

# **Three-Dimensional Nonlinear Dynamics of a Moored Cylinder to be Used as a Breakwater**

By

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Thesis submitted to the Faculty of  
Virginia Polytechnic Institute and State University  
in partial fulfillment of the requirements for the degree of

**MASTER OF SCIENCE  
IN  
CIVIL ENGINEERING**

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April 1999  
Blacksburg, Virginia

Keywords: nonlinear dynamics, chaos, vibration, breakwater, cylinder, mooring, snap load

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(ABSTRACT)

A three-dimensional, nonlinear dynamic analysis is conducted on a fully submerged, rigid, solid cylinder to be used as a breakwater. The breakwater could potentially be used as a single cylinder to protect small structures. Alternatively, multiple cylinders could be positioned in series to protect shorelines, harbors, or moored vessels from destructive incident water waves. The cylinder is positioned with its axis horizontal and is moored to the seafloor with four symmetrically placed massless mooring lines connected at the ends of the cylinder. The mooring lines are modeled as both linearly elastic (“regular”) springs and compressionless springs. All six degrees of freedom of the structure are considered. The breakwater is modeled in air with a net buoyant force acting through the cylinder’s center of gravity. The six “dry” natural frequencies of the structure are computed. Both linear and nonlinear free vibrations of the structure are considered. Linear damping is used to model the fluid and mooring damping effects. Normal and oblique harmonic wave forces at various frequencies and amplitudes are applied to the cylinder. The effects of the forcing amplitude and frequency, and the coefficient of damping, on the motion of the breakwater are studied. The results show that more erratic behavior occurs for the breakwater with compressionless springs, mainly due to the development of snap loads in the mooring lines.

## **Acknowledgements**

First, I would like to thank my advisor, Dr. Raymond H. Plaut, for all of his time and support throughout the completion of this thesis. His dedication towards assisting me at all stages of the project was above and beyond what I expected. I would also like to thank Dr. Richard M. Barker and Dr. Siegfried M. Holzer for all of their time and input, and for being members on my graduate committee. I also appreciate the assistance in FORTRAN given to me by Dr. Raul Andruet.

This research project was partially funded by the National Science Foundation under Grant No. BES-9521425. I would also like to thank The Center for Naval Analyses Corporation for selecting me as a GEM M.S. Engineering Fellow and for employing me during two summer internships. I am also grateful for the GEM fellowship stipend granted by the National Consortium for Graduate Degrees for Minorities in Engineering and Science, Inc. Finally, I appreciate the supplemental GEM funding from both the College of Engineering and the Graduate School in the form of a Graduate Teaching Assistantship and a Graduate Research Assistantship, respectively.

I thank my family and friends for all of their support and friendship over the past couple of years. I also would like to thank my fiancée, Daphne, for all of her love and encouragement. Finally, I would like to thank the God for all of the blessings he has provided me with.

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