd)^2}{\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)(kd) - \frac{1}{3}\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)^2 (kd)}$$

$$\bar{y} = \frac{\frac{2}{3}\left(\frac{0.000750}{0.00192}\right)(11.45in.)^2 - \frac{1}{4}\left(\frac{0.000750}{0.00192}\right)^2 (11.45in.)^2}{\left(\frac{0.000750}{0.00192}\right)(11.45in.) - \frac{1}{3}\left(\frac{0.000750}{0.00192}\right)^2 (11.45in.)}$$

$$\bar{y} = 7.49in.$$

$$C_c = b\int_0^{kd} f_c dy$$

$$= bf_c'\left[\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)(kd) - \frac{1}{3}\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)^2 (kd)\right]$$

$$= (45in.)(3ksi)\left[\left(\frac{0.000750}{0.00192}\right)(11.45in.) - \frac{1}{3}\left(\frac{0.000750}{0.00192}\right)^2 (11.45in.)\right]$$

$$= 525.19k$$

$$C_s = (8.89in.^2)(29{,}000ksi)(0.00207)\left(\frac{11.45in - 3.5in.}{43.05in. - 11.45in.}\right)$$

$$= 134.26k$$

$$M_y = (d - kd + \bar{y})C_c + C_s(d - d')$$

$$= (43.05in. - 11.45in. + 7.49in.)(525.19k) + (134.26k)(43.05in. - 3.5in.)$$

$$= 25{,}800kips - in.$$

$$EI_e = \frac{M_y}{\phi_y} = \frac{25{,}800 kips - in}{6.55 \times 10^{-5} / mm} = 394{,}000{,}000 kips - in.^2$$

$$I_e = \frac{394{,}000{,}000 kips - in.^2}{57\sqrt{3000} ksi} = 126{,}000 in.^4$$

$$\frac{I_e}{I_g} = \frac{126{,}000 in.^4}{\frac{1}{12}(45in.)(48in.)^3} = 0.305$$

<u>ACI Equation Method</u>

$$f_c' = 3ksi$$

$$f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{3000} psi = 411 psi$$

$$M_{cr} = \frac{f_r I}{y}$$

$$M_{cr} = \frac{(0.411ksi)\left(\frac{1}{12}\right)(45in.)(48in.)^3}{24in.}$$

$$= 7{,}102 kips - in.$$

This cracking moment must be compared with the maximum positive and negative moments in the pier cap beam (M_a), which are given in Table XVI-1.

Table XVI-1. The calculation for the maximum positive and negative moments in the pier cap beam.

	East Bound	Lane	West Bound	Lane
	Maximum Positive	Maximum Negative	Maximum Positive	Maximum Negative
Design Truck	356	-284	477	-351
Design Tandem	243	-194	326	-240
Two Design Trucks	476	-379	635	-468
Lane	327	-260	423	-312
Controlling Load	476	-379	635	-468
Live Load Effects	2204	-1754	2283	-1682
Dead Load Effects	5049	-3841	5237	-3881
LL+DL Effects	6151	-4718	6379	-4722

Note – Loads are in kips.

$M_{cr} = 7{,}102 kips - in > M_a = 6379 kips - in.$ for the maximum positive moment.

$M_{cr} = 7{,}102 kips - in > M_a = -4721 kips - in.$ for the maximum negative moment.

Therefore according to the ACI Equation method,

$$I_e = I_g$$
$$= \frac{1}{12}(45in.)(48in.)^3$$
$$= 415{,}000 in.^4$$

East Bound Pier Cap Beam

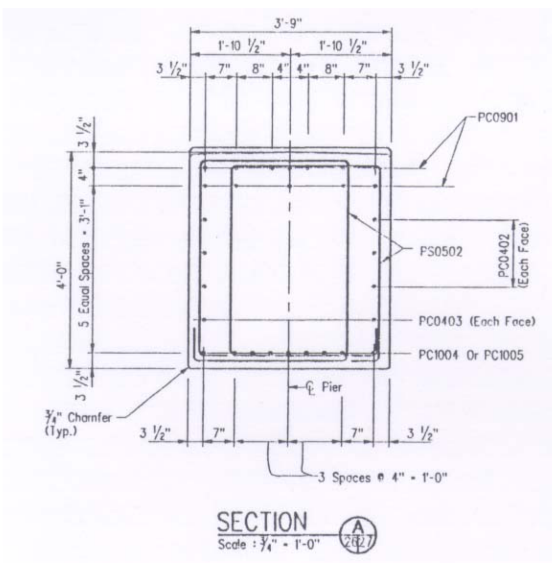

Figure XVI-3. The actual pier cap beam cross section [Brown, 1993]. The 3/4"' =1'-0" scale is no longer correct.

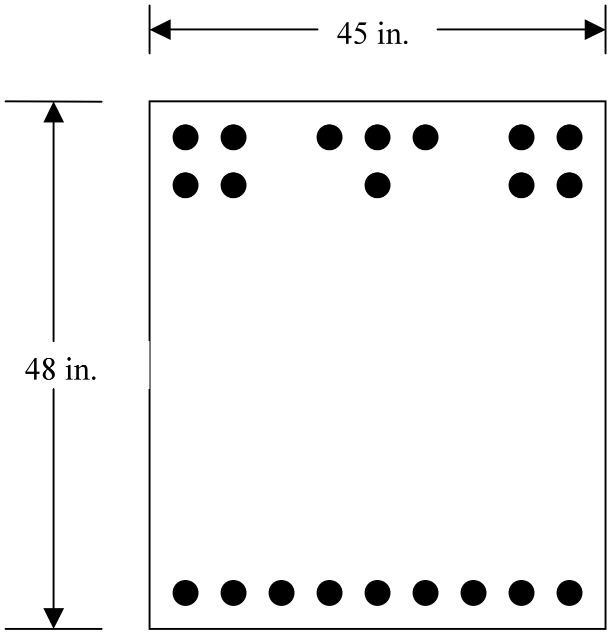

Figure XVI-4. The simplified pier cap beam cross section. This figure was not drawn to scale.

<u>Moment Curvature Method</u>

$$T = C_C + C_S$$

$$A_S f_y = b\int_0^{kd} f_c dy + A_s' f_s'$$

$$A_s f_y - A_s' f_s' = b\int_0^{kd} f_c dy$$

$$d = \frac{(5)(1.00in.^2)(40.5in.) + (7)(1.00in.^2)(44.5in.)}{(5)(1.00in.^2) + (7)(1.00in.^2)}$$
$$= 42.83in.$$

$$A_s = (12)(1.00in.^2) = 12.0in.^2$$
$$A_s' = (9)(1.27in.^2) = 11.43in.^2$$

Assume the compression steel will not yield:

$$\frac{\varepsilon_c}{kd} = \frac{\varepsilon_y}{d-kd} \Leftrightarrow \varepsilon_c = \varepsilon_y \frac{kd}{d-kd}$$

$$\frac{\varepsilon_s'}{kd-d'} = \frac{\varepsilon_y}{d-kd} \Leftrightarrow \varepsilon_s' = \varepsilon_y \frac{kd-d'}{d-kd}$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60ksi}{29{,}000ksi} = 0.00207$$

$$\varepsilon_s' = 0.00207\left(\frac{kd-3.5in.}{42.83in.-kd}\right)$$

$$\varepsilon_0 = \frac{2f_c'}{E_c} = \frac{2(3ksi)}{57\sqrt{3000}ksi} = 0.00192$$

$$A_s f_y - A_s' E_s \varepsilon_s' = bf_c'\left[\frac{\varepsilon_y}{\varepsilon_0}\frac{(kd)^2}{d-kd} - \frac{1}{3}\left(\frac{\varepsilon_y}{\varepsilon_0}\right)^2 \frac{(kd)^3}{(d-kd)^2}\right]$$

By plugging in these following values to the equation above,

$A_s = 12.0in.^2 (12\text{–}\#9bars)$

$f_y = 60ksi$

$A_s' = 11.43in.^2 (9\text{–}\#10bars)$

$E_s = 29{,}000ksi$

$\varepsilon_s' = 0.00207\left(\frac{kd-3.5in.}{42.83in.-kd}\right)$

$b = 45in.$

$f_c' = 3ksi$

$\varepsilon_y = 0.00207$

$\varepsilon_0 = 0.00192$

$d = 42.83in.$

then a third-degree equation in terms of kd is obtained. The solutions of the third-degree equation are

kd = 32.13 in. or kd = 11.59in. or kd = -19.32in.

kd = -19.32in. is obviously ruled out, and kd = 32.13in. is ruled out because the reinforcing area at the bottom is less than that at the top, therefore kd is supposed to be less than half of 42.83 in., which is equal to 21.415in. Thus kd = 11.45in. is the only possible solution.

Now the initial assumption that the compression steel will not yield must be checked:

$$\varepsilon_s' = \varepsilon_y \frac{kd - d'}{d - kd} = 0.00207\left(\frac{11.59in. - 3.5in.}{42.83in. - 11.59in.}\right) = 0.000536 < \varepsilon_y = 0.00207$$

Thus the initial assumption is correct.

Now $\varepsilon_{c\,max}$ must be checked to see if it is less than ε_0. If it is, then the stress block will have a parabolic shape.

$$\varepsilon_{c\max} = \varepsilon_y \frac{kd}{d - kd} = 0.00207\left(\frac{11.59in.}{42.83in. - 11.59in.}\right) = 0.000768 < \varepsilon_0 = 0.00192$$

Thus the stress block will be parabolic. The curvature at the yield point is

$$\phi_y = \frac{\varepsilon_{c\max}}{kd} = \frac{0.000768}{11.59in.} = 6.63 \times 10^{-5} / in.$$

Now the moment at yield can be obtained using Figure V-4.

$$\bar{y} = \frac{\frac{2}{3}\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)(kd)^2 - \frac{1}{4}\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)^2 (kd)^2}{\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)(kd) - \frac{1}{3}\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)^2 (kd)}$$

$$\bar{y} = \frac{\frac{2}{3}\left(\frac{0.000768}{0.00192}\right)(11.59in.)^2 - \frac{1}{4}\left(\frac{0.000768}{0.00192}\right)^2 (11.59in.)^2}{\left(\frac{0.000768}{0.00192}\right)(11.59in.) - \frac{1}{3}\left(\frac{0.000768}{0.00192}\right)^2 (11.59in.)}$$

$$\bar{y} = 7.49in.$$

$$C_c = b\int_0^{kd} f_c dy$$

$$= bf_c'\left[\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)(kd) - \frac{1}{3}\left(\frac{\varepsilon_{c\max}}{\varepsilon_0}\right)^2 (kd)\right]$$

$$= (45in.)(3ksi)\left[\left(\frac{0.000768}{0.00192}\right)(11.59in.) - \frac{1}{3}\left(\frac{0.000768}{0.00192}\right)^2 (11.59in.)\right]$$

$$= 542k$$

$$C_s = (11.43in.^2)(29{,}000ksi)(0.00207)\left(\frac{11.59in - 3.5in.}{42.83in. - 11.59in.}\right)$$

$$= 178k$$

$$M_y = (d - kd + \bar{y})C_c + C_s(d - d')$$

$$= (42.83in. - 11.59in. + 7.58in.)(542k) + (178k)(42.83in. - 3.5in.)$$

$$= 28{,}000kips - in.$$

$$EI_e = \frac{M_y}{\phi_y} = \frac{28{,}000kips - in}{6.63\times 10^{-5}/mm} = 423{,}000{,}000kips - in.^2$$

$$I_e = \frac{423{,}000{,}000kips - in.^2}{57\sqrt{3000}ksi} = 136{,}000in.^4$$

$$\frac{I_e}{I_g} = \frac{136{,}000in.^4}{\frac{1}{12}(45in.)(48in.)^3} = 0.327$$

ACI Equation Method

$$f_c' = 3ksi$$
$$f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{3000}psi = 411psi$$
$$M_{cr} = \frac{f_r I}{y}$$
$$M_{cr} = \frac{(0.411ksi)\left(\frac{1}{12}\right)(45in.)(48in.)^3}{24in.}$$
$$= 7{,}102kips - in.$$

This cracking moment must be compared with the maximum positive and negative moments in the pier cap beam (M_a), which are given in Table XVI-1.

$M_{cr} = 7{,}102kips - in > M_a = 6{,}151kips - in.$ for the maximum positive moment.

$M_{cr} = 7{,}102kips - in > M_a = -4{,}718kips - in.$ for the maximum negative moment.

Therefore according to the ACI Equation method,

$$I_e = I_g$$
$$= \frac{1}{12}(45in.)(48in.)^3$$
$$= 415{,}000in.^4$$