CHAPTER 3. EXISTING MULTIPLE OBJECTIVE-DEA TYPE METHODOLOGIES

This chapter gives an overview of the existing approaches as developed by Thanassoulis and Dyson (1992) and Athanassopoulos (1995). These approaches were considered relevant for the research because the general nature of the problem addressed by both the models are similar. Though the Thanassoulis and Dyson model does not address multiple objective issues, they estimate preferred targets for inputs and outputs. The structure of the model is a goal-programming model associating a penalty with each deviation from an ideal input or output level. In the paper by Athanassopoulos (1995), goal programming and data envelopment analysis are used as instruments for group decision making. The following section discusses the models in detail,

3.1 THANASSOULIS AND DYSON (1992)[4]

Some *DMUs* may be able to articulate the targets they would ideally wish to adopt. Such ideal targets would reflect the degree to which each *DMU* considers it desirable and/or feasible to improve each input and/or output level. The ideal targets need not contain only contain improvements to current input-output levels. The *DMU* may be willing to sacrifice the level of some input(s) and/or output(s) in order to improve the levels of others.

The ideal targets specified by a *DMU* may in general be neither 'feasible' nor efficient. Feasible input-output levels are those, which can be expressed as non-negative linear combinations of observed input-output levels. In order to determine feasible and efficient targets of a *DMU*, a two-stage process is followed. In the first stage feasible input-output levels are determined which are as close as possible to the ideal targets. During the second stage a set of efficient input-output levels is determined, if any, which dominates the input-output levels determined in the first stage. This second set of input-

[4]Thanassoulis, E., and Dyson, R.G., *Estimating Preferred Target Input-Output Levels using Data Envelopment Analysis*, European Journal of Operations Research, Vol. 56, 1992, pp. 80-97.

output level represent targets, which can be said to be compatible with the ideal targets, specified by the *DMU*. The model is presented below.

$$Min \sum_{i=1}^{m} w_i^{1-} k_i^{\,1} + \sum_{i=1}^{m} w_i^{2-} k_i^{\,2} + \sum_{r=1}^{s} w_r^{1+} c_r^{\,1} + \sum_{r=1}^{s} w_r^{2+} c_r^{\,2} \tag{3.1}$$

Subject to

$$\sum_{j=1}^{n} \gamma_j y_{rj} + c_r^{\,1} - c_r^{\,2} = y_r^{\,t} \qquad r = 1, \ldots, s$$

$$\sum_{j=1}^{n} \gamma_j x_{ij} + k_i^{\,1} - k_i^{\,2} = x_i^{\,t} \qquad i = 1, \ldots, m$$

$$\gamma_j \geq 0 \quad \forall j,\ c_r^{\,j},\ k_i^{\,j} \geq 0 \quad \forall\, i \text{ and } r \qquad j = 1, \ldots, n$$

y_{rj} is the amount of output r produced by DMU_j

x_{ij} is the amount if input i used by DMU_j

n is the total number of DMUs

s is the total number of output variables

m is the total number of input variables

γ_j is the vector of intensity factors that defines the hypothetical DMU to which DMU_{jo} is compared

k_i^1 and k_i^2 are the negative and positive deviation from the target level of input i

c_r^1 and c_r^2 are the negative and positive deviations from the target level of output r

w_i^{1-} and w_i^{2-} are user defined weights attached to the deviations k_i^1 and k_i^2 respectively

w_r^{1+} and w_r^{2+} are user defined weights attached to the deviations c_r^1 and c_r^2 respectively

x_i^t is the ideal input level specified by a DMU for input i

y_r^t is the ideal output level specified by a DMU for output r

The structure of the model ensures that there can be no simultaneous under achievement and over achievement of an ideal input or output level at an optimal solution.

Let $k_i^{2*}, k_i^{1*}, c_r^{2*}$ and c_r^{1*} be respectively the optimal values of k_i^2, k_i^1 and c_r^2, c_r^1 yielded by the model. Then

$$(x_i^f,\ i = 1, \ldots, m,\ y_r^f,\ r = 1, \ldots, s), \tag{3.2}$$

where

$$x_i^f = x_i^t + k_i^{2*} - k_i^{1*} \qquad i = 1, \ldots, m$$

$$y_r^f = y_r^t + c_r^{2*} - c_r^{1*} \qquad r = 1, \ldots, s$$

are the feasible input-output levels which are 'close' to the ideal targets and at the same time they are consistent with the relative desirability of the achievement of each ideal input or output level.

The structure of the model is a goal programming model associating a penalty with each deviation from an ideal input or output level.

The above model can be modified in a number of ways to reflect the preference structure of the *DMU* concerned. For example if the over achievement of the ideal output *r* or the under achievement of the ideal level of input *i* is desirable then the corresponding weights w_r^{2+} and w_i^{1-} can be set to zero. A preferential weighting structure could also be adopted over the deviation variables if compatible with the *DMUs* preferences. Such a weighting structure would ensure that some of the ideal input-output levels are met as closely as possible before the achievement of any other input-output levels is contemplated.

Once the feasible solutions have been determined, it is necessary to test them for efficiency. This might yield a set of efficient input-output levels dominating the feasible input-output levels in (3.2), if it exists. The following model can be used to test the input-output levels for relative efficiency.

$$Max \sum_{r=1}^{s} c_r^{'} + \sum_{i=1}^{m} k_i^{'} \qquad (3.3)$$

Subject to

$$\sum_{j=1}^{n} \delta_j y_{rj} - c_r^{'} = y_r^{f} \qquad r = 1, \ldots, s$$

$$\sum_{j=1}^{n} \delta_j x_{ij} + k_i^{'} = x_i^{f} \qquad i = 1, \ldots m$$

$$\delta_j \geq 0 \ \ \forall j, c_r^{'}, k_i^{'} \geq 0 \quad \forall r \text{ and } i$$

Here y_{rj}, x_{ij}, s, m and n are as defined in the preceding model.

Let $k_i^{'*}$ and $c_r^{'*}$ be the optimal values for $k_i^{'}, c_r^{'}$ yielded by the above model. If $k_i^{'*}$ and $c_r^{'*}$ are both zero then the input-output levels are relatively efficient and they are targets the *DMU* can adopt as feasible, efficient and compatible with its ideal targets.

If $k_i^{'*}$ and $c_r^{'*}$ are not all zero then the input-output levels

$$(\hat{x}_i^{f} , i = 1, \ldots, m, \ \hat{y}_r^{f} , r = 1, \ldots, s) \tag{3.4}$$

where

$$\hat{x}_i^{f} = x_i^{f} - k_i^{'*} , \qquad i = 1, \ldots, m,$$

$$\hat{y}_r^{f} = y_r^{f} + c_r^{'*} , \qquad r = 1, \ldots, s$$

dominate those in (3.2). In this case the *DMU* can adopt the input-output levels in (3.4) as feasible, efficient and compatible with its efficient targets.

The objective values in models (3.1) and (3.3) depend on ideal rather than the observed input-output levels of the *DMU*. They do not in any way yield a measure of its relative efficiency. Conceptually, relative efficiency is always evaluated with respect to the actual inputs-output levels.

The thrust of this model is that a *DMU* can have a preference structure over the potential improvements to input and output levels. The paper aims to illustrate that *DEA* models can be formulated to explore different targets which are consistent with a *DMUs* preference structure for improvement of input and output levels.

3.2 ATHANASSOPOULOS (1995)[5]

The formulation by Thanassoulis and Dyson (1992) is not sufficient, however to address planning and resource allocation problems where all the *DMUs* of the organization need to be considered simultaneously. The analysis does not cover the aspect of the global target achievement. The contribution of the individual processes to the overall target is not discussed. In the paper by Athanassopoulos (1995) an interface between goal programming and data envelopment analysis is developed in order to enable target setting and resource allocation in multi-level planning problems. This enhancement would give the opportunity to accommodate global targets, global resource constraints and finally to consider internal communication among the *DMUs* (resource reallocation).

The main problem of a multi-level system is to ensure that all decision-makers, acting according to their own goals, will achieve overall system goals. Nijkamp and Rietveld (1981) describe three principal problems of policy making in multi-level environments:

- Interdependencies among the components of the system;
- Conflicts among various priorities, objectives and targets within individual components of the system;
- Conflicts among the priorities, objectives and targets between the various components of the system.

Based on these three characteristics, they advocated the usefulness of multi-objective programming for addressing planning problems in multi level units (*MLUs*). The achievement of conflicting objectives, at the global level, can be compromised using multi-objective program methods. Conflicts between the organizational levels of the system require coordinating mechanisms so that all levels act in agreement.

The question of planning in multi-unit multi-level (*MULOs*) concerns the best deployment of all types of resources in order to meet the organizational objectives. Equity, efficiency and effectiveness are generally said to reflect the organizational

[5] Athanassopoulos, A., *Goal Programming and Data Envelopment Analysis (GoDEA) for Target-Based Multi-Level Planning: Allocating Central Grants to the Greek Local Authorities*, European Journal of Operations Research, Vol. 87, 1995, pp. 535-550.

mission in non-profit, multi-unit organizations. This representation would guide decision making towards:

- Maximizing the global achievements of the system (effectiveness)
- Maximizing the contribution of individual units to global targets (efficiency)
- Maximizing the share of each individual unit to the allocated resources (equity)

Centralized resource management is considered as the process where central management is responsible for the allocation and control of resources allocated to individual decision making units. A pictorial representation of this planning system is shown below.

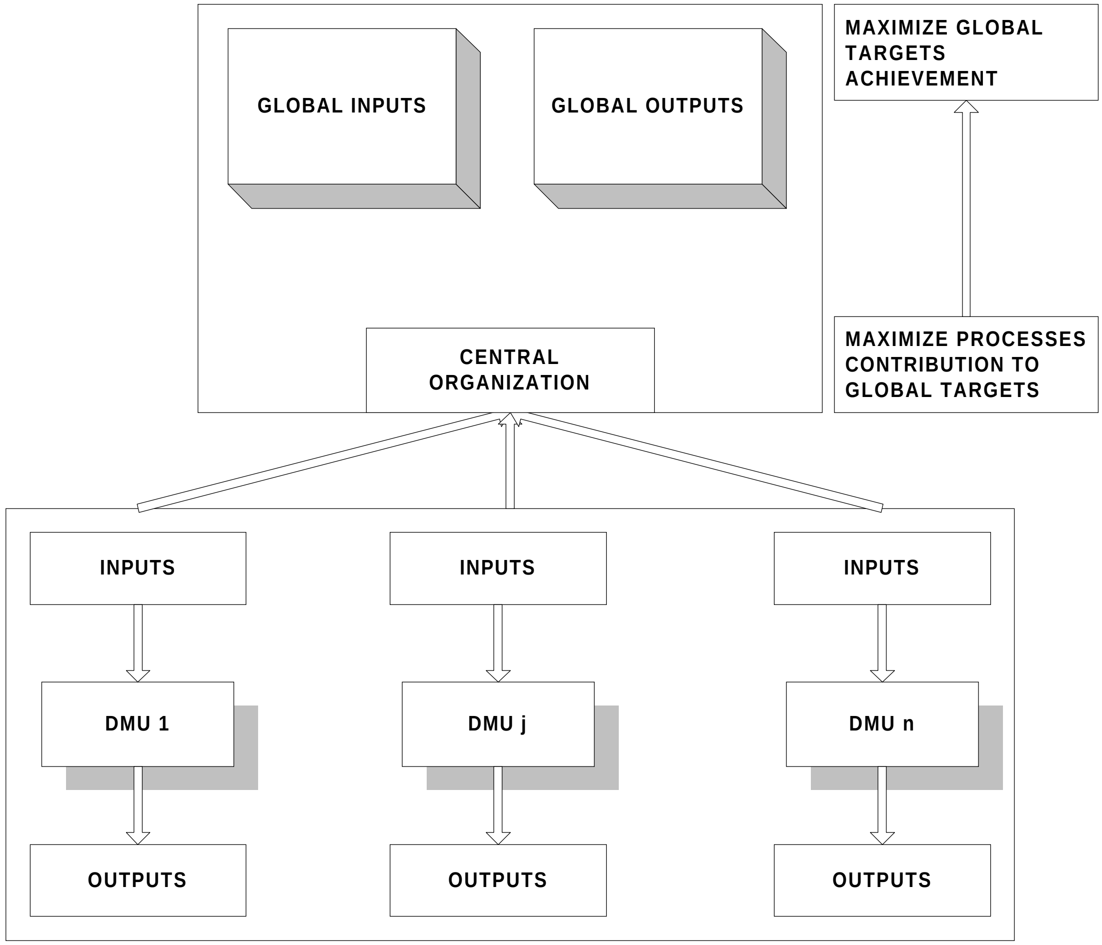

Figure 3.1 Centralized Planning System

The *MULO* represented in the figure 3.2 consists of a centralized coordinating mechanism that is responsible for controlling/allocating global resources to *DMU's* operating similar, but independent functions (Figure shows only two levels but an organization can have many levels). Central management seeks to maximize the achievement of global input/output targets. Individual *DMUs* are expected to maximize their contribution to the achievement of global organizational targets.

The activities of a multi-unit, multi-level organization can be aggregated and displayed by global levels of inputs/outputs, which are allocated among or produced by individual operating units**.** The extent to which the organization achieves these global targets is considered as a surrogate measure of its operational effectiveness, which can be supported by the efficient contribution of the individual *DMUs*. The operational effectiveness is represented in the planning model with the set of constraints:

There are three sets of constraints to this model:

1. A simultaneous representation of all DMU's within the planning process of the MULO, as advocated in Figure 3.2, is necessary.
2. At the global organizational level it is anticipated that for a subset of controllable inputs I_V and outputs O_V, management will be able to specify desired global targets. The global levels of the remaining inputs $\underline{I}_V$ and outputs $\underline{O}_V$ will be estimated by the solution process of the model. The distinction of the global controllability of inputs and outputs will apply to the inputs and outputs classified as controllable (I_C, O_C) at the individual unit level. Thus the distinction of the inputs/outputs can be stated as follows:

   $I_V \cup \underline{I}_V \equiv I_C$

   and

   $O_V \cup \underline{O}_V \equiv O_C$

3. The use of balance constraints to maximize the share of each individual unit to the allocated resources (equity)

The objective function for the *GoDEA* (Goal programming and data envelopment analysis) is given below:

$$
\begin{aligned}
&Min\{\sum_{j=1}^{n} \sum_{i \in I_C} (P_i^{-} \frac{n_i^{\,j}}{x_{ij}} + P_i^{+} \frac{p_i^{\,j}}{x_{ij}}) \\
&+ \sum_{j=1}^{n} \sum_{r \in O_c} (P_r^{-} \frac{n_r^{\,j}}{y_{rj}} + P_r^{+} \frac{p_r^{\,j}}{y_{rj}}), \\
&\sum_{i \in I_v} P_i^{g} \frac{d_i^{+}}{GX_i} + \sum_{r \in O_v} P_r^{g} \frac{d_r^{-}}{GY_r}\}
\end{aligned}
\qquad (3.6)
$$

subject to

Representation of individual DMUs:

$$\sum_{j=1}^{n} \delta_j^{\ k} y_{rj} - p_r^{\ k} + n_r^{\ k} = y_r^{\ k}, r \in O_c, \forall k$$

$$-\sum_{j=1}^{n} \delta_j^{\ k} x_{ij} + p_i^{\ k} - n_i^{\ k} = -x_i^{\ k}, i \in I_c, \forall k$$

$$\sum_{j=1}^{n} \delta_j^{\ k} y_{rj} \geq y_r^{\ k}, r \in O_f, \forall k$$

$$-\sum_{j=1}^{n} \delta_j^{\ k} x_{ij} \geq -x_i^{\ k}, i \in I_f, \forall k$$

Effectiveness and global targets achievement:

$$-\sum_{j=1}^{n} \delta_j^{\ 1} x_{ij} - \ldots - \sum_{j=1}^{n} \delta_j^{\ n} x_{ij} + d_i^{\ +} = -GX_i, \forall i \in I_v$$

$$-\sum_{j=1}^{n} \delta_j^{\ 1} x_{ij} - \ldots \sum_{j=1}^{n} \delta_j^{\ n} x_{ij} + VX_i = 0, \forall i \in \bar{I}_v$$

$$\sum_{j=1}^{n} \delta_j^{\ 1} y_{rj} + \ldots + \sum_{j=1}^{n} \delta_j^{\ n} y_{rj} + d_r^{\ -} = GY_r, \forall r \in O_v$$

$$\sum_{j=1}^{n} \delta_j^{\ 1} y_{rj} + \ldots + \sum_{j=1}^{n} \delta_j^{\ n} y_{rj} - VY_r = 0, \forall r \in \bar{O}_v$$

Budget balance:

$$\sum_{i\in I_B}\sum_{j=1}^{n}(\delta_j^{\ 1}+...+\delta_j^{\ n})x_{ij}$$

$$-\sum_{r\in O_B}\ \sum_{j=1}^{n}(\delta_j^{\ 1}+...+\delta_j^{\ n})y_{rj}\leq B,\forall i\in I_B \text{ and } \forall r\in O_B$$

$$\delta_j^{\ k},n_i^{\ j},n_r^{\ j},p_i^{\ j},p_r^{\ j},d_i,d_r\geq 0,$$

$$VX_i\geq 0,\forall i\in \bar{I}_{v,}$$

$$VY_r\geq 0,\forall r\in \bar{O}_v$$

where

n_i^j, p_i^j are the negative and positive deviation variables for the input *i* of DMU *j*

n_r^j, p_r^j are negative and positive deviation variables for output *r* of DMU *j*

d_i^+, d_r^- are the positive and negative deviation variables from the global target of input *i* and output *r*

P_i^-, P_i^+ preferences over the minimization of positive/negative goal deviations of input *i*

P_r^-, P_r^+ preferences over the minimization of positive/negative goal deviations of output *r*

P_i^g, P_r^g are the preference levels related to the global target of input *i* and output *r*

x_{ij}, y_{rj} is the input *i* and output *r* of DMU *j*

Gx_i, Gy_r are the *i*-th input and *r*-th output global target levels with prior knowledge

VX_i, VY_r are the *i*-th and *r*-th output global targets without prior knowledge of their values

B is a user specified constant concerning the balance between proportionate inputs and outputs in the planning model

I_B, O_B are the subsets of proportionate inputs and outputs

The model described above is goal programming one. The objective function of the *GoDEA* contains the deviation variables used in the constraint sets and seeks to minimize deviations that correspond to the global and individual *DMU* targets.

The first part of the objective function includes the deviation variables from the global input/output targets. The priorities used in the objective function reflect the penalty per unit of deviation from the global targets.

The second part of the objective function includes the deviation variables of inputs and outputs of individual *DMUs*. The priorities attached to these deviation variables can be used to monitor the contribution of individual *DMUs* to global organizational targets. The presence of two-way deviation variables implies that the problem can be solved by under - or over achieving the observed input/output values of individual units.

This is a fundamental departure from *DEA* models, which assume that the assessed targets should always contract inputs and expand outputs.

The constraint representing the individual *DMUs* represents *DMU k*. This constraint set represents the comparisons made between the inputs/outputs of the assessed *DMU k*, (x_i^k, y_r^k), and its composite units ($\sum_j \delta_j^{\,k} x_{ij}, \sum_j \delta_j^{\,k} y_{rj}$). Suitable formulations of the objective function (3.6) can yield input augmentation and output reduction targets in the light of achieving the global organizational targets.

The constraint representing global input/output targets seek to aggregate the contribution of the composite unit of assessed *DMUs*, say *DMU n*, for the *i*-th controllable input ($\sum_{j=1}^{n} \delta_j^{\,n} x_{ij}$) and *r*-th controllable output ($\sum_{j=1}^{n} \delta_j^{\,n} y_{rj}$). A distinction is taken into account by declaring inputs/outputs (I_v, O_v) with prior knowledge of their global targets levels and those, ($\bar{I}_v$, $\bar{O}_v$), that will be treated as variables in (3.6).

In the context of performance measurement, the goal deviation variables have stronger implications than in ordinary goal programming models. This is because the 'goal' levels in the right hand side of the individual constraints are the observed input/output values of individual units. These are effectively ‘undesirable’ goals in performance measurement context and, therefore, the solution process should aid units to move away from their current input/output levels to more efficient ones.